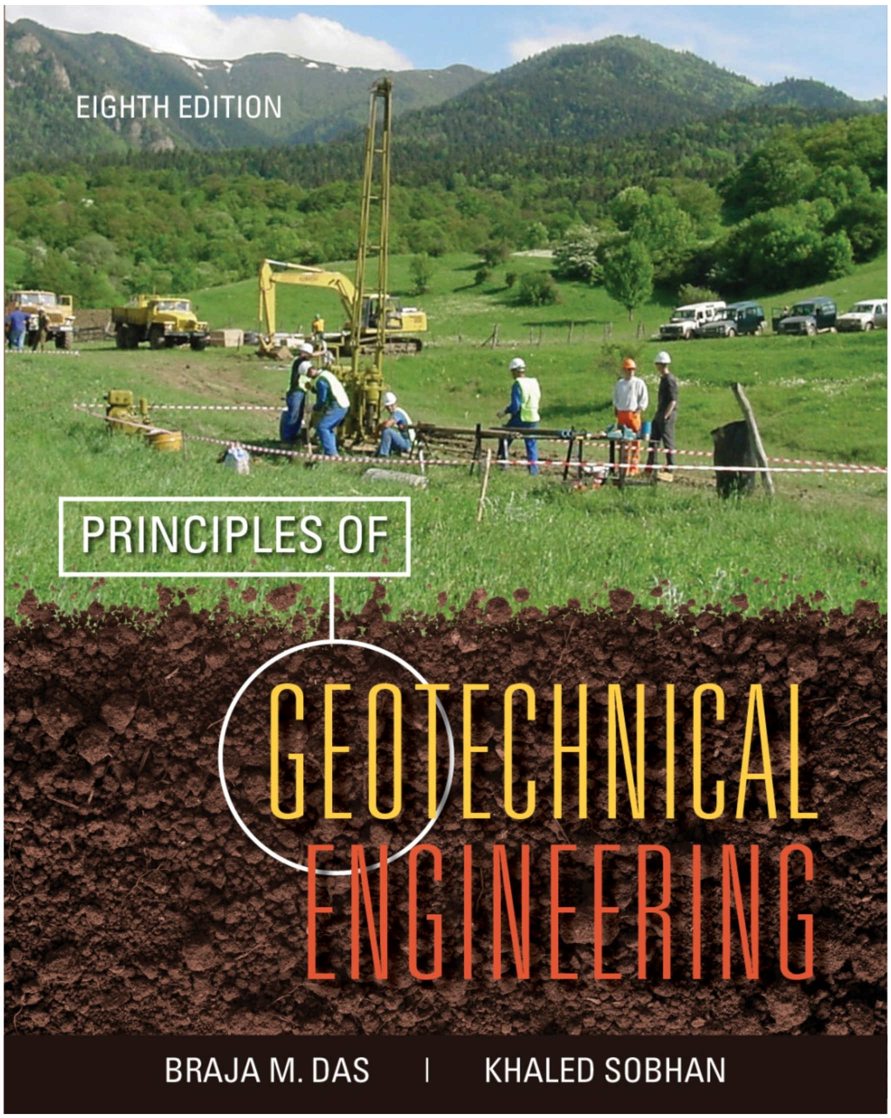
An Instructor's Solutions Manual to Accompany

PRINCIPLES OF GEOTECHNICAL ENGINEERING, 8TH EDITION

BRAJA M. DAS & KHALED SOBHAN



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INSTRUCTOR'S SOLUTIONS MANUAL
TO ACCOMPANY

PRINCIPLES OF GEOTECHNICAL ENGINEERING

Eighth Edition, SI

BRAJA M. DAS

KHALED SOBHAN

Contents

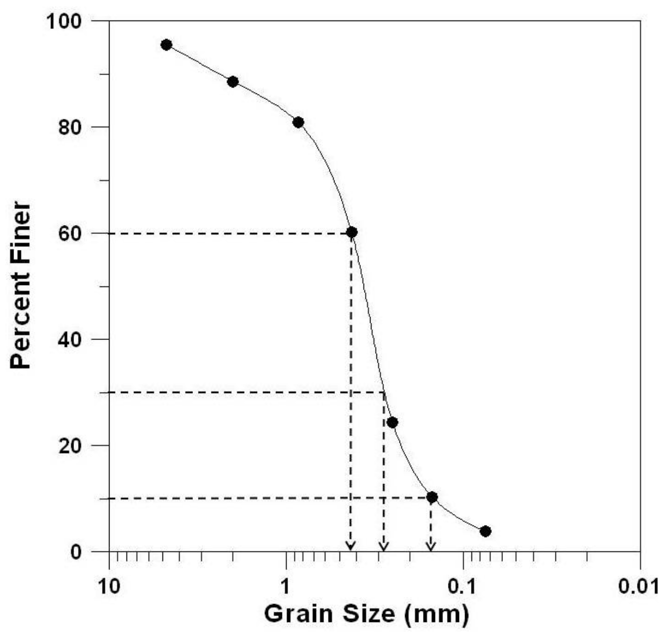
Chapter 2

2.1 $C_u = \frac{D_{60}}{D_{10}} = \frac{0.42}{0.16} = 2.625 \approx \mathbf{2.63}$; $C_c = \frac{D_{30}^2}{(D_{60})(D_{10})} = \frac{0.21^2}{(0.42)(0.16)} = 0.656 \approx \mathbf{0.66}$

2.2 $C_u = \frac{D_{60}}{D_{10}} = \frac{0.81}{0.27} = \mathbf{3.0}$; $C_c = \frac{D_{30}^2}{(D_{60})(D_{10})} = \frac{0.41^2}{(0.81)(0.27)} = 0.768 \approx \mathbf{0.77}$

2.3 a.

Sieve no.	Mass of soil retained on each sieve (g)	Percent retained on each sieve	Percent finer
4	28	4.54	**95.46**
10	42	6.81	**88.65**
20	48	7.78	**80.88**
40	128	20.75	**60.13**
60	221	35.82	**24.31**
100	86	13.94	**10.37**
200	40	6.48	**3.89**
Pan	24	3.89	**0.00**
	Σ617 g		



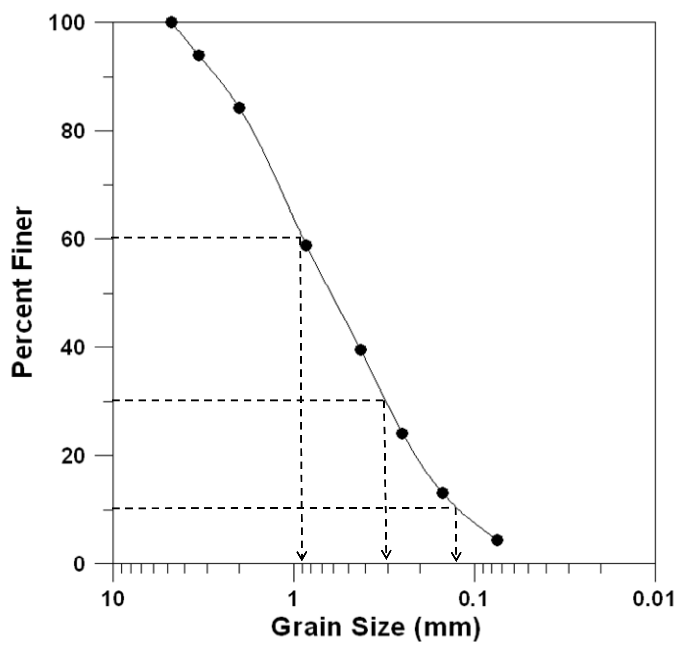
b. D_{10} = **0.16 mm**; D_{30} = **0.29 mm**; D_{60} = **0.45 mm**

c. $C_u = \frac{D_{60}}{D_{10}} = \frac{0.45}{0.16} = 2.812 \approx \mathbf{2.81}$

d. $C_c = \frac{D_{30}^2}{(D_{60})(D_{10})} = \frac{0.29^2}{(0.45)(0.16)} = 1.168 \approx \mathbf{1.17}$

2.4 a.

Sieve no.	Mass of soil retained on each sieve (g)	Percent retained on each sieve	Percent Finer
4	0	0.0	**100.00**
6	30	6.0	**94.0**
10	48.7	9.74	**84.26**
20	127.3	25.46	**58.80**
40	96.8	19.36	**39.44**
60	76.6	15.32	**24.12**
100	55.2	11.04	**13.08**
200	43.4	8.68	**4.40**
Pan	22	4.40	**0.00**
	Σ 500 g		



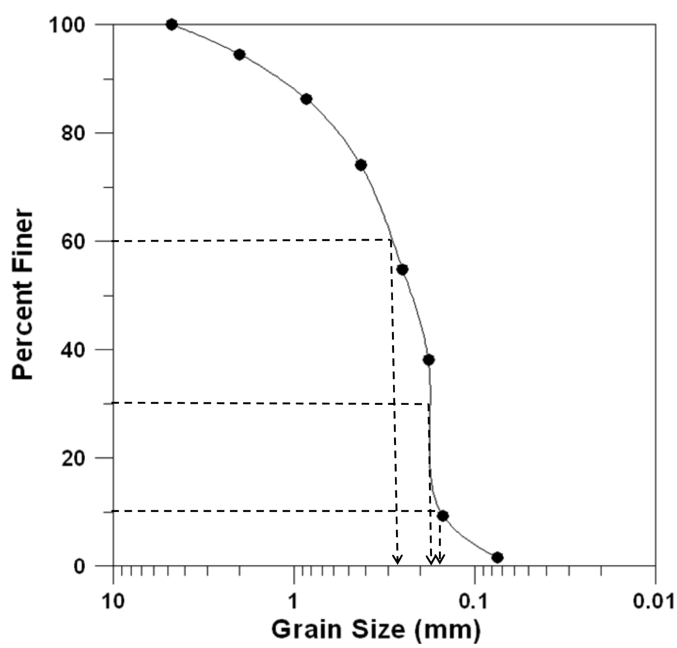
b. D_{10} = **0.13 mm**; D_{30} = **0.3 mm**; D_{60} = **0.9 mm**

c. $C_u = \frac{D_{60}}{D_{10}} = \frac{0.9}{0.13} = 6.923 \approx \mathbf{6.92}$

d. $C_c = \frac{D_{30}^2}{(D_{60})(D_{10})} = \frac{0.3^2}{(0.9)(0.13)} = 0.769 \approx \mathbf{0.77}$

2.5 a.

Sieve no.	Mass of soil retained on each sieve (g)	Percent retained on each sieve	Percent finer
4	0	0.0	**100.00**
10	40	5.49	**94.51**
20	60	8.23	**86.28**
40	89	12.21	**74.07**
60	140	19.20	**54.87**
80	122	16.74	**38.13**
100	210	28.81	**9.33**
200	56	7.68	**1.65**
Pan	12	1.65	**0.00**
	Σ 729 g		



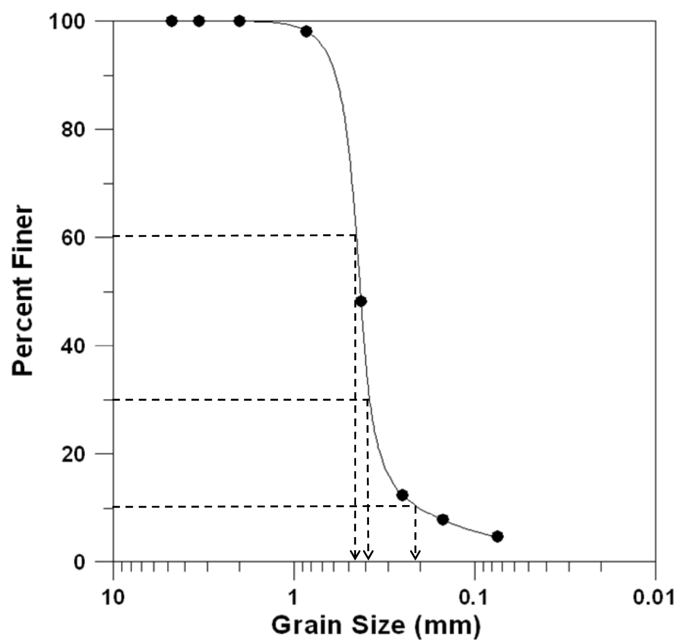
b. D_{10} = **0.17 mm**; D_{30} = **0.18 mm**; D_{60} = **0.28 mm**

c. $C_u = \frac{D_{60}}{D_{10}} = \frac{0.28}{0.17} = 1.647 \approx \mathbf{1.65}$

d. $C_c = \frac{D_{30}^2}{(D_{60})(D_{10})} = \frac{0.18^2}{(0.28)(0.17)} = \mathbf{0.68}$

2.6 a.

Sieve no.	Mass of soil retained on each sieve (g)	Percent retained on each sieve	Percent finer
4	0	0.0	**100.00**
6	0	0.0	**100.00**
10	0	0.0	**100.00**
20	9.1	1.82	**98.18**
40	249.4	49.88	**48.3**
60	179.8	35.96	**12.34**
100	22.7	4.54	**7.8**
200	15.5	3.1	**4.7**
Pan	23.5	4.7	**0.00**
	Σ 500 g		

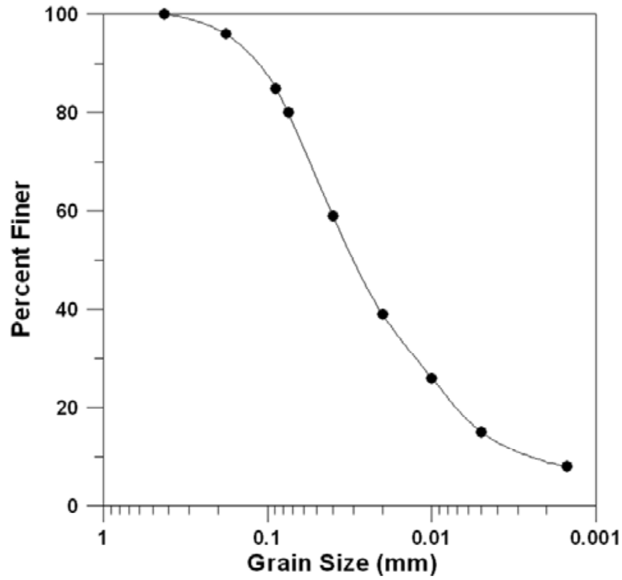


b. D_{10} = **0.21 mm**; D_{30} = **0.39 mm**; D_{60} = **0.45 mm**

c. $C_u = \frac{D_{60}}{D_{10}} = \frac{0.45}{0.21} = 2.142 \approx \mathbf{2.14}$

d. $C_c = \frac{D_{30}^2}{(D_{60})(D_{10})} = \frac{0.39^2}{(0.45)(0.21)} = 1.609 \approx \mathbf{1.61}$

2.7 a.

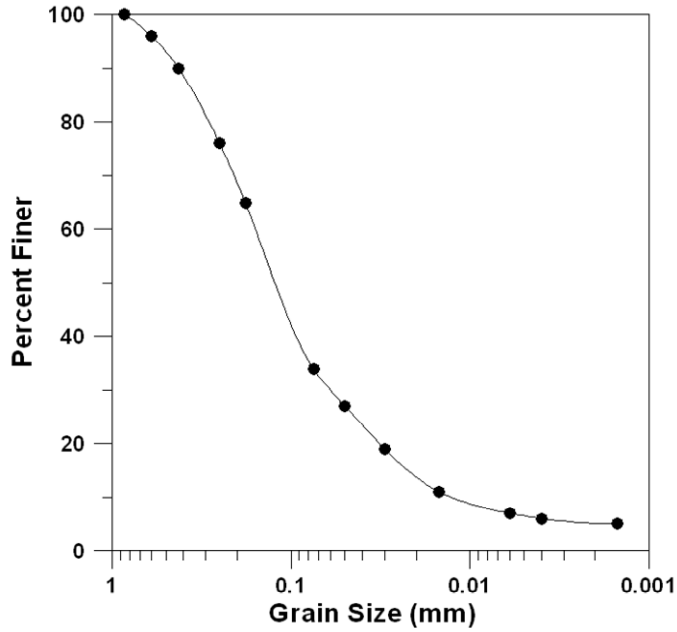


b. Percent passing 2 mm = 100 GRAVEL: 100 – 100 = **0%**
Percent passing 0.06 mm = 73 SAND: 100 – 73 = **27%**
Percent passing 0.002 mm = 9 SILT: 73 – 9 = **64%**
CLAY: 9 – 0 = **9%**

c. Percent passing 2 mm = 100 GRAVEL: 100 – 100 = **0%**
Percent passing 0.05 mm = 68 SAND: 100 – 68 = **32%**
Percent passing 0.002 mm = 9 SILT: 68 – 9 = **59%**
CLAY: 9 – 0 = **9%**

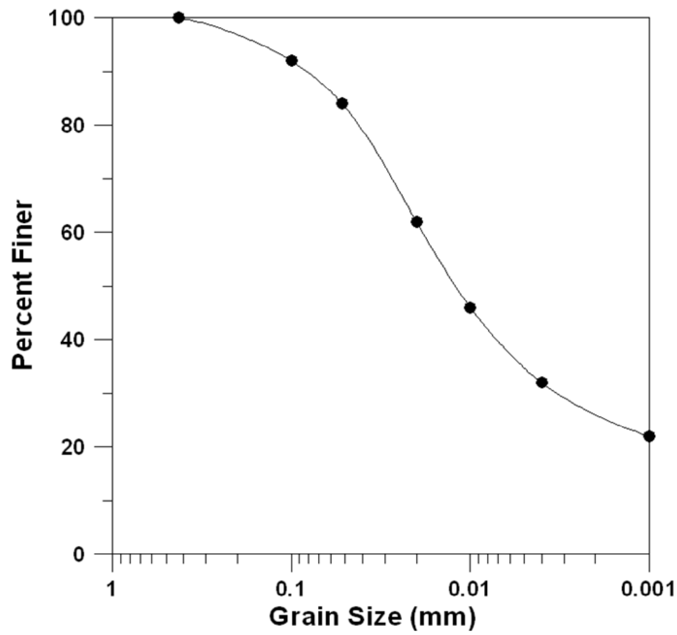
d. Percent passing 2 mm = 100 GRAVEL: 100 – 100 = **0%**
Percent passing 0.075 mm = 80 SAND: 100 – 80 = **20%**
Percent passing 0.002 mm = 9 SILT: 80 – 9 = **71%**
CLAY: 9 – 0 = **9%**

2.8 a.



b. Percent passing 2 mm = 100	GRAVEL: 100 – 100 = **0%**
Percent passing 0.06 mm = 30	SAND: 100 – 30 = **70%**
Percent passing 0.002 mm = 5	SILT: 70 – 5 = **65%**
	CLAY: 5 – 0 = **5%**
c. Percent passing 2 mm = 100	GRAVEL: 100 – 100 = **0%**
Percent passing 0.05 mm = 28	SAND: 100 – 28 = **72%**
Percent passing 0.002 mm = 5	SILT: 72 – 5 = **67%**
	CLAY: 5 – 0 = **5%**
d. Percent passing 2 mm = 100	GRAVEL: 100 – 100 = **0%**
Percent passing 0.075 mm = 34	SAND: 100 – 34 = **66%**
Percent passing 0.002 mm = 5	SILT: 66 – 5 = **61%**
	CLAY: 5 – 0 = **5%**

2.9 a.

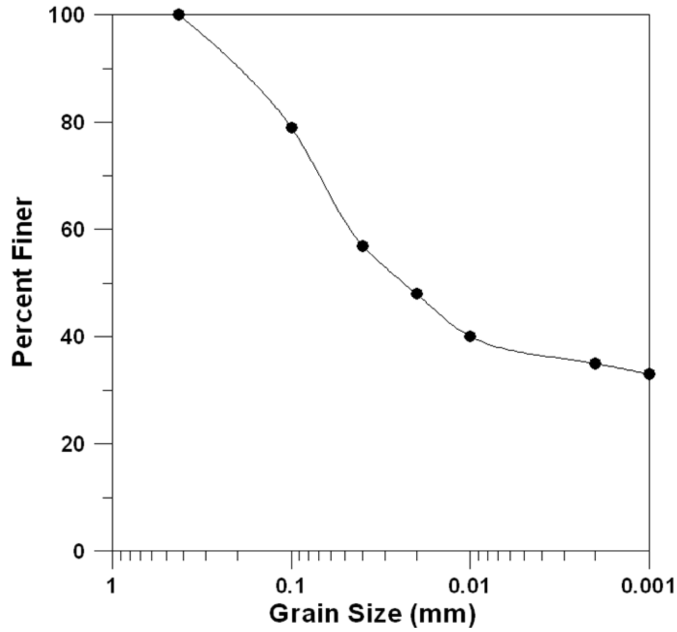


b. Percent passing 2 mm = 100 GRAVEL: 100 – 100 = **0%**
Percent passing 0.06 mm = 84 SAND: 100 – 84 = **16%**
Percent passing 0.002 mm = 28 SILT: 84 – 28 = **56%**
CLAY: 28 – 0 = **28%**

c. Percent passing 2 mm = 100 GRAVEL: 100 – 100 = **0%**
Percent passing 0.05 mm = 83 SAND: 100 – 83 = **17%**
Percent passing 0.002 mm = 28 SILT: 83 – 28 = **55%**
CLAY: 28 – 0 = **28%**

d. Percent passing 2 mm = 100 GRAVEL: 100 – 100 = **0%**
Percent passing 0.075 mm = 90 SAND: 100 – 90 = **10%**
Percent passing 0.002 mm = 28 SILT: 90 – 28 = **62%**
CLAY: 28 – 0 = **28%**

2.10 a.



b. Percent passing 2 mm = 100 GRAVEL: 100 – 100 = **0%**
Percent passing 0.06 mm = 65 SAND: 100 – 65 = **35%**
Percent passing 0.002 mm = 35 SILT: 65 – 35 = **30%**
CLAY: 35 – 0 = **35%**

c. Percent passing 2 mm = 100 GRAVEL: 100 – 100 = **0%**
Percent passing 0.05 mm = 62 SAND: 100 – 62 = **38%**
Percent passing 0.002 mm = 35 SILT: 62 – 35 = **27%**
CLAY: 35 – 0 = **35%**

d. Percent passing 2 mm = 100 GRAVEL: 100 – 100 = **0%**
Percent passing 0.075 mm = 70 SAND: 100 – 70 = **30%**
Percent passing 0.002 mm = 35 SILT: 70 – 35 = **35%**
CLAY: 35 – 0 = **35%**

2.11 G_s = 2.7; temperature = 24°; time = 60 min; L = 9.2 cm

Eq. (2.5): $D\,(\text{mm}) = K\sqrt{\dfrac{L\,(\text{cm})}{t\,(\text{min})}}$

From Table 2.6 for G_s = 2.7 and temperature = 24°, K = 0.01282

$$D = 0.01282\sqrt{\frac{9.2}{60}} = \mathbf{0.005\,mm}$$

2.12 G_s = 2.75; temperature = 23°C; time = 100 min; L = 12.8 cm

Eq. (2.5): $$D\,(\text{mm}) = K\sqrt{\frac{L\,(\text{cm})}{t\,(\text{min})}}$$

From Table 2.6 for G_s = 2.75 and temperature = 23°, K = 0.01279

$$D = 0.01279\sqrt{\frac{12.8}{100}} = \mathbf{0.0046\,mm}$$

CRITICAL THINKING PROBLEM

2.C.1 a. Soil A: $C_u = \frac{D_{60}}{D_{10}} = \frac{11}{0.6} = \mathbf{18.33}$; $C_c = \frac{D_{30}^2}{(D_{60})(D_{10})} = \frac{5^2}{(11)(0.6)} = \mathbf{3.78}$

Soil B: $C_u = \frac{D_{60}}{D_{10}} = \frac{7}{0.2} = \mathbf{35}$; $C_c = \frac{D_{30}^2}{(D_{60})(D_{10})} = \frac{2.1^2}{(7)(0.2)} = \mathbf{3.15}$

Soil C: $C_u = \frac{D_{60}}{D_{10}} = \frac{4.5}{0.15} = \mathbf{30}$; $C_c = \frac{D_{30}^2}{(D_{60})(D_{10})} = \frac{1^2}{(4.5)(0.15)} = \mathbf{1.48}$

b. Soil A is coarser than Soil C. A higher percentage of soil C is finer than any given size compared to Soil A. For example, about 15% is finer than 1 mm for Soil A, whereas almost 30% is finer than 1 mm in case of soil C.

c. Particle segregation may take place in aggregate stockpiles such that there is a separation of coarser and finer particles. This makes representative sampling difficult. Therefore Soils A, B, and C demonstrate quite different particle size distribution.

d. Soil A:

Percent passing 4.75 mm = 29 GRAVEL: 100 – 29 = **71%**

Percent passing 0.075 mm = 1 SAND: 29 – 1 = **28%**

FINES: 1 – 0 = **1%**

Soil B:

Percent passing 4.75 mm = 45 GRAVEL: 100 – 45 = **55%**

Percent passing 0.075 mm = 2 SAND: 45 – 2 = **43%**

FINES: 2 – 0 = **2%**

Soil C:

Percent passing 4.75 mm = 53 GRAVEL: 100 – 53 = **47%**

Percent passing 0.075 mm = 3 SAND: 47 – 3 = **44%**

FINES: 3 – 0 = **3%**

Chapter 3

3.1 $$\gamma_{\text{sat}} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{G_s\gamma_w}{1+e} + \frac{e\gamma_w}{1+e} = \frac{e\gamma_w}{(1+e)w_{\text{sat}}} + n\gamma_w = \boldsymbol{n}\left(\frac{\mathbf{1} + \boldsymbol{w}_{\text{sat}}}{\boldsymbol{w}_{\text{sat}}}\right)\boldsymbol{\gamma_w}$$

3.2 $$\gamma_{\text{sat}} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{G_s\gamma_w}{1+e} + \frac{e\gamma_w}{1+e} = \gamma_d + \left(\frac{e}{1+e}\right)\gamma_w$$

Rearranging, $\gamma_{\text{sat}}(1+e) = \gamma_d(1+e) + e\gamma_w$

Therefore, $e = \dfrac{\boldsymbol{\gamma_{\text{sat}} - \gamma_d}}{\boldsymbol{\gamma_d - \gamma_{\text{sat}} + \gamma_w}}$

3.3 $$\gamma_{\text{sat}} = \left(\frac{1 + w_{\text{sat}}}{1+e}\right)G_s\gamma_w = \left(\frac{1 + w_{\text{sat}}}{1+e}\right)\frac{e\gamma_w}{w_{\text{sat}}} = \frac{(1 + w_{\text{sat}})n\gamma_w}{w_{\text{sat}}}$$

Rearranging, $w_{\text{sat}}(\gamma_{\text{sat}} - n\gamma_w) = n\gamma_w$

Therefore, $w_{\text{sat}} = \dfrac{\boldsymbol{n\gamma_w}}{\boldsymbol{\gamma_{\text{sat}} - n\gamma_w}}$

3.4 a. $\gamma = \dfrac{W}{V} = \dfrac{12.5}{0.1} = \mathbf{125\ lb/ft^3}$

b. $\gamma_d = \dfrac{\gamma}{1+w} = \dfrac{125}{1+0.14} = \mathbf{109.64\ lb/ft^3}$

c. $e = \dfrac{G_s\gamma_w}{\gamma_d} - 1 = \dfrac{(2.71)(62.4)}{109.64} - 1 = \mathbf{0.54}$

d. $n = \dfrac{e}{1+e} = \dfrac{0.54}{1+0.54} = \mathbf{0.35}$

e. $S = \frac{(w)(G_s)}{e} = \frac{(0.14)(2.71)}{0.54} = 0.702 = \mathbf{70.2\%}$

f. Volume of water = $\frac{(\gamma - \gamma_d)V}{\gamma_w} = \frac{(125 - 109.64)(0.1)}{62.4} \approx \mathbf{0.024\,ft^3}$

3.5 a. $\gamma = \left(\frac{1+w}{1+e}\right) G_s \gamma_w$; $19.2 = \frac{(1+0.098)(2.69)(9.81)}{1+e}$; e = **0.51**

b. $\gamma_d = \frac{G_s \gamma_w}{1+e} = \frac{(2.69)(9.81)}{1+0.51} = \mathbf{17.48\,kN/m^3}$

c. $S = \frac{(w)(G_s)}{e} = \frac{(0.098)(2.69)}{0.51} = 0.517 = \mathbf{51.7\%}$

3.6 a. $\gamma = \frac{(G_s + Se)\gamma_w}{1+e} = \frac{(2.69)(9.81) + (0.9)(0.51)(9.81)}{1+0.51} = 20.45\,\text{kN/m}^3$

Water to be added = 20.45 – 19.2 = **1.25 kN/m³**

b. $\gamma_{\text{sat}} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{(2.69 + 0.51)(9.81)}{1+0.51} = 20.78\,\text{kN/m}^3$

Water to be added = 20.78 – 19.2 = **1.58 kN/m³**

3.7 a. $V = \frac{\pi}{4}(2.8)^2(22)\left(\frac{1}{12^3}\right) = 0.078\,\text{ft}^3$; $\gamma = \frac{W}{V} = \frac{9.56}{0.078} = \mathbf{122.56\,lb/ft^3}$

b. $w = \frac{W - W_s}{W_s} = \frac{9.56 - 8.51}{8.51} = 0.1233 = \mathbf{12.33\%}$

c. $\gamma_d = \frac{W_s}{V} = \frac{8.51}{0.078} = \mathbf{109.1\,lb/ft^3}$

d. $e = \dfrac{G_s \gamma_w}{\gamma_d} - 1 = \dfrac{(2.69)(62.4)}{109.1} - 1 \approx \mathbf{0.54}$

e. $S = \dfrac{wG_s}{e} = \dfrac{(0.1233)(2.69)}{0.54} = 0.614 = \mathbf{61.4\%}$

3.8 a. $\gamma = \dfrac{(1+w)G_s\gamma_w}{1+e} = \dfrac{(1+w)G_s\gamma_w}{1+\dfrac{wG_s}{S}}$; $108 = \dfrac{(1+0.26)(G_s)(62.4)}{1+\dfrac{(0.26)(G_s)}{0.72}}$; $G_s = \mathbf{2.72}$

b. $e = \dfrac{wG_s}{S} = \dfrac{(0.26)(2.72)}{0.72} = \mathbf{0.98}$

c. $\gamma_{\text{sat}} = \dfrac{(G_s + e)\gamma_w}{1+e} = \dfrac{(2.72+0.98)(62.4)}{1+0.98} = \mathbf{116.6\,lb/ft^3}$

3.9 a. $\gamma_d = \dfrac{\gamma}{1+w} = \dfrac{20.6}{1+0.166} = \mathbf{17.67\,kN/m^3}$

b. $e = \dfrac{G_s\gamma_w}{\gamma_d} - 1 = \dfrac{(2.74)(9.81)}{17.67} - 1 \approx \mathbf{0.52}$

c. $n = \dfrac{e}{1+e} = \dfrac{0.52}{1+0.52} = \mathbf{0.34}$

d. $S = \dfrac{wG_s}{e} = \dfrac{(0.166)(2.74)}{0.52} = 0.874 = \mathbf{87.4\%}$

3.10 a. $\gamma = \dfrac{(G_s + Se)\gamma_w}{1+e} = \dfrac{(2.74)(9.81)+(0.9)(0.52)(9.81)}{1+0.52} = 20.7\,\text{kN/m}^3$

Water to be added = 20.7 – 20.6 = **0.1 kN/m^3**

b. $\gamma_{\text{sat}} = \dfrac{(G_s + e)\gamma_w}{1+e} = \dfrac{(2.74+0.52)(9.81)}{1+0.52} = 21.04\,\text{kN/m}^3$

Water to be added = 21.04 – 20.6 = **0.44 kN/m³**

3.11 a. $\rho_d = \frac{\rho}{1+w} = \frac{1750}{1+0.23} = \mathbf{1422.76\,kg/m^3}$

b. $e = \frac{G_s \rho_w}{\rho_d} - 1 = \frac{(2.73)(1000)}{1422.76} - 1 = 0.92$; $n = \frac{e}{1+e} = \frac{0.92}{1+0.92} = \mathbf{0.48}$

c. $S = \frac{wG_s}{e} = \frac{(0.23)(2.73)}{0.92} = 0.682 = \mathbf{68.2\%}$

d. $\rho_{\text{sat}} = \frac{(G_s + e)\rho_w}{1+e} = \frac{(2.73+0.92)(1000)}{1+0.23} \approx 2967\ \text{kg/m}^3$

Water to be added = 2967 – 1750 = **1217 kg/m³**

3.12 a. $\gamma_d = \frac{\gamma}{1+w} = \frac{30.75}{0.25(1+0.098)} = \mathbf{112\,lb/ft^3}$

b. $e = \frac{G_s \gamma_w}{\gamma_d} - 1 = \frac{(2.66)(62.4)}{112} - 1 \approx \mathbf{0.48}$

c. Volume of water = $\frac{(\gamma - \gamma_d)V}{\gamma_w} = \frac{\left(\frac{30.75}{0.25} - 112\right)(0.25)}{62.4} \approx \mathbf{0.044\ ft^3}$

3.13 a. $e = \frac{n}{1-n} = \frac{0.3}{1-0.3} = \mathbf{0.43}$

b. $\rho_d = \frac{G_s \rho_w}{1+e}$; $G_s = \frac{\rho_d (1+e)}{\rho_w} = \frac{1800(1+0.43)}{1000} = \mathbf{2.57}$

3.14 $e = \frac{G_s \gamma_w}{\gamma_d} - 1 = \frac{(2.69)(62.4)}{105} - 1 \approx \mathbf{0.598}$

$$S = \frac{wG_s}{e} = \frac{(0.17)(2.69)}{0.598} = 0.764 = \mathbf{76.4\%}$$

3.15 a. $$\gamma = \frac{(1+w)G_s\gamma_w}{1+e} = \frac{(1+w)G_s\gamma_w}{1+\frac{wG_s}{S}} = \frac{(1+0.182)(2.67)(62.4)}{1+\frac{(0.182)(2.67)}{0.8}} = \mathbf{122.5\ lb/ft^3}$$

b. $$\gamma_d = \frac{\gamma}{1+w} = \frac{122.5}{1+0.182} = 103.6$$

$$\text{Volume of water} = \frac{(\gamma - \gamma_d)V}{\gamma_w} = \frac{(122.5-103.6)(1)}{62.4} = \mathbf{0.302\ ft^3/ft^3\ of\ soil}$$

3.16 a. $$\gamma = \frac{(G_s + Se)\gamma_w}{1+e};\ 106 = \frac{(G_s + 0.55e)(62.4)}{1+e}$$

$$G_s = 1.148e + 1.698 \qquad \text{(i)}$$

$$114 = \frac{(G_s + 0.822e)(62.4)}{1+e} \qquad \text{(ii)}$$

From (i) and (ii): G_s **= 2.73**

b. Using G_s = 2.73 in Equation (i), we get e **= 0.9**

3.17 a. $$D_r = \frac{e_{\max} - e}{e_{\max} - e_{\min}};\ 0.65 = \frac{0.75 - e}{0.75 - 0.52};\ e = \mathbf{0.6}$$

b. $$\gamma_d = \frac{G_s\gamma_w}{1+e} = \frac{(2.67)(9.81)}{1+0.6} = \mathbf{16.37\ kN/m^3}$$

3.18 $$D_r = \frac{e_{\max} - e}{e_{\max} - e_{\min}};\ 0.82 = \frac{0.72 - e}{0.72 - 0.46};\ e \approx 0.51$$

$$\gamma = \frac{(1+w)G_s\gamma_w}{1+e} = \frac{(1+0.11)(2.68)(9.81)}{1+0.51} = \mathbf{19.32\,kN/m^3}$$

3.19 $$\gamma_d = \frac{\gamma}{1+w} = \frac{115}{1+0.08} = 106.48\text{ lb/ft}^3$$

$$D_r = \frac{\left[\frac{1}{\gamma_{d(\min)}}\right]-\left[\frac{1}{\gamma_d}\right]}{\left[\frac{1}{\gamma_{d(\min)}}\right]-\left[\frac{1}{\gamma_{d(\max)}}\right]} = \frac{\left[\frac{1}{92}\right]-\left[\frac{1}{106.48}\right]}{\left[\frac{1}{92}\right]-\left[\frac{1}{108}\right]} = 0.918 = \mathbf{91.8\%}$$

CRITICAL THINKING PROBLEMS

3.C.1 a. $$e = \frac{V_v}{V_s};\quad e_1 + 1 = \frac{V_1}{V_s}$$

$$1+0.92 = \frac{V_1}{V_s};\quad V_1 = 1.92V_s$$

$$1+0.65 = \frac{V_2}{V_s};\quad V_2 = 1.65V_s$$

$$\frac{\Delta V}{V} = \frac{V_1 - V_2}{V_1} = \frac{1.92-1.65}{1.92} = 0.14 = \mathbf{14\%\ (decrease)}$$

b. $$\gamma_{d(1)} = \frac{G_s\gamma_w}{1+e_1} = \frac{G_s\gamma_w}{1+0.92} = \frac{G_s\gamma_w}{1.92}$$

$$\gamma_{d(2)} = \frac{G_s\gamma_w}{1.65}$$

$$\frac{\Delta\gamma_d}{\gamma_{d(1)}} = \frac{\gamma_{d(2)} - \gamma_{d(1)}}{\gamma_{d(1)}} = \frac{\frac{1}{1.65} - \frac{1}{1.92}}{\frac{1}{1.92}} = 0.163 = \mathbf{16.3\%\ \ (increase)}$$

c. $S_1 = \frac{wG_s}{e_1} = \frac{wG_s}{0.92}$; $S_2 = \frac{wG_s}{0.65}$

$$\frac{\Delta S}{S_1} = \frac{S_2 - S_1}{S_1} = \frac{\frac{1}{0.65} - \frac{1}{0.92}}{\frac{1}{0.92}} = 0.415 = \mathbf{41.5\%\ (increase)}$$

3.C.2 a. $D_r = \frac{e_{\max} - e}{e_{\max} - e_{\min}}$

$$e_1 = e_{\max} - D_r(e_{\max} - e_{\min}) = 0.92 - 0.47(0.92 - 0.53) = 0.736$$

$$\gamma_d = \frac{G_s\gamma_w}{1+e_1} = \frac{(2.65)(9.81)}{1+0.736} = \mathbf{14.97\,kN/m^3}\text{ (before compaction)}$$

$$e_2 = e_{\max} - D_r(e_{\max} - e_{\min}) = 0.92 - 0.8(0.92 - 0.53) = 0.608$$

$$\gamma_d = \frac{(2.65)(9.81)}{1+0.608} = \mathbf{16.17\ \ kN/m^3}\text{ (after compaction)}$$

b. $\frac{\Delta H}{H} = \frac{\Delta e}{1+e_1} = \frac{0.736 - 0.608}{1+0.736} = 0.074$; $\Delta H = 0.074H = (0.074)(2) = 0.148\ \text{m}$

Final Height = 2 – 0.148 = **1.852 m**

Chapter 4

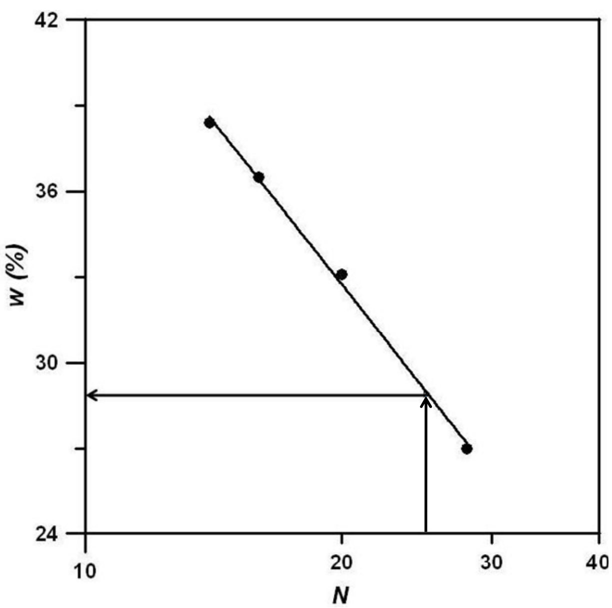
4.1 a. Refer to the plot of w versus N.

LL = **29.0**

b. $PI = LL - PL$

$= 29.0 - 13.4$

$= \mathbf{15.6}$



4.2 $$LI = \frac{w - PL}{LL - PL} = \frac{32 - 13.4}{15.6} = \mathbf{1.19}$$

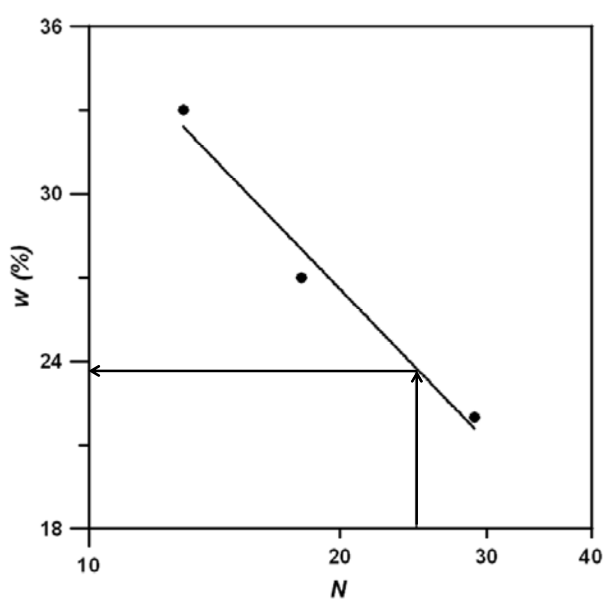
4.3 a. From the plot,

LL = **23.6**.

b. $PI = LL - PL$

$= 23.6 - 19.1$

$= \mathbf{4.5}$



4.4 $$LI = \frac{w - PL}{LL - PL} = \frac{21 - 19.1}{4.5} = \mathbf{0.422}$$

4.5 $$SL = \left(\frac{M_1 - M_2}{M_2}\right)(100) - \left(\frac{V_i - V_f}{M_2}\right)(\rho_w)(100)$$

$$= \left(\frac{34 - 24}{24}\right)(100) - \left(\frac{20.2 - 14.3}{24}\right)(1)(100) = \mathbf{17.08}$$

$$SR = \frac{M_2}{V_f \rho_w} = \frac{24}{(14.3)(1)} = \mathbf{1.68}$$

4.6 $$SL = \left(\frac{M_1 - M_2}{M_2}\right)(100) - \left(\frac{V_i - V_f}{M_2}\right)(\rho_w)(100)$$

$$= \left(\frac{44.6 - 32.8}{32.8}\right)(100) - \left(\frac{16.2 - 10.8}{32.8}\right)(1)(100) = \mathbf{19.51}$$

$$SR = \frac{M_2}{V_f \rho_w} = \frac{32.8}{(10.8)(1)} = \mathbf{3.03}$$

CRITICAL THINKING PROBLEMS

4.C.1 a. From Eq. (4.26): $A = \dfrac{PI}{(\text{\% of clay-size fraction, by weight})}$

The computed *PI* values are provided in the table on the following page.

Soil	% clay (< 0.002 mm in size)	A	$\tau_{f\text{-undisturbed}}$ (kN/m^2)	S_t	PI	$\tau_{f\text{-remolded}}$ (kN/m^2)
Beauharnois	79	0.52	18	14	**41.08**	**1.3**
Detroit I	36	0.36	17	2.5	**12.96**	**6.9**
Horten	40	0.42	41	17	**16.8**	**2.4**
Gosport	55	0.89	29	2.2	**48.95**	**13.0**
Mexico City	90	4.5	46	5.3	**405**	**8.7**
Shellhaven	41	1.33	36	7.6	**54.53**	**4.8**
St. Thuribe	36	0.33	38	150	**11.88**	**0.3**

b. From Table 4.2:

Beauharnois
A = 0.52; PI = 41.08; Mineral: **Illite**

Detroit I
A = 0.36; PI = 12.96; Mineral: **Kaolinite**

Horten
A = 0.42; PI = 16.8; Mineral: **Kaolinite**

Gosport
A = 0.89; PI = 48.95; Mineral: **Illite**

Mexico City
A = 4.5; PI = 405; Mineral: **Montmorillonite**

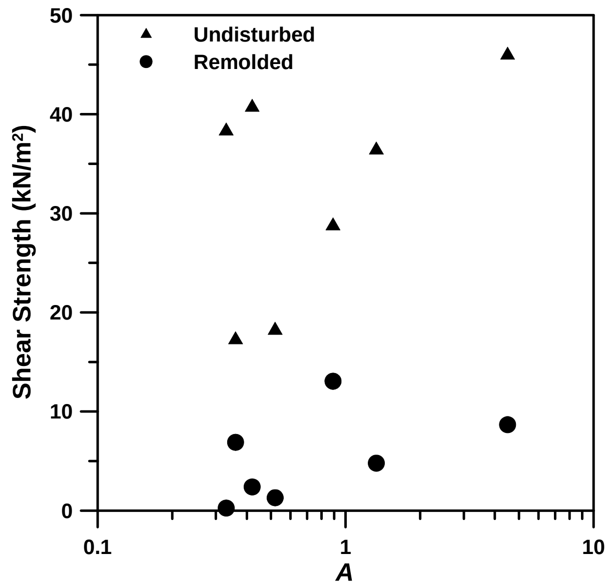
Shellhaven
A = 1.33; PI = 54.53; Mineral: **Illite**

St. Thuribe
A = 0.33; PI = 11.88; Mineral: **Kaolinite**

c. Sensitivity, $S_t = \dfrac{\tau_{f-\text{undisturbed}}}{\tau_{f-\text{remolded}}}$

$\tau_{f\text{-remolded}}$ is calculated using the above equation and listed in the table (Part a).

d. The plots are shown below.



Explanation: The shear strength of clay comes from two components: the cohesion, which is the cementing force between particles, and the frictional resistance, which is mainly due to the movement of one particle over another. The greater the activity of clay, the greater is the contribution of cohesion to shear strength. Although no reliable correlation can be developed from the above plots, both the undisturbed and remolded shear strengths certainly show increasing trends as the activity increases.

4.C.2 a. The liquidity index is given by: $LI = \dfrac{w - PL}{LL - PL}$

The range of liquidity index corresponding to the range in natural water content is calculated and listed in the table.

Soil	% clay (< 0.002 mm in size)	w_n (%)	LL	PL	LI
1	34	59-67	49	26	**1.43-1.78**
2	44	18-36	37	21	**-0.18-0.94**
3	54	51-56	61	26	**0.71-0.86**
4	81	61-70	58	24	**1.08-1.35**
5	28	441-600	511	192	**0.78-1.28**
6	67	98-111	132	49	**0.59-0.75**
7	72	51-65	89	31	**0.34-0.57**

b. **Soils 1 and 4**: Since *LI* range is greater than 1.0, the water content is greater than the liquid limit. From Figure 4.1, the soil behaves like a viscous fluid with practically no shearing resistance.

Soil 2: At a water content of 18%, the $LI < 0$, and the soil behaves like a brittle solid with high shear resistance. At water content of 36%, the soil is in a plastic state ($0 < LI < 1$) showing moderate shearing resistance and a ductile behavior.

Soils 3, 6, and 7: Since $0 < LI < 1$, the water content is less than the liquid limit. From Figure 4.1, the soil is in the plastic state showing moderate shearing resistance and a ductile behavior.

Soil 5: At a water content of 441%, the soil is in the plastic state ($0 < LI < 1$) with moderate shear resistance. At water content of 600%, the $LI > 1$, and the soil becomes a viscous fluid with practically no shearing resistance.

Chapter 5

5.1 Refer to Figure 5.1.

Soil	Classification
A	**Clay**
B	**Sandy clay**
C	**Loam**
D	**Sandy clay and sandy clay loam (borderline)**
E	**Sandy loam**

5.2 SOIL A: From Table 5.1, the soil is A-2-4. The *GI* for A-2-4 is zero. Classification: **A-2-4(0)**.

SOIL B: From Table 5.1, the soil is A-3. *GI* = 0. Classification: **A-3(0)**.

SOIL C: From Table 5.1, the soil is A-2-6. Equation 5.2:

$$GI = 0.01(F_{200} - 15)(PI - 10) = 0.01(12 - 15)(13 - 10) = -0.09 \approx 0$$

Classification: **A-2-6(0)**

SOIL D: From Table 5.1, the soil is A-2-7. Equation 5.2:

$$GI = 0.01(F_{200} - 15)(PI - 10) = 0.01(30 - 15)(18 - 10) = 1.2 \approx 1$$

Classification: **A-2-7(1)**

SOIL E: From Table 5.1, the soil is A-1-b. *GI* = 0. Classification: **A-1-b(0)**.

5.3 SOIL A: From Table 5.1, the soil is A-7-5. Note: $PI = 21 < LL - 30 = 22$

Eq. (5.1):

$$GI = (F_{200} - 35)[0.2 + 0.005(LL - 40)] + 0.01(F_{200} - 15)(PI - 10)$$

$$GI = (72 - 35)[0.2 + 0.005(52 - 40)] + 0.01(72 - 15)(21 - 10)$$

$$= 15.89 \approx 16$$

Classification: **A-7-5(16)**.

SOIL B: From Table 5.1, the soil is A-6. Eq. (5.1):

$$GI = (F_{200} - 35)[0.2 + 0.005(LL - 40)] + 0.01(F_{200} - 15)(PI - 10)$$

$$GI = (58 - 35)[0.2 + 0.005(38 - 40)] + 0.01(58 - 15)(12 - 10) = 5.23 \approx 5$$

Classification: **A-6(5)**

SOIL C: From Table 5.1, the soil is A-7-6. Note: $PI = 14 > LL - 30 = 11$
Eq. (5.1):

$$GI = (F_{200} - 35)[0.2 + 0.005(LL - 40)] + 0.01(F_{200} - 15)(PI - 10)$$

$$GI = (64 - 35)[0.2 + 0.005(41 - 40)] + 0.01(64 - 15)(14 - 10) = 7.9 \approx 8$$

Classification: **A-7-6(8)**

SOIL D: From Table 5.1, the soil is A-6. Eq. (5.1):

$$GI = (F_{200} - 35)[0.2 + 0.005(LL - 40)] + 0.01(F_{200} - 15)(PI - 10)$$

$$GI = (82 - 35)[0.2 + 0.005(32 - 40)] + 0.01(82 - 15)(12 - 10) = 8.86 \approx 9$$

Classification: **A-6(9)**

SOIL E: From Table 5.1, the soil is A-6. Eq. (5.1):

$$GI = (F_{200} - 35)[0.2 + 0.005(LL - 40)] + 0.01(F_{200} - 15)(PI - 10)$$

$$GI = (48 - 35)[0.2 + 0.005(30 - 40)] + 0.01(48 - 15)(11 - 10) = 2.28 \approx 2$$

Classification: **A-6(2)**

5.4 SOIL 1: Fine fraction = % passing No. 200 sieve = 30%
Coarse fraction = 100 -30 = 70%
Gravel fraction = 100 – 70 = 30%
Sand fraction = 70 – 30 = 40%
More than 50% of coarse fraction passing No. 4 sieve, so sandy soil.
Table 5.2 and Figure 5.3: **SC**
Figure 5.4: More than 15% gravel. **Clayey sand with gravel**.

SOIL 2: Coarse fraction = 200 – 20 = 80%
Gravel fraction = 100 – 48 = 52%
Sand fraction = 80 – 52 = 28%
Table 5.2 and Figure 5.3: **GC**
Figure 5.4: Greater than 15% sand. **Clayey gravel with sand**

SOIL 3: Coarse fraction = 100 – 30 = 30%
Gravel fraction = 100 – 95 = 5%
Sand fraction = 95 – 70 = 25%
Table 5.2: fine-grained soil; $LL = 52$; $PI = 28$.
Table 5.2 and Figure 5.3: **CH**
Figure 5.5: ≥ 30% plus 200, % sand > % gravel, < 15% gravel,
so **sandy fat clay**

SOIL 4: Coarse fraction = 100 – 82 = 18%
Gravel fraction = 100 – 100 = 0%
Sand fraction = 18 – 0 = 18%
Table 5.2: fine-grained soil; *LL* = 30; *PI* = 19.
Table 5.2 and Figure 5.3: **CL**
Figure 5.5: **lean clay with sand**

SOIL 5: Coarse fraction = 100 – 74 = 26%
Gravel fraction = 100 – 100 = 0%
Sand fraction = 26 – 0 = 26%
Table 5.2: fine-grained soil; *LL* = 35; *PI* = 21.
Table 5.2 and Figure 5.3: **CL**
Figure 5.5: **lean clay with sand**

SOIL 6: Coarse fraction = 100 – 26 = 74%
Gravel fraction = 100 – 87 = 13%
Sand fraction = 74 – 13 = 61%
Table 5.2: coarse-grained soil; *LL* = 38; *PI* = 18.
Table 5.2 and Figure 5.3: **SC**
Figure 5.4: < 15% gravel; **clayey sand**

SOIL 7: Coarse fraction = 100 – 78 = 22%
Gravel fraction = 100 – 88 = 12%
Sand fraction = 22 – 12 = 10%
Table 5.2: fine-grained soil; *LL* = 52; *PI* = 28.
Table 5.2 and Figure 5.3: **CH**
Figure 5.5: < 30% plus 200, % sand < % gravel; **fat clay with gravel**

SOIL 8: Coarse fraction = 100 – 57 = 43%
Gravel fraction = 100 – 99 = 1%
Sand fraction = 43 – 1 = 42%
Table 5.2: fine-grained soil; *LL* = 54; *PI* = 26.
Table 5.2 and Figure 5.3: **CH**
Figure 5.5: ≥ 30% plus 200, % sand > % gravel; **sandy fat clay**

SOIL 9: Coarse fraction = 100 – 11 = 89%
Gravel fraction = 100 – 71 = 29%
Sand fraction = 89 – 29 = 70%
LL = 32; *PI* = 16; C_u = 4.8; C_c = 2.9. Table 5.2 and Figure 5.3: **SP-SC**
Figure 5.4: **poorly graded sand with clay and gravel**

SOIL 10: Coarse fraction = 100 – 2 = 98%
Gravel fraction = 100 – 100 = 0%
Sand fraction = 98 – 0 = 98%
C_u = 7.2; C_c = 2.2. Table 5.2: **SW**
Figure 5.4: <15% gravel; **well graded sand**

SOIL 11: Coarse fraction = 100 – 65 = 35%
Gravel fraction = 100 – 89 = 11%
Sand fraction = 35 – 11 = 24%
Table 5.2: fine-grained soil; *LL* = 44; *PI* = 21.
Table 5.2 and Figure 5.3: **CL**
Figure 5.5: **sandy lean clay**

SOIL 12: Coarse fraction = 100 – 8 = 92%
Gravel fraction = 100 – 90 = 10%
Sand fraction = 92 – 10 = 82%
LL = 39; *PI* = 31; C_u = 3.9; C_c = 2.1. Table 5.2 and Figure 5.3: **SP-SC**
Figure 5.4: **poorly graded sand with clay**

5.5 a. 13% passing No. 200 sieve; 38% passing No. 40 sieve; 90% passing No. 10 sieve. *PI* = 23 – 19 = 4. Referring to Table 5.1, the soil is A-1-b. *GI* = 0. So the soil is **A-1-b(0)**.

b. Coarse fraction = 100 – 13 = 87%
Gravel fraction = 100 – 100 = 0%
Sand fraction = 87 – 0 = 87%
LL = 23; *PI* = 4. From Table 5.2 and Figure 5.3, the group symbol is **SC**.
From Figure 5.4, the group name is **clayey sand**.

CRITICAL THINKING PROBLEM

5.C.1 1. <u>Stratum 2</u>
18% passing No. 200 sieve; *PI* = 5. From Table 5.1, the soil is A-1-b.
GI = 0; Soil classification: **A-1-b(0)**

<u>Stratum 3</u>
8% passing No. 200 sieve; NP. From Table 5.1, the soil is A-3.
GI = 0; Soil classification: **A-3(0)**

Stratum 4

67% passing No. 200 sieve; LL = 52; PI = 10. From Table 5.1, the soil is A-5.

$$GI = (F_{200} - 35)[0.2 + 0.005(LL - 40)] + 0.01(F_{200} - 15)(PI - 10)$$

$$GI = (67 - 35)[0.2 + 0.005(52 - 40)] + 0.01(67 - 15)(10 - 10) = 8.3 \approx 8$$

Soil classification: **A-5(8)**

Stratum 5

52% passing No. 200 sieve; LL = 36; PI = 9. From Table 5.1, the soil is A-4.

$$GI = (52 - 35)[0.2 + 0.005(36 - 40)] + 0.01(52 - 15)(9 - 10) = 2.69 \approx 3$$

Soil classification: **A-4(3)**

2. Stratum 2

Coarse fraction: 100 – 18 = 82%; Table 5.2: coarse-grained soil. From Table 5.4, most probable soil classification (corresponding to A-1-b): SW, SP, GM, SM. Since it is a fine sand, and since C_c is not between 1 and 3, it is a poorly graded sand. Classification: **SP**

Stratum 3

Coarse fraction: 100 – 8 = 92%; Table 5.2: coarse-grained soil. From Table 5.4, most probable soil classification (corresponding to A-3): SP. Since it is a non-plastic fine sand, classification: **SP**

Stratum 4

Coarse fraction: 100 – 67 = 23%; Table 5.2: fine-grained soil. From Table 5.4, most probable soil classification (corresponding to A-5): OH, MH, ML, OL. Since the soil is an organic sandy silt, and since LL = 52 > 50, the classification is **OH**.

Stratum 5

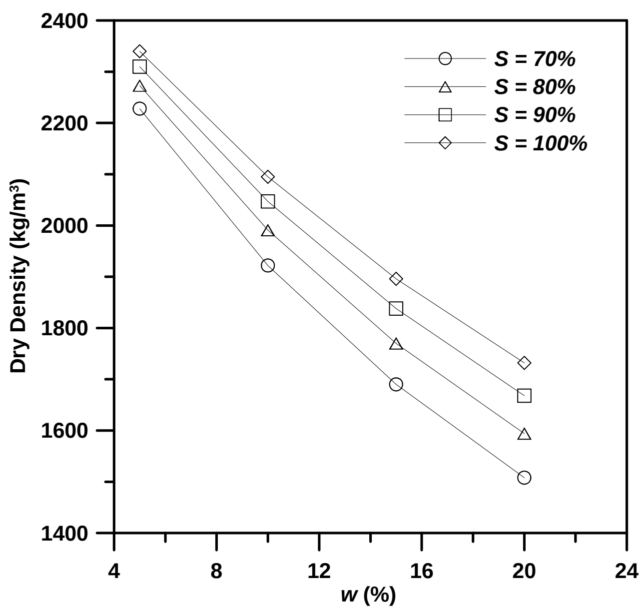
Coarse fraction: 100 – 52 = 48%; Table 5.2: fine-grained soil. From Table 5.4, most probable soil classification (corresponding to A-4): ML, OL. Since the soil is a sandy silt, and since LL = 36 < 50, the classification is **ML**.

Chapter 6

6.1 $\rho_d = \dfrac{G_s \rho_w}{1 + \dfrac{G_s w}{S}}$

G_s	ρ_w (kg/m^3)	w (%)	ρ_d @ S (kg/m^3) 70%	80%	90%	100%
2.65	1000	5	**2228**	**2273**	**2310**	**2340**
		10	**1922**	**1991**	**2047**	**2095**
		15	**1690**	**1770**	**1838**	**1896**
		20	**1508**	**1594**	**1668**	**1732**

The plot is shown below.



6.2 Eq. (6.4):

$$\gamma_{zav} = \frac{\gamma_w}{w + \dfrac{1}{G_s}} = \frac{9.81}{w + \dfrac{1}{2.68}} = \frac{9.81}{w + 0.3731}.$$

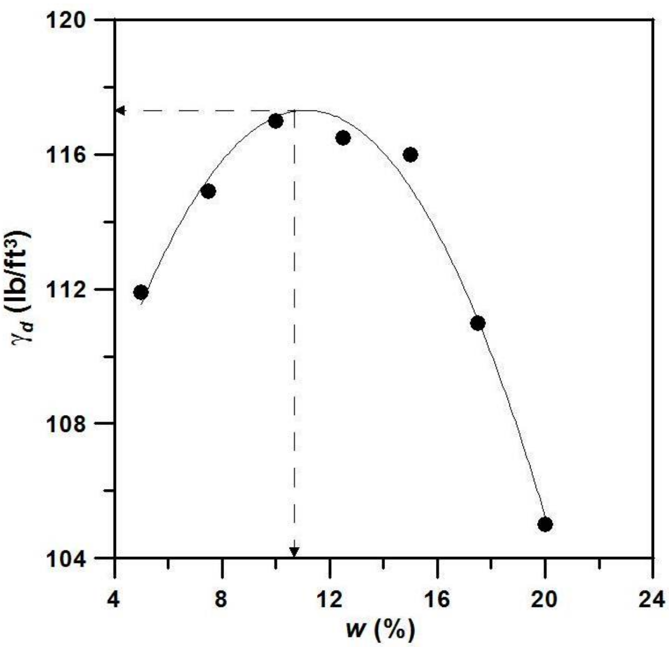
The table can now be prepared.

w (%)	γ_{zav} (kN/m^3)
5	**23.18**
10	**20.73**
15	**18.75**
20	**17.11**
25	**15.74**

6.3

Volume (ft^3)	Weight of soil mass, W (lb)	$\gamma = \frac{W}{V}$ (lb/ft^3)	w (%)	$\gamma_d = \frac{\gamma}{1 + \frac{w(\%)}{100}}$ (lb/ft^3)
$\frac{1}{30}$	3.92	117.6	5.0	111.9
$\frac{1}{30}$	4.12	123.6	7.5	114.9
$\frac{1}{30}$	4.29	128.7	10.0	117.0
$\frac{1}{30}$	4.37	131.1	12.5	116.5
$\frac{1}{30}$	4.45	133.5	15.0	116.0
$\frac{1}{30}$	4.35	130.5	17.5	111.0
$\frac{1}{30}$	4.20	126.0	20.0	105.0

a. The plot of γ_d versus w is shown. $\gamma_{d(\max)}$ **≈ 117.5 lb/ft³ @ w_{opt} = 10.8%**



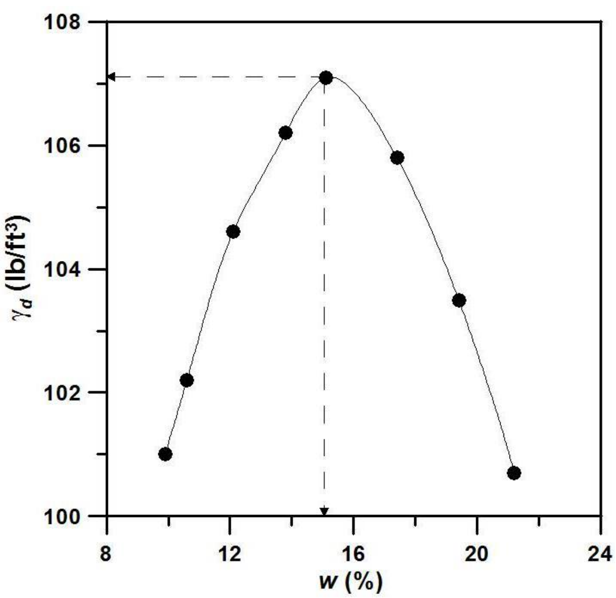
b. $\gamma_d = \frac{G_s \gamma_w}{1+e}$; $117.5 = \frac{(2.68)(62.4)}{1+e}$; $e \approx$ **0.42**

$$S = \frac{wG_s}{e} = \frac{(0.108)(2.68)}{0.423} = 0.684 = \mathbf{68.4\%}$$

6.4

Volume (ft^3)	Weight of soil mass, W (lb)	$\gamma = \frac{W}{V}$ (lb/ft^3)	w (%)	$\gamma_d = \frac{\gamma}{1+\frac{w(\%)}{100}}$ (lb/ft^3)
$\frac{1}{30}$	3.70	111.0	9.9	101.0
$\frac{1}{30}$	3.77	113.1	10.6	102.2
$\frac{1}{30}$	3.91	117.3	12.1	104.6
$\frac{1}{30}$	4.03	120.9	13.8	106.2
$\frac{1}{30}$	4.11	123.3	15.1	107.1
$\frac{1}{30}$	4.14	124.2	17.4	105.8
$\frac{1}{30}$	4.12	123.6	19.4	103.5
$\frac{1}{30}$	4.07	122.1	21.2	100.7

a. The plot of γ_d versus w is shown. $\gamma_{d(\max)}$ **≈ 107.1 lb/ft³ @ w_{opt} = 15%**



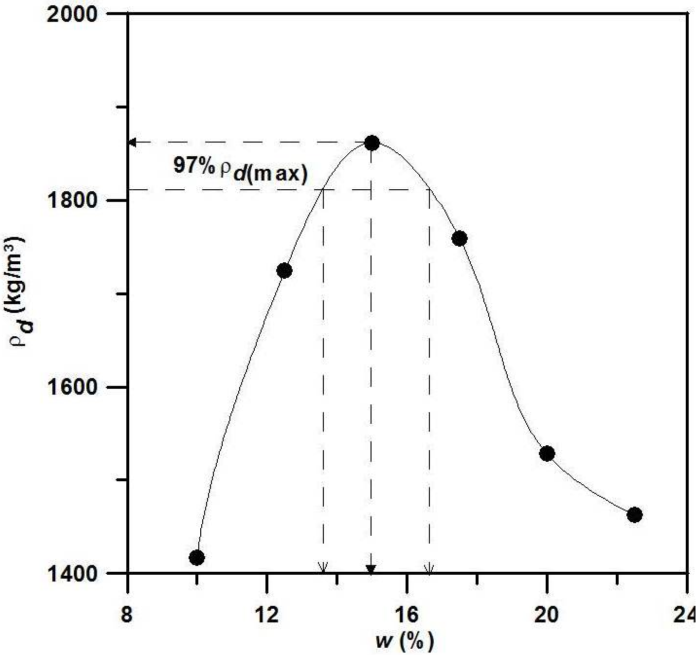
b. $\gamma_d = \frac{G_s \gamma_w}{1+e}$; $107.1 = \frac{(2.7)(62.4)}{1+e}$; $e \approx$ **0.57**

$$S = \frac{wG_s}{e} = \frac{(0.15)(2.7)}{0.57} = 0.71 = \mathbf{71\%}$$

6.5

Volume of mold V (cm^3)	Mass of wet soil M (kg)	$\rho = \frac{M}{V}$ (kg/m^3)	w (%)	ρ_d (kg/m^3)
943.3	1.47	1558.3	10.0	1416.6
943.3	1.83	1940.0	12.5	1724.4
943.3	2.02	2141.4	15.0	1862.0
943.3	1.95	2067.2	17.5	1759.3
943.3	1.73	1833.9	20.0	1528.2
943.3	1.69	1791.5	22.5	1462.4

a. The plot of ρ_d versus w is shown. $\rho_{d(\max)}$ **≈ 1870 kg/m^3 @ w_{opt} = 15%**



b. $$R(\%) = \frac{\rho_{d(\text{field})}}{\rho_{d(\max)}} \times 100 = \left(\frac{\rho_{d(\text{field})}}{1870}\right)(100)$$

Therefore, $\rho_{d(\text{field})} = (0.97)(1870) \approx$ **1814 kg/m^3**

From the graph, the acceptable range of moisture content is **13.5% – 16.5%.**

6.6 $\gamma_{\text{(in situ)}}$ = 17.3 kN/m^3; $\gamma_{d\,\text{(in situ)}} = \dfrac{17.3}{1+\dfrac{16}{100}} = 14.91\,\text{kN/m}^3$; $\gamma_{d\text{(compacted)}}$ = 18.1 kN/m^3

$$\text{Volume of soil to be excavated} = (2000)\left(\frac{18.1}{14.91}\right) = \mathbf{2{,}428\,m^3}$$

Weight of moist soil to be transported = (2428×17.3) = 42,004 kN

$$\text{Number of truck loads} = \frac{(42004)(1000)}{(20)(2000)(4.45)} = 235.9 \approx \mathbf{236}$$

6.7 Dry weight of solids required at the embankment site:

$$W_s = 5000\gamma_d \text{ kN} = (5000)\left(\frac{G_s \times 9.81}{1.75}\right) = 28{,}029 G_s \text{ kN}$$

Borrow Pit	W_s (kN)	γ_d at borrow pit (kN/m^3)	Volume to be excavated from borrow pit = [$W_s/\gamma_{d\text{ (borrow pit)}}$]	Cost/m^3 ($)	Total cost ($)
I	74276.8	$\dfrac{2.65 \times 9.81}{1+0.8} = 14.44$	**5143.8 m^3**	8	41,150
II	75117.7	$\dfrac{2.68 \times 9.81}{1+0.9} = 13.83$	**5431.5 m^3**	5	27,157
III	75958.6	$\dfrac{2.71 \times 9.81}{1+1.1} = 12.66$	**5999.8 m^3**	9	53,998
IV	76799.4	$\dfrac{2.74 \times 9.81}{1+0.85} = 14.53$	**5285.5 m^3**	12	63,427

a. Shown in table.

b. **Borrow Pit II**

6.8 From Eq. (6.20): $D_r = \left[\dfrac{\gamma_{d\,\text{(field)}} - \gamma_{d\,\text{(min)}}}{\gamma_{d\,\text{(max)}} - \gamma_{d\,\text{(min)}}}\right]\left[\dfrac{\gamma_{d\,\text{(max)}}}{\gamma_{d\,\text{(field)}}}\right]$

$$0.75 = \left[\frac{\gamma_{d\,\text{(field)}} - 15.5}{18.9 - 15.5}\right]\left[\frac{18.9}{\gamma_{d\,\text{(field)}}}\right]; \quad \gamma_{d\,\text{(field)}} = 17.91\,\text{kN/m}^3$$

$$R\,(\%) = \frac{\gamma_{d\,(\text{field})}}{\gamma_{d\,(\text{max})}}(100) = \left(\frac{17.91}{18.9}\right)(100) = \mathbf{94.8\%}$$

6.9 $R = 0.935 = \dfrac{\gamma_{d\,(\text{field})}}{\gamma_{d\,(\text{max})}} = \dfrac{\gamma_{d\,(\text{field})}}{16.98}$; $\gamma_{d\,(\text{field})} = \mathbf{15.87\ kN/m^3}$

$$D_r = \left[\frac{\gamma_{d\,(\text{field})} - \gamma_{d\,(\text{min})}}{\gamma_{d\,(\text{max})} - \gamma_{d\,(\text{min})}}\right]\left[\frac{\gamma_{d\,(\text{max})}}{\gamma_{d\,(\text{field})}}\right] = \left(\frac{15.87 - 14.46}{16.98 - 14.46}\right)\left(\frac{16.98}{15.87}\right) = 0.598 = \mathbf{59.8\%}$$

6.10 a. $R = 0.88 = \dfrac{\gamma_{d\,(\text{field})}}{\gamma_{d\,(\text{max})}} = \dfrac{\gamma_{d\,(\text{field})}}{118}$; $\gamma_{d\,(\text{field})} = \mathbf{103.84\ lb/ft^3}$

b. $D_r = \left[\dfrac{\gamma_{d\,(\text{field})} - \gamma_{d\,(\text{min})}}{\gamma_{d\,(\text{max})} - \gamma_{d\,(\text{min})}}\right]\left[\dfrac{\gamma_{d\,(\text{max})}}{\gamma_{d\,(\text{field})}}\right] = \left(\dfrac{103.84 - 98}{118 - 98}\right)\left(\dfrac{118}{103.84}\right) = 0.331 = \mathbf{33.1\%}$

$$\gamma = (1 + w)\gamma_{d\,(\text{field})} = (1 + 0.13)(103.84) = \mathbf{117.33\ \ lb/ft^3}$$

6.11 In the field:

Sand used to fill the hole and cone: 6.08 kg – 2.86 kg = 3.22 kg

Sand used to fill the hole: 3.22 kg – 0.118 kg = 3.102 kg

Volume of the hole: $\dfrac{3.102\,\text{kg}}{1731\,\text{kg/m}^3} = 0.001792\,\text{m}^3$

Moist density of compacted soil: $\dfrac{3.34}{0.001792} = 1863.84\ \text{kg/m}^3$

$$\gamma = \frac{(1863.84)(9.81)}{1000} = 18.28\,\text{kN/m}^3$$

a. $\gamma_d = \dfrac{\gamma}{1 + \dfrac{w(\%)}{100}} = \dfrac{18.28}{1 + \dfrac{12.1}{100}} = \mathbf{16.3\,kN/m^3}$

b. From Problem 6.5: $\rho_{d(\text{max})}$ = 1870 kg/m^3

$$\gamma_{d(\max)} = \frac{(1870)(9.81)}{1000} = 18.34\,\text{kN/m}^3$$

$$R = \frac{\gamma_{d(\text{field})}}{\gamma_{d(\max)}} = \frac{16.30}{18.34} = 0.888 = \mathbf{88.8\%}$$

6.12 $S_N = (1.7)\sqrt{\frac{3}{(D_{50})^2} + \frac{1}{(D_{20})^2} + \frac{1}{(D_{10})^2}} = (1.7)\sqrt{\frac{3}{(1.98)^2} + \frac{1}{(0.31)^2} + \frac{1}{(0.18)^2}} = \mathbf{11.02}$

Rating: **GOOD**

CRITICAL THINKING PROBLEM

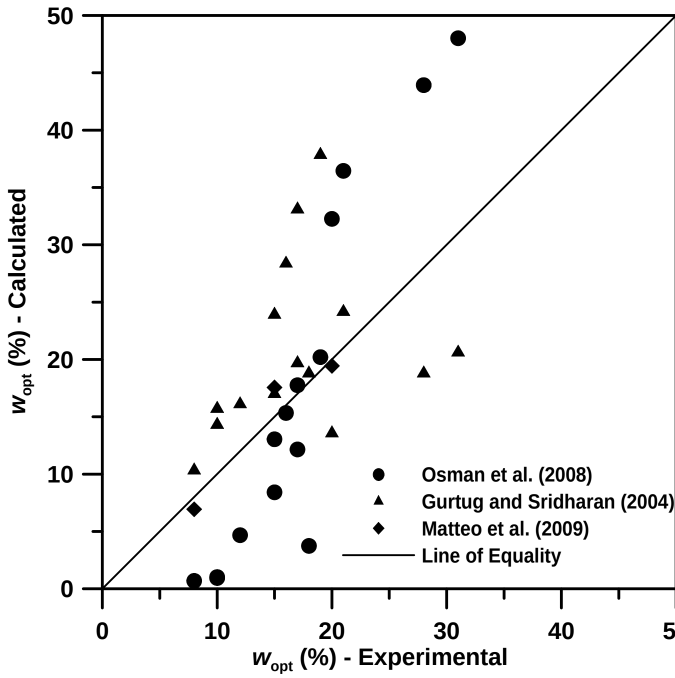
6.C.1 a. Osman et al. (2008) method: Eqs. (6.13) through (6.16) are used to calculate w_{opt} and $\gamma_{d(\max)}$. These values are listed in the following table.

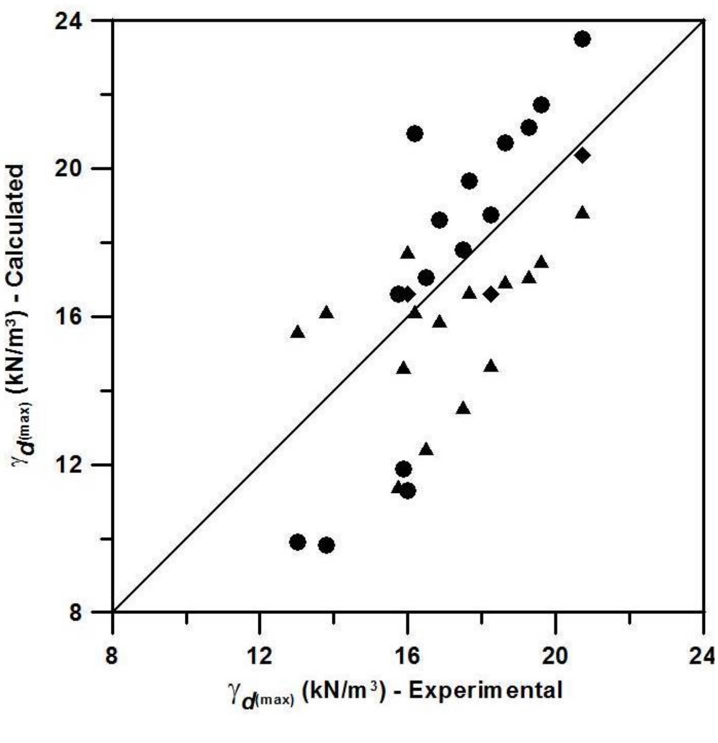
b. Gurtug and Sridharan (2004) method: Eqs. (6.11) and (6.12) are used to calculate w_{opt} and $\gamma_{d(\max)}$. These values are listed in the following table.

c. Matteo et al. (2009) method: Eqs. (6.17) and (6.18) are used to calculate w_{opt} and $\gamma_{d(\max)}$, only for the modified Proctor tests. These values are listed in the following table.

Soil	E (kN-m/m^3)	w_{opt} (Exp.) (%)	$\gamma_{d(\max)}$ (Exp.) (kN/m^3)	Part a		Part b		Part c	
				w_{opt} (%)	$\gamma_{d(\max)}$ (kN/m^3)	w_{opt} (%)	$\gamma_{d(\max)}$ (kN/m^3)	w_{opt} (%)	$\gamma_{d(\max)}$ (kN/m^3)
1	2700	8	20.72	0.69	23.52	10.34	18.77	6.94	20.37
	600	10	19.62	0.93	21.73	14.31	17.46		
	354	10	19.29	1.02	21.11	15.70	17.02		
2	2700	20	16.00	32.26	11.31	13.57	17.69	19.44	16.6
	600	28	13.80	43.92	9.82	18.78	16.08		
	354	31	13.02	48.01	9.90	20.61	15.55		
3	2700	15	18.25	13.04	18.74	23.91	14.64	17.56	16.62
	1300	16	17.5	15.33	17.80	28.37	13.50		
	600	17	16.5	17.76	17.07	33.09	12.38		
	275	19	15.75	20.20	16.61	37.85	11.34		
4	600	21	15.89	36.45	11.89	24.15	14.58		
5	600	18	16.18	3.74	20.95	18.78	16.08		
6	600	17	16.87	12.15	18.62	19.67	15.82		
7	600	12	18.63	4.67	20.69	16.10	16.89		
8	600	15	17.65	8.41	19.65	16.99	16.62		

d. The plots of calculated values against experimental results are shown in the figures below. The 45° line of equality is also drawn in both figures.





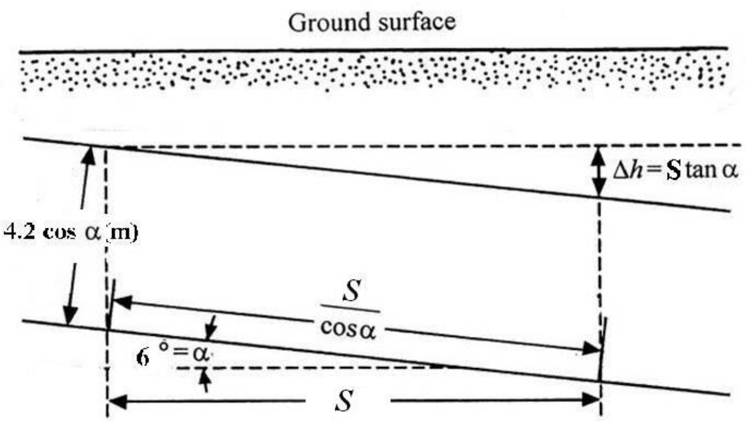
e. Prediction of w_{opt}: For both the Osman et al. (2008) and Gurtug and Sridharan (2004) models, several data points are closely packed around the 45° line suggesting reasonable agreement between the calculated and experimental values. For the remaining soils, a poor agreement is observed. All 3 points for the Matteo (2009) model plot near the equality line showing close agreement.

Prediction of $\gamma_{d(max)}$: Most data points for all models show good agreement between the calculated and the experimental values.

Empirical models are often limited to the materials, test methods, and environmental conditions (among other factors) under which the experiments were conducted and the models developed. For new materials and conditions, the predicted values may not be reliable.

Chapter 7

7.1



From the figure: $i = \dfrac{\text{head loss}}{\text{length}} = \dfrac{S \tan \alpha}{\left(\dfrac{S}{\cos \alpha}\right)} = \sin \alpha$

$q = k\,i\,A = (k)(\sin \alpha)(4.2 \cos \alpha)(1)$

$k = 4.8 \times 10^{-3}$ cm/sec $= 4.8 \times 10^{-5}$ m/sec

$$q = (4.8 \times 10^{-5})(\sin 6°)(4.2 \cos 6°)\underbrace{(3600)}_{\substack{\text{to change} \\ \text{to m/hr}}} = 0.0754 \text{ m}^3\text{/hr/m}$$

$\approx$ **7.54×10^{-2} m^3/hr/m**

7.2 $i = \dfrac{h}{\left(\dfrac{S}{\cos \alpha}\right)}$

$$q = kiA = k\left(\frac{h \cos \alpha}{S}\right)(H_1 \cos \alpha \times 1) = \left(\frac{0.075}{\underbrace{10^2}_{\text{m/sec}}}\right)\left(\frac{2.75 \cos 14}{30}\right)(2 \cos 14)$$

$=$ **1.29×10^{-4} m^3/sec/m**

7.3 a. $k = \frac{QL}{Aht} = \frac{(350\,\text{cm}^2)(30\,\text{cm})}{(176.71\,\text{cm}^2)(50\,\text{cm})(300\,\text{sec})} = \mathbf{3.96 \times 10^{-3}\ cm/sec}$

b. $v_s = v\left(\frac{1+e}{e}\right); \quad v = ki$

$$v_s = ki\left(\frac{1+e}{e}\right) = (0.00396)\left(\frac{50\,\text{cm}}{30\,\text{cm}}\right)\left(\frac{1+0.61}{0.61}\right) = \mathbf{0.0174\ cm/sec}$$

7.4 $k = \frac{QL}{Aht}; \quad 0.062 = \frac{(160)(15)}{(31.67)(h)(60)}$

h = **20.37 cm**

7.5 a. $k = 2.303\left(\frac{aL}{At}\right)\log_{10}\left(\frac{h_1}{h_2}\right) = (2.303)\left[\frac{(0.25)(15)}{(19.64)(8\times 60)}\right]\log_{10}\left(\frac{40}{20}\right)$

$= \mathbf{2.75 \times 10^{-4}\ cm/sec}$

b. $k = 2.303\left(\frac{aL}{At}\right)\log_{10}\left(\frac{h_1}{h_2}\right)$

$$0.000275 = (2.303)\left[\frac{(0.25)(15)}{(19.64)(6\times 60)}\right]\log_{10}\left(\frac{40}{h_2}\right) = 0.00122\log\left(\frac{40}{h_2}\right)$$

h_2 = **23.82 cm**

7.6 a. $k = 2.303\left(\frac{aL}{At}\right)\log_{10}\left(\frac{h_1}{h_2}\right)$

$$= (2.303)\left(\frac{0.97\times 50}{16\times 10}\right)\log_{10}\left(\frac{41}{18.5}\right) = 0.241\,\text{cm/min} = 0.00402\,\text{cm/sec}$$

$$\overline{K} = \frac{k\eta}{\gamma_w} = \frac{(4.02\times 10^{-5}\ \text{m/sec})(1.005\times 10^{-3}\ \text{N}\cdot\text{s/m}^2)}{9.789\times 10^3\ \text{N/m}^3} = \mathbf{4.13 \times 10^{-12}\ m^2}$$

b. $k = 2.303\left(\frac{aL}{At}\right)\log_{10}\left(\frac{h_1}{h_2}\right)$

$$0.241\,\text{cm/min} = (2.303)\left(\frac{0.97\times 50}{16\times 7}\right)\log_{10}\left(\frac{41}{h_2}\right)$$

h_2 = **23.5 cm**

7.7 From Eq. (7.15) and Table 7.2 for $T = 28^\circ$ C:

$$k_{20°C} = \left(\frac{\eta_{T°C}}{\eta_{20°C}}\right)k_{T°C} = (0.832)(0.009) = \mathbf{7.49\times 10^{-3}\ cm/sec}$$

7.8 Eq. (7.31): $\frac{k_1}{k_2} = \frac{\frac{e_1^3}{1+e_1}}{\frac{e_2^3}{1+e_2}}$, or $\frac{0.03}{k_2} = \frac{\left(\frac{0.62^3}{1+0.62}\right)}{\left(\frac{0.48^3}{1+0.48}\right)} = \frac{0.1471}{0.0747}$

k_2 = **0.015 cm/sec**

7.9 $e = e_{max} - (e_{max} - e_{min})D_r = 0.68 - (0.68 - 0.42)(0.52) = 0.544$

Eq. (7.32): $k\ (\text{cm/sec}) = 2.4622\left[D_{10}^2\frac{e^3}{1+e}\right]^{0.7825}$

$$= (2.4622)\left[(0.4)^2\left(\frac{0.544^3}{1+0.544}\right)\right]^{0.7825} = \mathbf{0.1\ cm/sec}$$

7.10 $e_1 = e_{max} - (e_{max} - e_{min})D_r = 0.72 - (0.72 - 0.46)(0.8) = 0.512$

$e_2 = 0.72 - (0.72 - 0.46)(0.67) = 0.545$

$$\frac{k_1}{k_2} = \frac{\frac{e_1^3}{1+e_1}}{\frac{e_2^3}{1+e_2}},\ \text{or}\ \frac{0.006}{k_2} = \frac{\left(\frac{0.512^3}{1+0.512}\right)}{\left(\frac{0.545^3}{1+0.545}\right)} = \frac{0.0887}{0.1047}$$

k_2 = **7.08 × 10⁻³ cm/sec**

7.11 $n_1 = 0.36;\ \ e_1 = \dfrac{n_1}{1-n_1} = \dfrac{0.36}{1-0.36} = 0.562$

$n_2 = 0.48;\ \ e_2 = \dfrac{0.48}{1-0.48} = 0.923$

$$\text{Eq. (7.31):}\quad k_2 = k_1\left(\frac{e_2^3}{1+e_2}\right)\left(\frac{1+e_1}{e_1^3}\right) = k_1\left(\frac{1+e_1}{1+e_2}\right)\left(\frac{e_2}{e_1}\right)^3 = (0.072)\left(\frac{1.562}{1.923}\right)\left(\frac{0.923}{0.562}\right)^3$$

k_2 = **0.259 cm/sec**

7.12 $\gamma_{d(\text{field})} = R\gamma_{d(\max)} = (0.9)(16) = 14.4\ \text{kN/m}^3$

$$e = \frac{G_s\gamma_w}{\gamma_{d(\text{field})}} - 1 = \frac{(2.7)(9.81)}{14.4} - 1 = 0.839$$

$$\text{Eq. (7.34):}\quad k\ (\text{cm/sec}) = 35\left(\frac{e^3}{1+e}\right)(C_u^{0.6})(D_{10}\ \text{mm})^{2.32}$$

$$= 35\left(\frac{0.839^3}{1+0.839}\right)(3.1)^{0.6}(0.23)^{2.32} = \mathbf{0.732\ cm/sec}$$

7.13

Sieve no.	Opening (cm)	Percent passing	Fraction between two consecutive sieves (%)
30	0.06	100	
			{– – – – 27
40	0.0425	73	
			{– – – – 14
60	0.02	59	
			{– – – – 36
100	0.015	23	
			{– – – – 23
200	0.0075	0	

For fraction between sieve Nos. 30 and 40 $\Bigg\}$ $\dfrac{f_i}{D_{li}^{0.404} \times D_{si}^{0.595}} = \dfrac{27}{0.06^{0.404} \times 0.0425^{0.595}} = 550.93$

For fraction between sieve Nos. 40 and 60 $\left.\right\}$ $\dfrac{f_i}{D_{li}^{0.404} \times D_{si}^{0.595}} = \dfrac{14}{0.0425^{0.404} \times 0.02^{0.595}} = 514.21$

For fraction between sieve Nos. 60 and 100 $\left.\right\}$ $\dfrac{f_i}{D_{li}^{0.404} \times D_{si}^{0.595}} = \dfrac{36}{0.02^{0.404} \times 0.015^{0.595}} = 2127.71$

For fraction between sieve Nos. 100 and 200 $\left.\right\}$ $\dfrac{f_i}{D_{li}^{0.404} \times D_{si}^{0.595}} = \dfrac{23}{0.015^{0.404} \times 0.0075^{0.595}} = 2306.35$

$$\sum \frac{\sum 100\%}{D_{li}^{0.404} \times D_{si}^{0.595}} = \frac{100}{550.93 + 514.21 + 2127.71 + 2306.35} = 0.0182$$

$$k = (1.99 \times 10^4)(0.0182)^2 \left(\frac{1}{7.5}\right)^2 \left(\frac{0.68^3}{1 + 0.68}\right) = \mathbf{0.0219\ cm/sec}$$

7.14 $\dfrac{k_1}{k_2} = \left(\dfrac{e_1^n}{1 + e_1}\right)\left(\dfrac{1 + e_2}{e_2^n}\right) = \left(\dfrac{1 + e_2}{1 + e_1}\right)\left(\dfrac{e_1}{e_2}\right)^n$

$$\frac{0.2 \times 10^{-6}}{0.91 \times 10^{-6}} = \left(\frac{2.6}{1.95}\right)\left(\frac{0.95}{1.6}\right)^n; \ 0.1648 = 0.593^n$$

$$n = \frac{\log 0.1648}{\log 0.593} = 3.45$$

$$k_1 = C\left(\frac{e_1^n}{1 + e_1}\right); \quad C = \frac{(0.2 \times 10^{-6})(1 + 0.95)}{0.95^{3.45}} = 4.655 \times 10^{-7} \text{ cm/sec}$$

$$k_3 = \left(\frac{1.1^{3.45}}{2.1}\right)(4.655 \times 10^{-7}) = \mathbf{3.08 \times 10^{-7}\ cm/sec}$$

7.15 Eq. (7.37): $\log k = A' \log e + B'$

$$A' = \frac{\log k_1 - \log k_2}{\log e_1 - \log e_2} = \frac{\log(0.2 \times 10^{-6}) - \log(0.91 \times 10^{-6})}{\log(0.95) - \log(1.6)} = 2.9$$

$$B' - \log k_1 - A' \log e_1 = \log(0.2 \times 10^{-6}) - 2.9 \log(0.95) = -6.634$$

$$\log(k_3) = (2.9)\log(1.1) - 6.634 = -6.514$$

k_3 = **3.062 × 10^{-7} cm/sec**

7.16 $k_{H(\text{eq})} = \dfrac{1}{H}(k_1H_1 + k_2H_2 + ...)$

$$k_{H(\text{eq})} = \frac{1}{4}[(10^{-4})(1) + (2.8\times10^{-2})(1) + (3.5\times10^{-5})(2)] = 7.042\times10^{-3}\text{ cm/sec}$$

$$k_{V(\text{eq})} = \frac{H}{\dfrac{H_1}{k_1} + \dfrac{H_2}{k_2} + ...} = \frac{4}{\dfrac{1}{10^{-4}} + \dfrac{1}{2.8\times10^{-2}} + \dfrac{2}{3.5\times10^{-5}}}$$

$$= 5.95\times10^{-5}\text{ cm/sec}$$

$$\frac{k_{H(\text{eq})}}{k_{V(\text{eq})}} = \frac{7.042\times10^{-3}}{5.95\times10^{-5}} = \mathbf{118.35}$$

7.17 $q = kiA$

$$i = \frac{160-150}{125} = 0.08$$

$$q = 250\text{ m}^3\text{/day}$$

$$A = 2\times500 = 1000\text{ m}^2$$

$$k = \frac{q}{iA} = \frac{250}{(0.08)(1000)} = \mathbf{3.125\,m/day}$$

CRITICAL THINKING PROBLEM

7.C.1 a. $k_{v(\text{eq})} = \dfrac{H}{\dfrac{H_1}{k_1} + \dfrac{H_2}{k_2} + \dfrac{H_3}{k_3}} = \dfrac{60}{\dfrac{20}{5\times10^{-3}} + \dfrac{20}{4.2\times10^{-2}} + \dfrac{20}{3.9\times10^{-4}}} = 1.076\times10^{-3}\text{ cm/sec}$

$$q = k_{v(\text{eq})}iA = (0.001076)\left(\frac{47}{60}\right)\left(\frac{\pi}{4}\right)(15^2) = 0.149\text{ cm}^3\text{/sec} = \mathbf{536.4\ cm^3/hr}$$

b. <u>$x = 0$ mm</u>

Z = **–220 mm**;

$$\frac{u}{\gamma_w} = 470 + 220 = \mathbf{690\,mm}$$

$$h = \frac{u}{\gamma_w} + Z = 690 - 220 = \mathbf{470\,mm}$$

<u>$x = 200$ mm</u>

Z = **–220 mm**

$$k_{v(\text{eq})} i = k_1 i_1$$

$$(0.001076)\left(\frac{47}{60}\right) = (0.005)\left(\frac{\Delta H}{20}\right)$$

$$\Delta H = 3.371 \text{ cm} = 33.71 \text{ mm}$$

Therefore, $h = 470 - 33.71 = \mathbf{436.29\ mm}$

$$\frac{u}{\gamma_w} = 436.29 - (-220) = \mathbf{656.29\,mm}$$

<u>$x = 400$ mm</u>

Z = **–220 mm**

$$k_{v(\text{eq})} i = k_2 i_2$$

$$(0.001076)\left(\frac{47}{60}\right) = (0.042)\left(\frac{\Delta H}{20}\right)$$

$$\Delta H = 0.4 \text{ cm} = 4 \text{ mm}$$

Therefore, $h = 436.29 - 4 = \mathbf{432.29\ mm}$

$$\frac{u}{\gamma_w} = 432.29 - (-220) = \mathbf{652.29\,mm}$$

$\underline{x = 600 \text{ mm}}$

Z = **–220 mm**

$k_{v(\mathrm{eq})} i = k_3 i_3$

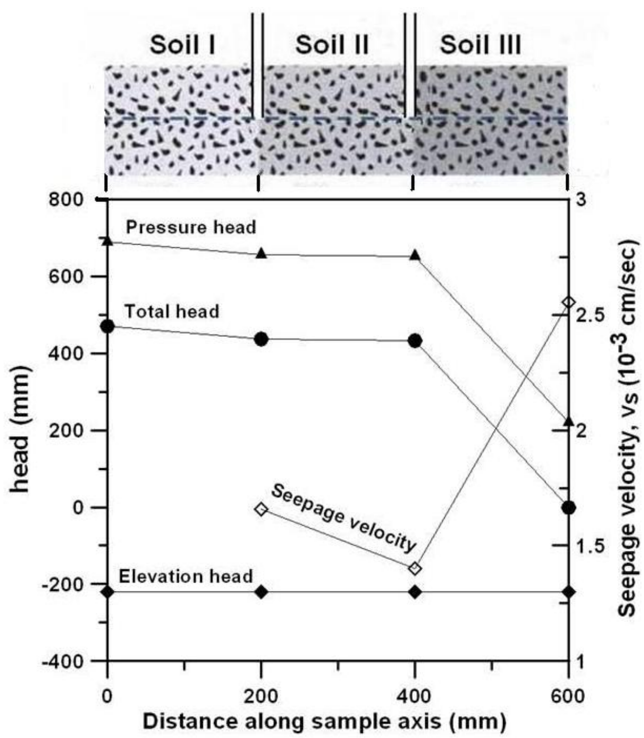
$$(0.001076)\left(\frac{47}{60}\right) = (0.00039)\left(\frac{\Delta H}{20}\right)$$

$\Delta H = 43.224 \text{ cm} = 432.24 \text{ mm}$

Therefore, $h = 432.29 - 432.24 \approx \mathbf{0\,mm}$

$$\frac{u}{\gamma_w} = 0 - (-220) = \mathbf{220\,mm}$$

c. The variation of heads with distance is shown in the following figure.



d. $v = k_{v(\mathrm{eq})} i = (0.001076)\left(\frac{47}{60}\right) = 0.000843 \text{ cm/sec}$

$$v_s = \frac{v}{n}$$

Soil I: $v_s = \frac{0.000843}{0.5} = \mathbf{0.00168\ cm/sec}$

Soil II: $v_s = \frac{0.000843}{0.6} = \mathbf{0.0014\ cm/sec}$

Soil III: $v_s = \frac{0.000843}{0.33} = \mathbf{0.00255\ cm/sec}$

These values are plotted in the figure on the previous page.

e. Height of water column is equal to the piezometric or pressure head at a point.
Therefore: height of water in *A* = pressure head at x = 200 mm = **656.29 mm**
height of water in *B* = pressure head at x = 400 mm = **652.29 mm**

Chapter 8

8.1 Eq. (8.14): $h_2 = \dfrac{h_1 k_1}{H_1\left(\dfrac{k_1}{H_1} + \dfrac{k_2}{H_2}\right)}$

$$8\text{ cm} = \frac{(20\text{ cm})(0.004\text{ cm/sec})}{(10\text{ cm})\left(\dfrac{0.004\text{ cm/sec}}{10\text{ cm}} + \dfrac{k_2\text{ cm/sec}}{15\text{ cm}}\right)}$$

k_2 = **0.009 cm/sec**

8.2 The flow net is shown.

$k = 4 \times 10^{-4}$ cm/sec

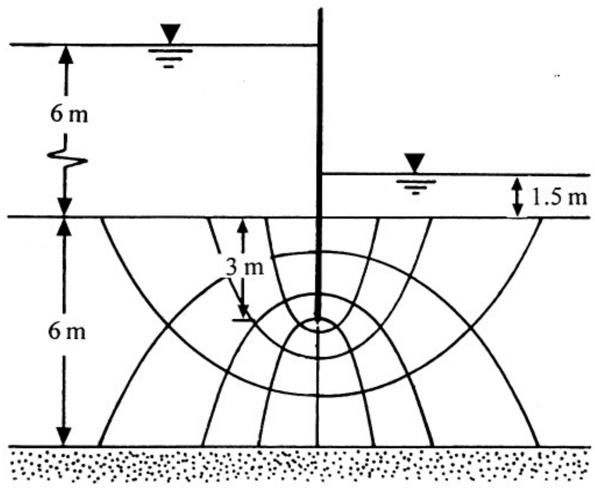
$H = H_1 - H_2$
$= 6.0 - 1.5 = 4.5$ m.

So

$$q = \left(\frac{4\times10^{-4}}{10^2}\right)\left(\frac{4.5\times4}{8}\right)$$

$$= 9\times10^{-6}\ \text{m}^3\text{/m/sec}$$

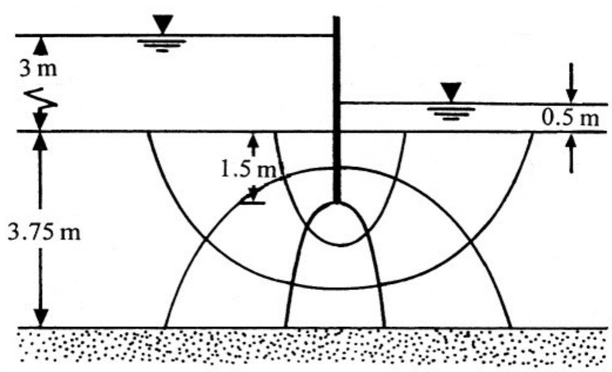
$$= \mathbf{77.76\times10^{-6}\ m^3/m/day}$$



8.3 The flow net is shown.

$N_f = 3$; $N_d = 5$

$$q = kH\left(\frac{N_f}{N_d}\right)$$



$$q = \left(\frac{4 \times 10^{-4}}{10^2} \text{ m/sec}\right)(3 - 0.5)\left(\frac{3}{5}\right) = 6 \times 10^{-6} \text{ m}^3\text{/m/sec} = \mathbf{0.518\ m^3/m/day}$$

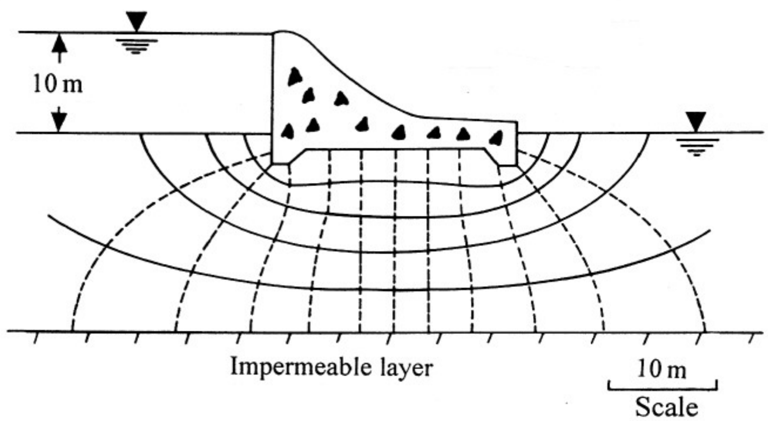
8.4 Based on the notations in Figure 8.10:

$H = (4 - 1.5)$ m = 2.5 m; $S = D = 3.6$ m; $T' = D_1 = 6$ m; $S/T' = 3.6/6 = 0.6$

From the figure, $\frac{q}{kH} \approx 0.44$

$$q = (0.44)(2.5)\left(\frac{4 \times 10^{-4}}{10^2} \times 60 \times 60 \times 24 \text{ m/day}\right) = \mathbf{0.38\ m^3/m/day}$$

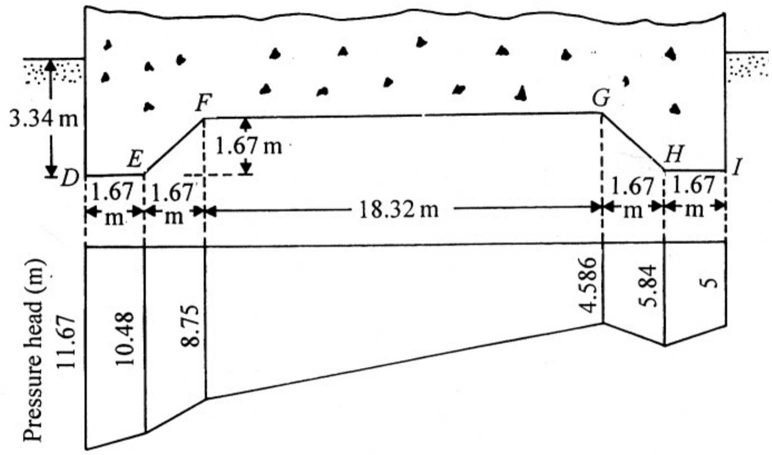
8.5 The flow net is shown.



$$q = kH\left(\frac{N_f}{N_d}\right) = \left(\frac{0.002}{10^2} \times 60 \times 60 \times 24 \text{ m/day}\right)(10)\left(\frac{5}{12}\right) = \mathbf{7.2\ m^3/m/day}$$

8.6 Refer to the flow net given in Problem 8.5 and the figure on the next page.

The flow net has 12 potential drops. Also, $H = 10$ m. So the head loss for each drop = (10/12) m. Thus,



Pressure head at $D = (10 + 3.34) - (2)(10/12) = 11.67$ m

Pressure head at $E = (10 + 3.34) - (3)(10/12) = 10.84$ m

Pressure head at $F = (10 + 1.67) - (3.5)(10/12) = 8.75$ m

Pressure head at $G = (10 + 1.67) - (8.5)(10/12) = 4.586$ m

Pressure head at $H = (10 + 3.34) - (9)(10/12) = 5.84$ m

Pressure head at $I = (10 + 3.34) - (10)(10/12) = 5$ m

The pressure heads calculated are shown in the figure. The hydraulic uplift force per unit length of the structure can now be calculated to be

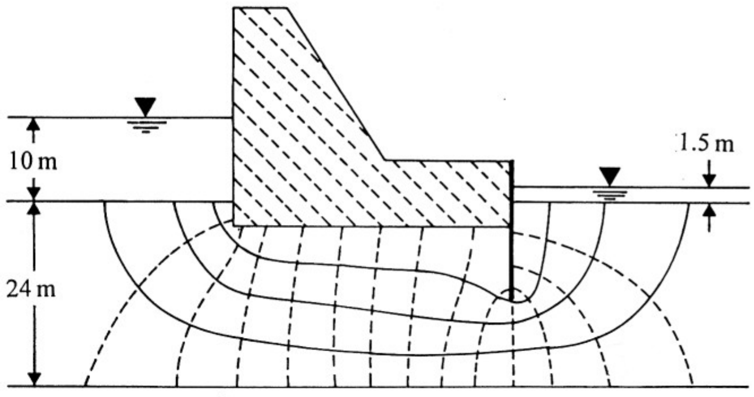
$$= \gamma_w(\text{area of the pressure head diagram})(1)$$

$$= \left[\begin{array}{l} \left(\dfrac{11.67+10.84}{2}\right)(1.67) + \left(\dfrac{10.84+8.754}{2}\right)(1.67) \\ + \left(\dfrac{8.75+4.586}{2}\right)(18.32) + \left(\dfrac{4.586+5.84}{2}\right)(1.67) + \left(\dfrac{5.84+5}{2}\right)(1.67) \end{array}\right]$$

$$= (9.81)(18.8 + 16.36 + 122.16 + 8.71 + 9.05)$$

$= \mathbf{1717.5\ kN/m}$

8.7 The flow net is shown. $N_f = 3$; $N_d = 5$.



$$q = kH\left(\frac{N_f}{N_d}\right) = \left(\frac{10^{-3}}{10^2}\right)(10-1.5)\left(\frac{4}{14}\right) = (10^{-5})(8.5)\left(\frac{4}{14}\right)$$

$$= 2.429\times10^{-5}\ \text{m}^3/\text{m/sec} \approx \mathbf{2.1\ m^3/m/day}$$

8.8 For this case, $T' = 8$ m; $S = 4$ m; $H = H_1 - H_2 = 6$ m; $B = 8$ m; $b = B/2 = 4$ m.

a. $\frac{S}{T'} = \frac{4}{8} = 0.5$; $x = b - x' = 4 - 1 = 3$ m; $\frac{x}{b} = \frac{3}{4} - 0.75$; $\frac{b}{T'} = \frac{4}{8} = 0.5$

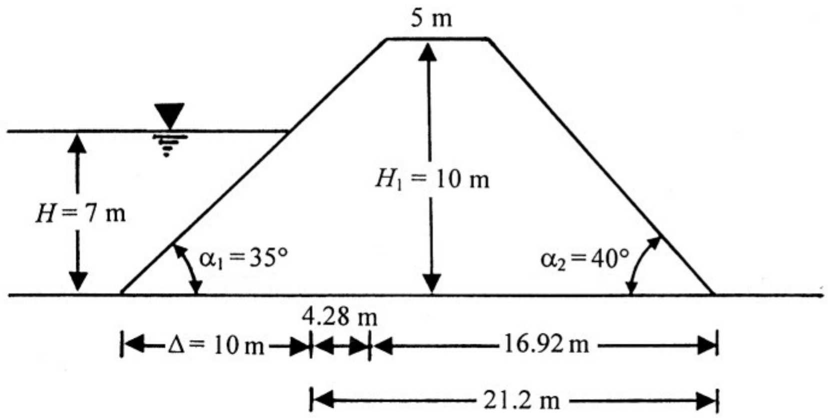
From Figure 8.11, $q/kH = 0.37$.

$$q = (0.37)\left(\frac{0.001}{10^2}\times 60\times 60\times 24\right)(6) \approx \mathbf{1.92\ m^3/m/day}$$

b. $\frac{S}{T'} = 0.5$; $\frac{b}{T'} = 0.5$; $x = b - x' = 4 - 2 = 2$ m; $\frac{x}{b} = \frac{2}{4} = 0.5$. So $q/kH = 0.4$.

$$q = (0.4)\left(\frac{0.001}{10^2}\times 60\times 60\times 24\right)(6) \approx \mathbf{2.07\ m^3/m/day}$$

8.9



$\alpha_1 = 35°$; $\alpha_2 = 40°$; $H = 7$ m; $\Delta = 7\cot 35 = 10$ m. $0.3\Delta = 3$ m.

$$d = H_1 \cot\alpha_2 + L_1 + (H_1 - H)\cot\alpha_1 + 0.3\Delta$$
$$= (10)(\cot 40) + 5 + (10-7)\cot 34 + 3 = 24.2 \text{ m}$$

$$L = \frac{d}{\cos\alpha_2} - \sqrt{\frac{d^2}{\cos^2\alpha_2} - \frac{H^2}{\sin^2\alpha_2}} = \frac{24.2}{\cos 40} - \sqrt{\left(\frac{24.2}{\cos 40}\right)^2 - \left(\frac{7}{\sin 40}\right)^2}$$
$$= 1.94 \text{ m}$$

$$q = kL\tan\alpha_2 \sin\alpha_2 = \left[\left(\frac{3\times10^{-4}}{10^2}\right)(1.94)\right](\tan 40)(\sin 40)$$
$$= 3.139\times10^{-6} \text{ m}^3\text{/sec/m} \approx \mathbf{0.271\ m^3/m/day}$$

8.10 From Problem 8.9, $d = 24.2$ m; $H = 7$ m; $\alpha_2 = 40°$

$$\frac{d}{H} = \frac{24.2}{7} = 3.46;\ m \approx 0.25 \text{ (Figure 8.14)}$$

$$L = \frac{mH}{\sin\alpha_2} = \frac{(0.25)(7)}{\sin 40} = 2.72 \text{ m}$$

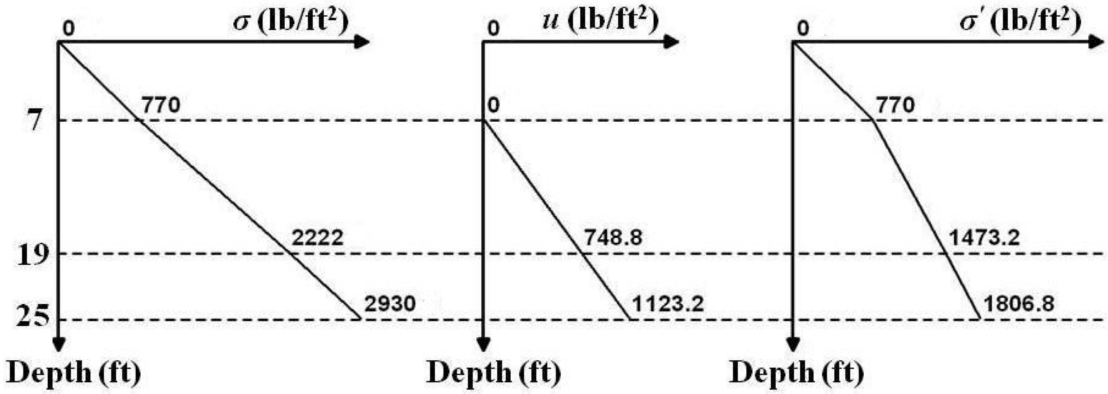
$$q = kL\sin^2\alpha_2 = \left(\frac{3\times10^{-4}}{10^2}\right)(2.72)(\sin^2 40)$$
$$= 3.37\times10^{-6} \text{ m}^3\text{/sec/m} \approx \mathbf{0.291\ m^3/m/day}$$

Chapter 9

9.1

Point	lb/ft^2		
	σ	u	σ'
A	**0**	**0**	**0**
B	(7)(110) = **770**	**0**	**770**
C	770 + (12)(121) = **2222**	(62.4)(12) = **748.8**	**1473.2**
D	2222 + (6)(118) = **2930**	748.8 + (62.4)(6) = **1123.2**	**1806.8**

The plot is given below.



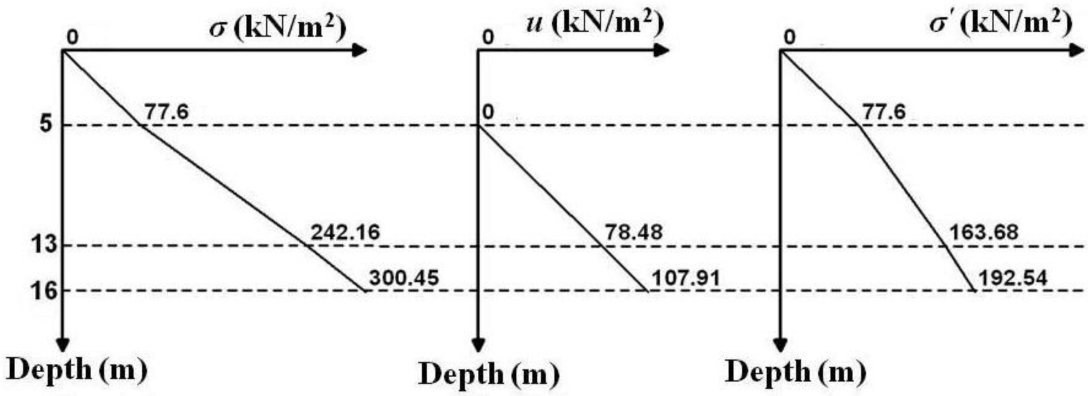
9.2 $$\gamma_{d\,(\text{layer 1})} = \frac{G_s\gamma_w}{1+e} = \frac{(2.69)(9.81)}{1+0.7} = 15.52 \text{ kN/m}^3$$

$$\gamma_{\text{sat(layer 2)}} = \frac{\gamma_w(G_s+e)}{1+e} = \frac{(9.81)(2.7+0.55)}{1+0.55} = 20.57 \text{ kN/m}^3$$

$$\gamma_{\text{sat(layer 3)}} = \frac{\gamma_w(G_s+e)}{1+e} = \frac{(9.81)\left(\dfrac{1.2}{0.38}+1.2\right)}{1+1.2} = 19.43 \text{ kN/m}^3$$

Point	kN/m^2		
	σ	u	σ'
A	**0**	**0**	**0**
B	(5)(15.52) = **77.6**	**0**	**77.6**
C	77.6 + (8)(20.57) = **242.16**	(9.81)(8) = **78.48**	**163.68**
D	242.16 + (3)(19.43) = **300.45**	78.48 + (9.81)(3) = **107.9**	**192.54**

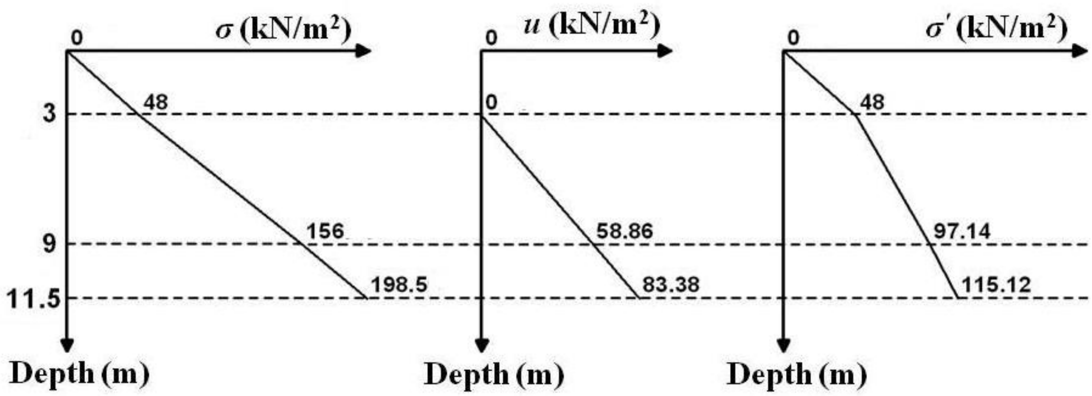
The plot is shown below.



9.3

Point	kN/m²		
	σ	u	σ'
A	**0**	**0**	**0**
B	(3)(16) = **48**	**0**	**48**
C	48 + (6)(18) = **156**	(9.81)(6) = **58.86**	**97.14**
D	156 + (2.5)(17) = **198.5**	58.86 + (9.81)(2.5) = **83.38**	**115.12**

The plot is shown below.



9.4 a. Water table drops 2 m within layer 2. Assuming dry condition for 2 m:

$$\gamma_{d\,(\text{layer 2})} = \frac{G_s \gamma_w}{1+e} = \frac{(2.7)(9.81)}{1+0.55} = 17.08 \text{ kN/m}^3$$

σ'(at point C) = (5)(15.52) + (2)(17.08) + (6)(20.57) – (6)(9.81)

= 176.32 kN/m^2

Increase in σ': 176.32 – 163.68 = **12.64 kN/m²**

b. Water table rises to the surface. Layer 1 is saturated.

$$\gamma_{\text{sat(layer1)}} = \frac{\gamma_w(G_s + e)}{1+e} = \frac{(9.81)(2.69+0.7)}{1+0.7} = 19.56\,\text{kN/m}^3$$

σ'(at point C) = (5)(19.56) + (8)(20.57) – (13)(9.81) = 134.83 kN/m^2

Decrease in σ': 163.68 – 134.83 = **28.85 kN/m²**

c. Water level rises 3 m above ground. All layers are saturated

σ'(at point C) = (3)(9.81) + (5)(19.56) + (8)(20.57) – (16)(9.81)

= 134.83 kN/m^2

Decrease in σ': 163.68 – 134.83 = **28.85 kN/m²** (same as Part b)

9.5 a. $$\gamma_{d\text{(sand)}} = \frac{G_s\gamma_w}{1+e} = \frac{(2.66)(9.81)}{1+0.61} = 16.2\,\text{kN/m}^3$$

$$\gamma_{\text{sat(sand)}} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{(2.67+0.48)(9.81)}{1+0.48} = 20.88\,\text{kN/m}^3$$

Point	kN/m^2		
	σ	u	σ'
A	**0**	**0**	**0**
B	(16.2)(4) = **64.8**	**0**	**64.8**
C	64.8 + (20.88)(5) = **169.2**	(9.81)(5) = **49.05**	**120.15**

b. Let the height of rise be h. Portions of the top sand layer will be saturated.

$$\gamma_{\text{sat(top sand)}} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{(2.66+0.61)(9.81)}{1+0.61} = 19.92\ \text{kN/m}^3$$

So, at any time, the stresses at *C* are:

σ = (4 – *h*)(16.2) + (*h*)(19.92) + (5)(20.88) = 169.2 + 3.72*h*

u = (5 + *h*)(9.81) = 49.05 + 9.81*h*

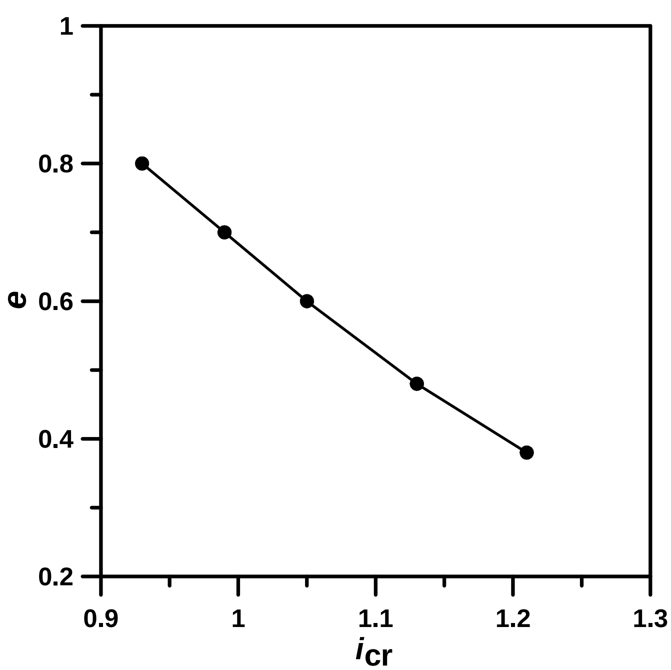
σ' = (169.2 + 3.72*h*) – (49.05 + 9.81*h*) = 120.15 – 6.09*h*

New σ' at *C*: 111 = 120.15 – 6.09*h*; *h* = **1.5 m**

9.6 $$i_{cr} = \frac{\gamma'}{\gamma_w} = \frac{G_s - 1}{1+e} = \frac{2.68-1}{1+e} = \frac{1.68}{1+e}$$

e	i_{cr}
0.38	1.21
0.48	1.13
0.6	1.05
0.7	0.99
0.8	0.93

The plot is shown below.



9.7 $\gamma_{\text{sat(clay)}} = \dfrac{(1+w)G_s\gamma_w}{1+wG_s} = \dfrac{(1+0.29)(2.68)(9.81)}{1+(0.29)(2.68)} = 19.08\ \text{kN/m}^3$

Let the depth of the excavation be H.

So, $(10 - H)(19.08) - (6)(9.81) = 0 = \sigma'$.

$H \approx$ **6.91 m**

9.8 Consider the stability of point A in terms of heaving.

$\gamma_{\text{sat(clay)}} = \dfrac{(1925)(9.81)}{1000} = 18.88\ \text{kN/m}^3$

$\sigma_A = (10 - 5.75)(18.88) = 80.24\ \text{kN/m}^2$

$u_A = (6)(9.81) = 58.86\ \text{kN/m}^2$

For heaving to occur, $\sigma' = 0$; or $\sigma = u$

Therefore, factor of safety = $\dfrac{\sigma_A}{u_A} = \dfrac{80.24}{58.86} =$ **1.36**

9.9 Let the maximum permissible depth of cut be H.

$\sigma_A = (10 - H)(18.88)$

$u_A = (6)(9.81) = 58.86\ \text{kN/m}^2$

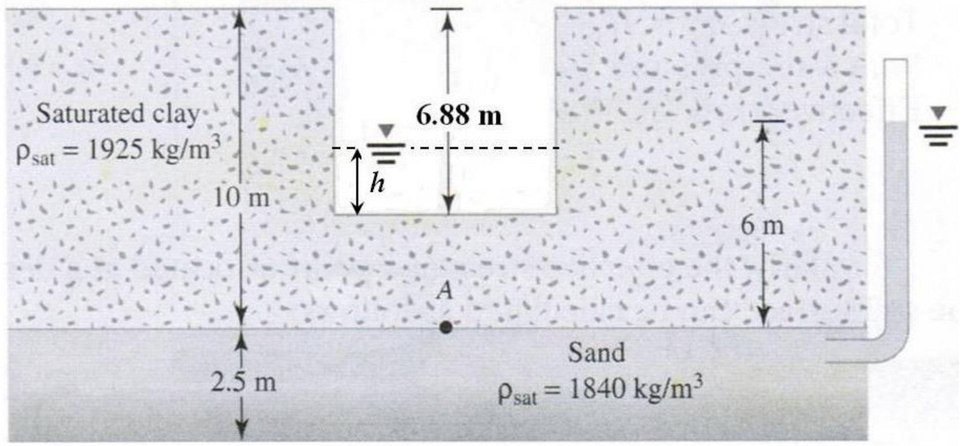
For heaving to occur, $\sigma' = 0$; or $\sigma_A - u_A = 0$

$(10 - H)(18.88) - 58.86 = 0$; $H =$ **6.88 m**

9.10 Let the height of water inside the cut be h (see figure on following page)

$\sigma_A = (10 - 6.88)(18.88) + (h)(9.81) = 58.9 + 9.81h$

$u_A = (6)(9.81) = 58.86\ \text{kN/m}^2$



Factor of safety: $\frac{\sigma_A}{u_A} = \frac{58.9 + 9.81h}{58.86} = 1.5$

h = **3.0 m**

9.11 a. $i = \frac{h}{H_2} = \frac{1.5}{2.5} = 0.6$

$q = kiA = (0.21)(0.6)(0.62 \times 100^2 \text{ cm}^2) =$ **781.2 cm³/sec**

b. $i_{\text{cr}} = \frac{\gamma'}{\gamma} = \frac{G_s - 1}{1 + e} = \frac{2.66 - 1}{1 + 0.49} = 1.11$

Since $i < i_{\text{cr}}$, **no boiling**.

c. $i = i_{\text{cr}} = \frac{h}{H_2}; \quad 1.11 = \frac{h}{2.5}$

h = **2.77 m**

9.12 a. $i = \frac{h}{H_2} = \frac{1.5}{4.5} = 0.33$

$q = kiA = (0.31)(0.33)(6.2) =$ **0.634 ft³/min**

b. Refer to Figure 9.4 (a). Since C is located at the middle of the soil layer,

$z = H_2/2 = 4.5/2 = 2.25$ ft

Eq. (9.7): $\sigma'_c = z\gamma' - iz\gamma_w =$ (2.25)(119-62.4) – (0.33)(2.25)(62.4) = **81 lb/ft²**

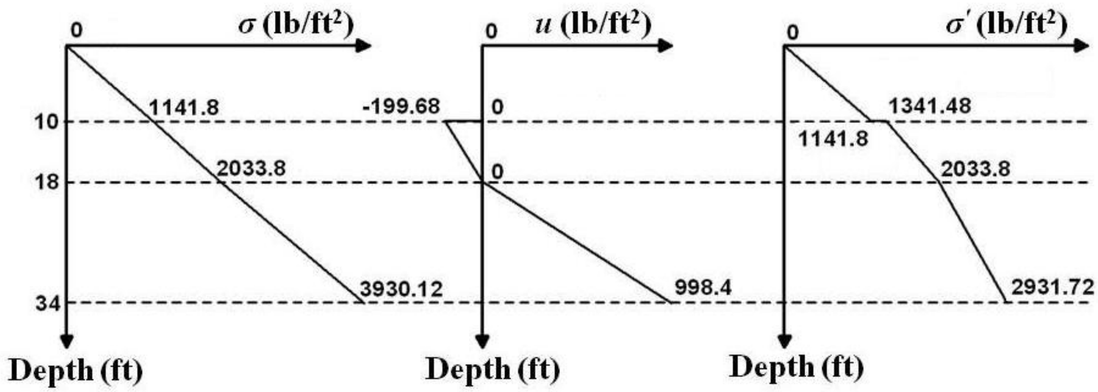
9.13 $$\gamma_{d\,(\text{sand})} = \frac{G_s\gamma_w}{1+e} = \frac{(2.69)(62.4)}{1+0.47} = 114.18\ \text{lb/ft}^3$$

$$\gamma_{\text{sat(clay: capillary zone)}} = \frac{\gamma_w(G_s+Se)}{1+e} = \frac{(62.4)[2.73+(0.4)(0.68)]}{1+0.68} = 111.5\ \text{lb/ft}^3$$

$$\gamma_{\text{sat(clay)}} = \frac{\gamma_w(G_s+e)}{1+e} = \frac{(62.4)(2.7+0.89)}{1+0.89} = 118.52\ \text{lb/ft}^3$$

Depth (ft)	lb/ft² σ	u	σ'
0	**0**	**0**	**0**
10	(114.18)(10) = **1141.8**	**0**	**1141.8**
		(–0.4)(62.4)(8) = **–199.68**	**1341.48**
10+8=18	1141.8+ (111.5)(8) = **2033.8**	**0**	**2033.8**
10+8+16=34	2033.8 + (118.52)(16) = **3930.12**	(16)(62.4) = **998.4**	**2931.72**

The plot is given below.



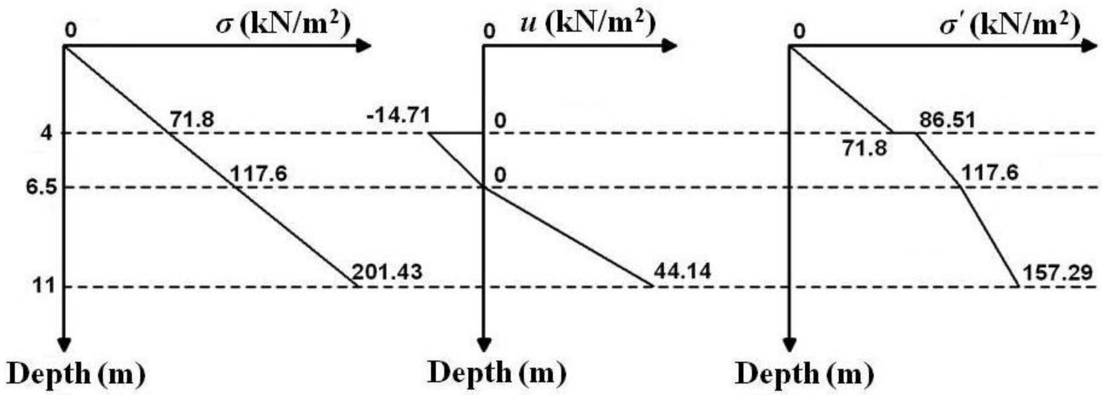
9.14 $\gamma_{d\,(\text{sand})} = \dfrac{(2.69)(9.81)}{1+0.47} = 17.95\ \text{kN/m}^3$

$$\gamma_{\text{sat(clay: capillary zone)}} = \frac{(9.81)[2.73+(0.6)(0.68)]}{1+0.68} = 18.32\ \text{kN/m}^3$$

$$\gamma_{\text{sat(clay)}} = \frac{(9.81)(2.7+0.89)}{1+0.89} = 18.63\ \text{kN/m}^3$$

Depth (m)	kN/m² σ	u	σ'
0	**0**	**0**	**0**
4	(17.95)(4) = **71.8**	**0**	**71.8**
		(–0.6)(9.81)(2.5) = **–14.71**	**86.51**
4+2.5=6.5	71.8 + (18.32)(2.5) = **117.6**	**0**	**117.6**
4+2.5+4.5=11	117.6 + (18.63)(4.5) = **201.43**	(4.5)(9.81) = **44.14**	**157.29**

The plot is given.



9.15 From Eq. (9.22), $FS = \dfrac{D\gamma'}{C_o \gamma_w (H_1 - H_2)}$

D = 4.5 m; γ' = 17 – 9.81 = 7.19 kN/m³; $H_1 - H_2$ = 7 – 3 = 4 m;

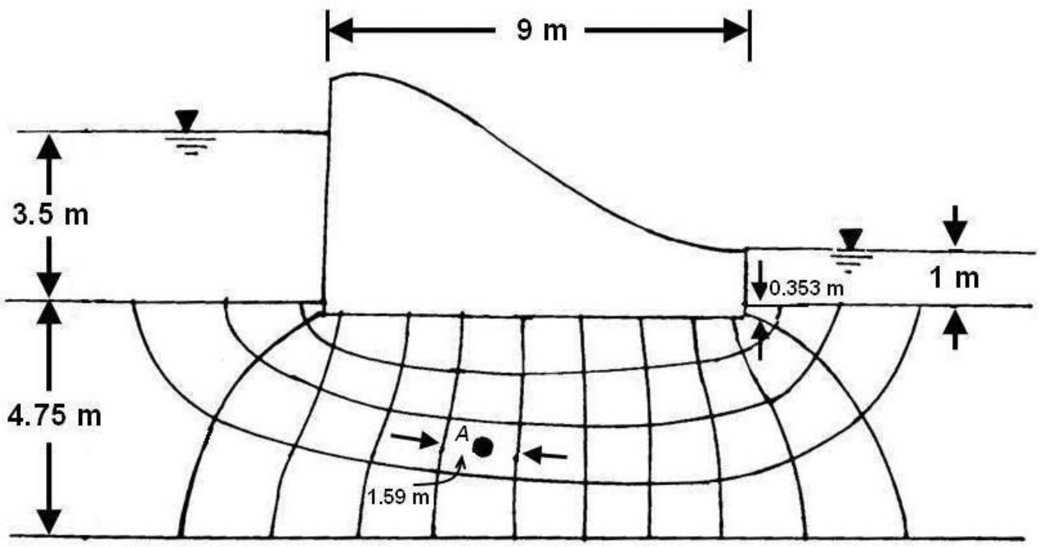
D/T = 4.5/12 = 0.375. From Table 9.1, C_o = 0.354 (by linear interpolation).

$$FS = \frac{(4.5)(7.19)}{(0.354)(9.81)(4)} = \mathbf{2.33}$$

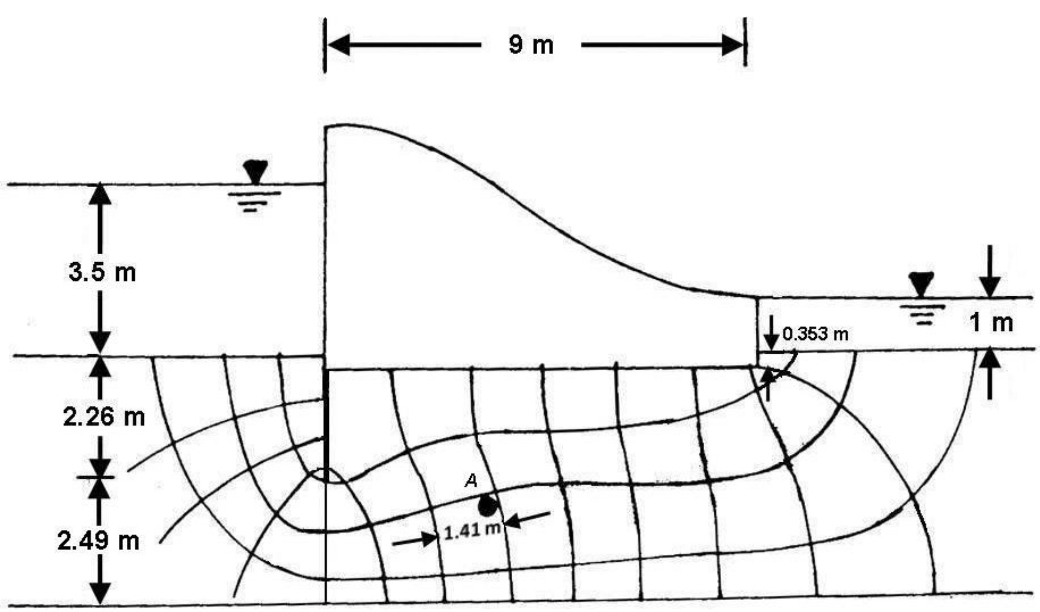
CRITICAL THINKING PROBLEM

9.C.1 a. The flow nets for both cases are given below:

Case 1



Case 2



b. Determination of $\frac{q}{k}$:

From Eq. (8.21): $q = kH\frac{N_f}{N_d}$

Case 1: $N_f = 4$, $N_d = 11$, $H = 3.5 - 1 = 2.5$ m

$$\frac{q}{k} = (2.5)\left(\frac{4}{11}\right) = \mathbf{0.909\,m}$$

Case 2: $N_f = 3.5$, $N_d = 13$, $H = 2.5$ m

$$\frac{q}{k} = (2.5)\left(\frac{3.5}{13}\right) = \mathbf{0.673\,m}$$

c. $FS = \frac{i_{\text{cr}}}{i_{\text{exit}}}$

$$i_{\text{cr}} = \frac{G_s - 1}{1+e} = \frac{2.66-1}{1.55} = 1.071$$

Case 1: Refer to the flow net and Eq. (9.24a):

$$i_{\text{exit}} = \frac{H}{N_d\, l} = \frac{2.5}{(11)(0.353)} = 0.643$$

$$FS = \frac{1.071}{0.643} \approx \mathbf{1.67}$$

Case 2: Refer to the flow net and Eq. (9.24a):

$$i_{\text{exit}} = \frac{H}{N_d\, l} = \frac{2.5}{(13)(0.353)} = 0.545$$

$$FS = \frac{1.071}{0.545} \approx \mathbf{1.97}$$

d. From Eq. (9.18), seepage force per unit volume is $\gamma_w i$

Case 1: Refer to the flow net. At *A*,

$$i = \frac{\Delta H}{l} = \frac{(2.5/11)}{1.59} = 0.143$$

Seepage force = $\gamma_w i$ = (9.81)(0.143) = **1.4 kN/m^3**

Case 2: Refer to the flow net. At *A*,

$$i = \frac{\Delta H}{l} = \frac{(2.5/13)}{1.41} = 0.136$$

Seepage force = $\gamma_w i$ = (9.81)(0.136) = **1.33 kN/m^3**

Installation of the sheet pile cut-off wall reduced the exit gradient and increased the factor of safety against heaving. Accordingly, at any point *A*, the seepage force also decreased due to a drop in the hydraulic gradient.

Chapter 10

10.1 a. $$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{\sigma_y + \sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

σ_x = 162 kN/m^2; σ_y = 128 kN/m^2; τ_{xy} = +32 kN/m^2

$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{128+162}{2} \pm \sqrt{\left(\frac{128-162}{2}\right)^2 + (32)^2}$$

σ_1= **181.23 kN/m^2;** σ_3 = **108.76 kN/m^2**

b. $$\sigma_n = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau_{xy}\sin 2\theta; \quad \theta = 35°$$

$$\sigma_n = \frac{128+162}{2} + \frac{128-162}{2}\cos[(2)(35)] + 32\sin[(2)(35)] = \mathbf{169.25\ kN/m^2}$$

$$\tau_n = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$= \frac{128-162}{2}\sin[(2)(35)] - 32\cos[(2)(35)] = \mathbf{-26.92\ kN/m^2}$$

10.2 a. σ_x = 72 kN/m^2; σ_y = 121 kN/m^2; τ_{xy} = 39 kN/m^2; θ = 147°

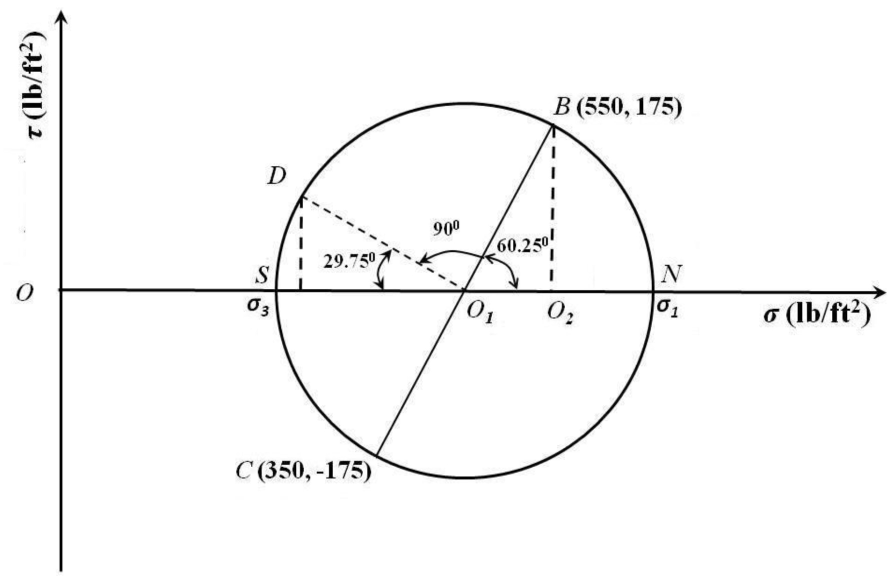
$$\left.\begin{matrix}\sigma_1\\ \sigma_3\end{matrix}\right\} = \frac{121+72}{2} \pm \sqrt{\left(\frac{121-72}{2}\right)^2 + (39)^2}$$

σ_1= **142.55 kN/m^2;** σ_3 = **50.45 kN/m^2**

b. $$\sigma_n = \frac{121+72}{2} + \frac{121-72}{2}\cos[(2)(147)] + 39\sin[(2)(147)] = \mathbf{131.33\ kN/m^2}$$

$$\tau_n = \frac{121-72}{2}\sin[(2)(147)] - (39)\cos[(2)(147)] = -\mathbf{38.24\ kN/m^2}$$

10.3 a. The Mohr's circle is shown below.



$$\overline{OO_1} = \frac{550+350}{2} = 450\ \text{lb/ft}^2;\ O_1O_2 = 550 - 450 = 100\ \text{lb/ft}^2$$

$$\overline{O_1B} = \sqrt{\left(\frac{350-550}{2}\right)^2 + (-175)^2} = 201.5\ \text{lb/ft}^2$$

$$\sigma_3 = \overline{OS} = 450 - 201.5 = \mathbf{248.4\ lb/ft^2\ (+)}$$

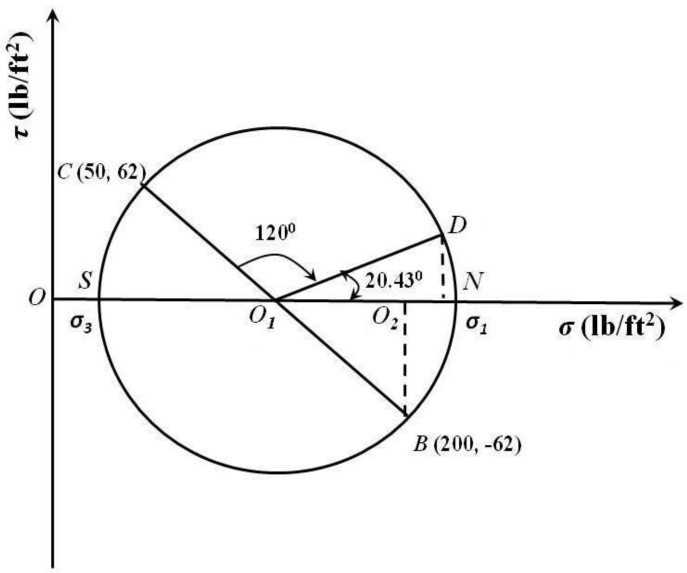
$$\sigma_1 = \overline{ON} = 450 + 201.5 = \mathbf{651.5\ lb/ft^2\ (+)}$$

$$\angle BO_1O_2 = \tan^{-1}\left(\frac{175}{100}\right) = 60.25°$$

b. $\sigma_n = \overline{OO_1} - \overline{O_1D}\cos(29.75) = 450 - 201.5\cos(29.75) = \mathbf{275.1\ lb/ft^2\ (+)}$

$$\tau_n = \overline{O_1D}\sin(29.75) = \mathbf{99.98\ lb/ft^2}\ (+)$$

10.4 a. The Mohr's circle is shown below.



$$\overline{OO_1} = \frac{200+50}{2} = 125\ \text{lb/ft}^2;\ O_1O_2 = 200-125 = 75\ \text{lb/ft}^2$$

$$\overline{O_1B} = \sqrt{(75)^2 + (62)^2} = 97.3\ \text{lb/ft}^2$$

$$\sigma_1 = \overline{ON} = 125 + 97.3 = \mathbf{222.3\ lb/ft^2}$$

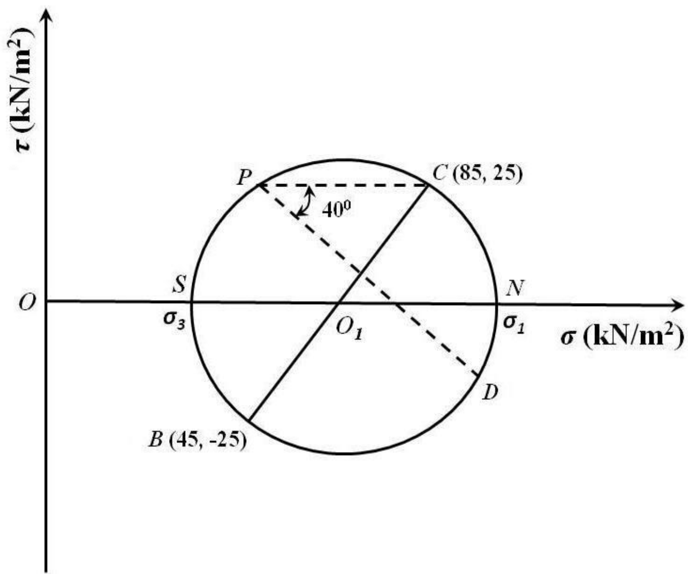
$$\sigma_3 = \overline{OS} = 125 - 97.3 = \mathbf{27.7\ lb/ft^2}$$

$$\angle BO_1O_2 = \tan^{-1}\left(\frac{62}{75}\right) = 39.57°$$

b. $$\sigma_n = \overline{OO_1} + \overline{O_1D}\cos(20.43) = 125 + 97.3\cos(20.43) = \mathbf{216\ lb/ft^2}$$

$$\tau_n = 97.13\sin(20.43) = \mathbf{33.9\ lb/ft^2}$$

10.5 a. The Mohr's circle is shown below.

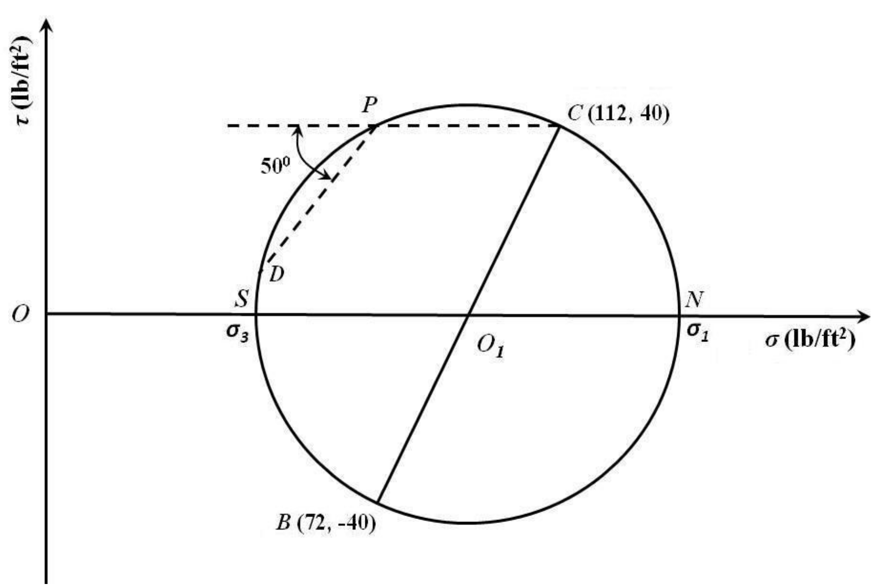


$$\sigma_1 = \overline{ON} = \mathbf{99\,kN/m^2}\,;\;\; \sigma_3 = \overline{OS} = \mathbf{33\,kN/m^2}$$

b. σ_n and τ_n are coordinates of *D*. So

$\sigma_n \approx$ **96 kN/m²**; $\tau_n \approx$ **15.8 kN/m² (–)**

10.6 a. The Mohr's circle is shown below.



$\sigma_1 = \overline{ON} = \mathbf{136.5\ lb/ft^2}$; $\sigma_3 = \overline{OS} = \mathbf{47.3\ lb/ft^2}$

b. σ_n and τ_n are coordinates of D. So

$\sigma_n \approx \mathbf{48\ lb/ft^2}$; $\tau_n \approx \mathbf{10\ lb/ft^2}$

10.7

Load @	P (kN)	r (m)	z (m)	$\frac{r}{z}$	I_1 (Table 10.1)	$\Delta\sigma_z = \frac{P}{z^2} I_1$ (kN/m^2)
B	100	6	6	1.0	0.0844	0.234
C	200	$(6^2 + 6^2)^{0.5} = 8.48$	6	1.41	0.0311	0.173
D	400	$(6^2+3^2) = 6.708$	6	1.118	0.0648	0.72

$\Delta\sigma_z = \sum \mathbf{1.127\ kN/m^2}$

10.8 Eq. (10.15):

$$\Delta\sigma_z = \frac{2q_1 z^3}{\pi[(x_1 + x_2)^2 + z^2]^2} + \frac{2q_2 z^3}{\pi[x_2^2 + z^2]^2} = \frac{(2)(90)(3)^3}{\pi[(6.5)^2 + (3)^2]^2} + \frac{(2)(325)(3)^3}{\pi[2.5^2 + 3^2]^2}$$

$$= \mathbf{24.6\ kN/m^2}$$

10.9 Eq. (10.15): In this case, $x_2 = 0$

$$\Delta\sigma_z = \frac{2q_1 z^3}{\pi[(x_1 + x_2)^2 + z^2]^2} + \frac{2q_2 z^3}{\pi[x_2^2 + z^2]^2}$$

$$= \frac{(2)(90)(3)^3}{\pi[(4+0)^2 + (3)^2]^2} + \frac{(2)(325)(3)^3}{\pi[0^2 + 3^2]^2} = \mathbf{71.44\ kN/m^2}$$

10.10 $$\Delta\sigma_z = \frac{2q_1 z^3}{\pi[(x_1 + x_2)^2 + z^2]^2} + \frac{2q_2 z^3}{\pi[x_2^2 + z^2]^2}$$

$$48 = \frac{(2)(q_1)(6)^3}{\pi[19^2 + 6^2]^2} + \frac{2(930)(6)^3}{\pi[5^2 + 6^2]^2} = 34.36 + 0.00087 q_1$$

$q_1 = \mathbf{15{,}678\ lb/ft}$

10.11 $\Delta\sigma_z$ at A due to $q_1 = \dfrac{2q_1 z^3}{\pi[x^2 + z^2]^2}$, or $(\Delta\sigma_z)_1 = \dfrac{(2)(292)(3)^3}{\pi[(3)^2 + (3)^2]^2} = 15.49\ \text{kN/m}^2$

Vertical component of $q_2 = q_2 \sin 45°$

$$(\Delta\sigma_z)_2 = \frac{2q_2(\sin 45)(3)^3}{\pi[(7.5)^2 + (3)^2]^2};\ (\Delta\sigma_z)_2 = 0.00285q_2$$

Horizontal component of $q_2 = q_2 \cos 45°$

From Eq. (10.17): $(\Delta\sigma_z)_3 = \dfrac{2q_2 xz^2}{\pi(x^2 + z^2)^2} = \dfrac{2q_2(\cos 45)(7.5)(3)^2}{\pi[7.5^2 + 3^2]^2} = 0.007136q_2$

Total vertical stress,

$\Delta\sigma_z = 42\ \text{kN/m}^2 = (\Delta\sigma_z)_1 + (\Delta\sigma_z)_2 + (\Delta\sigma_z)_3$

$42 = 15.49 + 0.00285q_2 + 0.007136q_2$

$q_2 = \mathbf{2656.3\ kN/m}$

10.12 B = 36 ft; q = 900 lb/ft^2; x = 21 ft; z = 15 ft

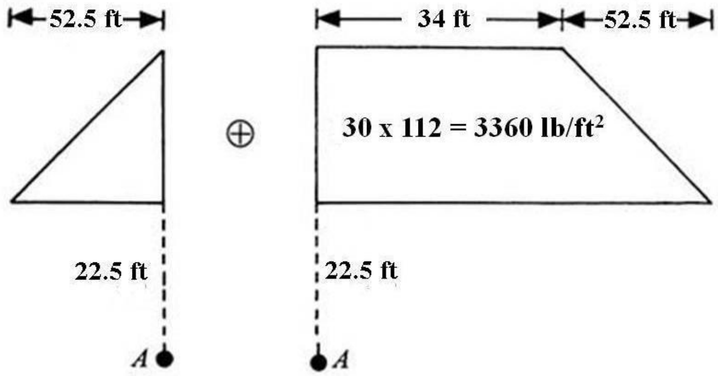
$$\frac{2x}{B} = \frac{(2)(27)}{36} = 1.5;\ \frac{2z}{B} = \frac{(2)(15)}{36} = 0.833.\ \text{From Table 10.4, } \frac{\Delta\sigma_z}{q} = 0.18$$

$\Delta\sigma_z = (0.18)(900) =$ **162 lb/ft^2**

10.13 $\dfrac{2x}{B} = \dfrac{(2)(0)}{6} = 0;\ \dfrac{2z}{B} = \dfrac{(2)(5)}{6} = 1.67.$ From Table 10.4, $\dfrac{\Delta\sigma_z}{q} = 0.61$

$\Delta\sigma_z = (120)(0.61) =$ **73.2 kN/m^2**

10.14 Refer to the figure.



For the left side (with the notations given in Figure 10.19):

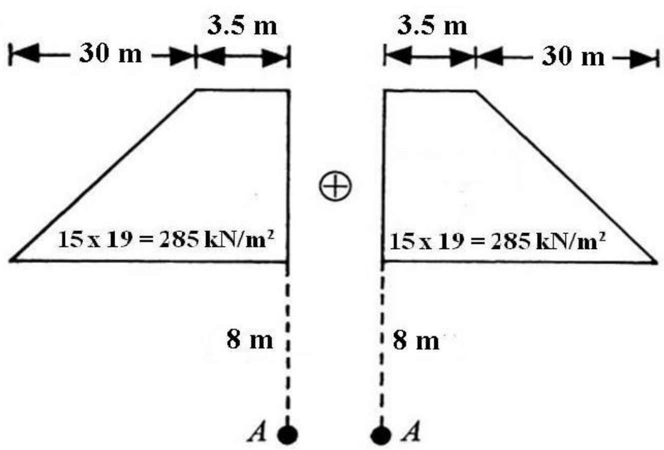
$$\frac{B_1}{z} = \frac{0}{22.5} = 0; \ \frac{B_2}{z} = \frac{52.5}{22.5} = 2.33. \text{ From Figure 10.20, } I_{2(L)} = 0.375$$

For the right side:

$$\frac{B_1}{z} = \frac{34}{22.5} = 1.51; \ \frac{B_2}{z} = \frac{52.5}{22.5} = 2.33. \text{ From Figure 10.20, } I_{2(R)} = 0.48$$

$\Delta\sigma_z = q[I_{2(L)} + I_{2(R)}] = (3360)(0.375 + 0.48) =$ **2872.8 lb/ft^2**

10.15 At *A*:



For the left side:

$$\frac{B_1}{z} = \frac{3.5}{8} = 0.437$$

$$\frac{B_2}{z} = \frac{30}{8} = 3.75$$

$I_2 = 0.46$

For the right side:

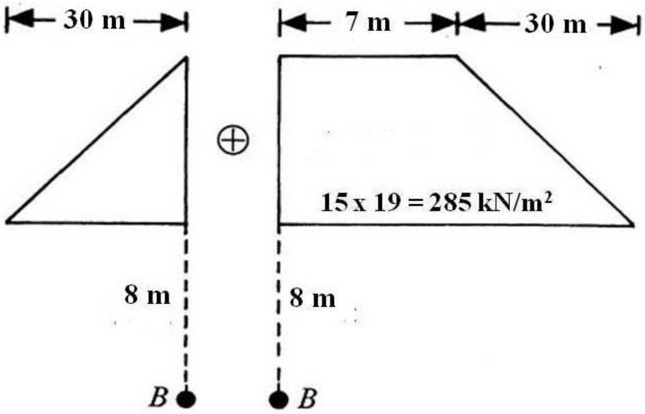
$$\frac{B_1}{z} = 0.437$$

$$\frac{B_2}{z} = 3.75$$

$I_2 = 0.46$

$\Delta\sigma_z = (15)(19)(0.46 + 0.46) =$ **262.2 kN/m^2**

At *B*:



For the left side:

$\frac{B_1}{z} = \frac{0}{8} = 0$

$\frac{B_2}{z} = \frac{30}{8} = 3.75$

$I_2 = 0.41$

For the right side:

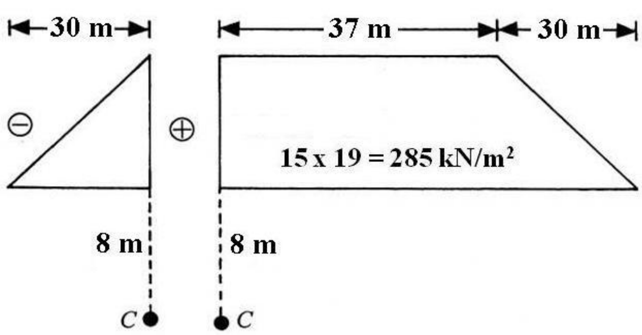
$\frac{B_1}{z} = \frac{7}{8} = 0.875$

$\frac{B_2}{z} = \frac{30}{8} = 3.75$

$I_2 = 0.48$

$\Delta\sigma_z = (15)(19)(0.41 + 0.48) = \mathbf{253.65\ kN/m^2}$

At *C*:



For the left side:

$\frac{B_1}{z} = 0$

$\frac{B_2}{z} = \frac{30}{8} = 3.75$

$I_2 = 0.41$

For the right side:

$\frac{B_1}{z} = \frac{37}{8} = 4.625$

$\frac{B_2}{z} = \frac{30}{8} = 3.75$

$I_2 = 0.5$

$\Delta\sigma_z = (15)(19)(0.5 - 0.41) = \mathbf{25.65\ kN/m^2}$

10.16 Eq. (10.26) and Table 10.6: $q = 2200$ lb/ft^2

R (ft)	z (ft)	$\frac{z}{R}$	$\frac{\Delta\sigma_z}{q}$	$\Delta\sigma_z$ (lb/ft^2)
12	0	0	1	**2200**
12	4	0.333	0.9634	**2119.5**
12	8	0.666	0.8251	**1815.2**
12	16	1.333	0.4983	**1096.2**
12	32	2.667	0.1809	**397.9**

10.17 Eq. (10.27) and Tables 10.7 and 10.8: q = 380 kN/m^2

z (m)	r (m)	R (m)	$\frac{z}{R}$	$\frac{r}{R}$	A'	B'	$\Delta\sigma_z$ (kN/m^2)
3	0	5	0.6	0	0.48550	0.37831	**328.24**
3	1	5	0.6	0.2	0.47691	0.37531	**323.84**
3	3	5	0.6	0.6	0.40427	0.32822	**278.34**
3	5	5	0.6	1	0.25588	0.14440	**152.1**
3	7	5	0.6	1.4	0.12657	0.00085	**48.42**

10.18 Refer to the Newmark's chart.

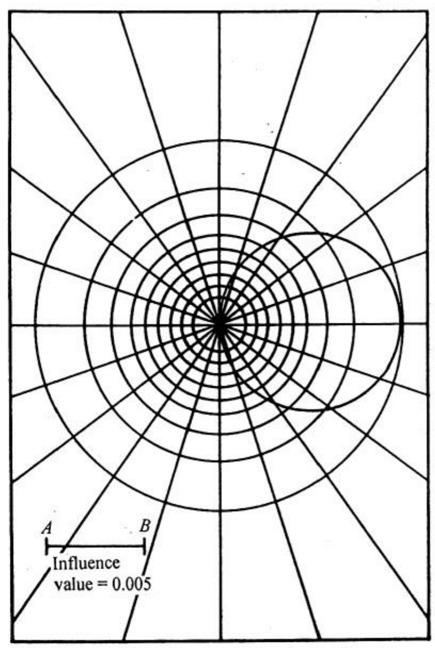
The plan is drawn to scale.

$\overline{AB} = 6$ m. $M \approx 65$.

$\Delta\sigma_z = (IV)\, q\, M$

$= (0.005)(450)(65)$

$=$ **146.25 kN/m^2**



10.19 Point A:

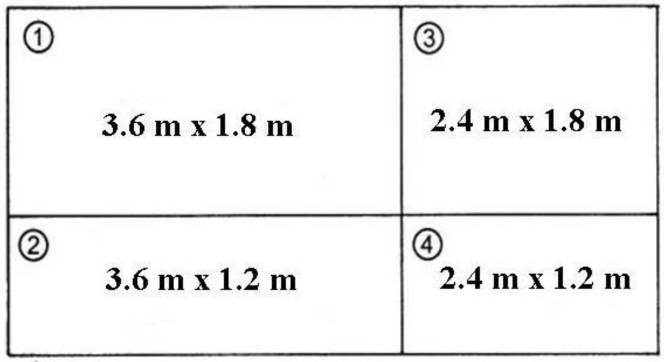
Eqs. (10.32) and (10.33): $n = \frac{L}{z} = \frac{6}{3} = 2;\ m = \frac{B}{z} = \frac{3}{3} = 1$

Eq. (10.30): $\Delta\sigma_z = q I_3$; Table 10.9: $I_3 = 0.1999$

$\Delta\sigma_z = (225)(0.1999) = 44.97$ kN/m^2 $\approx$ **45 kN/m^2**

Point B:

Refer to the figure on the next page.



For rectangle 1: $m = \frac{3.6}{3} = 1.2;\quad n = \frac{1.8}{3} = 0.6;\quad I_3 = 0.1431$

For rectangle 2: $m = \frac{3.6}{3} = 1.2;\quad n = \frac{1.2}{3} = 0.4;\quad I_3 = 0.1063$

For rectangle 3: $m = \frac{2.4}{3} = 0.8;\quad n = \frac{1.8}{3} = 0.6;\quad I_3 = 0.1247$

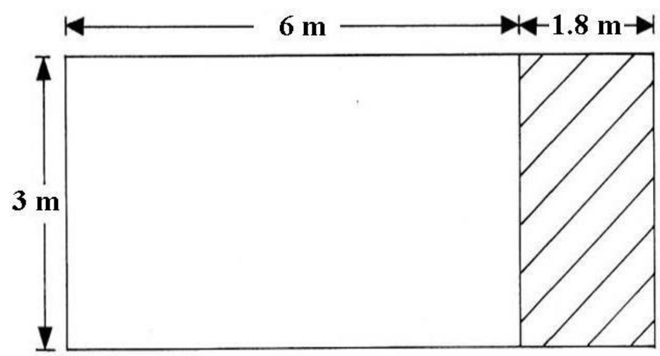
For rectangle 4: $m = \frac{2.4}{3} = 0.8;\quad n = \frac{1.2}{3} = 0.4;\quad I_3 = 0.0931$

$\Delta\sigma_z = q[I_{3(1)} + I_{3(2)} + I_{3(3)} + I_{3(4)}] = (225)(0.1431 + 0.1063 + 0.1247 + 0.0931)$

$= \mathbf{105.12\ kN/m^2}$

Point C:

Refer to the figure.



$$\Delta\sigma_z = \begin{pmatrix}\text{stress at } C \text{ due to}\\ \text{area } 7.8\,\text{m} \times 3\,\text{m}\end{pmatrix} - \begin{pmatrix}\text{stress at } C \text{ due to}\\ \text{area } 1.8\,\text{m} \times 3\,\text{m}\end{pmatrix}$$

For rectangular area 7.8 m × 3 m: $m = \frac{7.8}{3} = 2.6;\ \ n = \frac{3}{3} = 1;\ \ I_3 = 0.2026$

For rectangular area 1.8 m × 3 m: $m = \frac{1.8}{3} = 0.6;\ \ n = \frac{3}{3} = 1;\ \ I_3 = 0.1361$

$\Delta\sigma_z = q(0.2026 - 0.1361) = (225)(0.2026 - 0.1361) =$ **14.96 kN/m^2**

10.20 Eqs. (10.35), (10.37), (10.38), and (10.39):

$$b = \frac{B}{2} = \frac{3}{2} = 1.5\text{ m}$$

$$m_1 = \frac{L}{B} = \frac{6}{3} = 2$$

	z (m)				
	2	4	6	8	10
$n_1 = \frac{z}{b}$	1.33	2.66	4	5.33	6.66
I_4 (Table 10.10)	0.682	0.356	0.190	0.119	0.079
$\Delta\sigma_z = qI_4$ (kN/m^2)	**153.4**	**80.1**	**42.7**	**26.8**	**17.8**

CRITICAL THINKING PROBLEM

10.C.1

1. Vertical stress increase due to wheel load:

   y = 0.305 m; R = 0.15 m; q = 565 kN/m^2

Element	r (m)	$\frac{y}{R}$	$\frac{r}{R}$	A'	B'	$\Delta\sigma_y$ (kN/m^2)
A	0.457	2.03	3.05	0.02221	0.00028	**12.7**
B	0.267	2.03	1.78	0.05278	0.04391	**54.63**
C	0	2.03	0	0.10557	0.17889	**160.71**

Overburden pressure at the middle of the layer = 0.305 × 19.4 = 5.92 kN/m^2

Total vertical pressure, $\Delta\sigma_y$:

At A: $\sigma_{y\text{-}A}$ = 12.7 + 5.92 = **18.62 kN/m^2**

At B: $\sigma_{y\text{-}B}$ = 54.63 + 5.92 = **60.55 kN/m^2**

At C: $\sigma_{y\text{-}C}$ = 160.71 + 5.92 = **166.63 kN/m^2**

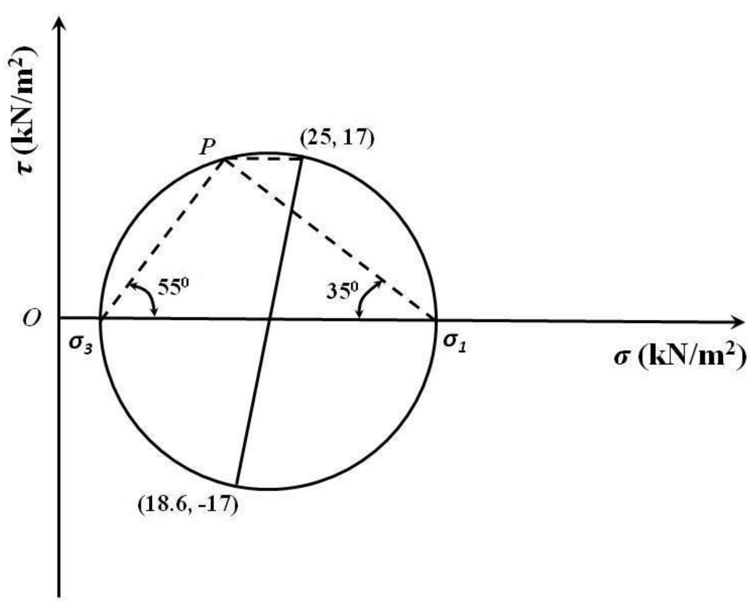
These values are entered into the following table.

Element at	Horizontal stress, σ_x (kN/m^2)	Shear stress, τ (kN/m^2)	Vertical stress, σ_y (kN/m^2)	σ_1 (kN/m^2)	σ_3 (kN/m^2)	α_i (deg)
A	25	17	**18.62**	**39**	**4.5**	**55**
B	32	45	**60.55**	**93**	**1**	**48**
C	7	0	**166.63**	**167**	**7**	**0**

2. Element at A:

The Mohr's circle is shown. $\sigma_1 \approx$ **39 kN/m^2**; $\sigma_3 \approx$ **4.5 kN/m^2**

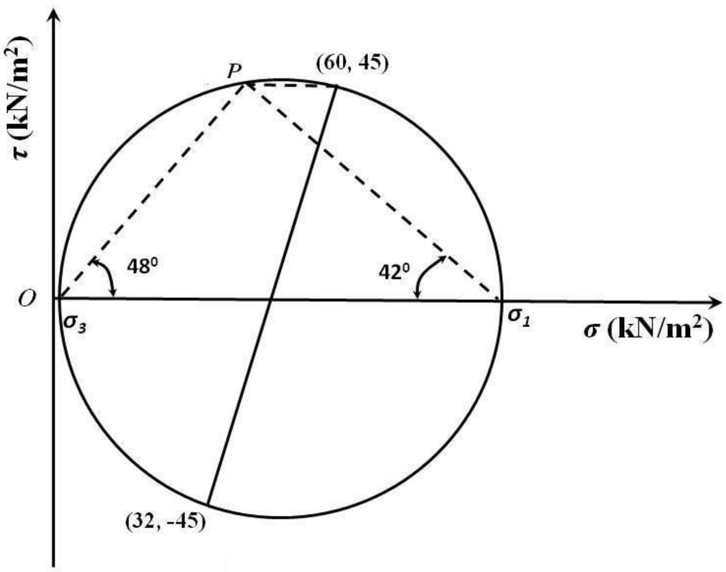
These values are entered in the above table. The pole is located at point P. The maximum principal stress acts on a plane which is inclined at 35° with the horizontal. Therefore, α_A = 90 – 35 = 55°.



Element at *B*:

The Mohr's circle is shown. $\sigma_1 \approx$ **93 kN/m²**; $\sigma_3 \approx$ **1 kN/m²**

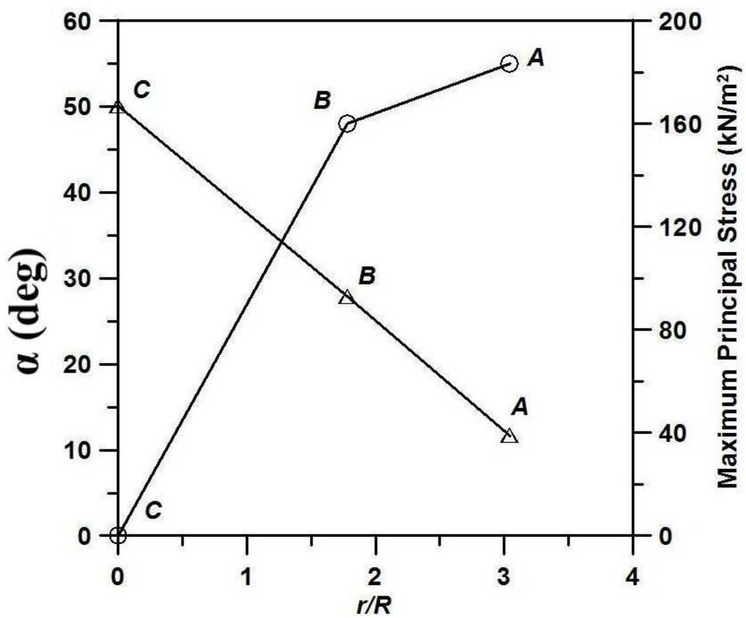
These values are entered in the table on the previous page. The pole is located at point *P*. The maximum principal stress acts on a plane which is inclined at 42° with the horizontal. Therefore, $\alpha_B = 90 - 42 =$ **48°**.



Element at *C*:

Since there is no shear stress, the horizontal and vertical stresses are principal stresses. Therefore, $\sigma_1 \approx$ **167 kN/m²**; $\sigma_3 \approx$ **7 kN/m²;** $\alpha_C =$ **0°.** These values are entered in the table on the previous page.

3. The plot is shown in the figure.



Chapter 11

11.1 $$S_{e(\text{flexible, center})} = \Delta\sigma(\alpha B')\frac{1-\mu_s^2}{E_s} I_s I_f$$

$$\Delta\sigma = \frac{355}{(2)(3)} = 59.16 \text{ kN/m}^2$$

Given: $\alpha = 4$; $B' = \frac{2}{2} = 1$; $\mu_s = 0.35$; $E_s = 13{,}500$ kN/m^2

$$I_s = F_1 + \frac{1-2\mu_s}{1-\mu_s} F_2$$

$$m' = \frac{L}{B} = \frac{3}{2} = 1.5; \quad n' = \frac{H}{\left(\frac{B}{2}\right)} = \frac{4}{\left(\frac{2}{2}\right)} = 4$$

From Table 11.1, $F_1 = 0.454$; from Table 11.2, $F_2 = 0.054$.

$$I_s = 0.454 + \frac{1-(2)(0.4)}{1-0.4}(0.054) = 0.472$$

Also, with $\frac{D_f}{B} = \frac{1.5}{2} = 0.75$ and $\frac{L}{B} = 1.5$, Table 11.3 gives $I_f = 0.765$. Hence,

$$S_{e(\text{flexible, center})} = (59.16)[(4)(1)]\left(\frac{1-0.4^2}{13500}\right)(0.472)(0.765) = 0.0053 \text{ m} = 5.3 \text{ mm}$$

$S_{e(\text{rigid})} = (0.93)(5.3) \approx$ **4.93 mm**

11.2 As in Problem 11.1, $S_{e(\text{rigid})} = 0.93\Delta\sigma(\alpha B')\frac{1-\mu_s^2}{E_s} I_s I_f$

$$E_s = \frac{\sum E_{s(i)}\Delta z}{\bar{z}} = \frac{(3000)(6)+(1100)(8)+(8500)(10)}{24} = 4658 \text{ lb/in}^2 = 670{,}752 \text{ lb/ft}^2$$

Given: $B = L = 6$ ft; $\mu_s = 0.3$; $\alpha = 4$

$$\Delta\sigma = \frac{100000}{(6)(6)} = 2778 \text{ lb/ft}^2$$

$$B' = \frac{B}{2} = \frac{6}{2} = 3 \text{ ft.}$$

$$I_s = F_1 + \frac{1-2\mu_s}{1-\mu_s} F_2$$

$$m' = \frac{L}{B} = 1; \quad n' = \frac{H}{\left(\frac{B}{2}\right)} = \frac{24}{\left(\frac{6}{2}\right)} = 8$$

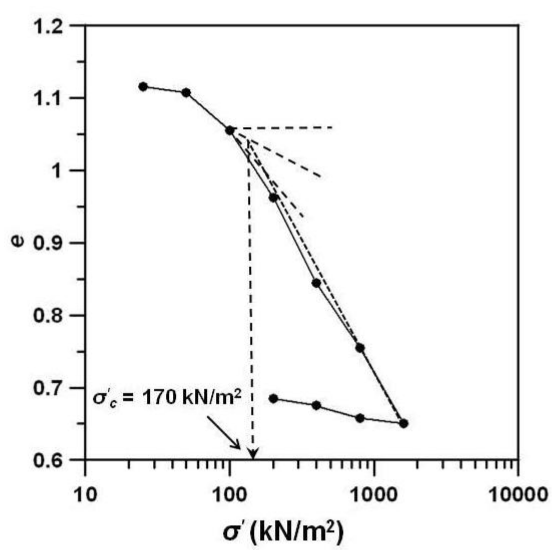
From Table 11.1, F_1 = 0.482; from Table 11.2, F_2 = 0.02

$$I_s = 0.482 + \frac{1-(2)(0.3)}{1-0.3}(0.02) = 0.493$$

Also, $\frac{D_f}{B} = \frac{3}{6} = 0.5.$ From Table 11.3, $I_f \approx 0.77$. So,

$$S_{e(\text{rigid})} = (0.93)(2778)(4\times 3)\left(\frac{1-0.3^2}{670752}\right)(0.493)(0.77) = 0.016 \text{ ft} \approx \mathbf{0.2\ in}$$

11.3 a. The plot of e vs. σ' is shown.



b. $\sigma'_c = \mathbf{170\ kN/m^2}$

c. $$C_c = \frac{e_1 - e_2}{\log\left(\frac{\sigma'_2}{\sigma'_1}\right)} = \frac{0.755 - 0.65}{\log\left(\frac{16}{8}\right)} \approx 0.35$$

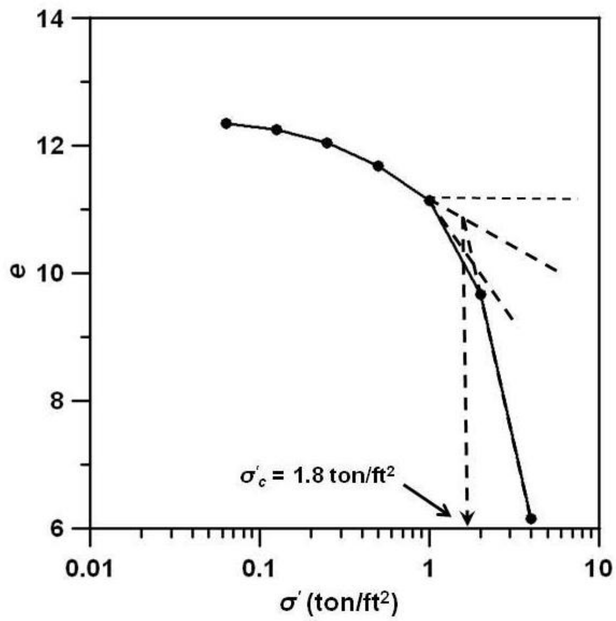
$$C_s = \frac{e_1 - e_2}{\log\left(\frac{\sigma'_2}{\sigma'_1}\right)} = \frac{0.658 - 0.65}{\log\left(\frac{16}{8}\right)} = 0.026$$

$$\frac{C_s}{C_c} = \frac{0.026}{0.35} = \mathbf{0.074}$$

11.4 a. Height of solids: $H_s = \frac{W_s}{AG_s\gamma_w} = \frac{12\ g}{(4.91)(2.54)^2(2.49)(1)} = 0.152\text{ cm} \approx 0.06\text{ in.}$

σ' (ton/ft^2)	Change in dial reading (in.)	Final height, H (in.)	H_s (in.)	$H_v = H - H_s$ (in.)	$e = \frac{H_v}{H_s}$
0.063	0.0112	0.8013	0.06	0.7413	12.35
0.125	0.0059	0.7954	0.06	0.7354	12.25
0.250	0.0124	0.7830	0.06	0.723	12.05
0.500	0.0222	0.7608	0.06	0.7008	11.68
1.000	0.0324	0.7284	0.06	0.6684	11.14
2.000	0.0886	0.6398	0.06	0.5798	9.66
4.000	0.2105	0.4293	0.06	0.3693	6.15

The e-log σ' graph is plotted on the following page.



b. From the graph, $\sigma'_c = \mathbf{1.8\ ton/ft^2}$

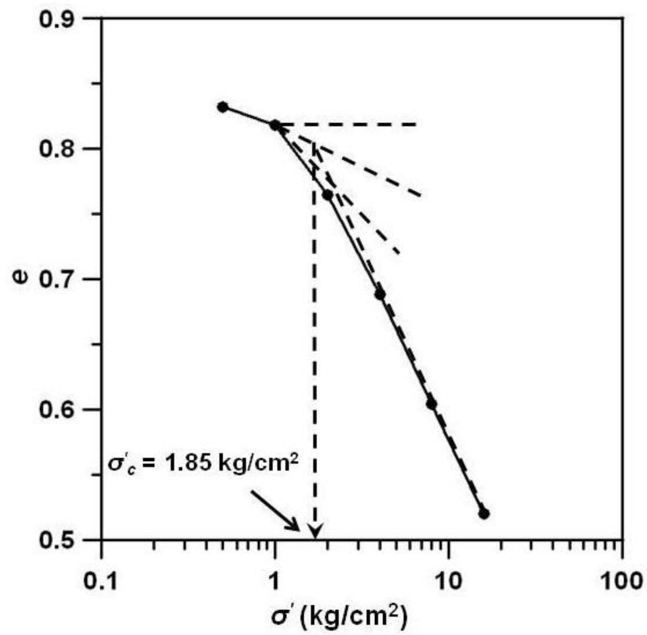
c. $$C_c = \frac{e_1 - e_2}{\log\left(\frac{\sigma'_2}{\sigma'_1}\right)} = \frac{9.66 - 6.15}{\log\left(\frac{4}{2}\right)} = \mathbf{11.66}$$

11.5 $$W_s = \frac{W}{1+w} = \frac{140}{1+0.19} = 117.6\ \text{g}$$

$$H_s = \frac{W_s}{AG_s\gamma_w} = \frac{117.6}{(\pi)(3.175)^2(2.7)(1)} = 1.375\ \text{cm}$$

σ' (kg/cm^2)	Final height, H (cm)	H_s (cm)	$H_v = H - H_s$ (cm)	$e = \frac{H_v}{H_s}$
0.5	2.519	1.375	1.144	0.832
1.0	2.5	1.375	1.125	0.818
2.0	2.428	1.375	1.053	0.765
4.0	2.322	1.375	0.947	0.688
8.0	2.206	1.375	0.831	0.604
16.0	2.09	1.375	0.715	0.576

The e-log σ' graph is plotted.



From the graph, $\sigma'_c = \mathbf{1.85\ kg/cm^2}$

$$C_c = \frac{e_1 - e_2}{\log\left(\frac{\sigma'_2}{\sigma'_1}\right)} = \frac{0.688 - 0.604}{\log\left(\frac{8}{4}\right)} = \mathbf{0.28}$$

11.6 a. $$S_c = \frac{C_c H}{1 + e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right)$$

$C_c = 0.009(LL - 10) = (0.009)(42 - 10) = 0.288$

$$\Delta\sigma = \frac{150}{(3)(3)} = 16.67\ \text{kN/m}^2$$

$$\gamma_d = \frac{(2.72)(9.81)}{1 + 0.7} = 15.7\ \text{kN/m}^3$$

$$\gamma_{\text{sat}} = \frac{(2.72+0.7)(9.81)}{1+0.7} = 19.73\ \text{kN/m}^3$$

$$\sigma_o' = (15.7)(1.5) + (19.73 - 9.81)\left(\frac{8}{2}\right) = 63.23\ \text{kN/m}^2$$

$$S_c = \frac{(0.288)(8)}{1+0.7}\log\left(\frac{63.23+16.67}{63.23}\right) = \mathbf{0.137\ m}$$

11.7 Eq. (11.68): $\Delta\sigma'_{\text{av}} = \dfrac{\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b}{6}$

Eq. (10.35): $\Delta\sigma' = qI_4$

$$m_1 = \frac{L}{B} = \frac{3}{3} = 1;\ b = \frac{B}{2} = 1.5\ \text{m};\ n_1 = \frac{z}{b}$$

$$q = \frac{150\ \text{kN}}{(3)(3)} = 16.67\ \text{kN/m}^2$$

m_1	z (m)	$b = \frac{B}{2}$ (m)	$n_1 = \frac{z}{b}$	q (kN/m^2)	I_4 (Table 10.9)	$\Delta\sigma' = qI_4$ (kN/m^2)
1	0.0	1.5	0	16.67	≈ 1.0	16.67
1	4.0	1.5	2.67	16.67	0.231	3.85
1	8.0	1.5	5.33	16.67	0.065	1.08

$$\Delta\sigma'_{\text{av}} = \frac{16.67 + (4\times 3.85) + 1.08}{6} = 5.52\ \text{kN/m}^2$$

$$S_c = \frac{(0.288)(8)}{1+0.7}\log\left(\frac{63.23+5.52}{63.23}\right) = \mathbf{0.049\ m}$$

11.8 a. $\sigma_o' = (6)(114) + (12)(118) + \left(\dfrac{18}{2}\right)(117) - (12+9)(62.4) = 1842.6\ \text{lb/ft}^2$

$C_c = 0.009(LL - 10) = (0.009)(38 - 10) = 0.252$

$$S_c = \frac{(0.252)(18)}{1+0.73}\log\left(\frac{1842.6+550}{1842.6}\right) = \mathbf{0.297\,ft. \approx 3.5\,in.}$$

b. $$S_c = \frac{C_s H}{1+e_o}\log\left(\frac{\sigma'_c}{\sigma'_o}\right) + \frac{C_c H}{1+e_o}\log\left(\frac{\sigma'_o+\Delta\sigma'}{\sigma'_c}\right)$$

$$= \frac{\left(\frac{0.252}{5}\right)(18)}{1.73}\log\left(\frac{2200}{1842.6}\right) + \frac{(0.252)(18)}{1.73}\log\left(\frac{1842.6+550}{2200}\right)$$

$$= 0.135\text{ ft} \approx \mathbf{1.63\,in.}$$

11.9 $$\gamma_{d(\text{sand})} = \frac{G_s\gamma_w}{1+e} = \frac{(2.66)(9.81)}{1+0.65} = 15.81\,\text{kN/m}^3$$

$$\gamma'_{\text{sand}} = \frac{(G_s-1)\gamma_w}{1+e} = \frac{(2.66-1)(9.81)}{1.65} = 9.87\,\text{kN/m}^3$$

$$\gamma'_{\text{clay}} = \frac{(G_s-1)\gamma_w}{1+e} = \frac{(2.74-1)(9.81)}{1+0.98} = 8.62\,\text{kN/m}^3$$

$$\sigma'_o = (2)(15.81) + (4)(9.87) + \left(\frac{6}{2}\right)(8.62) = 96.96\,\text{kN/m}^2$$

$$C_c = 0.009(LL - 10) = (0.009)(54 - 10) = 0.396$$

$$S_c = \frac{C_s H}{1+e_o}\log\left(\frac{\sigma'_c}{\sigma'_o}\right) + \frac{C_c H}{1+e_o}\log\left(\frac{\sigma'_o+\Delta\sigma'}{\sigma'_c}\right)$$

$$= \frac{\left(\frac{0.396}{6}\right)(6)}{1.98}\log\left(\frac{150}{96.96}\right) + \frac{(0.396)(6)}{1.98}\log\left(\frac{96.96+85}{150}\right)$$

$$= 0.138\text{ m} \approx \mathbf{13.8\,cm}$$

11.10 $$C_c = \frac{e_1 - e_2}{\log\left(\frac{\sigma'_2}{\sigma'_1}\right)} = \frac{1.22-0.97}{\log\left(\frac{225}{108}\right)} = 0.784$$

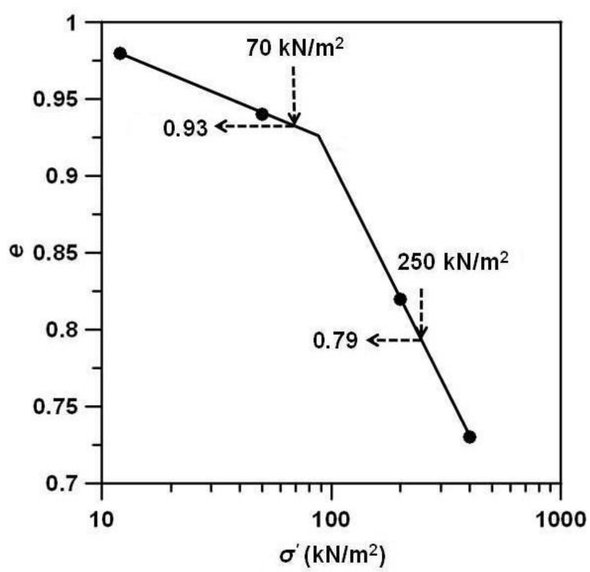
$$C_c = \frac{e_1 - e_3}{\log\left(\frac{\sigma'_3}{\sigma'_1}\right)}$$

$$e_3 = e_1 - C_c \log\left(\frac{\sigma'_3}{\sigma'_1}\right) = 1.22 - 0.784\log\left(\frac{300}{108}\right) = \mathbf{0.872}$$

11.11 $C_c = \frac{0.92 - 0.77}{\log\left(\frac{3}{1.5}\right)} = 0.498$

$$e_3 = 0.92 - 0.498\log\left(\frac{4.5}{1.5}\right) = \mathbf{0.682}$$

11.12 The plot of e–log σ' is shown.



$\sigma'_o = 70$ kN/m^2; $e_1 = 0.93$;

$\sigma'_o + \Delta\sigma' = 250$ kN/m^2; $e = 0.79$

$\Delta e = 0.93 - 0.79 = 0.14$

$$S_c = \frac{H\Delta e}{1+e_o} = \frac{(2)(0.14)}{1+0.93}$$

$$= \mathbf{0.145\,m}$$

11.13 $T_v = \frac{c_v t}{H_{\text{dr}}^2}$; $U = 75\%$; $T_v = 0.477$ (Table 11.7)

$$0.477 = \frac{(0.24\,\text{cm}^2/\text{min})(t)}{\left(\frac{600}{2}\,\text{cm}\right)^2};\ t = 178{,}875 \text{ min} = \mathbf{124.2\ days}$$

11.14 a. $m_v = \dfrac{a_v}{1+e_{av}} = \dfrac{\left(\dfrac{\Delta e}{\Delta\sigma'}\right)}{1+e_{av}}$

$\Delta e = e_1 - e_2 = 0.92 - 0.77 = 0.15$

$\Delta\sigma' = \sigma_2' - \sigma_1' = 3 - 1.5 = 1.5 \text{ ton/ft}^2$

$e_{\mathbf{av}} = \dfrac{0.92 + 0.77}{2} = 0.845$

$m_v = \dfrac{a_v}{1+e_{\mathbf{av}}} = \dfrac{\left(\dfrac{0.15}{1.5}\right)}{1+0.845} = \mathbf{0.054\ ft^2/ton}$

b. $c_v = \dfrac{k}{m_v \gamma_w} = 0.001085 \text{ in.}^2\text{/sec} = 7.534 \times 10^{-6} \text{ ft}^2\text{/sec}$

$7.534 \text{ x } 10^{-6} \text{ ft}^2\text{/sec} = \dfrac{k}{0.054 \text{ ft}^2\text{/ton}\left(\dfrac{62.4}{2000}\right)\text{ton/ft}^3}$

$k = \mathbf{1.27 \times 10^{-8}\ ft/sec}$

11.15 In the laboratory:

$T_{65} = \dfrac{c_v t_{65}}{H_{dr}^2}$

$0.304 = \dfrac{(c_v)(10 \text{ min})}{\left(\dfrac{0.019}{2}\text{m}\right)^2}; \; c_v = 2.74 \times 10^{-6} \text{ m}^2\text{/min}$

$U = 40\%$; $T_{40} = 0.126$ (Table 11.7)

In the field:

$T_{40} = \dfrac{c_v t_{40}}{H_{dr}^2}$

$$0.126 = \frac{(2.74 \times 10^{-6}\ \text{m}^2/\text{min})(t_{40})}{(4\ \text{m})^2}$$

t_{40} = 735,766 min = **511 days**

11.16 a. $$m_v = \frac{a_v}{1+e_{\text{av}}} = \frac{\left(\frac{\Delta e}{\Delta \sigma'}\right)}{1+e_{\text{av}}}$$

Δe = 0.9 – 0.75 = 0.15

$$\Delta\sigma' = 1.5 - 0.5 = 1\ \text{ton/ft}^2$$

$$e_{\text{av}} = \frac{0.9+0.75}{2} = 0.825$$

$$m_v = \frac{\left(\frac{0.15}{1}\right)}{1+0.825} = 0.082\ \text{ft}^2/\text{ton}$$

$$c_v = \frac{k}{m_v \gamma_w} = \frac{(6\times10^{-7})(0.0328)\ \text{ft/sec}}{(0.082\ \text{ft}^2/\text{ton})\left(\frac{62.4}{2000}\ \text{ton/ft}^3\right)}$$
$$= 7.69 \times 10^{-6}\ \text{ft}^2/\text{sec} = 0.664\ \text{ft}^2/\text{day}$$

$$t_{50} = \frac{T_v H_{\text{dr}}^2}{c_v} = \frac{(0.197)(12)^2}{0.644} = \mathbf{44\ days}$$

b. $$C_c = \frac{e_1 - e_2}{\log\left(\frac{\sigma_2'}{\sigma_1'}\right)} = \frac{0.9-0.75}{\log\left(\frac{1.5}{0.5}\right)} = 0.31$$

$$S_c = \frac{C_c H}{1+e_o}\log\left(\frac{\sigma_o' + \Delta\sigma'}{\sigma_o'}\right) = \frac{(0.31)(12)}{1+0.9}\log\left(\frac{1.5}{0.5}\right) = 0.934\ \text{ft}$$

S_c at 50% = (0.5)(0.934) = 0.467 ft ≈ **5.6 in.**

11.17 $m_v = \dfrac{a_v}{1+e_{\text{av}}} = \dfrac{\left(\dfrac{\Delta e}{\Delta\sigma'}\right)}{1+e_{\text{av}}} = \dfrac{\left(\dfrac{0.85-0.71}{250-125}\right)}{1+\left(\dfrac{0.85+0.71}{2}\right)} = 6.29\times10^{-4}\ \text{m}^2/\text{kN}$

$$c_v = \frac{T_{70}H_{\text{dr}}^2}{t_{70}} = \frac{(0.403)\left(\frac{0.025}{2}\right)^2}{4.8} = 1.31\times10^{-5}\ \text{m}^2/\text{min}$$

$k = c_v\, m_v\, \gamma_w = (6.29\times10^{-4}\ \text{m}^2/\text{kN})(1.31\times10^{-5}\ \text{m}^2/\text{min})(9.81\ \text{kN/m}^3)$

$= \mathbf{8.08\times10^{-8}\ m/min}$

11.18 a. $T_{90} = \dfrac{c_v t_{90}}{H_{\text{dr}}^2}$; $0.848 = \dfrac{c_v(180)}{\left(\dfrac{15}{2}\right)^2}$

$c_v = \mathbf{0.265\ ft^2/day}$

b. $T_{65} = \dfrac{c_v t_{65}}{H_{\text{dr}}^2}$

From Table 11.7, $U = 65\%$; $T_{65} = 0.119$

$$t_{65} = \frac{T_{65}H_{\text{dr}}^2}{c_v} = \frac{(0.119)\left(\frac{0.75}{(2)(12)}\ \text{ft}\right)^2}{0.265} = \mathbf{4.38\times10^{-4}\ day \approx 38\ sec}$$

11.19 a. Eq. (11.68): $\Delta\sigma'_{\text{av}} = \dfrac{\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b}{6}$

Eq. (10.35): $\Delta\sigma' = qI_4$

$$m_1 = \frac{L}{B} = \frac{2}{2} = 1;\ b = \frac{B}{2} = 1.0\ \text{m};\ n_1 = \frac{z}{b}$$

$$q = \frac{300\ \text{kN}}{(2)(2)} = 75\ \text{kN/m}^2$$

m_1	z (m)	$b = \frac{B}{2}$ (m)	$n_1 = \frac{z}{b}$	q (kN/m^2)	I_4 (Table 10.9)	$\Delta\sigma' = qI_4$ (kN/m^2)
1	1.0	1.0	1.0	75	0.701	52.57
1	2.0	1.0	2.0	75	0.336	25.2
1	3.0	1.0	3.0	75	0.179	13.42

$$\Delta\sigma'_{av} = \frac{52.57 + (4 \times 25.2) + 13.42}{6} = \mathbf{27.8\ kN/m^2}$$

b. $$\gamma_{sat-clay} = \frac{(1 + w)\gamma_w G_s}{1 + wG_s} = \frac{(1 + 0.24)(9.81)(2.74)}{1 + (0.24)(2.74)} = 20.1\,\text{kN/m}^3$$

$$\sigma'_o = (1)(14) + (1)(17 - 9.81) + \left(\frac{2}{2}\right)(20.1 - 9.81) = 31.48\,\text{kN/m}^2$$

$C_c = 0.009(LL - 10) = (0.009)(46 - 10) = 0.324$

Since $\sigma'_o \le \sigma'_c$, the clay is overconsolidated

$$S_c = \frac{C_s H}{1 + e_o}\log\left(\frac{\sigma'_c}{\sigma'_o}\right) + \frac{C_c H}{1 + e_o}\log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_c}\right)$$

$$= \frac{\left(\frac{0.324}{5}\right)(2)}{1 + 0.657}\log\left(\frac{40}{31.48}\right) + \frac{(0.324)(2)}{1 + 0.657}\log\left(\frac{31.48 + 27.8}{40}\right)$$

$= 0.0749\text{ m} \approx \mathbf{75\ mm}$

c. $$U(\%) = \left(\frac{19}{75}\right)(100) = \mathbf{25.3\%}$$

d. $T_v = \frac{c_v t}{H_{dr}^2}$; $U = 25.3\%$; $T_v = 0.0503$ (Table 11.7)

$$0.0503 = \frac{(c_v)(365)}{(1\,\text{m})^2}$$

$c_v = \mathbf{1.378 \times 10^{-4}\ m^2/day}$

e. $T_v = \dfrac{(0.0001378)(2)(365)}{\left(\dfrac{2}{2}\right)^2} = 0.1$

Table (11.7): $U\,(\%) \approx 36\%$

$$U(\%) = \left(\frac{S_{c(t)}}{S_c}\right)(100);\; S_{c(t)} = (0.36)(75) = \mathbf{27\,mm}$$

CRITICAL THINKING PROBLEM

11.C.1 a. For the clay layer:

Eq. (11.68): $\Delta\sigma'_{\text{av}} = \dfrac{\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b}{6}$

Eq. (10.35): $\Delta\sigma' = qI_4$

$$m_1 = \frac{L}{B} = \frac{10}{10} = 1;\; b = \frac{B}{2} = 5\text{ m};\; n_1 = \frac{z}{b}$$

$$q = (2)(19) = 38\,\text{kN/m}^2$$

m_1	z (m)	$b = \dfrac{B}{2}$ (m)	$n_1 = \dfrac{z}{b}$	q (kN/m^2)	I_4 (Table 10.9)	$\Delta\sigma' = qI_4$ (kN/m^2)
1	4	5	0.8	38.0	0.8	30.4
1	6	5	1.2	38.0	0.606	23.02
1	8	5	1.6	38.0	0.449	17.06

$$\Delta\sigma'_{\text{av}} = \frac{30.4 + (4 \times 23.02) + 17.06}{6} = 23.25\,\text{kN/m}^2$$

$$\sigma'_o = (2)(15) + (2)(17) + \left(\frac{4}{2}\right)(18) - (2+2)(9.81) = 60.76\,\text{kN/m}^2$$

$$S_{c-\text{clay}} = \frac{(0.36)(4)}{1+1.1}\log\left(\frac{60.76 + 23.25}{60.76}\right) = \mathbf{0.096\,m}$$

For the peat layer:

$$m_1 = \frac{L}{B} = \frac{10}{10} = 1;\ b = \frac{B}{2} = 5\text{ m};\ n_1 = \frac{z}{b}$$

$$q = (2)(19) = 38\,\text{kN/m}^2$$

m_1	z (m)	$b = \frac{B}{2}$ (m)	$n_1 = \frac{z}{b}$	q (kN/m^2)	I_4 (Table 10.9)	$\Delta\sigma' = qI_4$ (kN/m^2)
1	8	5	1.6	38.0	0.449	17.06
1	9	5	1.8	38.0	0.388	14.74
1	10	5	2.0	38.0	0.336	12.76

$$\Delta\sigma'_{\text{av}} = \frac{17.06 + (4 \times 14.74) + 12.76}{6} = 14.8\ \text{kN/m}^2$$

$$\sigma'_o = (2)(15) + (2)(17) + (4)(18) + (1)(16) - (7)(9.81) = 83.33\,\text{kN/m}^2$$

$$S_{c-\text{peat}} = \frac{(6.6)(2)}{1+5.9}\log\left(\frac{83.33+14.8}{83.33}\right) = \mathbf{0.136\,m}$$

Total consolidation settlement, S_c = 0.096 + 0.136 = **0.232 m**

b. For the clay layer, a double drainage condition is assumed since the bottom peat layer has high void ratio and considered permeable.

$$t_{99-\text{clay}} = \frac{(H_{\text{dr}}^2)(T_{99})}{c_v} = \frac{(200^2)(1.781)}{0.003} = 23{,}746{,}666\,\text{sec} \approx \mathbf{275\ days}$$

For the peat layer, a single drainage is assumed since the top layer is considered to have relatively low permeability.

$$t_{99-\text{peat}} = \frac{(H_{\text{dr}}^2)(T_{99})}{c_v} = \frac{(200^2)(1.781)}{0.025} = 2{,}849{,}600\,\text{sec} \approx \mathbf{33\ days}$$

c. Secondary compression in clay:

$$\Delta e_{\text{primary}} = C_c \log\left(\frac{\sigma'_0 + \Delta\sigma'}{\sigma'_0}\right) = 0.36\log\left(\frac{60.76+23.25}{60.76}\right) = 0.0506$$

$$e_p = e_0 - \Delta e_{\text{primary}} = 1.1 - 0.0506 = 1.049$$

$$C'_\alpha = \frac{C_\alpha}{1+e_p} = \frac{0.03}{1+1.049} = 0.0146$$

$$S_{s-\text{clay}} = C'_\alpha H \log\left(\frac{t_2}{t_1}\right) = (0.0146)(4)\log\left(\frac{(2)(365)}{275}\right) = \mathbf{0.0247\ m}$$

Secondary compression in peat:

$$\Delta e_{\text{primary}} = C_c \log\left(\frac{\sigma'_0 + \Delta\sigma'}{\sigma'_0}\right) = 6.6\log\left(\frac{83.33+14.8}{83.33}\right) = 0.468$$

$$e_p = e_0 - \Delta e_{\text{primary}} = 5.9 - 0.468 = 5.432$$

$$C'_\alpha = \frac{C_\alpha}{1+e_p} = \frac{0.263}{1+5.432} = 0.0408$$

$$S_{s-\text{peat}} = C'_\alpha H \log\left(\frac{t_2}{t_1}\right) = (0.0408)(2)\log\left(\frac{(2)(365)}{33}\right) = \mathbf{0.109\ m}$$

d. Total settlement:

$S_c + S_{s\text{-clay}} + S_{s\text{-peat}} = 0.232 + 0.0247 + 0.109 =$ **0.365 m**

e. Time factor in 3 months:

$$T_v = \frac{c_v t}{H_{\text{dr}}^2} = \frac{(0.003)(3)(30)(24)(3600)}{200^2} = 0.583$$

Determine the degree of consolidation, U_z from Figure 11.25:

$$\frac{z}{H_{\text{dr}}} = \frac{3}{2} = 1.5\,; \quad U_z \approx 0.85$$

Eq. 11.57: $U_z = 1 - \dfrac{u_z}{u_0}$

Initial excess pore water pressure, $u_0 \approx \Delta\sigma' = 23.25\ \text{kN/m}^2$

$u_z = (1 - U_z)u_0 = (1 - 0.85)(23.25) = 3.487\ \text{kN/m}^2$ = remaining excess pore water pressure at point *A* after 3 months.

The increase in effective stress after 3 months = 23.25 – 3.487 = 19.76 kN/m^2

$$\sigma_o' = (2)(15) + (2)(17) + (3)(18) - (2 + 3)(9.81) = 68.95\ \text{kN/m}^2$$

Therefore, the final effective stress after 3 months = 68.95 + 19.76

= **88.71 kN/m^2**

Chapter 12

12.1 a. $c' = 0$. From Eq. (12.3): $\tau_f = \sigma' \tan \phi'$

$$\tau = \frac{300}{(1000)(0.063)^2} = 75 \text{ kN/m}^2$$

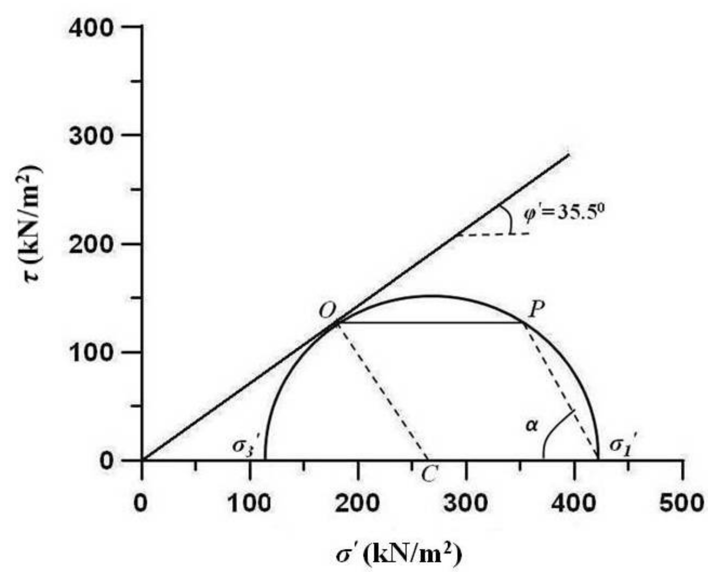
So, $75 = 105 \tan \phi'$

$$\phi' = \tan^{-1}\left(\frac{75}{105}\right) = \mathbf{35.5°}$$

b. For $\sigma' = 180$ kN/m^2, $\tau_f = 180 \tan 35.5° = 128.39$ kN/m^2

Shear force, $S = (128.39)(1000)(0.063)^2 =$ **509.5 N**

12.2 The point O (180, 128.4) represents the failure stress conditions on the Mohr-Coulomb failure envelope. The perpendicular line OC to the failure envelope determines the center, C of the Mohr's circle. With the center at C, and the radius as OC, the Mohr's circle is drawn by trial and error such that the circle is tangent to the failure envelope at O. From the graph,



a. $\sigma_3' \approx$ **115 kN/m²**; $\sigma_1' \approx$ **420 kN/m²**

b. The horizontal line *OP* drawn from *O* determines the pole *P*. Therefore, the orientation or the major principal plane with the horizontal is given by the angle $\alpha \approx$ **65°**.

12.3 For $\sigma' = 28$ lb/in², $\tau_f = 28 \tan 33° = 18.18$ lb/in²

Shear force, $S = (18.18)(2.5)^2 =$ **113.65 lb**

12.4 Area of specimen $A = \left(\frac{\pi}{4}\right)(2)^2 = 3.14 \text{ in.}^2$

Test No.	Normal force N (lb)	$\sigma' = \frac{N}{A}$ (lb/in.²)	Shear force S (lb)	$\tau_f = \frac{S}{A}$ (lb/in.²)	$\phi' = \tan^{-1}\left(\frac{\tau_f}{\sigma'}\right)$ (deg)
1	15	4.77	5.25	1.67	19.29
2	30	9.55	10.5	3.34	19.27
3	48	15.28	16.8	5.35	19.29
4	83	26.43	29.8	9.5	19.77

A graph of τ_f vs. σ' will yield $\phi' =$ **19.5°**.

12.5 Area of specimen $A = \left(\frac{\pi}{4}\right)(0.05)^2 = 0.00196 \text{ m}^2$

Test No.	Normal force N (N)	$\sigma' = \frac{N}{A}$ (N/m²)	Shear force S (N)	$\tau_f = \frac{S}{A}$ (N/m²)	$\phi' = \tan^{-1}\left(\frac{\tau_f}{\sigma'}\right)$ (deg)
1	250	79.6	139	44.26	29.07
2	375	119.4	209	66.56	29.13
3	450	143.3	250	79.61	29.05
4	540	171.9	300	95.54	29.06

A graph of τ_f vs. σ' will yield $\phi' \approx$ **29°**.

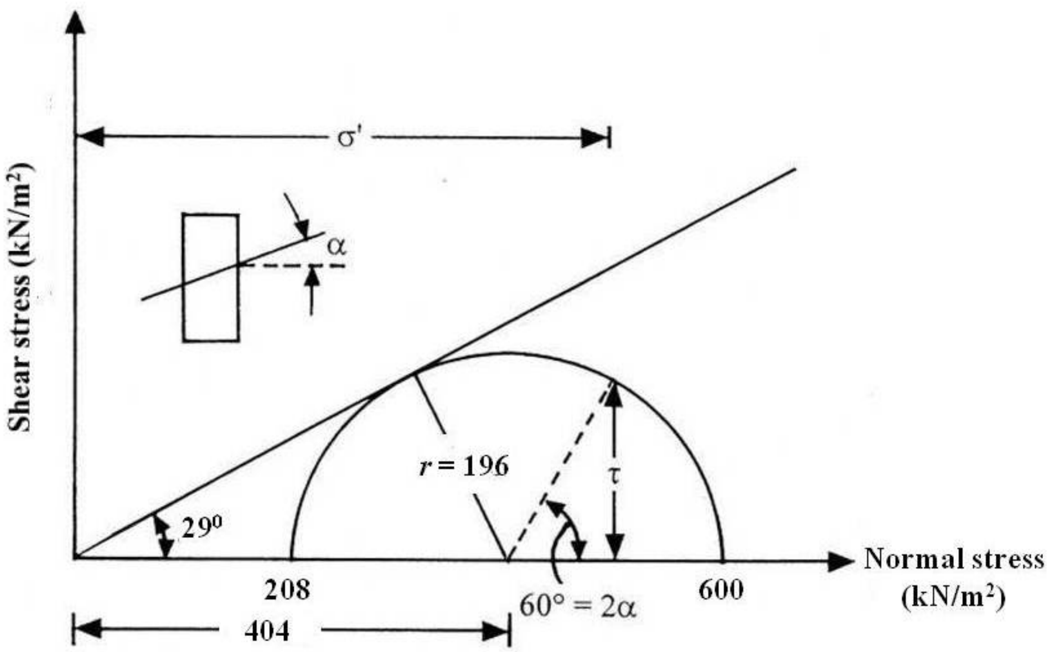
12.6 $c' = 0$. From Eq. (12.8): $\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right)$; $\phi' = 30°$

$$\sigma_1' = 208\tan^2\left(45 + \frac{29}{2}\right) \approx 600\,\text{kN/m}^2$$

$$\Delta\sigma_{d(\text{failure})} = \sigma_1' - \sigma_3' = 600 - 208 = \mathbf{392\ kN/m^2}$$

12.7 a. From Eq. (12.4): $\theta = 45 + \frac{\phi'}{2} = 45 + \frac{29}{2} = \mathbf{59.5}°$

b. Refer to the figure.



$\tau = 196\sin 60° = \mathbf{169.7\ kN/m^2}$

$\sigma' = 404 + r\cos 60 = 404 + 196\cos 60 = \mathbf{502\ kN/m^2}$

For failure, $\tau_f = \sigma'\tan\phi' = 502\tan 29 = 278.26\ \text{kN/m}^2$. Since the developed shear stress = 169.5 kN/m² (which is less than 278.26 kN/m²), the specimen did not fail along this plane.

12.8 $\phi' = 28 + 0.18D_r = 28 + (0.18)(68) = 40.24°$

$$\sigma_1' = \sigma_3'\tan^2\left(45 + \frac{\phi'}{2}\right) = 150\tan^2\left(45 + \frac{40.24}{2}\right) = \mathbf{697.43\ kN/m^2}$$

12.9 $\sigma'_3 = 125\,\text{kN/m}^2;\ \ \sigma'_1 = \sigma'_3 + \Delta\sigma_{d(f)} = 125 + 175 = 300\,\text{kN/m}^2$

$$\sigma'_1 = \sigma'_3 \tan^2\left(45 + \frac{\phi'}{2}\right);\quad 300 = 125\tan^2\left(45 + \frac{\phi'}{2}\right)$$

$\phi' \approx$ **24.3°**

12.10 $\sigma'_3 + \Delta\sigma'_3 = \sigma'_3 \tan^2\left(45 + \frac{\phi'}{2}\right) = \sigma'_3 \tan^2(60.5°) = 3.12\sigma'_3$

$$1 + \frac{\Delta\sigma'_3}{\sigma'_3} = 3.12;\ \ \frac{\Delta\sigma'_3}{\sigma'_3} = 2.12$$

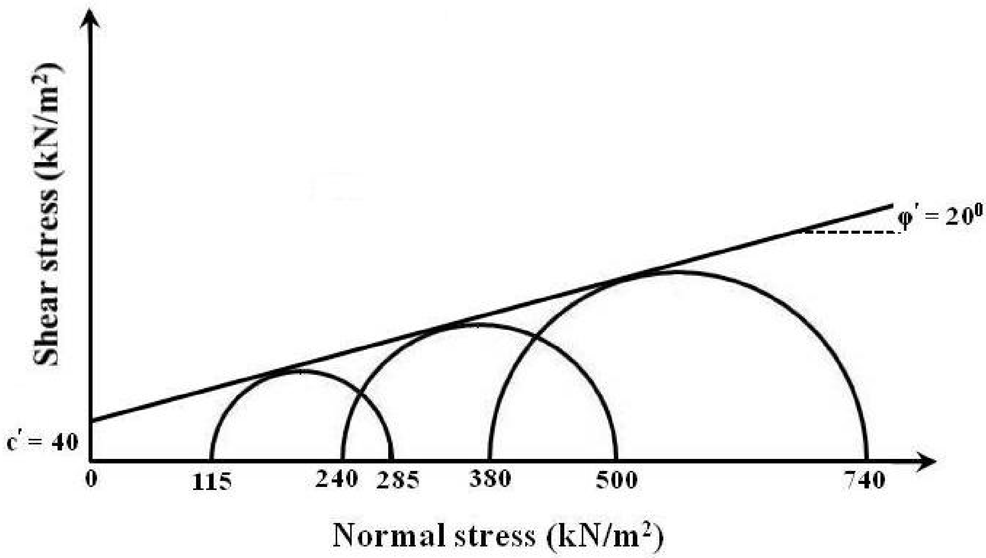
$\sigma'_3 = \frac{18}{2.12} =$ **8.49 lb/in.²**

12.11 $\sigma'_1 = \sigma'_3 \tan^2\left(45 + \frac{\phi'}{2}\right) = \left(\sigma_3 - \Delta u_{d(f)}\right)\tan^2\left(45 + \frac{31}{2}\right)^2$

$\sigma'_1 = (15 - 4.8)\tan^2(60.5)^2 =$ **31.86 lb/in.²**

12.12 a. The effective principal stresses at failure are calculated as follows and the Mohr-Coulomb failure envelope is drawn from the Mohr's circles on the following page.

Test no.	σ_3 (kN/m²)	$(\Delta\sigma_d)_f$ (kN/m²)	$(\Delta u_d)_f$ (kN/m²)	$\sigma'_3 = \sigma_3 - (\Delta u_d)_f$ (kN/m²)	$\sigma'_1 = \sigma'_3 + (\Delta\sigma_d)_f$ (kN/m²)
1	100	170	–15	115	285
2	200	260	–40	240	500
3	300	360	–80	380	740



From the graph: $c' \approx$ **40 kN/m²** and $\phi' \approx$ **20°**

b. Effective stress in the middle of the clay layer:

$$\sigma_0' = (2)(19 - 9.8) = 18.4 \text{ kN/m}^2$$

$$\tau = c' + \sigma' \tan\phi' = 40 + 18.4\tan(20) = \mathbf{46.7\ kN/m^2}$$

12.13 a. $\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right)$; $(25 + 33) = 25\tan^2\left(45 + \frac{\phi'}{2}\right)$

$\phi' \approx$ **23.4°**

b. $\theta = 45 + \frac{\phi'}{2} = 45 + \frac{23.4}{2} = \mathbf{56.7}°$

c. From Eqs. (10.8) and (10.9):

$$\sigma_f' = \frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2}\cos 2\theta = \frac{58 + 25}{2} + \frac{58 - 25}{2}\cos(2 \times 56.7)$$

$$= \mathbf{34.94\ lb/in.^2}$$

$$\tau_f = \sigma' \tan\phi' = 34.94\tan 23.4 = \mathbf{15.12\ lb/in.^2}$$

12.14 $\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right)$

Specimen I: $(105 + 220) = 325 = 105 \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right)$ (a)

Specimen II: $(210 + 400) = 610 = 210 \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right)$ (b)

Subtracting Eq. (a) from Eq. (b): $610 - 325 = 105 \tan^2\left(45 + \frac{\phi'}{2}\right)$; $\phi' = \mathbf{27.48}°$

From Eq. (b):

$$c' = \frac{610 - 210 \tan^2\left(45 + \frac{27.48}{2}\right)}{2 \tan\left(45 + \frac{27.48}{2}\right)} = \mathbf{12.48\ kN/m^2}$$

12.15 a. From Eqs. (10.8) and (10.9): $\theta = 40°$

$$\sigma' = \frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2} \cos 2\theta = \frac{105 + 325}{2} + \frac{325 - 105}{2} \cos(2 \times 40)$$

$$= \mathbf{234.1\ kN/m^2}$$

$$\tau = \frac{\sigma_1' - \sigma_3'}{2} \sin 2\theta = \frac{325 - 105}{2} \sin(2 \times 40) = \mathbf{108.32\ kN/m^2}$$

b. The angle of inclination of the failure plane:

$$\theta = 45 + \frac{\phi'}{2} = 45 + \frac{27.48}{2} = \mathbf{58.73}°$$

$$\sigma' = \frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2} \cos 2\theta = \frac{610 + 210}{2} + \frac{610 - 210}{2} \cos(2 \times 58.73)$$

$$= \mathbf{317.74\ kN/m^2}$$

$$\tau = \frac{\sigma_1' - \sigma_3'}{2} \sin 2\theta = \frac{610 - 210}{2} \sin(2 \times 58.73) = \mathbf{177.46\ kN/m^2}$$

12.16 $\sigma_1 = \sigma_3 \tan^2\left(45 + \frac{\phi}{2}\right)$; $\phi = 2\left[\tan^{-1}\left(\frac{22+28}{22}\right)^{0.5} - 45\right] = \mathbf{22.9°}$

$\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right)$; $\phi' = 2\left[\tan^{-1}\left(\frac{22+28+4}{22+4}\right)^{0.5} - 45\right] = \mathbf{20.48°}$

12.17 a. $\sigma_3 = 150$ kN/m^2; $\sigma_1 = 150 + 120 = 270$ kN/m^2

$\sigma_1 = \sigma_3 \tan^2\left(45 + \frac{\phi}{2}\right)$; $\frac{270}{150} = \tan^2\left(45 + \frac{\phi}{2}\right)$

$\phi = \mathbf{16.6°}$

b. $\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right)$

$$\frac{\sigma_1 - \Delta u_{d(f)}}{\sigma_3 - \Delta u_{d(f)}} = \tan^2\left(45 + \frac{27}{2}\right) = 2.662$$

$$\frac{270 - \Delta u_{d(f)}}{150 - \Delta u_{d(f)}} = 2.662$$

Or, $270 - \Delta u_{d(f)} = 399.3 - 2.662\Delta u_{d(f)}$

$\Delta u_{d(f)} = \mathbf{77.8\ kN/m^2}$

12.18 $\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right) = 150 \tan^2 (45 + 27/2) = 399.4$ kN/m^2

$\Delta\sigma_{d(f)} = \sigma_1' - \sigma_3' = 399.4 - 150 = \mathbf{249.4\ kN/m^2}$

12.19 $\sigma_1 = \sigma_3 \tan^2\left(45 + \frac{\phi}{2}\right) = 20 \tan^2\left(45 + \frac{31}{2}\right) = 62.48$ lb/in.^{2}

$\Delta\sigma_{d(f)} = 62.48 - 20 = \mathbf{42.48\ lb/in.^2}$

$$\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right)$$

$$\frac{62.48 - \Delta u_{d(f)}}{20 - \Delta u_{d(f)}} = \tan^2\left(45 + \frac{24}{2}\right)$$

$\Delta u_{d(f)}$ = **–10.98 lb/in.^{2}**

A dense sand tends to expand during shear. Due to undrained condition, it creates a negative pore water pressure. A loose sand tends to contract during shear, and a positive pore water pressure is developed in undrained conditions. Therefore, for loose sand, $\sigma' < \sigma$, and so, $\phi' > \phi$.

12.20 $$\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right)$$

$$\frac{148 - \Delta u_{d(f)}}{0 - \Delta u_{d(f)}} = \tan^2\left(45 + \frac{28}{2}\right)$$

$\Delta u_{d(f)}$ = **–83.6 kN/m^2**

12.21 a.

Test no.	$\frac{\sigma_1' + \sigma_3'}{2} = p'$ (kN/m^2)	$\frac{\sigma_1' - \sigma_3'}{2} = q'$ (lb/in.^{2})
1	212	108
2	362	155

$q' = m + p' \tan\alpha$

$108 = m + 212 \tan\alpha$ (a)

$155 = m + 362 \tan\alpha$ (b)

m = **41.56 kN/m^2**, α = **17.4°**

b. ϕ' = sin^{-1}(tan α) = sin^{-1}(tan 17.4) = **18.26°**

$$c' = \frac{m}{\cos\alpha} = \frac{41.56}{\cos(17.4)} = \mathbf{43.55\ kN/m^2}$$

12.22 $\frac{c_{u(\text{VST})}}{\sigma'_o} = 0.11 + 0.0037PI$

$\sigma' = (2)(18) + (7)(19.5 - 9.81) = 103.83$ kN/m^2

$c_{u(\text{VST})} = [0.11 + (0.0037)(23)](103.83) =$ **20.25 kN/m^2**

CRITICAL THINKING PROBLEM

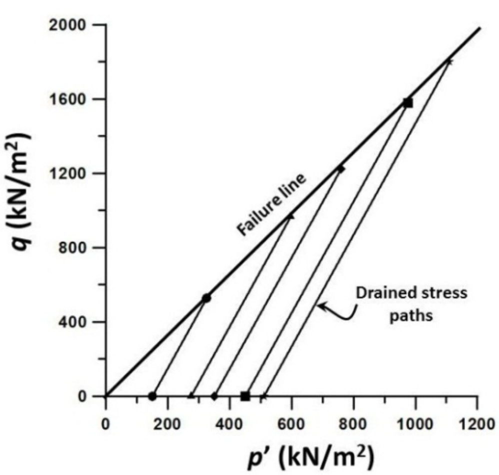
12.C.1 Task 1

$$p' = \frac{1}{3}(\sigma'_1 + 2\sigma'_3) = \sigma'_3 + \frac{1}{3}\Delta\sigma_d \; ; q = \Delta\sigma_d$$

At the end of consolidation, $p'_0 = \sigma'_3; \; q_0 = 0$

σ_3 (kN/m^2)	p'_0 (kN/m^2)	$(\Delta\sigma_d)_f = q_f$ (kN/m^2)	p'_f (kN/m^2)
150	150	527	325.67
275	275	965	596.67
350	350	1225	758.33
450	450	1580	976.67
510	510	1800	1110

The drained stress paths and the failure line are shown below.



Task 2

At point *O*: $p' = 0$; $q = 0$

At point *P*: $p' = \sigma'_3 = 250\,\text{kN/m}^2$; $q = 0$

At point *A*: $q = 1000 = \Delta\sigma_d$

$$p' = 675 = \sigma'_3 + \frac{1}{3}\Delta\sigma_d = \sigma'_3 + \frac{1}{3}(1000)$$

Therefore, $\sigma'_3 = 341.67\,\text{kN/m}^2$

Stress path *O* to *P*: Increase confining pressure from 0 to 250 kN/m^2 under drained condition (effective stress).

Stress path *P* to *A*: Increase confining pressure from 250 kN/m^2 to 341.67 kN/m^2 under drained condition (effective stress). Simultaneously increase axial stress from 0 to 1000 kN/m^2.

Chapter 13

13.1 — 13.4 $K_o = (1-\sin\phi')(\text{OCR})^{\sin\phi'}$

Problem	ϕ'(deg)	K_o	$P_o = \frac{1}{2}K_o\gamma H^2$	$\bar{z} = H/3$
13.1	35	0.634	**143.44 kN/m**	**1.67 m**
13.2	33	0.627	**8,607.1 lb/ft**	**5.67 ft**
13.3	29	0.515	**176.13 kN/m**	**2 m**
13.4	40	0.463	**8,625.7 lb/ft**	**6 ft**

13.5 — 13.8 $K_a = \tan^2(45 - \phi'/2)$

Problem	ϕ'(deg)	K_a	$\sigma'_{a(z=H)} = K_a\gamma H$	$P_a = \frac{1}{2}K_a\gamma H^2$	$\bar{z} = H/3$
13.5	32	0.307	(0.307)(110)(14) = **472.7 lb/ft²**	($\frac{1}{2}$)(0.307)(110)(14)² = **3309.4 lb/ft**	**4.66 ft**
13.6	28	0.361	(0.361)(99)(22) = **786.2 lb/ft²**	($\frac{1}{2}$)(0.361)(99)(22)² = **8648.8 lb/ft**	**7.33 ft**
13.7	37	0.248	(0.248)(17.6)(5) = **21.8 kN/m²**	($\frac{1}{2}$)(0.248)(17.6)(5)² = **54.56 kN/m**	**1.67 m**
13.8	41	0.207	(0.207)(19.5)(9) = **36.32 kN/m²**	($\frac{1}{2}$)(0.207)(19.5)(9)² = **163.47 kN/m**	**3 m**

Note: 1. Pressure distribution is similar to that shown in Figure 13.11a., i.e., $\sigma'_a = 0$ at $z = 0$ and $\sigma'_a = K_a\gamma H$ at $z = H$

2. $\bar{z}$ = distance measured from the bottom of the wall

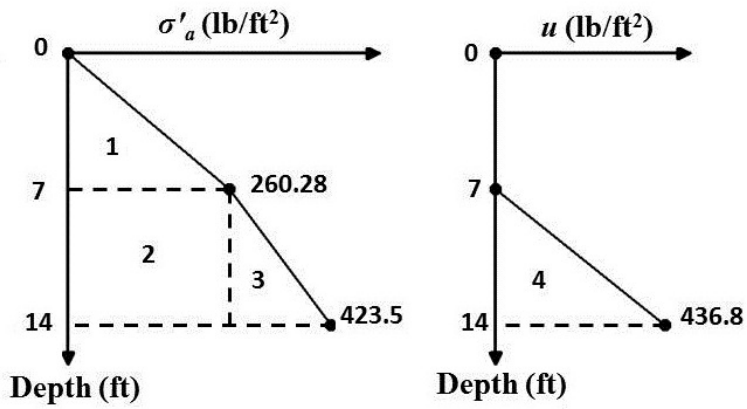
13.9 — 13.12 $K_p = \tan^2(45 + \phi'/2)$

Problem	ϕ'(deg)	K_p	$\sigma'_{p(z=H)} = K_p\gamma H$	$P_p = \frac{1}{2}K_p\gamma H^2$	$\bar{z} = H/3$
13.9	32	3.254	(3.254)(117)(11) = **4187.9 lb/ft²**	($\frac{1}{2}$)(3.254)(117)(11)² = **23,033 lb/ft**	**3.67 ft**
13.10	38	4.203	(4.203)(101)(16) = **6792 lb/ft²**	($\frac{1}{2}$)(4.203)(101)(16)² = **54,336 lb/ft**	**5.33 ft**
13.11	30	3.0	(3)(16.6)(7) = **348.6 kN/m²**	($\frac{1}{2}$)(3)(16.6)(7)² = **1220.1 kN/m**	**2.33 m**
13.12	27	2.662	(2.662)(20.5)(12) = **654.8 kN/m²**	($\frac{1}{2}$)(2.662)(20.5)(12)² = **3929.1 kN/m**	**4 m**

Note: 1. $\sigma'_{p(z=0)} = 0$; triangular pressure distribution

2. $\bar{z}$ = distance measured from the bottom of the wall

13.13 $K_a = \tan^2\left(45 - \frac{\phi'}{2}\right) = \tan^2\left(45 - \frac{28}{2}\right) = 0.361$. Refer to the figure.



$z = 0$ ft: $\sigma'_a = \sigma'_a K_a = 0$; $u = 0$

$z = 7$ ft: $\sigma'_a = (103)(7)(0.361) = 260.28$ lb/ft^2; $u = 0$

$z = 14$ ft: $\sigma'_a = [(103)(7) + (127 - 62.4)(7)](0.361) = 423.5$ lb/ft^2

$u = (62.4)(7) = 436.8$ lb/ft^2

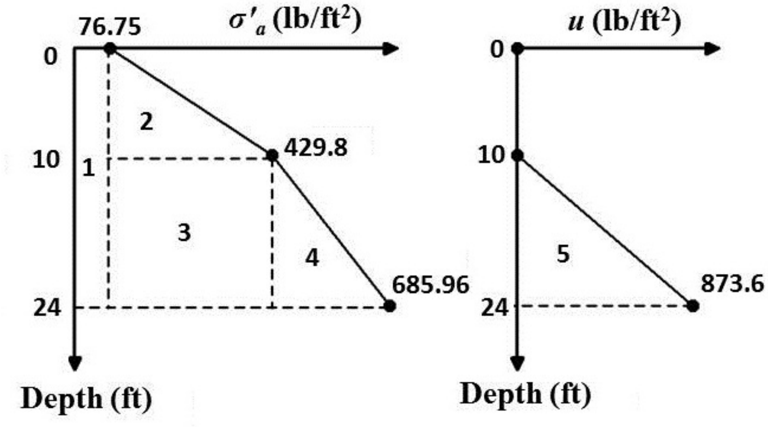
Area No.	Area	
1	$(\frac{1}{2})(7)(260.28) = 910.98 =$	910.98
2	$(260.28)(7) =$	1,821.96
3	$(\frac{1}{2})(7)(423.5 - 260.28) =$	571.27
4	$(\frac{1}{2})(7)(436.8) =$	1,528.8
		Σ4,833 lb/ft

Resultant: Taking the moment about the bottom of the wall,

$$\bar{z} = \frac{\left[(910.98)\left(7 + \frac{7}{3}\right) + (1821.96)\left(\frac{7}{2}\right) + (571.27)\left(\frac{7}{3}\right) + (1528.8)\left(\frac{7}{3}\right)\right]}{4833}$$

$= \mathbf{4.09\ ft}$

13.14 $K_a = \tan^2\left(45 - \frac{32}{2}\right) = 0.307.$ Refer to the figure.



$z = 0$ ft: $\sigma'_a = qK_a = (250)(0.307) = 76.75$ lb/ft^2; $u = 0$

$z = 10$ ft: $\sigma'_a = [250 + (10)(115)](0.307) = 429.8$ lb/ft^2; $u = 0$

$z = 24$ ft: $\sigma'_a = [250 + (10)(115) + (122 - 62.4)(14)](0.307) = 685.96$ lb/ft^2

$u = (62.4)(14) = 873.6$ lb/ft^2

Area No.	Area	
1	(76.75)(24) =	1,842
2	$(\frac{1}{2})(10)(429.8 - 76.75)$ =	1,765.25
3	(14)(429.8 – 76.75) =	4,942.7
4	$(\frac{1}{2})(14)(685.96 - 429.8)$ =	1,793.12
5	$(\frac{1}{2})(14)(873.6)$ =	6,115.20

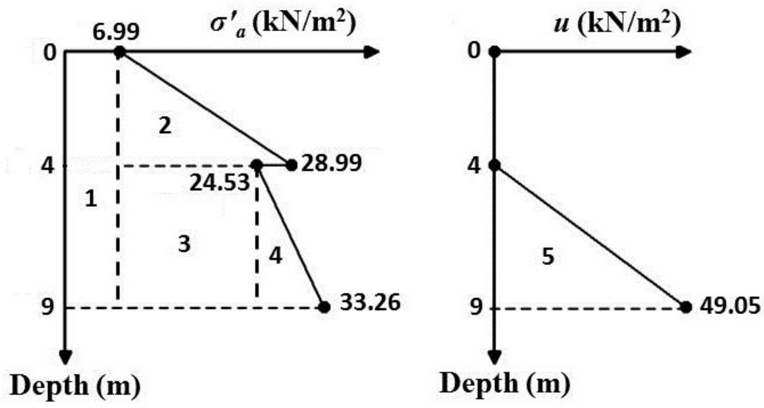
$P_a = \Sigma$**16,458 lb/ft**

Location of resultant: Taking the moment about the bottom of the wall,

$$\bar{z} = \frac{\left[(1842)\left(\frac{24}{2}\right) + (1765.25)\left(14 + \frac{10}{3}\right) + (4942.7)\left(\frac{14}{2}\right) + (1{,}793.12)\left(\frac{14}{3}\right) + (6{,}115.2)\left(\frac{14}{3}\right)\right]}{16458} = \mathbf{7.55\ ft}$$

13.15 $K_{a(1)} = \tan^2\left(45 - \frac{30}{2}\right) = 0.333;\;\; K_{a(2)} = \tan^2\left(45 - \frac{34}{2}\right) = 0.282.$

Refer to the figure.



$z = 0$ m: $\sigma'_a = q\,K_{a(1)} = (21)(0.333) = 6.99 \text{ kN/m}^2;\;\; u = 0$

$z = 4$ m: $\sigma'_a = [(16.5)(4) + 21](0.333) = 28.99 \text{ kN/m}^2$

$\sigma'_a = [(16.5)(4) + 21](0.282) = 24.53 \text{ kN/m}^2$

$u = 0$

$z = 9$ m: $\sigma'_a = [(16.5)(4) + (20.2 - 9.81)(5)](0.282) = 33.26 \text{ kN/m}^2$

$u = (9.81)(5) = 49.05 \text{ kN/m}^2$

Area No.	Area	
1	(6.99)(9) =	62.91
2	($\frac{1}{2}$)(4)(28.99 – 6.99) =	44
3	(5)(24.53 – 6.99) =	87.7
4	($\frac{1}{2}$)(5)(33.26 –24.53) =	21.82
5	($\frac{1}{2}$)(5)(49.05) =	122.62

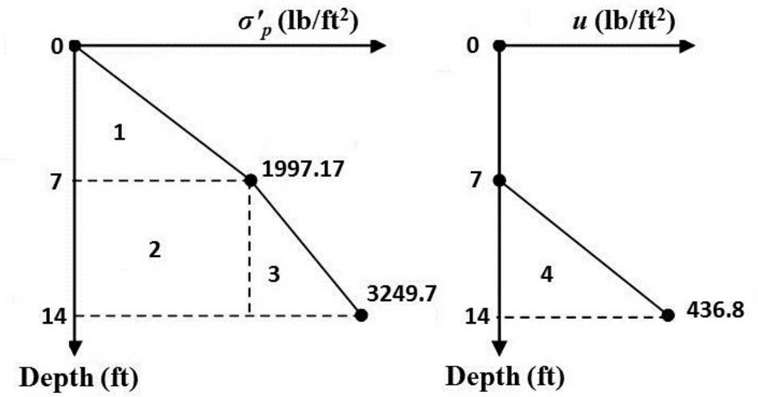
$P_a = \Sigma$**339.05 kN/m**

Location of resultant: Taking the moment about the bottom of the wall,

$$\bar{z} = \frac{\left[(62.91)\left(\frac{9}{2}\right) + (44)\left(5 + \frac{4}{3}\right) + (87.7)\left(\frac{5}{2}\right) + (21.82)\left(\frac{5}{3}\right) + (122.62)\left(\frac{5}{3}\right)\right]}{339.05}$$

$= \mathbf{3.01\,m}$

13.16 $K_p = \tan^2\left(45 + \frac{28}{2}\right) = 2.77.$ Refer to the figure.



$z = 0$ ft: $\sigma'_p = 0$; $u = 0$

$z = 7$ ft: $\sigma'_p = \gamma_1 z K_p = (103)(7)(2.77) = 1997.17 \text{ lb/ft}^2$; $u = 0$

$z = 14$ ft: $\sigma'_p = [(103)(7) + (127 - 62.4)(7)](2.77) = 3249.7 \text{ lb/ft}^2$

$u = (62.4)(7) = 436.8 \text{ lb/ft}^2$

Area No.	Area	
1	$(\frac{1}{2})(7)(1997.17) =$	6,990.09
2	$(7)(1997.17) =$	13,980.19
3	$(\frac{1}{2})(7)(3249.7 - 1997.17) =$	4,383.8
4	$(\frac{1}{2})(7)(436.8) =$	1,528.8
		$P_a = \mathbf{\Sigma 26{,}883\ lb/ft}$

Location of the resultant: Taking the moment about the bottom of the wall,

$$\bar{z} = \frac{\left[(6990.09)\left(7 + \frac{7}{3}\right) + (13980.19)\left(\frac{7}{2}\right) + (4383.8)\left(\frac{7}{3}\right) + (1528.8)\left(\frac{7}{3}\right)\right]}{26883} = \mathbf{4.76\ ft}$$

13.17 a. Use Table 13.2: For $\alpha = 10°$ and $\phi' = 36°$, $K_a = 0.270$

$\sigma'_a = \gamma z K_a = (19)(4)(0.270) = \mathbf{20.52\ kN/m^2}$

b. Equation (13.24):

$$P_a = \frac{1}{2}K_a\gamma H^2 = \frac{1}{2}(0.27)(19)(4)^2 = \mathbf{41.04\ kN/m}$$

Location and Direction of Resultant: **At a distance of *H*/3 = 4/3 = 1.33 m above the bottom of the wall inclined at an angle *α* = 10° to the horizontal.**

13.18 a. Use Table 13.3: For $\alpha = 10°$ and $\phi' = 36°$, $K_p = 3.598$

$$\sigma_a' = \gamma z K_p = (19)(4)(3.598) = 273.44\ \text{kN/m}^2$$

b. Equation (13.25):

$$P_p = \frac{1}{2}\gamma H^2 K_p = \frac{1}{2}(19)(4)^2(3.598) = \mathbf{546.89\ kN/m}$$

Location and Direction of Resultant: **At a distance of *H*/3 = 4/3 = 1.33 m above the bottom of the wall inclined at an angle *α* = 10° to the horizontal.**

13.19 a. $H = 5$ m; $c_u = 17$ kN/m^2; $\gamma = 21$ kN/m^2; $\phi = 0$

$$K_a = \tan^2\left(45 - \frac{\phi}{2}\right) = 1; \sigma_a' = \gamma z - 2c_u$$

At the top ($z = 0$ m):

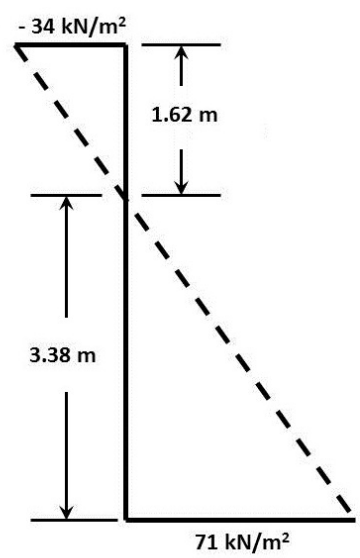
$$\sigma_a' = -2c_u = (-2)(17)$$

$$= -34\ \text{kN/m}^2$$

At the bottom ($z = 5$ m):

$$\sigma_a' = (21)(5) - (2)(17)$$

$$= 71\ \text{kN/m}^2$$



The pressure diagram is shown.

b. Eq. (13.41): $z_o = \frac{2c_u}{\gamma} = \frac{(2)(17)}{21} = \mathbf{1.62\ m}$

c. Eq. (13.43): $P_a = \frac{1}{2}\gamma H^2 - 2c_u H = \frac{1}{2}(21)(5)^2 - (2)(17)(5) = \mathbf{92.5\ kN/m}$

d. Eq. (13.45):

$$P_a = \frac{1}{2}\gamma H^2 - 2c_u H + \frac{2c_u^2}{\gamma}$$
$$= \frac{1}{2}(21)(5)^2 - (2)(17)(5) + \frac{(2)(17)^2}{21} = \mathbf{120\ kN/m}$$

Resultant measured from the bottom:

$$\bar{z} = \frac{H - z_o}{3} = \frac{5 - 1.62}{3} = 1.126\text{ m} \approx \mathbf{1.13\ m}$$

13.20 a. $\sigma_a = \sigma_o K_a - 2c\sqrt{K_a}$

$\sigma_o = \gamma z + q$; $K_a = 1$.

At $z = 0$ ft:

$\sigma_o = 11$ kN/m^2

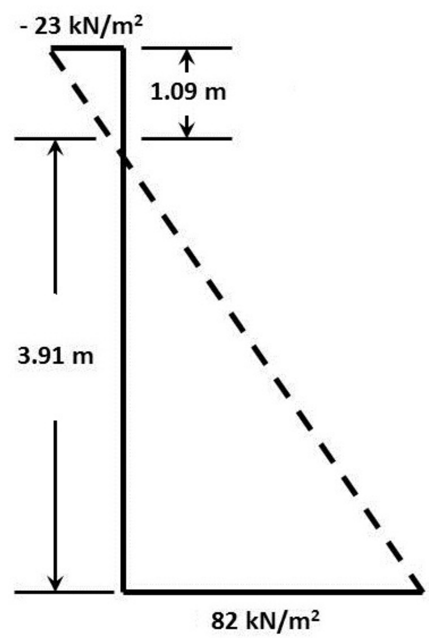
$\sigma_a = 11 - (2)(17) = -23$ kN/m^2

At $z = 5$ m:

$\sigma_o = (21)(5) + 11 = 116$ kN/m^2

$\sigma_a = 116 - (2)(17) = 82$ kN/m^2

The pressure diagram is shown.



b. $\sigma_a = 0$; $(\gamma z_o + q) - 2c = 0$

$$z_o = \frac{2c_u - q}{\gamma} = \frac{34 - 11}{21} = \mathbf{1.09\ m}$$

c. Referring to the diagram in Part a,

$$P_a = \frac{1}{2}(3.91)(82) - \frac{1}{2}(23)(1.09) = \mathbf{147.8\,kN/m}$$

d. $P_a = \frac{1}{2}(3.91)(82) = \mathbf{160.3\,kN/m}$

Location of the resultant from the bottom of the wall: $\frac{3.91}{3} = \mathbf{1.3\,m}$

13.21 $K_a = \tan^2\left(45 - \frac{\phi'}{2}\right) = \tan^2\left(45 - \frac{28}{2}\right) = 0.361;\ \sqrt{K_a} = 0.6.$ Eq. (13.44):

$$P_a = \frac{1}{2}K_a\gamma H^2 - 2\sqrt{K_a}c'H + \frac{2c'^2}{\gamma}$$

$$= \frac{1}{2}(0.361)(122)(33)^2 - (2)(0.6)(750)(33) + \frac{(2)(750)^2}{122} = \mathbf{3502.18\,lb/ft}$$

13.22 Use Eqs. (13.53) and (13.54). $\alpha = 0$; $\theta = 12^\circ$; $\phi' = 34^\circ$; $\gamma = 119$ lb/ft^3; $H = 32$ ft

Part	δ' (deg)	K_a [Eq. (13.54)]	$P_a = \frac{1}{2}K_a\gamma H^2$ [Eq. (13.53)]
1	14	0.3511	**21,392 lb/ft**
2	21	0.3509	**21,381 lb/ft**

***P_a* is located at a vertical distance of 32/3 = 10.67 ft above the bottom of the wall and is inclined at an angle δ' to the normal drawn to the back face of the wall.**

13.23 a. $\phi' = 38^\circ$; $\psi = 90 - \theta - \delta' = 90 - 5 - 20 = 65^\circ$

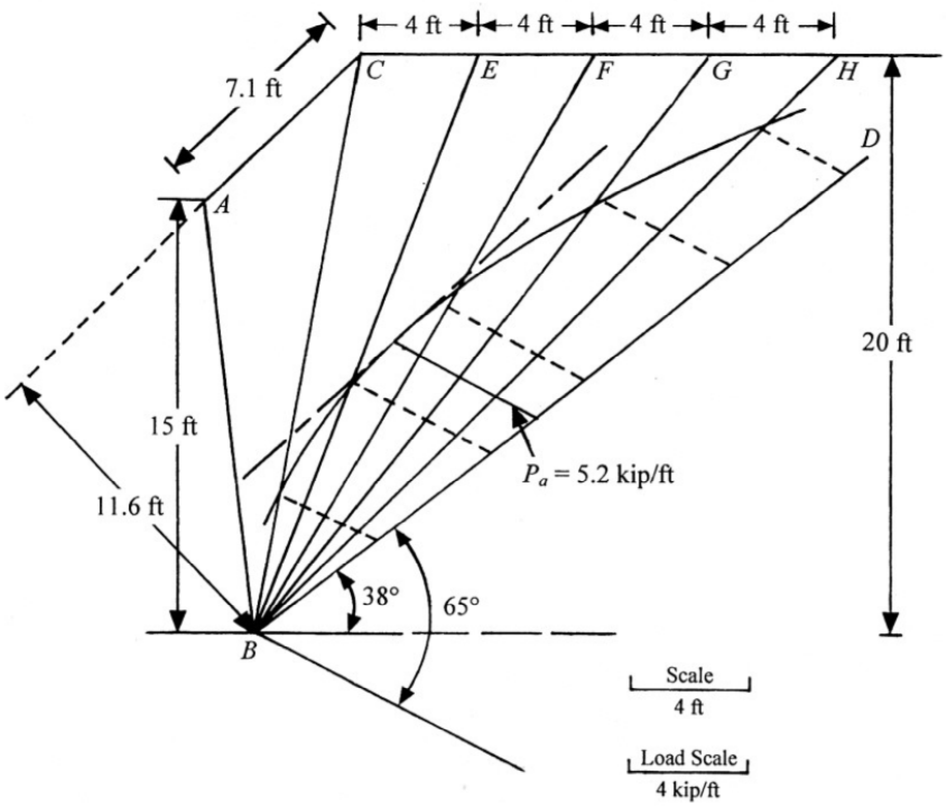
Weight of wedge $ABC = \frac{1}{2}(11.6)(7.1)\underbrace{(128)}_{\gamma} = 5271\,\text{lb/ft} = 5.271\,\text{kip/ft}$

The weight of each of the wedges

$$CBE, EBF, FBG, GBH = \frac{1}{2}(20)(4)(128) = 5120 \text{ lb/ft} = 5.12 \text{ kip/ft}$$

Wedge	Weight (kip/ft)
ABC	5.271
ABE	5.271 + 5.12 = 10.391
ABF	10.391 + 5.12 = 15.511
ABG	15.511 + 5.12 = 20.631
ABH	20.631 + 5.12 = 25.751

The graphical construction is shown. P_a = **5.2 kip/ft**



b. $\gamma = \frac{(1680)(9.81)}{1000} = 16.48 \text{ kN/m}^3; \phi' = 30°; \ \psi = 90 - 10 - 30 = 50°$

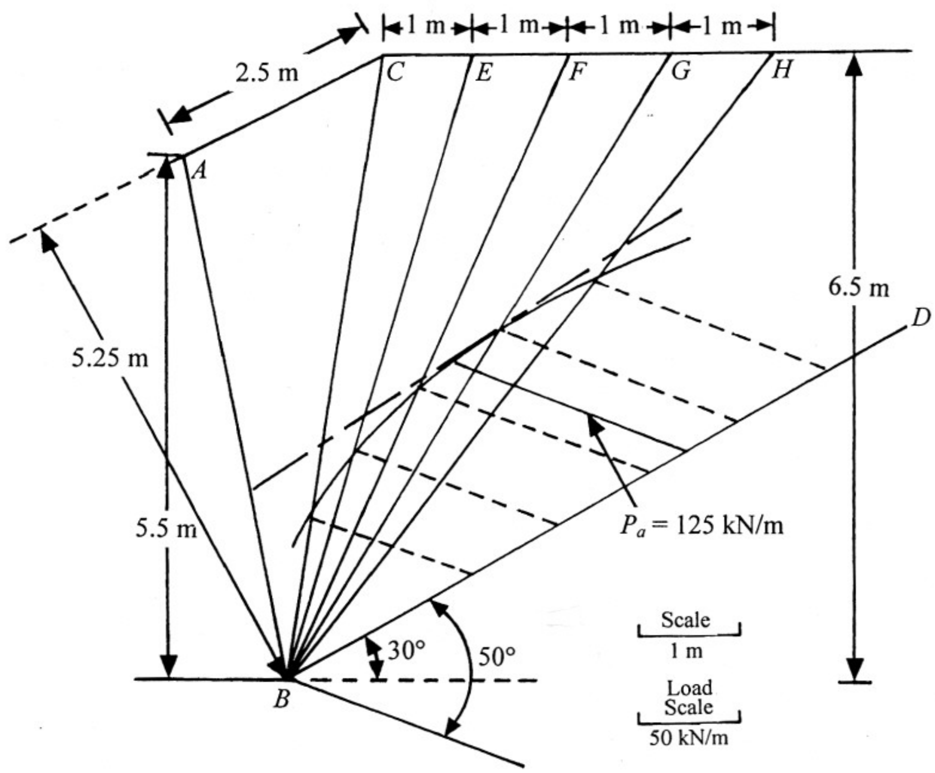
Weight of wedge $ABC = \frac{1}{2}(5.25)(2.5)(16.48) = 108.15 \text{ kN/m}$

The weight of each of the wedges

$$CBE, EBF, FBG, GBH = \frac{1}{2}(1)(6.5)(16.48) = 53.56 \text{ kN/m}$$

Wedge	Weight (kip/ft)
ABC	108.15
ABE	108.15 + 53.56 = 161.71
ABF	161.71 + 53.56 = 215.27
ABG	215.27 + 53.56 = 268.83
ABH	268.83 + 53.56 = 322.39

The graphical construction is shown. P_a = **125 kN/m**



13.24 From Eqs. (13.66) and (13.67), $\theta^* = \theta + \bar{\beta}$ and $\alpha^* = \alpha + \bar{\beta}$.

$$\bar{\beta} = \tan^{-1}\left(\frac{k_h}{1-k_v}\right) = \tan^{-1}\left(\frac{0.1}{1-0}\right) = 5.71°$$

θ^* = 9º + 5.71º = 14.71º

$\alpha^* = 12^\circ + 5.71^\circ = 17.71^\circ$

$$P_a(\theta^*, \alpha^*) = \frac{1}{2}\gamma H^2 K_a$$

$$\frac{\delta'}{\phi'} = \frac{2}{3}$$

From Table 13.5, for $\theta^* = 14.71^\circ$ and $\alpha^* = 17.71^\circ$, the value of $K_a \approx 0.582$.

From Eq. (13.70):

$$P_{ae} = P_a(\theta^*, \alpha^*)(1 - k_v)\left[\frac{\cos^2(\theta + \overline{\beta})}{\cos\theta \cos^2\overline{\beta}}\right]$$

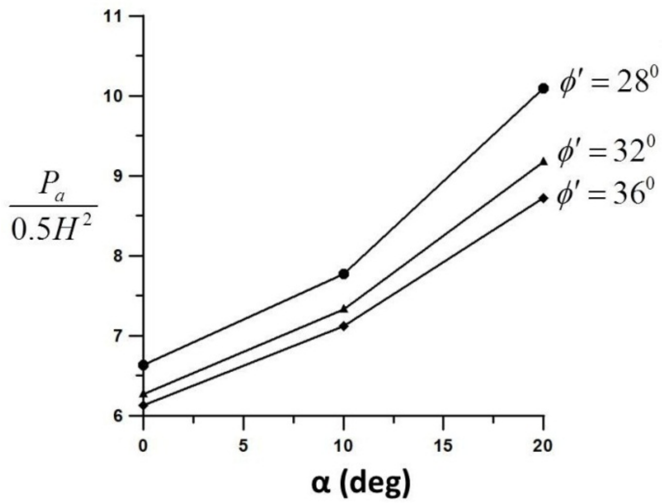
$$= \left[\left(\frac{1}{2}\right)(0.582)(19)(6)^2\right](1-0)\left[\frac{\cos^2(9+5.71)}{\cos(9)\cos^2(5.71)}\right] = \mathbf{190.41\,kN/m}$$

CRITICAL THINKING PROBLEM

13.C.1 Refer to Table A.1 to prepare the following table:

$\theta = 10^\circ$	γ (kN/m^3)	ϕ'	$K_{a(R)}$	$K_{a(R)}\gamma$
$\alpha = 0^\circ$	16.5	28°	0.402	6.633
	17.7	32°	0.354	6.265
	19.5	36°	0.314	6.123
$\alpha = 10^\circ$	16.5	28°	0.471	7.771
	17.7	32°	0.414	7.327
	19.5	36°	0.365	7.117
$\alpha = 20^\circ$	16.5	28°	0.612	10.09
	17.7	32°	0.518	9.168
	19.5	36°	0.447	8.716

The graph of $\frac{P_a}{0.5H^2}$ versus backfill inclination angle, α, is shown on the next page.



The above chart shows that for any backfill inclination α and any wall height H, the active force P_a per unit length of the wall decreases as the soil friction angle (or the compacted unit weight) increases. For a desired level of P_a (at a given α), a compaction unit weight could be estimated from the chart for field specification.

Chapter 14

14.1 $P_p = \frac{1}{2}K_p\gamma H^2$; $K_p = K_{p(\delta'=0)}R$. With $\phi' = 30°$, θ = 15°, and $\alpha = 0$, the value of $K_{p(\delta'=0)} = 2.34$ (Table 14.2). With $\theta = 15°$, $\delta' = 18°$, $\delta'/\phi' = 18/30 = 0.6$, the value of R is 1.55 (Table 14.3). So,

$$P_p = \frac{1}{2}(2.34 \times 1.55)(17.8)(5)^2 \approx \mathbf{807\ kN/m}$$

14.2 $P_p = \frac{1}{2}K_p\gamma H^2$. From Figure 14.5, for $\phi' = 35°$ and $\delta' = 23.33°$, $\delta'/\phi' = 0.666$, the value of K_p is about 6.2. So,

$$P_p = \frac{1}{2}(6.2)(112)(14)^2 = \mathbf{68{,}051\ lb/ft}$$

14.3 $P_p = \frac{1}{2}K_p\gamma H^2$. From Figure 14.4, for $\phi' = 30°$ and δ′ = 20°, the value of K_p is about 5.5. So,

$$P_p = \frac{1}{2}(5.5)(19)(4)^2 = \mathbf{836\ kN/m}$$

14.4 From Table 14.2, for $\phi' = 30°$ and $\theta = 0$, the value of $K_{p(\delta'=0)} = 3.0$. For $\theta = 0$ and $\delta'/\phi' = 20/30 = 0.667$, the value of R is about 1.75. So

$$P_p = \frac{1}{2}(3 \times 1.75)(19)(4)^2 \approx \mathbf{798\ kN/m}$$

14.5 From Equation (14.15): $P_p = \frac{1}{2}\gamma H^2 K_p$

Step 1: $\dfrac{\alpha}{\phi'} = \dfrac{10}{30} = 0.333$

Step 2: From Figure 14.7: $K_{p(\delta'=\phi')} \approx 8.5$

Step 3: $\dfrac{\delta'}{\phi'} = \dfrac{18}{30} = 0.6$

Step 4: From Table 14.4: $R' = 0.811$

Step 5: $K_p = (R')\left[K_{p(\delta'=\phi')}\right] = (0.811)(8.5) = 6.893$

$$P_p = \frac{1}{2}(16.8)(4.75)^2(6.893) \approx \mathbf{1306\ kN/m}$$

14.6 Eq. (14.16): $P_{pe} = \left[\dfrac{1}{2}\gamma H^2 K_{p\gamma(e)}\right]\dfrac{1}{\cos\delta'}$

For $k_v = 0$, $k_h = 0.25$, $\delta'/\phi' = 15/30 = 0.5$, the value of $K_{p\gamma(e)} \approx 3.95$.

$$P_{pe} = \left[(0.5)(118)(15)^2(3.95)\right]\frac{1}{\cos 15} = \mathbf{54{,}286\ lb/ft}$$

14.7 $n_a = \dfrac{2.75\text{ m}}{5.5\text{ m}} = 0.5$. $\phi' = 40^\circ$; $\delta' = 15^\circ$. Table 14.5: $\dfrac{P_a}{0.5\gamma H^2} = 0.216$

$P_a = (0.216)(0.5)(15.8)(5.5)^2 = \mathbf{51.61\ kN/m}$

14.8 $n_a = \dfrac{6.5\text{ ft}}{21\text{ ft}} \approx 0.3$; $\dfrac{c'}{\gamma H} = \dfrac{255}{(121)(21)} = 0.1$

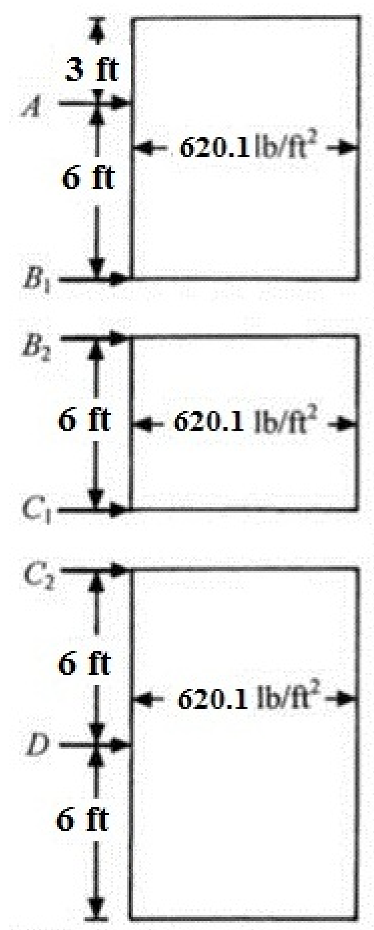
From Table 14.6 for $\phi' = 25^\circ$ and $\delta' = 15^\circ$, $\dfrac{P_a}{0.5\gamma H^2} = 0.085$.

$P_a = (0.085)(0.5)(121)(21)^2 \approx \mathbf{2268\ lb/ft}$

14.9 $\sigma_a = 0.65\gamma H \tan^2\left(45 - \frac{\phi'}{2}\right)$

$$= (0.65)(115)(27)\tan^2\left(45 - \frac{32}{2}\right)$$

$$= \mathbf{620.1\ lb/ft^2}$$



$\sum M_{B_1} = 0$

$$A = \left(\frac{1}{6}\right)\left[(620.1)(9)\left(\frac{9}{2}\right)\right] = 4185.6 \text{ lb/ft}$$

$B_1 = (620.1)(9) - 4185.6 = 1395.3$ lb/ft

$$B_2 = C_1 = \frac{(620.1)(6)}{2} = 1860.3 \text{ lb/ft}$$

$\sum M_{C_2} = 0$

$$D = \left(\frac{1}{6}\right)\left[(620.1)(12)\left(\frac{12}{2}\right)\right] = 7441.2 \text{ lb/ft}$$

$C_2 = (620.1)(12) - 7441.2 = 0$ lb/ft

Strut Loads:

$A = (4185.6)(10) \approx$ **41,856 lb**

$B = (B_1 + B_2)(10)$

$= (1395.3 + 1860.3)(10) \approx$ **32,556 lb**

$C = (C_1 + C_2)(10)$

$= (1860.3 + 0)(10) =$ **18,603 lb**

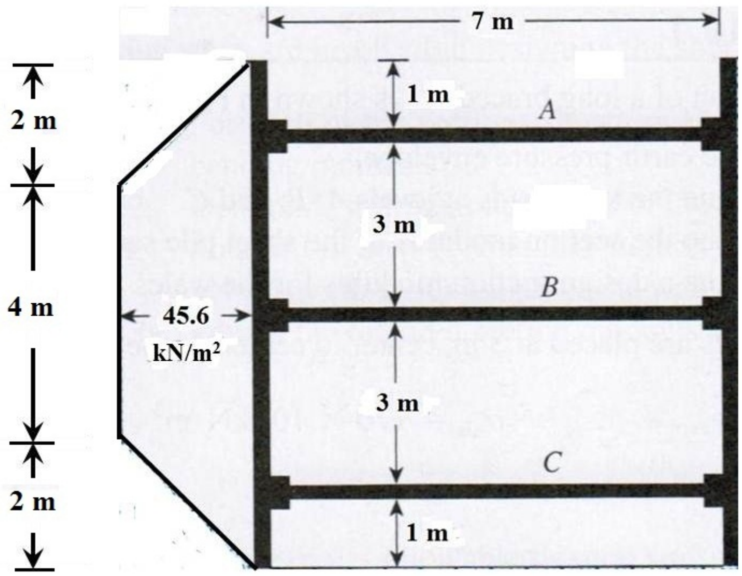
$D = (7441.2)(10) =$ **74,412 lb**

14.10 a. $\dfrac{\gamma H}{c} = \dfrac{(19)(8)}{42} = 3.61 < 4$

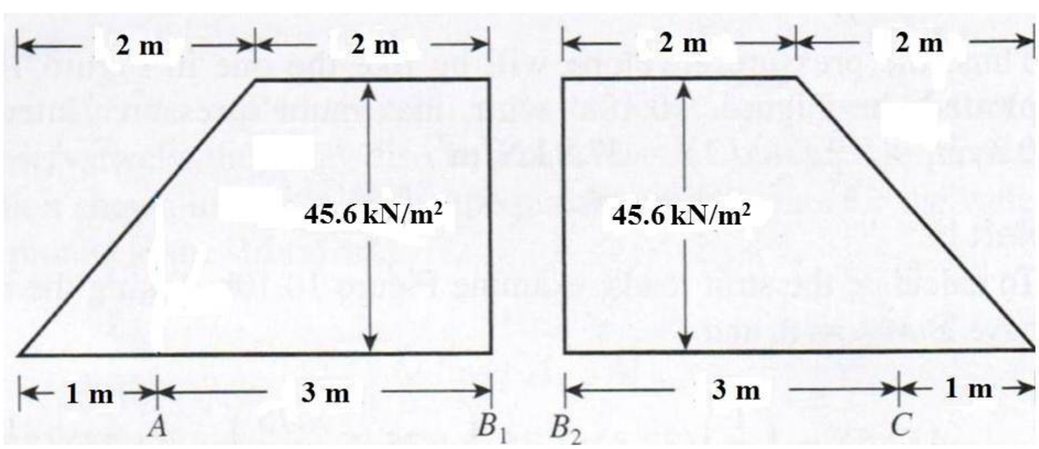
Use Figure 14.14(c) to determine the earth pressure envelope.

Maximum pressure intensity: $\sigma_a = 0.3\gamma H = (0.3)(19)(8) = 45.6\ \text{kN/m}^2$

The pressure envelope is shown below.



b. To determine the strut loads, refer to the following diagram.



$$\sum M_{B_1} = 0$$

$$A(3) - (0.5)(45.6)(2)\left(2 + \frac{2}{3}\right) - (2)(45.6)\left(\frac{2}{2}\right) = 0$$

Therefore, A = 70.93 kN/m

Also, sum of vertical forces, $\sum V = 0$

$$A + B_1 = (0.5)(45.6)(2) + (45.6)(2)$$

or

$$70.93 + B_1 = 136.8$$

Therefore, $B_1 = 65.87$ kN/m

Due to symmetry, $B_2 = 65.87$ kN/m

and

C = 70.93 kN/m

Strut load at A = (70.93)(s) = (70.93)(3.5) = **248.25 kN**

Strut load at B = (B_1 + B_2)(s) = (65.87+65.87)(3.5) = **461.09 kN**

Strut load at C = (70.93)(3.5) = **248.25 kN**

Chapter 15

15.1 Eq. (15.15):

$$F_s = \frac{c'}{\gamma H \cos^2 \beta \tan \beta} + \frac{\tan \phi'}{\tan \beta}$$

$$2.75 = \frac{31}{(17.8)(H)(\cos^2 25)(\tan 25)} + \frac{\tan 28}{\tan 25}$$

$$2.75 = \frac{4.546}{H} + 1.14$$

$H =$ **2.82 m**

15.2 Eq. (15.16):

$$H_{\text{cr}} = \frac{c'}{\gamma} \frac{1}{\cos^2 \beta(\tan \beta - \tan \phi')} = \frac{300}{115} \frac{1}{(\cos^2 30)(\tan 30 - \tan 21)} = \mathbf{17.97\ ft}$$

15.3 $\gamma' = 19.2 - 9.81 = 9.39$ kN/m^3

Eq. (15.28):

$$F_s = \frac{c'}{\gamma_{\text{sat}} H \cos^2 \beta \tan \beta} + \frac{\gamma'}{\gamma_{\text{sat}}} \frac{\tan \phi'}{\tan \beta} = \frac{46}{(19.2)(11)(\cos 18)^2 (\tan 18)} + \frac{9.39}{19.2} \frac{\tan 22}{\tan 18}$$

$$= 0.741 + 0.608 \approx \mathbf{1.35}$$

15.4 $$\gamma_{\text{sat}} = \frac{(G_s + e)\gamma_w}{1 + e} = \frac{(2.73 + 0.69)(62.4)}{1 + 0.69} = 126.27 \text{ lb/ft}^3$$

$\gamma' = 126.27 - 62.4 = 63.87$ lb/ft^3

$$F_s = \frac{c'}{\gamma_{sat} H \cos^2 \beta \tan \beta} + \frac{\gamma'}{\gamma_{sat}} \frac{\tan \phi'}{\tan \beta}$$

$$= \frac{1000}{(126.27)(27)(\cos^2 28)(\tan 28)} + \frac{63.87}{126.27} \frac{\tan 18}{\tan 28}$$

$$= 0.707 + 0.309 \approx \mathbf{1.016}$$

15.5 a. Eq. (15.15):

$$F_s = \frac{c'}{\gamma H \cos^2 \beta \tan \beta} + \frac{\tan \phi'}{\tan \beta}$$

$$F_s = \frac{21}{\left[\frac{(1950)(9.81)}{1000}\right](5)(\cos^2 18)(\tan 18)} + \frac{\tan 26}{\tan 18}$$

$$\approx \mathbf{2.25}$$

b. Eq. (15.15):

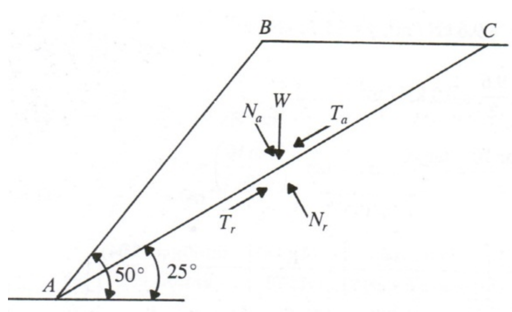
$$F_s = \frac{c'}{\gamma H \cos^2 \beta \tan \beta} + \frac{\tan \phi'}{\tan \beta}$$

$$1.75 = \frac{21}{\left[\frac{(1950)(9.81)}{1000}\right](H)(\cos^2 27)(\tan 27)} + \frac{\tan 26}{\tan 27}$$

$$1.75 = \frac{2.713}{H} + 0.957$$

$$H = \mathbf{3.42\ m}$$

15.6 Consider a 1-ft length of the wedge *ABC*



Eq. (15.29):

$$W = \frac{1}{2}H^2\gamma(\cot\theta - \cot\beta) = (0.5)(25^2)(115)(\cot 25 - \cot 50) = 46{,}913\text{ lb}$$

$$T_a = 46913\sin 25 = 19826.3\text{ lb}$$

$$N_a = 46913\cos 25 = 42517.6\text{ lb}$$

$$T_r = \frac{1}{F_s}\left(\overline{AC}\,c' + N_a\tan\phi'\right) = \frac{1}{F_s}\left[\left(\frac{25}{\sin 25}\right)(400) + (42517.6)\tan 25\right] = \frac{43488.3}{F_s}$$

Since $T_r = T_a$,

$$\frac{43488.3}{F_s} = 19826.3$$

Therefore, $F_s =$ **2.19**

15.7 Eq. (15.42):

$$H_{\text{cr}} = \frac{4c'}{\gamma}\left[\frac{\sin\beta\cos\phi'}{1-\cos(\beta-\phi')}\right] = \left[\frac{(4)(28)}{16.5}\right]\left[\frac{(\sin 58)(\cos 14)}{1-\cos(58-14)}\right] = \mathbf{19.9\text{ m}}$$

15.8 $F_s = 2.5$; $c'_d = \frac{c'}{F_s} = \frac{28}{2.5} = 11.2$ kN/m^2; $\phi'_d = \tan^{-1}\left(\frac{\tan 14}{2.5}\right) = 5.69°$

$$H = \frac{4c'_d}{\gamma}\left[\frac{\sin\beta\cos\phi'_d}{1-\cos(\beta-\phi'_d)}\right] = \left[\frac{(4)(11.2)}{16.5}\right]\left[\frac{(\sin 58)(\cos 5.69)}{1-\cos(58-5.69)}\right] = \mathbf{5.89\text{ m}}$$

15.9 $H = \frac{4c'_d}{\gamma}\left[\frac{\sin\beta\cos\phi'_d}{1-\cos(\beta-\phi'_d)}\right]$; $\gamma = 118$ lb/ft^3

$F_{s(\text{assumed})}$	$\phi'_d = \tan^{-1}\left(\frac{\tan 22}{F_s}\right)$ (deg)	$c'_d = \frac{c'}{F_s}$ (lb/ft^2)	β (deg)	H (ft)
2.0	11.42	350	45	49.27
2.5	9.18	280	45	35.02
3.0	7.67	233.3	45	27
2.75	8.36	254.5	45	30.54

$F_s \approx \mathbf{2.75}$

15.10 $\rho = 1800\ \text{kg/m}^3$; $\gamma = \frac{(1800)(9.81)}{1000} = 17.65\ \text{kN/m}^3$; $c' = 20\ \text{kN/m}^2$; $\phi' = 17°$

$$\beta = \tan^{-1}\left(\frac{1}{2}\right) = 26.57°;\ F_s = 2.3;\ c'_d = \frac{c'}{F_s} = \frac{20}{2.3} = 8.69\ \text{kN/m}^2$$

$$\phi'_d = \tan^{-1}\left(\frac{\tan\phi'}{F_s}\right) = \tan^{-1}\left(\frac{\tan 17}{2.3}\right) = 7.57°$$

$$H = \frac{4c'_d}{\gamma}\left[\frac{\sin\beta\cos\phi'_d}{1-\cos(\beta-\phi'_d)}\right] = \left[\frac{(4)(8.69)}{17.65}\right]\left[\frac{(\sin 26.57)(\cos 7.57)}{1-\cos(26.57-7.57)}\right] \approx \mathbf{16\ m}$$

15.11 $m \approx 0.185$ (From Figure 15.13). Eq. (15.48):

$$H_{\text{cr}} = \frac{c_u}{\gamma m} = \frac{26}{(18.5)(0.185)} = \mathbf{7.59\ m\text{ - -Toe circle}}$$

15.12 $m \approx 0.185$ for $\beta = 55°$ (Figure 15.13)

$$c_d = \frac{c_u}{F_s} = \frac{26}{2.5} = 10.4\ \text{kN/m}^2$$

$$H = \frac{c_d}{\gamma m} = \frac{10.4}{(18.5)(0.185)} = \mathbf{3.04\ m}$$

15.13 $\beta = \tan^{-1}\left[\frac{1}{2}\right] = 26.56°$. For $\beta = 26.56°$ and $D = 1.5$, $m = 0.16$ (Figure 15.13).

$$c_d = \frac{c_u}{F_s} = \frac{800}{2.5} = 320 \text{ lb/ft}^2$$

$$H = \frac{c_d}{\gamma m} = \frac{320}{(119)(0.16)} = \mathbf{16.8 \text{ ft}}$$

15.14 $H_{cr} = \frac{c_u}{\gamma m} = \frac{800}{(119)(0.16)} = \mathbf{42 \text{ ft.}}$ It is a **midpoint circle.**

15.15 a. $D = \frac{14}{10} = 1.4$; γ_{sat} = 17 kN/m^3. For β = 48° and D = 1.4, m = 0.18.

$H_{cr} = \frac{c_u}{\gamma m}$; c_u = (10)(17)(0.18) = **30.6 kN/m^2**

b. From Figure 15.13, **midpoint circle**

c. From Figure 15.15, $n \approx 0.85$.

Distance = nH = (0.85)(10) = **8.5 m**

15.16 a. $\beta = \tan^{-1}\left(\frac{1}{2}\right) = 26.57°$

$$\frac{F_s}{\tan\phi'} = \frac{1}{\tan 12} = 4.7$$

From Figure 15.25

$$\frac{c'}{\gamma H_{cr} \tan\phi'} \approx 0.2$$

Or,

$$H_{cr} = \frac{750}{(118)(0.2)(\tan 12)} = \mathbf{149.5 \text{ ft}}$$

b. $\frac{F_s}{\tan\phi'} = \frac{1}{\tan 18} = 3.07$. $\beta = \tan^{-1}\left(\frac{1}{1}\right) = 45°$.

Figure 15.25:

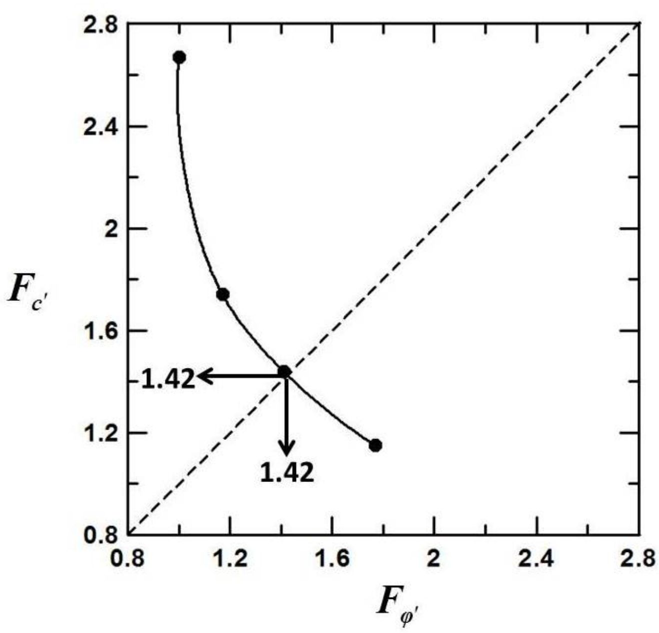
$$\frac{c'}{\gamma H_{\text{cr}} \tan \phi'} \approx 0.22; \quad \frac{30}{(17)(H_{\text{cr}})(\tan 18)} = 0.22$$

H_{cr} = **24.7 m**

15.17 $\beta = \tan^{-1}\left(\frac{1}{2.5}\right) = 21.8°$; $\phi' = 14°$; $c' = 500$ lb/ft^2; $\gamma = 120$ lb/ft^3.

ϕ'_d (deg)	$F_{\phi'} = \frac{\tan \phi'}{\tan \phi'_d}$	m	$c'_d = m\gamma H$ (lb/ft^2)	$F_{c'} = \frac{c'}{c'_d}$
8	1.77	0.06	432	1.15
10	1.41	0.048	346	1.44
12	1.17	0.04	288	1.74
14	1.00	0.026	187	2.67

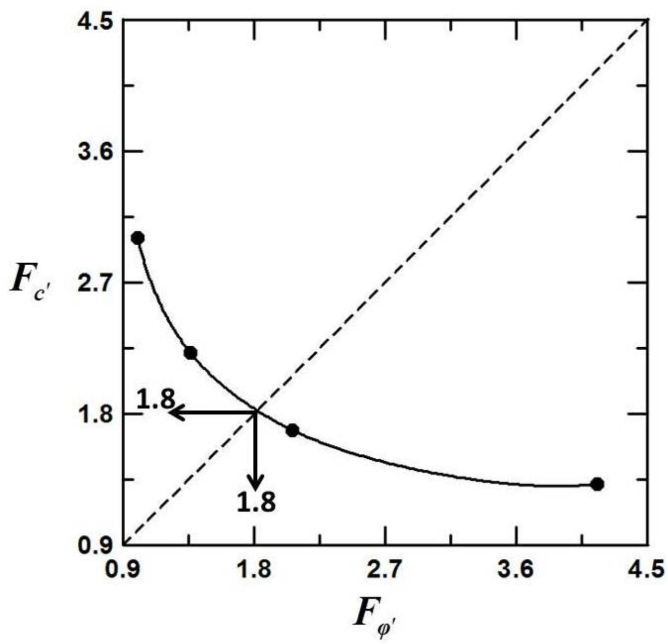
The plot of $F_{c'}$ vs. $F_{\phi'}$ is shown. From the figure, $F_{c'} = F_{\phi'} = F_s =$ **1.42**.



15.18 $n' = 1$; $\beta = \tan^{-1}\left(\frac{1}{1}\right) = 45°$; $\phi' = 20°$; $c' = 32$ kN/m^2; $\gamma = 18$ kN/m^3; $H = 10$ m

ϕ'_d (deg)	$F_{\phi'} = \frac{\tan\phi'}{\tan\phi'_d}$	m	$c'_d = m\gamma H$ (kN/m^2)	$F_{c'} = \frac{c'}{c'_d}$
5	4.16	0.134	24.12	1.32
10	2.06	0.105	18.9	1.69
15	1.36	0.08	14.4	2.22
20	1.00	0.059	10.62	3.01

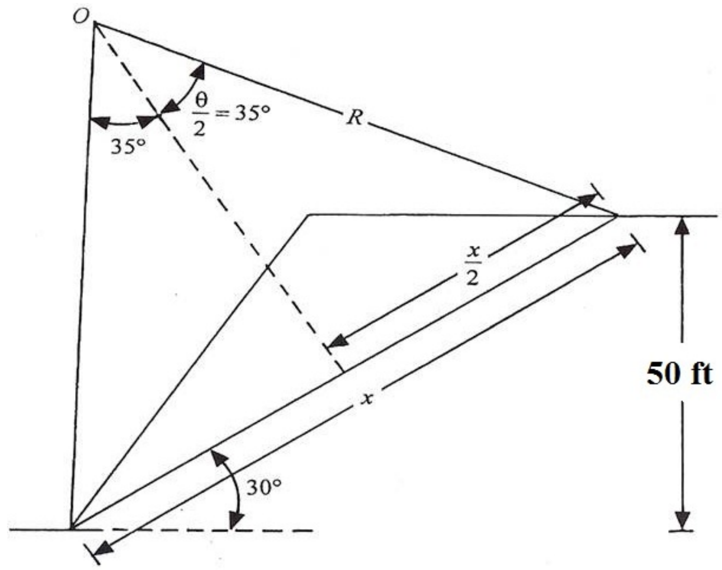
The plot of $F_{c'}$ vs. $F_{\phi'}$ is shown below. From the figure, $F_{\phi'} = F_{c'} = F_s =$ **1.8**.



15.19 For $\beta = 21.8°$ and $\frac{c'}{\gamma H \tan\phi'} = \frac{500}{(120)(60)\tan 14} = 0.278$, the value of $\frac{\tan\phi}{F_s}$ obtained from Figure (15.27) is about 0.15. Therefore,

$$F_s = \frac{\tan 14}{0.15} = \mathbf{1.66}$$

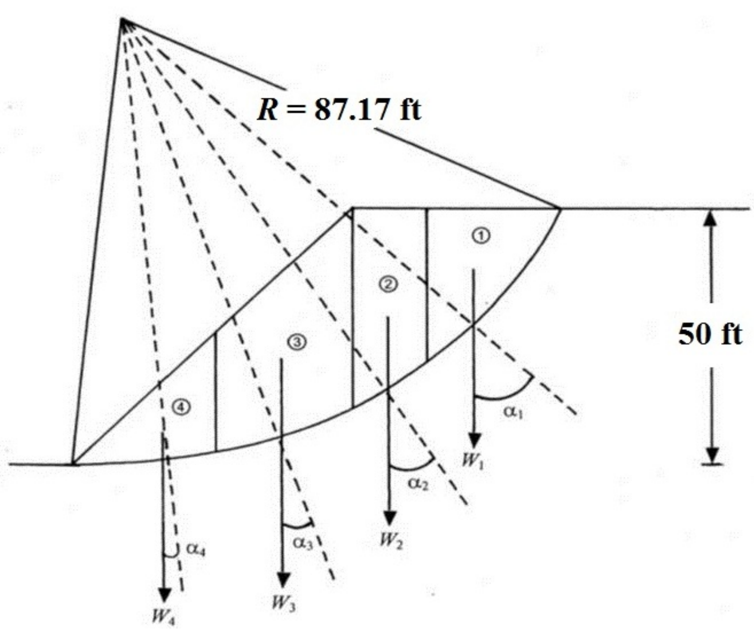
15.20 a. Refer to the figure.



$$\frac{50}{x} = \sin 30° ; x = 100 \text{ ft}$$

$$\frac{50}{\text{Radius}, R} = \sin 35° ; \; R = \frac{50}{\sin 35} = 87.17 \text{ ft}$$

With radius R = 87.17 ft, the trial surface circle has been drawn.



Now the following table can be prepared.

Slice	Area of slices (ft^2)	Weight of slice $W_n = A \times \gamma$ (kip/ft)	α_n (deg)	$W_n \cos \alpha_n$ (kip/ft)	$W_n \sin \alpha_n$ (kip/ft)
1	$\frac{(30)(23)}{2} = 345$	41.75	47	28.47	30.52
2	$\frac{(13)(30+41)}{2} = 462$	55.9	32	47.4	29.62
3	$\frac{(25)(41+25)}{2} = 825$	99.82	20	93.8	34.14
4	$\frac{(25)(25)}{2} = 312$	37.75	5	37.6	3.29
				$\sum 207.27$	$\sum 97.57$

$$F_s = \frac{R\theta c' + (\sum W_n \cos \alpha_n) \tan \phi'}{\sum W_n \sin \alpha_n}$$

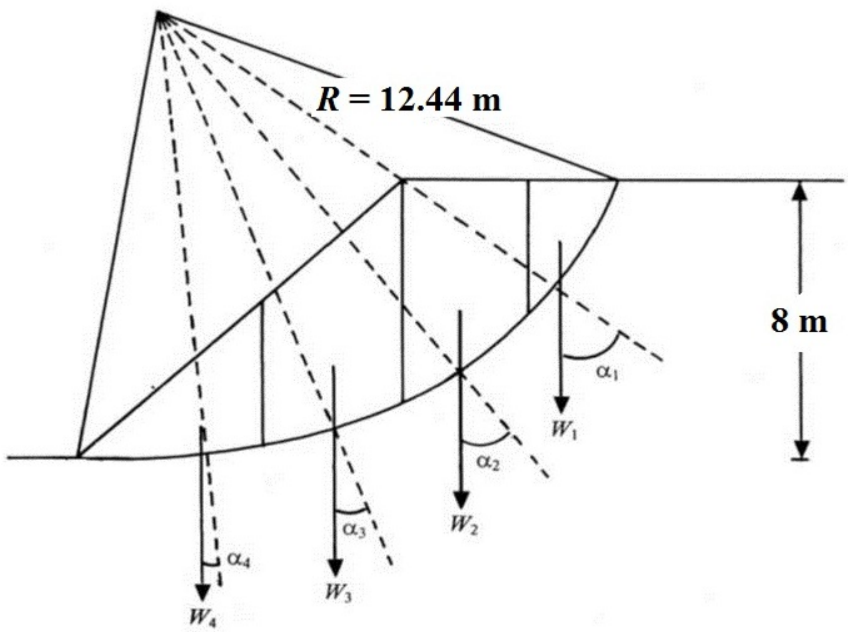
$$= \frac{(87.17)\left[\left(\frac{\pi}{180}\right)(70)\right](0.65) + (207.27)(\tan 18)}{97.57} = \mathbf{1.4}$$

Note: The accuracy can be increased by increasing the number of slices.

b. As in Part a, $\frac{H}{x} = \sin \alpha$; $x = \frac{H}{\sin \alpha} = \frac{8}{\sin 30} = 16 \text{ m}$

$$\frac{\left(\frac{x}{2}\right)}{R} = \sin \frac{\theta}{2}, \text{ or } \frac{8}{\sin 40} = R = 12.44 \text{ m}$$

With a radius $R = 12.44$ m, the trial surface has been drawn on the next page.



The following table can now be prepared.

Slice	Area of slices (m^2)	Weight of slice $W_n = A \times \gamma$ (kN/m)	α_n (deg)	$W_n \cos \alpha_n$ (kN/m)	$W_n \sin \alpha_n$ (kN/m)
1	$\frac{(2.36)(3.85)}{2} = 4.54$	77.18	54	45.36	62.44
2	$\frac{(3.85+6.51)(3.25)}{2} = 16.83$	286.11	38	225.45	176.14
3	$\frac{(6.51+4.14)(3.55)}{2} = 18.9$	321.3	20	301.92	109.89
4	$\frac{(4.14)(4.74)}{2} = 9.81$	166.77	6	165.85	17.43
				∑738.58	∑365.9

$$F_s = \frac{R\theta c' + (\sum W_n \cos \alpha_n) \tan \phi'}{\sum W_n \sin \alpha_n}$$

$$= \frac{(12.44)\left[\left(\frac{\pi}{180}\right)(80)\right](27) + (738.58)(\tan 20)}{365.9} = \mathbf{2.01}$$

Note: The accuracy will improve with smaller slices.

15.21 $\phi' = 25^\circ$; $\beta = 26.56^\circ$; $r_u = 0.5$; $\dfrac{c'}{\gamma H} = \dfrac{20}{(19)(14)} \approx 0.075$.

Using Table 15.3, the following table can be prepared.

D	m'	n'	$F_s = m' - n'r_u$
Toe circle	1.853	1.430	1.138
1.00	1.872	1.386	1.179
1.25	2.004	1.641	1.183
1.50	2.308	1.914	1.351

$F_s \approx$ **1.14**.

15.22 $\phi' = 20^\circ$; $\beta = 18.43^\circ$; $r_u = 0.5$; $\dfrac{c'}{\gamma H} = \dfrac{475}{(118)(40)} = 0.1$

Using Table 15.3, the following table can be prepared.

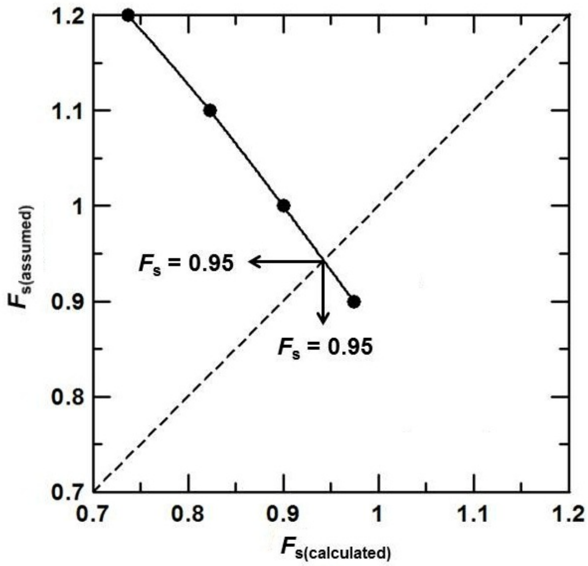
D	m'	n'	$F_s = m' - n'r_u$
Toe circle	2.286	1.588	1.49
1.00	2.421	1.472	1.68
1.25	2.283	1.558	1.50
1.50	2.387	1.742	1.51

$F_s \approx$ **1.49**.

15.23 $\beta = 26^\circ$; $\phi' = 21^\circ$; $r_u = 0.5$; $\gamma = 19$ kN/m^3; $c' = 21$ kN/m^2; $H = 17$ m

$F_{s(\text{assumed})}$	$\dfrac{c'}{\gamma H F_s}$	ϕ'_d (deg)	$F_{s(\text{calculated})} = \dfrac{\tan\phi'}{\tan\phi'_d}$
1.2	0.0541	27.5	0.737
1.1	0.0591	25	0.823
1.0	0.065	23	0.9
0.9	0.0722	21.5	0.974

From the plot on the next page, $F_s \approx$ **0.95**.



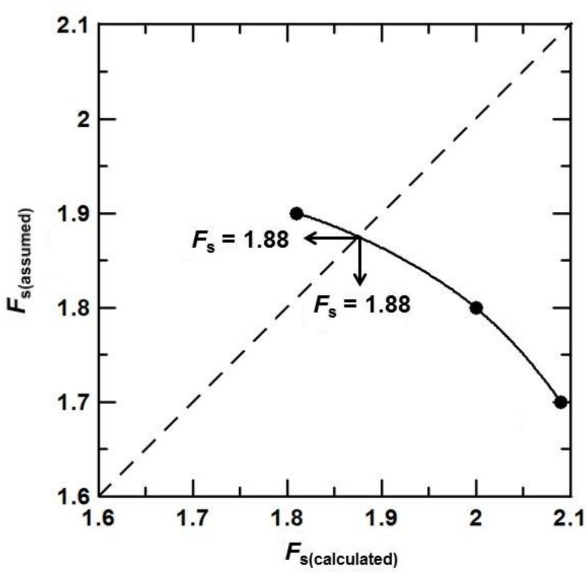
15.24 $\beta = \tan^{-1}\left(\frac{1}{3}\right) = 18.43°$; $\phi' = 24°$; $c' = 27$ kN/m^2; $\gamma = 17.5$ kN/m^3; $r_u = 0.25$; $H = 18$ m

a. Spencer's solution (Figure 15.35):

$F_{s(\text{assumed})}$	$\frac{c'}{\gamma H F_s}$	ϕ'_d (deg)	$F_{s(\text{calculated})} = \frac{\tan\phi'}{\tan\phi'_d}$
1.7	0.0504	12	2.09
1.8	0.0476	12.5	2.0
1.9	0.045	13.8	1.81

From the graph,

$F_s \approx$ **1.88**.



b. Michalowski's solution (Figure 15.36)

$\beta = \tan^{-1}\left(\frac{1}{3}\right) = 18.43°$; $\phi' = 24°$; $c' = 27$ kN/m^2; $\gamma = 17.5$ kN/m^3; $r_u = 0.25$;

$H = 18$ m

$$\frac{c'}{\gamma H \tan\phi'} = \frac{27}{(17.5)(18)\tan 24} 0.192$$

Figure (15.36): $\frac{F_s}{\tan 24} \approx 4.5$; Therefore, $F_s \approx \mathbf{2.0}$

Chapter 16

16.1 $\phi' = 32^\circ$; $N_c = 44.14$; $N_q = 28.52$; $N_\gamma = 26.87$ (Table 16.1)

$$q_{\text{all}} = \frac{q_u}{F_s} = \frac{1}{3}\left(c'N_c + qN_q + \frac{1}{2}\gamma BN_\gamma\right)$$

$$= \frac{1}{3}\left[(21)(44.14) + (1)(17.5)(28.52) + \frac{1}{2}(17.5)(1.5)(26.87)\right]$$

$$= \mathbf{593\ kN/m^2}$$

16.2 $\phi' = 24^\circ$; $N_c = 23.36$; $N_q = 11.40$; $N_\gamma = 7.08$ (Table 16.1)

$$q_{\text{all}} = \frac{q_u}{F_s} = \frac{1}{4}\left(c'N_c + qN_q + \frac{1}{2}\gamma BN_\gamma\right)$$

$$= \frac{1}{4}\left[(1500)(23.36) + (118)(4)(11.40) + \frac{1}{2}(6)(118)(7.08)\right]$$

$$= \mathbf{10{,}732\ lb/ft^2}$$

16.3 $\phi' = 0^\circ$; $N_c = 5.7$; $N_q = 1$; $N_\gamma = 0$ (Table 16.1)

$$q_{\text{all}} = \frac{q_u}{F_s} = \frac{1}{6}\left(c_u N_c + qN_q\right) = \frac{1}{6}[(37)(5.7) + (19.5)(0.75)(1)] = \mathbf{37.58\ kN/m^2}$$

16.4 For a continuous foundation with vertical loading, all inclination factors and shape factors are equal to one. So,

$$q_{\text{all}} = \frac{q_u}{F_s} = \frac{1}{F_s}\left(c'N_c\lambda_{cd} + qN_q\lambda_{qd} + \frac{1}{2}\gamma BN_\gamma\lambda_{\gamma d}\right)$$

$\phi' = 32^\circ$; $N_c = 35.49$; $N_q = 23.18$; $N_\gamma = 30.22$ (Table 16.2)

$$\lambda_{cd} = 1 + 0.4\left(\frac{D_f}{B}\right) = 1 + 0.4\left(\frac{1}{1.5}\right) = 1.266$$

$$\lambda_{qd} = 1 + 2\tan\phi'(1-\sin\phi')^2 \frac{D_f}{B} = 1 + 2\tan 32(1-\sin 32)^2\left(\frac{1}{1.5}\right) = 1.184$$

$$\lambda_{\gamma d} = 1$$

$$q_{\text{all}} = \frac{1}{3}\begin{bmatrix}(21)(35.49)(1.266) + (17.5)(1)(23.18)(1.184) \\ +\frac{1}{2}(17.5)(1.5)(30.22)(1)\end{bmatrix} = \mathbf{606.8\ kN/m^2}$$

16.5 $\phi' = 24^\circ$; $N_c = 19.32$; $N_q = 9.60$; $N_\gamma = 9.44$

$$\lambda_{cd} = 1 + 0.4\left(\frac{4}{6}\right) = 1.266$$

$$\lambda_{qd} = 1 + 2\tan\phi'(1-\sin\phi')^2 \frac{D_f}{B} = 1 + 2\tan 24(1-\sin 24)^2\left(\frac{4}{6}\right) = 1.209$$

$$\lambda_{\gamma d} = 1$$

$$q_{\text{all}} = \frac{1}{4}\begin{bmatrix}(1500)(19.32)(1.266) + (4)(118)(9.60)(1.209) \\ +\frac{1}{2}(118)(6)(9.44)(1)\end{bmatrix} = \mathbf{11{,}377\ lb/ft^2}$$

16.6 $$q_{\text{all}} = \frac{1}{F_s}\left(c'N_c\lambda_{cd} + qN_q\lambda_{qd} + \frac{1}{2}\gamma BN_\gamma\lambda_{\gamma d}\right)$$

$\phi' = 0^\circ$; $N_c = 5.14$; $N_q = 1.0$; $N_\gamma = 0$

$$\lambda_{cd} = 1 + 0.4\left(\frac{D_f}{B}\right) = 1 + 0.4\left(\frac{0.75}{2.5}\right) = 1.12$$

$$\lambda_{qd} = 1 + 2\tan\phi'(1-\sin\phi')^2 \frac{D_f}{B} = 1$$

$$\lambda_{\gamma d} = 1$$

$$q_{\text{all}} = \frac{1}{6}[(37)(5.14)(1.12) + (0.75)(19.5) + 0] = \mathbf{37.94\ kN/m^2}$$

16.7 Eq. (16.12): $q_u = 1.3c'N_c + qN_q + 0.4\gamma' BN_\gamma$

$\phi' = 32°$; $N_c = 44.04$; $N_q = 28.52$; $N_\gamma = 26.87$

$q = \gamma h + \gamma'(D_f - h) = (16 \times 0.9) + (18.9 - 9.81)(1.2 - 0.9) = 17.12 \text{ kN/m}^2$

$$Q_{\text{all}} = \frac{q_u B^2}{F_s} = \frac{B^2}{F_s}(1.3c'N_c + qN_q + 0.4\gamma' BN_\gamma)$$

$$Q_{\text{all}} = \frac{1.75^2}{3.5}\left[(1.3)(17)(44.04) + (17.12)(28.52) + (0.4)(18.9 - 9.81)(1.75)(26.87)\right]$$

$$= \mathbf{1428\ kN}$$

16.8 $q = \gamma D_f = (16 \times 1.2) = 19.2 \text{ kN/m}^2$

$D = h - D_f = (1.2 + 0.5) - 1.2 = 0.5 \text{ m}$

Eq. (16.7):

$$\gamma_{av} = \frac{1}{B}\left[\gamma D + \gamma'(B - D)\right] = \frac{1}{1.75}\left[(16)(0.5) + (18.9 - 9.81)(1.75 - 0.5)\right] = 11.06 \text{ kN/m}^3$$

$$F_s = \frac{q_u B^2}{Q_{\text{all}}} = \frac{B^2}{Q_{\text{all}}}(1.3c'N_c + qN_q + 0.4\gamma' BN_\gamma)$$

$$F_s = \frac{1.75^2}{1428}[(1.3)(17)(44.04) + (17.12)(28.52) + (0.4)(11.06)(1.75)(26.87)] = \mathbf{3.58}$$

16.9 $\phi' = 22°$. From Table 16.1, $N_c = 20.27$; $N_q = 9.19$; $N_\gamma = 5.09$

$$\gamma = \frac{(1750)(9.81)}{1000} = 17.16 \text{ kN/m}^3$$

$$q_u = 1.3c'N_c + qN_q + 0.4\gamma_{\text{av}} BN_\gamma$$

$q = \gamma D_f = (1.5)(17.16) = 25.74 \text{ kN/m}^2$

$$\gamma_{\text{av}} = \frac{1}{B}[\gamma D + \gamma'(B - D)]$$

$D = h - D_f = 2.5 - 1.5 = 1 \text{ m}$

$$\gamma_{\text{sat}} = \frac{(1950)(9.81)}{1000} = 19.13\ \text{kN/m}^3$$

$$\gamma_{\text{av}} = \frac{1}{2}[(17.16)(1) + (19.13 - 9.81)(2 - 1)] = 13.24\ \text{kN/m}^3$$

$$q_u = (1.3)(28)(20.27) + (25.74)(9.19) + (0.4)(13.24)(2)(5.09)$$

$$= 1028.3\ \text{kN/m}^2$$

$$Q_{\text{all}} = \frac{(q_u)B^2}{F_s} = \frac{(1028.3)(2)^2}{3.5} = \mathbf{1175\ kN}$$

16.10 From Eq. (16.12): $q_{\text{all}} = \frac{1}{F_s}(1.3c'N_c + qN_q + 0.4\gamma BN_\gamma)$

$\phi' = 29^\circ$; $N_c = 34.24$; $N_q = 19.98$; $N_\gamma = 16.18$ (Table 16.1)

$$q_{\text{all}} = \frac{1}{4}[(1.3)(900)(34.24) + (4.5)(116)(19.98) + (0.4)(116)(B)(16.18)]$$

$$= 12622.6 + 187.7B \qquad \text{(a)}$$

$$q_{\text{all}} = \frac{250000}{B^2} \qquad \text{(b)}$$

From Eqs. (a) and (b), $\frac{250{,}000}{B^2} = 12{,}622.6 + 187.7B$. By trial and error,

$B \approx$ **4.31 ft**

16.11 $\phi' = 25^\circ$. From Table 16.1, $N_q = 12.72$; $N_\gamma = 8.34$.

$$q_{\text{all}} = \frac{1}{2.5}[(2.1 \times 19)(12.72) + (0.4)(19)(B)(8.34)] = 203 + 25.35B$$

$$q_{\text{all}} = \frac{550}{B^2} = 203 + 25.35B$$

$B \approx$ **1.5 m**

16.12 $$q_{\text{all}} = \frac{q_u}{F_s} = \frac{1}{F_s}\left(c'N_c\lambda_{cs}\lambda_{cd} + qN_q\lambda_{qs}\lambda_{qd} + \frac{1}{2}\gamma BN_\gamma\lambda_{\gamma s}\lambda_{\gamma d}\right)$$

$\phi' = 32^\circ$; $N_c = 35.49$; $N_q = 23.18$; $N_\gamma = 30.22$ (Table 16.2)

$$\lambda_{cs} = 1 + \left(\frac{B}{L}\right)\left(\frac{N_q}{N_c}\right) = 1 + \left(\frac{23.18}{35.49}\right) = 1.653$$

$$\lambda_{cd} = 1 + 0.4\left(\frac{D_f}{B}\right) = 1 + 0.4\left(\frac{1.2}{1.75}\right) = 1.274$$

$$\lambda_{qs} = 1 + \left(\frac{B}{L}\right)\tan\phi' = 1 + \tan 32 = 1.624$$

$$\lambda_{qd} = 1 + 2\tan\phi'(1-\sin\phi')^2\,\frac{D_f}{B} = 1 + 2\tan 32(1-\sin 32)^2\left(\frac{1}{1.75}\right) = 1.157$$

$$\lambda_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right) = 0.6$$

$$\lambda_{\gamma d} = 1$$

$q = \gamma h + \gamma'(D_f - h) = (16 \times 0.9) + (18.9 - 9.81)(1.2 - 0.9) = 17.12$ kN/m^2

$$Q_{\text{all}} = \frac{(1.75)^2}{3.5}\left[\begin{array}{c}(17)(35.49)(1.653)(1.274) + (17.12)(23.18)(1.624)(1.157) \\ + \frac{1}{2}(18.9 - 9.81)(1.75)(30.22)(0.6)(1)\end{array}\right]$$

$\approx$ **1890 kN/m^2**

16.13 a. For vertical load, Eq. (16.44): $q_u = qN_q\lambda_{qd}\lambda_{qs} + \frac{1}{2}\gamma B'N_\gamma\lambda_{\gamma d}\lambda_{\gamma s}$

$c' = 0$, $\phi' = 31^\circ$. Table 16.2: $N_q = 20.63$; $N_\gamma = 25.99$.

$B' = B - 2x = 2.5 - (2)(0.2) = 2.1$ m; $L' = 2.5$ m

$$\lambda_{qs} = 1 + \left(\frac{B'}{L'}\right)\tan\phi' = 1 + \left(\frac{2.1}{2.5}\right)\tan 31 = 1.504$$

$$\lambda_{\gamma s} = 1 - 0.4\left(\frac{B'}{L'}\right) = 1 - 0.4\left(\frac{2.1}{2.5}\right) = 0.664$$

$$\lambda_{qd} = 1 + 2\tan\phi'(1-\sin\phi')^2 \frac{D_f}{B'} = 1 + 2\tan 31(1-\sin 31)^2\left(\frac{1}{2.1}\right) = 1.134$$

$$\lambda_{\gamma d} = 1$$

$$q_u = (19)(1)(1.504)(1.134)(20.63) + \frac{1}{2}(19)(2.1)(25.99)(1)(0.664)$$

$$= 1012.8 \text{ kN/m}^2$$

$$Q_{\text{all}} = \frac{q_u B'L'}{F_s} = \frac{(1012.8)(2.1)(2.5)}{(5)} = \mathbf{1063.4\,kN}$$

b. $B' = 6 - (2)(0.9) = 4.2$ ft; $L' = 6$ ft. $\phi' = 26^\circ$

$N_c = 22.25$; $N_q = 11.85$; $N_\gamma = 12.54$

$$q_u = c'N_c\lambda_{cs}\lambda_{cd} + qN_q\lambda_{qs}\lambda_{qd} + \frac{1}{2}\gamma B'N_\gamma\lambda_{\gamma s}\lambda_{\gamma d}$$

$$\lambda_{cs} = 1 + \left(\frac{B'}{L'}\right)\left(\frac{N_q}{N_c}\right) = 1 + \left(\frac{4.2}{6}\right)\left(\frac{11.85}{22.25}\right) = 1.373$$

$$\lambda_{qs} = 1 + \left(\frac{B'}{L'}\right)\tan\phi' = 1 + \left(\frac{4.2}{6}\right)\tan 26 = 1.341$$

$$\lambda_{\gamma s} = 1 - 0.4\left(\frac{B'}{L'}\right) = 1 - 0.4\left(\frac{4.2}{6}\right) = 0.72$$

$$\lambda_{cd} = 1 + 0.4\tan^{-1}\left(\frac{D_f}{B'}\right) = 1 + 0.4\tan^{-1}\left(\frac{4}{4.2}\right) = 1.006$$

$$\lambda_{qd} = 1 + 2\tan\phi'(1-\sin\phi')^2 \frac{D_f}{B'} = 1 + 2\tan 26(1-\sin 26)^2\left(\frac{4}{4.2}\right) = 1.293$$

$$\lambda_{\gamma d} = 1$$

$$q_u = (900)(22.25)(1.373)(1.006) + (115)(4)(11.85)(1.341)(1.293)$$

$$+(0.5)(115)(4.2)(12.54)(0.72)(1)$$

$$= 39{,}291 \text{ lb/ft}^2 = 39.29 \text{ kip/ft}^2$$

$$Q_{\text{all}} = \frac{(39.29)(4.2)(6)}{5} = \mathbf{198\ kip}$$

c. $\phi' = 38^\circ$; $c' = 0$. $N_q = 48.93$; $N_\gamma = 78.03$.

$B' = 1.5 - (2)(0.1) = 1.3$ m; $L' = 1.5$ m

$$\gamma = \frac{(1800)(9.81)}{1000} = 17.66\ \text{kN/m}^3$$

For vertical load, Eq. (16.44): $q_u = qN_q \lambda_{qd} \lambda_{qs} + \frac{1}{2}\gamma B' N_\gamma \lambda_{\gamma d} \lambda_{\gamma s}$

$$\lambda_{qs} = 1 + \left(\frac{B'}{L'}\right)\tan\phi' = 1 + \left(\frac{1.3}{1.5}\right)\tan 38 = 1.677$$

$$\lambda_{\gamma s} = 1 - 0.4\left(\frac{B'}{L'}\right) = 1 - 0.4\left(\frac{1.3}{1.5}\right) = 0.653$$

$$\lambda_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2 \frac{D_f}{B'} = 1 + 2\tan 38(1 - \sin 38)^2\left(\frac{1.5}{1.3}\right) = 1.266$$

$$\lambda_{\gamma d} = 1$$

$$q_u = (17.66)(1.5)(1.266)(1.677)(48.93) + \frac{1}{2}(17.66)(1.3)(78.03)(1)(0.653)$$

$$= 3336.7\ \text{kN/m}^2$$

$$Q_{\text{all}} = \frac{q_u B' L'}{F_s} = \frac{(3336.7)(1.3)(1.5)}{(5)} = \mathbf{1301.3\ kN}$$

16.14 Eq. (16.57): $q_{u(F)} = q_{u(P)}\left(\frac{B_F}{B_P}\right) = (6800)\left(\frac{5}{2}\right) = 170{,}000\ \text{lb/ft}^2$

$$Q_{\text{all}} = \frac{Aq_{u(F)}}{5} = \left(\frac{170{,}000}{5}\right)(5)^2 = 85{,}000\ \text{lb} = \mathbf{85\ kip}$$

16.15 $q_{u(P)} = 320\ \text{kN/m}^2$. Eq. (16.56): $q_{u(F)} = q_{u(P)}$; $q_{u(F)} = 320\ \text{kN/m}^2$

$$Q_{\text{all}} = \frac{Aq_{u(F)}}{F_s} = \frac{\left(\frac{\pi}{4}\right)(2.5)^2(320)}{4} = \mathbf{392.7\ kN}$$

CRITICAL THINKING PROBLEM

C.16.1 The footing is placed at a depth of 1.5 m.

Part (a)

<u>B = 1 m</u>

Eq. (16.49): $q_{\text{net}} = \frac{N_{60}}{0.05} F_d \left[\frac{S_e(\text{mm})}{25}\right]$

2 × B = 2 × 1 = 2 m

N_{60} should be averaged up to a distance of 2 m below the foundation or up to a depth = 1.5 + 2 = 3.5 m

Therefore, $N_{60\text{-avg}}$ = (12 + 7)/2 ≈ 10; S_e = 20 mm

$F_d = 1 + 0.33\left(\frac{D_f}{B}\right) = 1 + 0.33\left(\frac{1.5}{1}\right) = 1.495 \leq 1.33$ So, F_d = 1.33

$$q_{\text{net}} = \frac{10}{0.05}(1.33)\left[\frac{20}{25}\right] = 212.8\ \text{kN/m}^2$$

$$Q_{\text{all-net}} = \frac{(q_{\text{net}})(A)}{3} = \frac{(212.8)(1)^2}{3} \approx 71\ \text{kN}$$

<u>B = 1.5 m</u>

Eq. (16.50): $q_{\text{net}} = \frac{N_{60}}{0.08}\left(\frac{B+0.3}{B}\right)^2 F_d \left[\frac{S_e(\text{mm})}{25}\right]$

2 × B = 2 × 1.5 = 3 m

N_{60} should be averaged up to a distance of 3 m below the foundation or up to a depth = 1.5 + 3 = 4.5 m

Therefore, $N_{60\text{-avg}} = (12 + 7)/2 \approx 10$; $S_e = 20$ mm

$$F_d = 1 + 0.33\left(\frac{D_f}{B}\right) = 1 + 0.33\left(\frac{1.5}{1.5}\right) \approx 1.33$$

$$q_{\text{net}} = \frac{8}{0.08}\left(\frac{1.5+0.3}{1.5}\right)^2 (1.33)\left[\frac{20}{25}\right] = 153.2 \text{ kN/m}^2$$

$$Q_{\text{all-net}} = \frac{(q_{\text{net}})(A)}{3} = \frac{(153.2)(1.5)^2}{3} \approx 115 \text{ kN}$$

<u>$B = 2$ m</u>

Eq. (16.50): $$q_{\text{net}} = \frac{N_{60}}{0.08}\left(\frac{B+0.3}{B}\right)^2 F_d\left[\frac{S_e(\text{mm})}{25}\right]$$

$2 \times B = 2 \times 2 = 4$ m

N_{60} should be averaged up to a distance of 4 m below the foundation or up to a depth = 1.5 + 4 = 5.5 m

Therefore, $N_{60\text{-avg}} = (12 + 7 + 8)/3 = 9$; $S_e = 20$ mm

$$F_d = 1 + 0.33\left(\frac{D_f}{B}\right) = 1 + 0.33\left(\frac{1.5}{2}\right) = 1.247$$

$$q_{\text{net}} = \frac{8}{0.08}\left(\frac{2+0.3}{2}\right)^2 (1.247)\left[\frac{20}{25}\right] = 114.7 \text{ kN/m}^2$$

$$Q_{\text{all-net}} = \frac{(q_{\text{net}})(A)}{3} = \frac{(114.7)(2)^2}{3} \approx 229 \text{ kN}$$

<u>$B = 3$ m</u>

Eq. (16.50): $$q_{\text{net}} = \frac{N_{60}}{0.08}\left(\frac{B+0.3}{B}\right)^2 F_d\left[\frac{S_e(\text{mm})}{25}\right]$$

$2 \times B = 2 \times 3 = 6$ m

N_{60} should be averaged up to a distance of 6 m below the foundation or up to a depth = 1.5 + 6 = 7.5 m

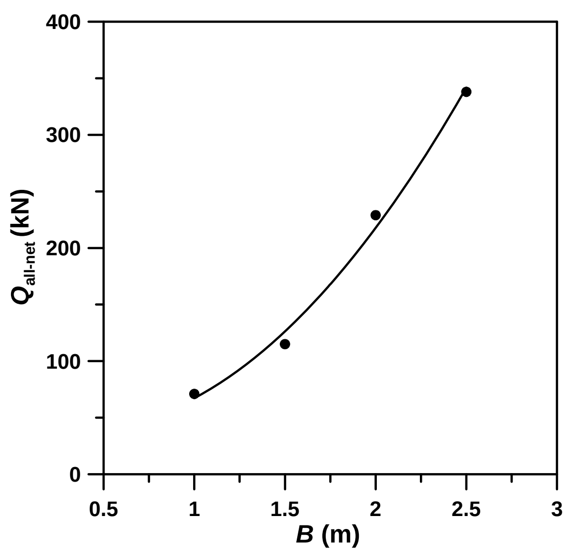
Therefore, $N_{60\text{-avg}} = (12 + 7 + 8 + 19)/4 \approx 12$; $S_e = 20$ mm

$$F_d = 1 + 0.33\left(\frac{D_f}{B}\right) = 1 + 0.33\left(\frac{1.5}{3}\right) = 1.165$$

$$q_{\text{net}} = \frac{8}{0.08}\left(\frac{3+0.3}{3}\right)^2 (1.165)\left[\frac{20}{25}\right] = 112.77 \text{ kN/m}^2$$

$$Q_{\text{all-net}} = \frac{(q_{\text{net}})(A)}{3} = \frac{(112.77)(3)^2}{3} \approx 338 \text{ kN}$$

The design chart is shown below.



Part (b)

For any design footing size *B*,

$Q_{\text{all-net}} \geq Q_{\text{applied}}$ ………………………………………………………….. (1)

$Q_{\text{applied}} = 250$ kN

From the chart it is found that condition (1) is satisfied when $B \approx 2.25$ m

Part (c)

$\phi' = 33^{\circ}$; $c' = 0$; $N_q = 32.33$; $N_\gamma = 31.94$ (Table 16.1); $B = 2.25$ m

$$q_{\text{all-net}} = \frac{q_{\text{u-net}}}{F_s} = \frac{q_u - q}{3} = \frac{1}{3}\left(1.3c'N_c + qN_q + 0.4\gamma BN_\gamma - q\right)$$

$$= \frac{1}{3}\left[(1.5)(17)(32.33) + 0.4(17)(2.25)(31.94) - (1.5)(17)\right]$$

$$= 429.2 \text{ kN/m}^2$$

$$Q_{\text{all-net}} = q_{\text{all-net}}(B)^2 = (429.2)(2.25)^2 \approx \mathbf{2173\ kN}$$

Part (d)

Net allowable column load calculated by the Terzaghi's bearing capacity equation (Part c) is significantly higher than that calculated by the method based on N_{60} and limting settlement value (Part b). In actual design of foundations, both bearing capacity (based on shear strength) and settlement criteria need to be satisfied. It is possible that the allowable column load calculated in Part (c) will change (decrease) if a settlement limit is imposed. Considering the uncertainty in sampling and testing of granular materials for the determination of shear strength, the method based on N_{60} and settlement criteria may be preferable for these conditions.

Chapter 17

17.1 Shelby tube *A*:

Eq. (17.6): $$A_R(\%) = \frac{D_o^2 - D_i^2}{D_i^2}(100) = \frac{(76.2)^2 - (73)^2}{(73)^2} \times 100 = \mathbf{8.96\% \leq 10\%}$$

Samples can be considered undisturbed. Suitable for grain size distribution, Atterberg limits, consolidation and unconfined compression tests.

Shelby tube *B*:

Eq. (17.6): $$A_R(\%) = \frac{D_o^2 - D_i^2}{D_i^2}(100) = \frac{(3.5)^2 - (3.375)^2}{(3.375)^2} \times 100 = \mathbf{7.54\% \leq 10\%}$$

Samples can be considered undisturbed. Suitable for grain size distribution, Atterberg limits, consolidation and unconfined compression tests.

Split spoon sampler:

Eq. (17.6): $$A_R(\%) = \frac{D_o^2 - D_i^2}{D_i^2}(100) = \frac{(50.8)^2 - (35)^2}{(35)^2} \times 100 = \mathbf{110\% \geq 10\%}$$

Samples can be considered as highly disturbed. Suitable for grain size distribution and Atterberg limits tests, but not for consolidation and unconfined compression tests.

17.2

Depth (m)	σ'_0 (kN/m^2)	$C_N = \left[\frac{\sigma'_0}{p_a}\right]^{-0.5}$	N_{60}	$(N_1)_{60} = C_N N_{60}$
2	34	1.71	7	**≈12**
4	68	1.21	10	**≈12**
6	102	0.99	11	**≈11**
8	136	0.857	14	**≈12**
10	170	0.767	9	**≈7**

17.3 $$\phi' = \tan^{-1}\left[\frac{N_{60}}{12.2 + 20.3\left(\frac{\sigma'_o}{p_a}\right)}\right]; \; p_a \approx 100 \text{ kN/m}^2$$

Depth (m)	σ'_o (kN/m^2)	p_o (kN/m^2)	N_{60}	ϕ' (deg) [Eq. (17.20)]
2	34	100	7	20.1
4	68	100	10	21.0
6	102	100	11	18.48
8	136	100	14	19.37
10	170	100	9	10.9

Average $\phi' \approx$ **18°**

17.4 Eq. (17.18): $$D_r\ (\%) = \left[\frac{N_{60}\left(0.23 + \frac{0.06}{D_{50}}\right)^{1.7}}{9}\left(\frac{98}{\sigma'_o}\right)\right]^{0.5}(100)$$

Given $\gamma = 15.7$ kN/m^3. The following table can now be prepared.

Depth z (m)	$\sigma'_o = \gamma z$ (kN/m^2)	D_{50} (mm)	N_{60}	D_r (%)
1.5	23.55	0.3	9	**99.5 ≈ 100**
3.0	47.1	0.3	10	**74.2 ≈ 74**
4.5	70.65	0.3	14	**71.7 ≈ 72**
6.0	94.2	0.3	18	**70.4 ≈ 70**
7.5	117.75	0.3	20	**66.4 ≈ 66**

17.5 $$(N_1)_{60} = C_N N_{60} = \left(\frac{\sigma'_0}{p_a}\right)^{-0.5} N_{60}\ ;\quad \phi' = 27.1 + 0.3(N_1)_{60} - 0.00054(N_1)^2_{60}$$

Depth (m)	σ'_o (kN/m^2)	N_{60}	C_N	$(N_1)_{60}$	ϕ' (deg)
1.5	$1.5 \times 18 = 27$	8	1.924	$15.4 \approx 15$	31.4
3	$3 \times 18 = 54$	9	1.36	$12.24 \approx 12$	30.6
4.5	$4.5 \times 18 = 81$	11	1.11	$12.2 \approx 12$	30.6
6	$\left[\begin{array}{l}(5.3 \times 18)\\ + 0.7(18.8 - 9.8)\end{array}\right] = 101.7$	12	0.99	$11.89 \approx 12$	30.6
7.5	$101.7 + 1.5(18.8 - 9.8) = 115.2$	15	0.931	$13.96 \approx 14$	31.2
9	$115.2 + 1.5(18.8 - 9.8) = 128.7$	17	0.881	$14.97 \approx 15$	31.4

Average $\phi' \approx$ **31°**

17.6 a. The properties should be averaged over a distance of 2*B* or 4 m below the footing, i.e. up to a depth of 5.5 m.

Therefore, design N_{60} = (8 + 9 + 11) / 3 = **9.33 ≈ 9**

and design ϕ' (deg) = (31.4 + 30.6 + 30.6) / 3 = **30.8**

b. $F_d = 1 + 0.33\left(\frac{D_f}{B}\right) = 1 + (0.33)\left(\frac{1.5}{2}\right) = 1.247$

$$q_{\text{net}} = \frac{N_{60}}{0.08}\left(\frac{B+0.3}{B}\right)^2 F_d\left(\frac{S_e}{25}\right) = \frac{9}{0.08}\left(\frac{2+0.3}{2}\right)^2 (1.247)\left(\frac{25}{25}\right) = 185.53\ \text{kN/m}^2$$

$Q_{\text{net}} = q_{\text{net}} \times B^2 = 185.53 \times 4 = \mathbf{742\ kN}$

17.7 Eq. (17.39): $\frac{\left(\frac{q_c}{p_a}\right)}{N_{60}} = 7.64 D_{50}^{0.26}$. Use $p_a \approx 100$ kN/m^2.

Depth (m)	N_{60}	D_{50} (mm)	q_c (kN/m^2)
1.5	8	0.28	**4,390**
3	9	0.28	**4,938**
4.5	11	0.28	**6,036**
6	12	0.28	**6,585**
7.5	15	0.28	**8,231**
9	17	0.28	**9,328**

17.8 Eq. (17.33): $E_s = 3q_c$

Using q_c from Problem 17.7:

Depth (m)	q_c (kN/m^2)	E_s (kN/m^2)
1.5	4,390	**13,170**
3	4,938	**14,814**
4.5	6,036	**18,108**
6	6,585	**19,755**
7.5	8,231	**24,693**
9	9,328	**27,984**

17.9 Eq. (17.35): $c_u = \dfrac{q_c - \sigma_c}{N_k}$; $N_k \approx 18.3$

$$c_u = \frac{26000 - (22)(118)}{18.3} = \mathbf{1278.9\ lb/ft^2}$$

17.10 From Eq. (17.43):

$$\text{Recovery ratio, } R = \left(\frac{4.5}{8}\right)(100) = \mathbf{56.25\%}$$