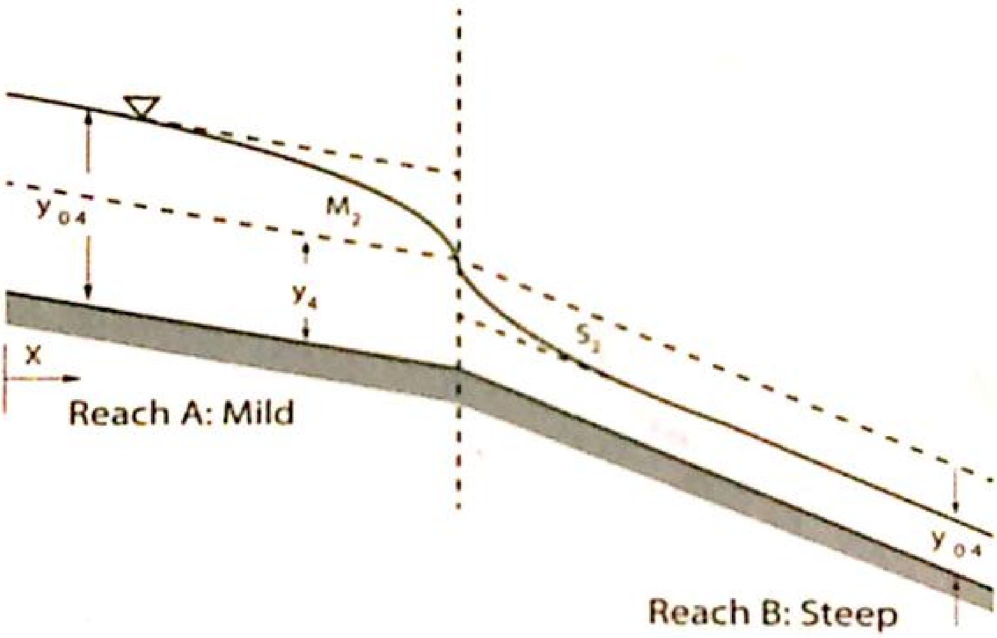
Lecture note on

OPEN CHANNEL FLOW

(for undergraduate Students)

Md. Abdul Halim



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This lecture note has been compiled for the third year students of the Department of Civil Engineering and the Department of Water Resources Engineering, Bangladesh University of Engineering and Technology (BUET). There is a 4-credit theory course (WRE 301 Open Channel Flow) for them in a term of 15 weeks duration.

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LIST OF SYMBOLS

The main symbols used in this lecture note are included in the following list. When other symbols have been used, they are defined in the text.

Symbol	Meaning	Unit(s)
A	flow area	m^2
b	bottom width	m
B	top or free surface width	m
c	celerity of wave	m/s
C	Chezy coefficient	$m^{1/2}/s$
C_c	coefficient of contraction	-
C_d	coefficient of discharge	-
d	depth of flow section	m
	grain diameter	m
d_0	diameter of circular section	m
d_{50}, d_{75}	grain diameters	m
D	hydraulic depth	m
E	specific energy	m
f	Darcy-Weisbach friction factor	-
f_s	Lacey silt factor	-
F	specific force	m^3
F_b	freeboard	m
F_f	friction or resistance force	N
F_g	gravity force	N
F_p	pressure force	N
F_r	Froude number	-
g	acceleration due to gravity	m/s^2
h	depth of flow	m
h_c	critical depth	m
h_e	eddy loss	m
h_f	frictional head loss	m
h_L	total head loss	m
h_n	normal depth	m
h_t	tailwater depth	m
H	total energy or head	m
k_s	Nikuradse equivalent sand grain roughness	m
K	conveyance	m^3/s
L	channel length	m
	length of flow profile	m
	length of weir	m
L_j	length of hydraulic jump	m
M	hydraulic exponent for critical flow computation	-
n	Manning roughness coefficient	$s/m^{1/3}$
N	hydraulic exponent for uniform flow computation	-
p	intensity of pressure	N/m^2
P	wetted perimeter	m

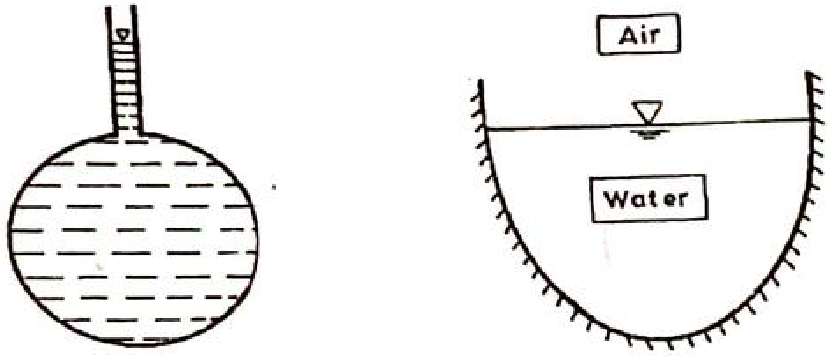
Symbol	Meaning	Unit(s)
q	discharge per unit width	$m^3/s/m$
q^*	transverse or lateral inflow	$m^3/s/m$
Q	discharge	m^3/s
R	hydraulic radius	m
Re	Reynolds number	
s	side slope (sH:1V)	
s_s	specific gravity of sediment	-
S	submergence factor	-
S_c	critical slope	-
S_f	friction slope	-
S_n	normal slope	-
S_w	water surface slope	-
S_0	bottom slope	-
t	time	s
u	point velocity in the x direction	m/s
u_0	velocity at free surface	m/s
u_*	shear or friction velocity	m/s
U	cross sectional mean velocity	m/s
$\bar{U}$	depth averaged velocity	m/s
W	weight	N
x	longitudinal coordinate	m
z	vertical coordinate	m
$\bar{z}$	centroidal distance from the free surface	m
z_b	datum or elevation head	m
z_0	roughness height	m
z_w	stage	m
Z	section factor for critical flow computation	$m^{5/2}$
α	kinetic energy coefficient	
β	momentum coefficient	
γ	specific weight	N/m^3
δ_v	thickness of viscous or laminar sublayer	m
ω	angle subtended by top width at centre of a circular section	deg or rad
ϕ	side slope angle	deg
ψ	angle of repose	deg
θ	bottom slope angle	deg
κ	von Karman constant	-
μ	dynamic viscosity	$N.s/m^2$
ν	kinematic viscosity	m^2/s
ρ	mass density of fluid	kg/m^3
ρ_w	mass density of water	kg/m^3
ρ_s	mass density of sediment	kg/m^3
τ	shear stress	N/m^2
τ_c	critical shear stress	N/m^2
τ_0	average shear stress	N/m^2

Chapter 1

BASIC CONCEPTS OF OPEN CHANNEL FLOW

1.1 INTRODUCTION

The flow of a liquid in a conduit may be either open channel flow or pipe flow (Fig.1.1). In pipe flow the flowing liquid is completely enclosed by solid boundary and flow occurs under pressure. In open channel flow the flowing liquid is not completely enclosed by solid boundary and flow occurs with a free surface. A free surface is subjected to atmospheric pressure. Also, water is the most common liquid in civil engineering applications. Therefore, open channel flow may be defined as the flow of water in a conduit with a free surface. Open channel flow is also known as the *free surface flow*.



(a) Pipe flow (b) Open channel flow

Fig. 1.1 Pipe flow and open channel flow

Flows in rivers and canals are the familiar examples of open channel flow. Flow of water in a closed conduit, e.g. in an underground sewer or in a culvert, may be open channel flow if the flow occurs with a free surface (Fig.1.2). The flow of groundwater with a free surface is also an example of open channel flow.

Open channel flow Pipe flow

Fig.1.2 Flow in an underground sewer

Open channel flow occurs under the action of gravity and at atmospheric pressure. The component of the gravity force or weight of water along the bottom slope acts as the driving force. The boundary friction over the channel perimeter acts as the resistance force. Obviously, for open channel flow to occur, the total energy at an upstream section must be more than the total energy at a downstream section.

In this lecture note, we will follow the SI system of units. We will also follow a Cartesian coordinate system in which the x-axis is along the channel bottom, the y-axis is the

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lateral direction and the z-axis is vertically upward (Fig.1.3). The mean direction of flow is taken to be parallel to the channel bottom and along the x-axis.

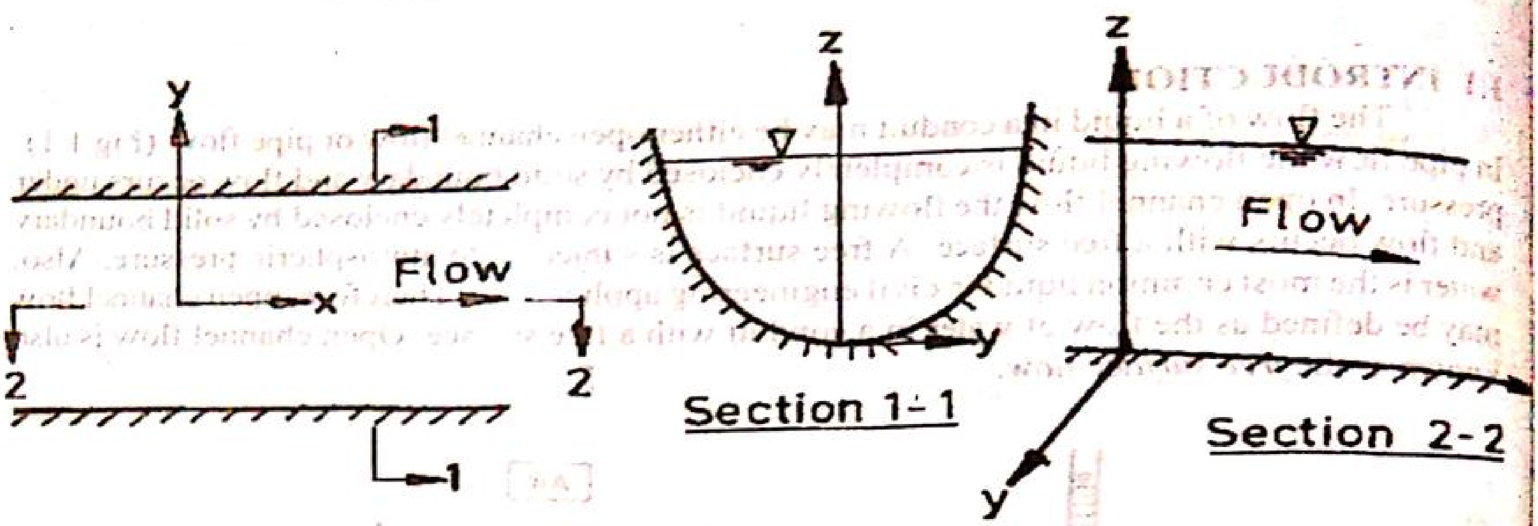


Fig.1.3 The coordinate system

1.2 KINDS OF OPEN CHANNEL

Types of channel

An open channel is a conduit in which water flows with a free surface. Open channels are classified on different criteria as follows.

(a) Natural and Artificial Channels

Natural open channels are the channels which exist naturally on the earth, e.g. rivers and tidal estuaries. They are generally very irregular in shape.

Artificial open channels are the channels developed by men, e.g. irrigation canals, laboratory flumes, spillway chutes, drops, culverts, roadside gutters, etc. They are usually designed with regular geometric shapes.

(b) Prismatic and Non-prismatic Channels

A channel in which the cross-section does not change along the channel and the bottom slope is constant is called a prismatic channel; otherwise it is non-prismatic. The artificial channels are usually prismatic and the natural channels are generally non-prismatic.

(c) Rigid and Mobile Boundary Channels

A channel with immovable bed and sides is known as a rigid boundary channel, e.g. lined canals and sewers. If the channel boundary is composed of loose sedimentary particles moving under the action of flowing water, the channel is called a mobile boundary channel. An alluvial channel is a mobile boundary channel transporting the same type of material as that comprising the channel perimeter.

(d) Small and Large Slope Channels

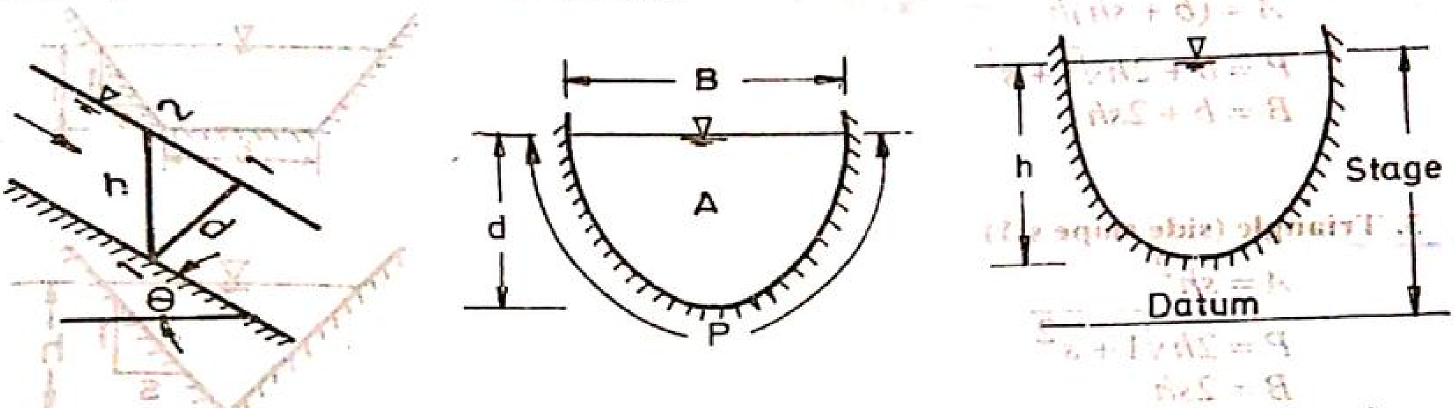
A channel with a bottom (or longitudinal) slope less than or equal to 10:1 (10 horizontal to 1 vertical) or angle of bottom slope less than or equal to 6^0, is called a channel of small slope; otherwise it is a channel of large slope. The angle of bottom slope θ is the angle made by the channel bottom with the horizontal (Fig. 1.4a). The slopes of many natural and artificial channels, like rivers and canals, are less than 10:1 or $\theta < 6^0$. However, some artificial channels like drops and chutes have slopes more than 10:1 or $\theta > 6^0$.

1.3 CHANNEL GEOMETRY AND SECTION ELEMENTS

(a) Prismatic Channels

The rectangle, trapezoid, triangle, parabola and circle are the most commonly used shapes of prismatic or regular or uniform channels. Table 1.1 gives the formulas for the cross-sectional area A, the wetted perimeter P and the top width B for these channel shapes.

The cross-section of a channel taken normal to the direction of flow is called a *channel section* (Fig. 1.4b). A *vertical channel section* is the vertical section passing through the bottom or lowest point of a channel section (Fig. 1.4c).



(a) Channel section and vertical channel section

(b) Channel section (section 1-1)

(c) Vertical channel section (section 1-2)

Fig.1.4 Channel section and vertical channel section

The properties of a channel section which are determined by the geometric shape of the channel and the depth of flow are known as the *geometric elements of a channel section*. They are defined below.

Depth of flow h and depth of flow section d: The depth of flow h is the vertical distance from the lowest point of a channel section to the water surface. The depth of flow section d is the depth of flow normal to the direction of flow. The relationship between h and d is

$$d = h\cos\theta \tag{1.1}$$

Stage: The stage is the elevation of the water surface from a horizontal datum (Fig. 1.4c) and may be positive or negative.

Flow area A: The flow area is the cross-sectional area of flow normal to the direction of flow.

Wetted perimeter P: The wetted perimeter is the length of the interface between water and channel boundary.

Top width B: The top width is the width of a channel section at the water surface.

Hydraulic radius R: The hydraulic radius is the ratio of the flow area to the wetted perimeter, i. e.

$$R = A/P \tag{1.2}$$

Hydraulic depth D: The hydraulic depth is the ratio of the flow area to the top width, i.e.

$$D = A/B \tag{1.3}$$

Table 1.1 Geometric elements of some channel sections

1. Rectangle

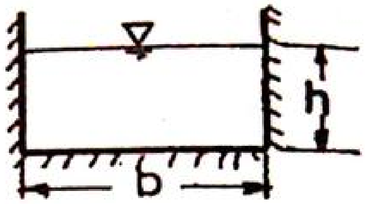
$A = bh$

$P = b + 2h$

$B = b$

$R = \frac{A}{P} = \frac{bh}{b+2h}$

$D = \frac{A}{B} = \frac{bh}{b} = h$

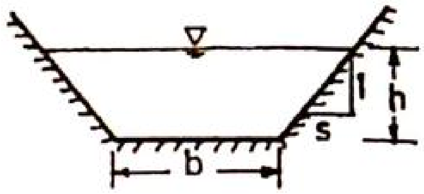


2. Trapezoid (side slope s:1)

$A = (b + sh)h = bh + sh^2$

$P = b + 2h\sqrt{1+s^2}$

$B = b + 2sh$

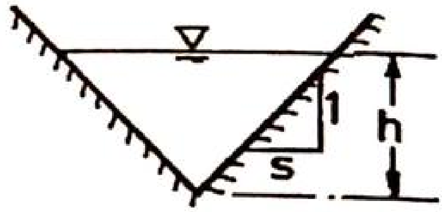


3. Triangle (side slope s:1)

$A = sh^2$

$P = 2h\sqrt{1+s^2}$

$B = 2sh$



4. Parabola (perimeter equation $z = cy^2$)

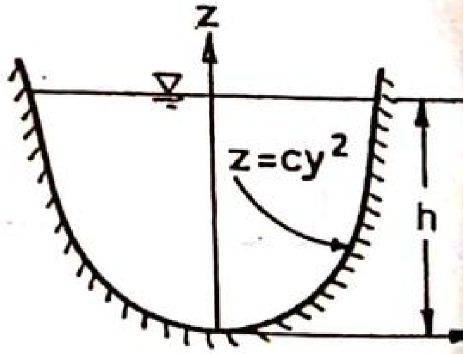
$$A = \frac{2}{3}Bh = \frac{4h^{3/2}}{3\sqrt{c}}$$

$$P = \frac{B}{2}\left[\sqrt{1+x^2} + \frac{1}{x}\ln\left(x + \sqrt{1+x^2}\right)\right]$$

$$(where\ x = \frac{4h}{B})$$

$$\approx B + \frac{8h^2}{3B}(when\ \ x \le 1)$$

$$B = \frac{3A}{2h} = \frac{2h^{1/2}}{\sqrt{c}}$$



5. Circle

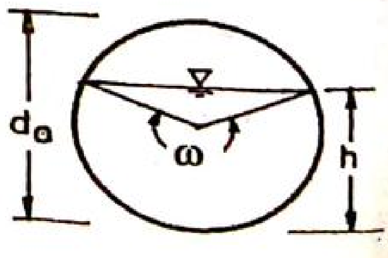
$h = d_0[1 - \cos(\omega/2)]/2$

$\omega = 2\cos^{-1}(1 - 2h/d_0)$

$A = (\omega - \sin\omega)d_0^{\,2}/8$

$P = \omega d_0/2$

$B = d_0\sin(\omega/2)$



Note that ω is the angle made by the top width at the center of the circle ($\omega = 0$ when h = 0, $\omega = \pi$ when h = $d_0/2$ and $\omega = 2\pi$ when h = d_0)

Obviously, the geometric elements A, P, R, etc. of an open channel flow section depend on the depth of flow (Table 1.1). As a result, the solution of an open channel flow problem usually becomes difficult when the depth of flow is unknown and may require a trial-and-error procedure.

Wide Channel

When the width of a rectangular channel is very large compared to the depth, i.e. b >> h (generally $b \geq 10h$), the channel is known as a wide channel (Chow, 1959). The alluvial rivers are usually treated as a wide channel. For a wide channel

$$R = \frac{A}{P} = \frac{bh}{b+2h} \approx \frac{bh}{b} \approx h \tag{1.4}$$

and the discharge is usually expressed per unit width of the channel which is given by q (= Q/b, where Q is the total discharge and b is the bottom width) and the units of q are $m^3/s/m$ or simply m^2/s.

(b) Non-prismatic Channels

Non-prismatic or irregular or non-uniform channels like rivers are often encountered in practice and in such cases it is necessary to compute area, top width, wetted perimeter, etc. of the channel section. The stream-gauging procedure used by the U.S. Geological Survey involves dividing the total cross-sectional area into (N-1) pockets or segments or vertical strips by N number of successive verticals as shown in Fig. 1.5. The depth of flow at each vertical is measured and is taken to be the depth of the associated segment. The segment cross-sectional area extends laterally to half the distance between the preceding and the following verticals according to the *mid-section rule*. The total cross-sectional area is computed as

$$A = \sum_{i=1}^{N-1} \Delta A_i \tag{1.5}$$

where ΔA_i = area of the ith segment

= depth at the ith segment × (1/2 width to the left + 1/2 width to the right)

= $0.5 h_i (W_i + W_{i+1})$

The top width is computed as

$$B = \sum_{i=1}^{N} W_i \tag{1.6}$$

where W_i is the width between the two adjacent verticals.

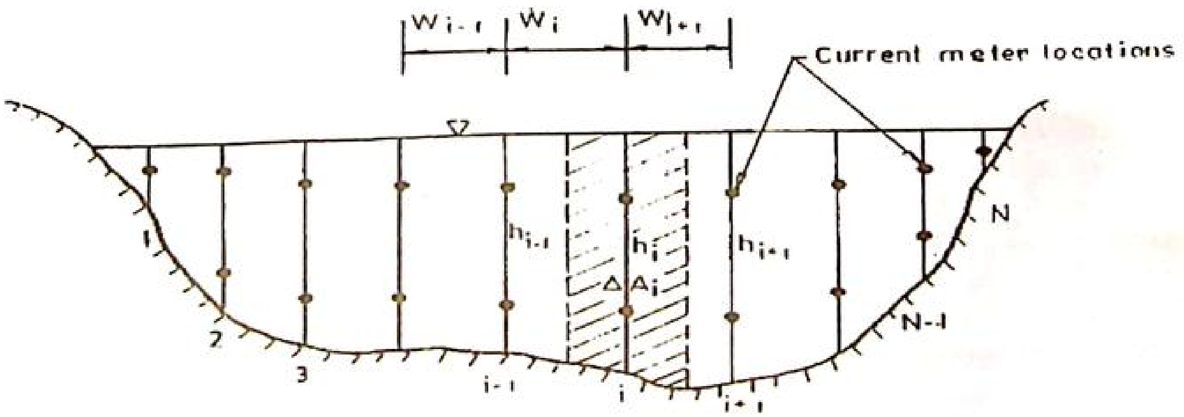


Fig. 1.5 U.S. Geological Survey procedure for stream-gauging

The wetted perimeter P is calculated by the expression

$$P = \sum_{i=2}^{N} \Delta P_i \tag{1.7}$$

where

$$\Delta P_i = \sqrt{W_i^2 + (h_i - h_{i-1})^2} \tag{1.8}$$

and it is assumed that the channel bed is linear between the two adjacent verticals.

1.4 TYPES OF OPEN CHANNEL FLOW

Open channel flows are classified on the basis of different criteria as follows.

(a) Steady and Unsteady Flows (time is the criterion)

Flow in an open channel is said to be steady if the depth of flow h, mean velocity U and discharge Q at a channel section do not change with time. The flow is unsteady if these quantities at a channel section change with time. Mathematically, for steady flow

$$\partial h / \partial t = \partial U / \partial t = \partial Q / \partial t = 0 \quad \textit{for fixed } x$$

True steady flow is rare in nature. *Flow in a channel may be steady only when the discharge in the channel is constant, i.e. for steady flow not only* $\partial Q/\partial t = 0$, *but also* $\partial Q/\partial x = 0$. The flow of water in a straight prismatic channel with a constant discharge (e.g. in a laboratory flume in which a constant discharge is circulated) and the dry-season flow of a river may be considered as steady flow. The flood flow in a river and the tidal flow in an estuary are two familiar examples of unsteady flow.

(b) Uniform and Varied Flows (space is the criterion)

Flow in an open channel is said to be uniform if the depth, mean velocity and discharge do not change along the length of the channel at a given instant of time. When these quantities change along the length of the channel at any instant, the flow is termed as varied or non-uniform. Mathematically, for uniform flow

$$\partial h / \partial x = \partial U / \partial x = \partial Q / \partial x = 0 \quad \textit{for fixed } t$$

In uniform flow, the channel bottom, the water surface and the total energy line are parallel to one another, i.e. their slopes are equal (Fig. 1.6).

True uniform flow is rare in nature. The flow of water in a straight prismatic channel (e.g. in a laboratory flume) with a constant discharge and a constant velocity and without any structure like weir or sluice gate may be considered as uniform flow.

Uniform flow can be steady only. The unsteady uniform flow requires that the water surface fluctuates from time to time remaining parallel to the channel bottom. This is practically impossible and hence "uniform flow" is used to mean "steady uniform flow".

Varied or non-uniform flows are further classified into

(i) gradually varied flow,

(ii) rapidly varied flow, and

(iii) spatially varied flow.

Gradually Varied Flow

If the depth of flow and the mean velocity change gradually along the length of the channel ($\partial h/\partial x \approx 0$, $\partial U/\partial x \approx 0$), the flow is gradually varied (Fig. 1.6). It may again be steady or unsteady. For steady gradually varied flow, $\partial h/\partial t = \partial U/\partial t = \partial Q/\partial t = \partial Q/\partial x = 0$, $\partial h/\partial x \approx 0$, $\partial U/\partial x \approx 0$. For unsteady gradually varied flow, $\partial h/\partial t \neq 0$, $\partial U/\partial t \neq 0$, $\partial Q/\partial t \neq 0$, $\partial Q/\partial x \neq 0$, $\partial h/\partial x \approx 0$, $\partial U/\partial x \approx 0$. The flow upstream of a dam in a river or upstream of a sluice gate in a canal is an example of steady gradually varied flow. The flood flow in a river and the tidal flow in an estuary are two familiar examples of unsteady gradually varied flow. Friction plays an important role in gradually varied flows.

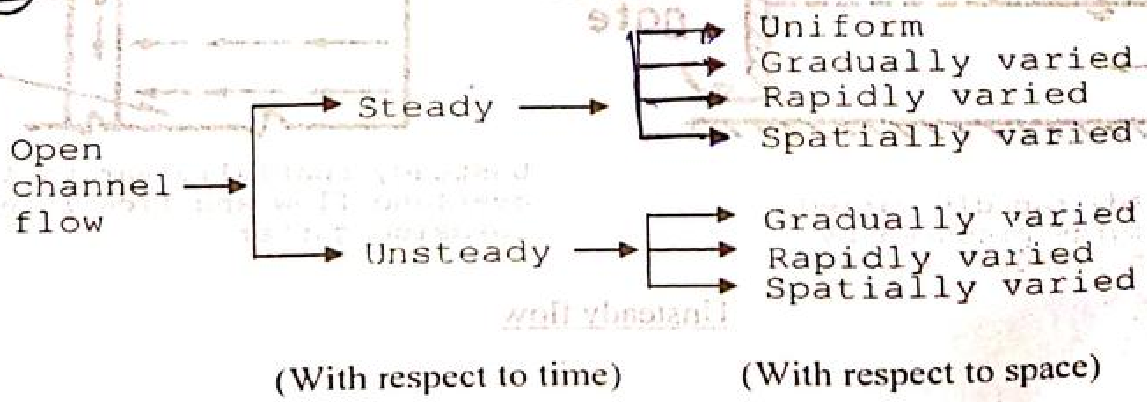
Rapidly Varied Flow

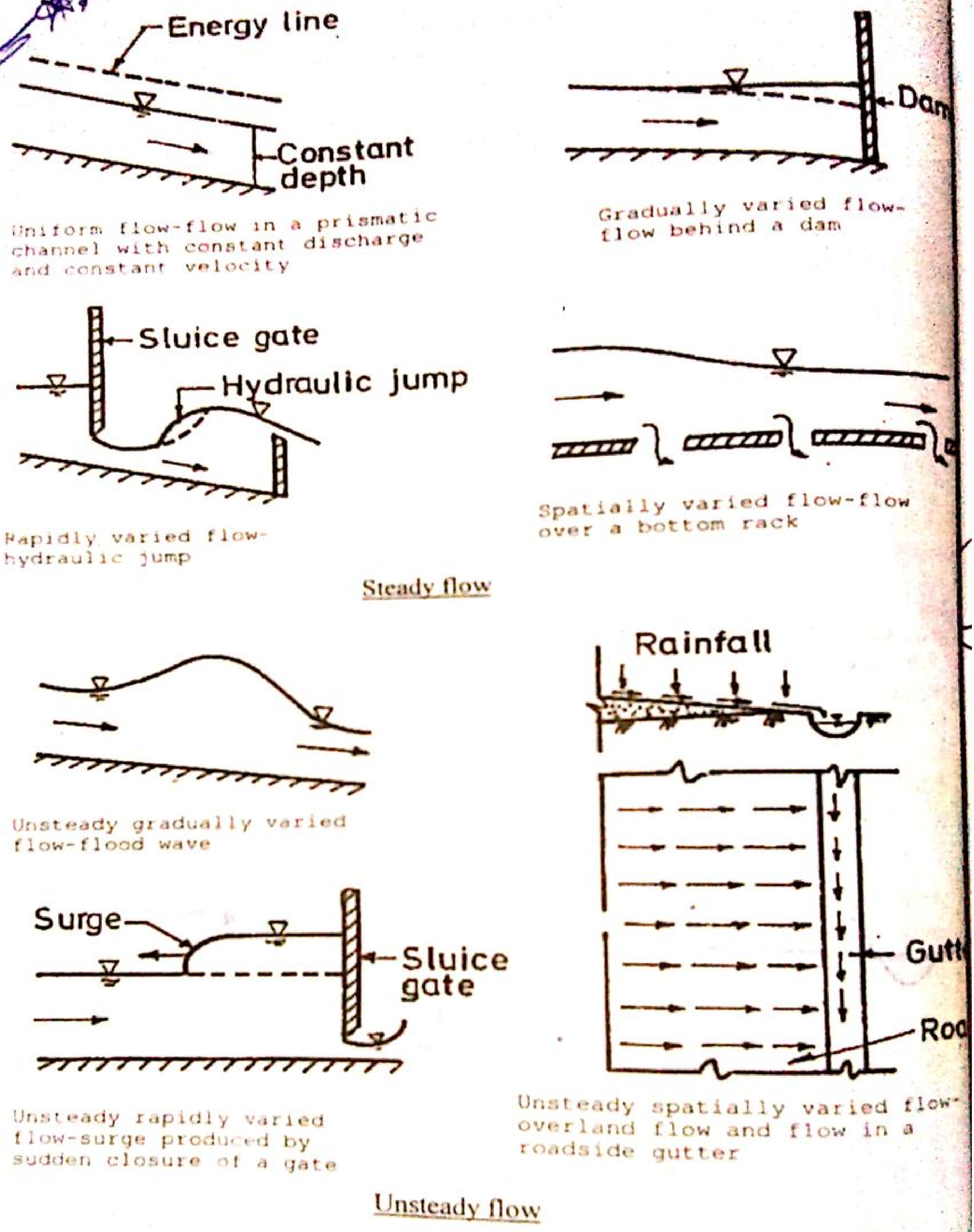
The flow is rapidly varied if the depth of flow and the mean velocity change abruptly over a comparatively short distance ($\partial h/\partial x >> 0$, $\partial U/\partial x >> 0$). Rapidly varied flow is also known as a *local phenomenon*. It may be steady or unsteady. For steady rapidly varied flow, $\partial h/\partial t = \partial U/\partial t = \partial Q/\partial t = \partial Q/\partial x = 0$, $\partial h/\partial x >> 0$, $\partial U/\partial x >> 0$. For unsteady rapidly varied flow, $\partial h/\partial t \neq 0$, $\partial U/\partial t \neq 0$, $\partial Q/\partial t \neq 0$, $\partial Q/\partial x \neq 0$, $\partial h/\partial x >> 0$, $\partial U/\partial x >> 0$. Hydraulic jumps and hydraulic drops are two familiar examples of steady rapidly varied flow and surges in canals and tidal bores are examples of unsteady rapidly varied flow (Fig. 1.6). A surge is a moving wave front that is produced whenever there is an abrupt change in the discharge or depth of flow or both, i.e. during the sudden closure of a sluice gate, in a channel. The tidal bore is a moving wave front by which the tide enters a river. Friction can usually be neglected in rapidly varied flows.

Spatially Varied Flow

A flow in which the discharge varies along the length of the channel resulting from the lateral addition or withdrawal of water so that $\partial Q/\partial x \neq 0$ is known as a spatially varied flow. It may be either steady or unsteady. For steady spatially varied flow, $\partial h/\partial t = \partial U/\partial t = \partial Q/\partial t = 0$, $\partial Q/\partial x \neq 0$, $\partial h/\partial x \neq 0$, $\partial U/\partial x \neq 0$. For unsteady spatially varied flow, $\partial h/\partial t \neq 0$, $\partial U/\partial t \neq 0$, $\partial Q/\partial t \neq 0$, $\partial Q/\partial x \neq 0$, $\partial h/\partial x \neq 0$, $\partial U/\partial x \neq 0$. The flow over a bottom rack is an example of steady spatially varied flow (Fig. 1.6). The production of surface run-off due to rainfall, known as *overland flow*, and the flow over a roadside gutter are examples of unsteady spatially varied flow.

The classification of open channel flow is summarized below and the various types of flow are sketched in Fig. 1.6.





Uniform flow-flow in a prismatic channel with constant discharge and constant velocity

Gradually varied flow-flow behind a dam

Rapidly varied flow-hydraulic jump

Spatially varied flow-flow over a bottom rack

Steady flow

Unsteady gradually varied flow-flood wave

Unsteady rapidly varied flow-surge produced by sudden closure of a gate

Unsteady spatially varied flow-overland flow and flow in a roadside gutter

Unsteady flow

Fig.1.6 Various types of open channel flow

1.5 EFFECTS OF VISCOSITY AND GRAVITY

(a) Effect of Viscosity

The effect of the viscous forces relative to the inertial forces on open channel flow is determined by the *Reynolds number*, which may be written as

$$\text{Re} = \frac{\textit{Inertial forces}}{\textit{Viscous forces}} = \frac{\textit{Mass} * \textit{acceleration}}{\textit{Viscous shear stress} * \textit{area}} = \frac{\rho L^3\left(L/T^2\right)}{\tau . A}$$

$$= \frac{\rho L^3\left(L/T^2\right)}{\mu(du/dz)L^2} = \frac{\rho L^3\left(L/T^2\right)}{\mu(L/T/L)L^2} = \frac{\rho L^2}{\mu T} = \frac{(L/T)L}{\mu/\rho} = \frac{UL}{\nu} \qquad (1.9)$$

where U is a characteristic or representative velocity, taken as the mean velocity of flow, L is a characteristic length and ν is the kinematic viscosity of water. For water at 20^0C, $\nu = 10^{-6}\ m^2/s$. This value of ν is normally used.

In open channel flow, the characteristic length commonly used is the hydraulic radius R (= A/P), so that

$$\text{Re} = \frac{UR}{\nu} \qquad (1.10)$$

Then, when

i) Re < 500, the flow is *laminar*,
ii) Re > 12,500, the flow is *turbulent*, and
iii) 500 ≤ Re ≤ 12,500, the flow is *transitional*.

In laminar flow the viscous forces are strong relative to the inertial forces and dominate the flow. When the flow is laminar, the water particles appear to move in definite smooth paths or laminas with only a small-scale mixing. In turbulent flow the viscous forces are very weak relative to the inertial forces and the inertial forces dominate the flow. The water particles move in irregular paths in a random fashion with large-scale mixing due to generated eddies.

By injecting a fine stream of dye, it is possible to identify whether a flow is laminar, transitional or turbulent. The stream of dye looks like a thread and does not mix with water in laminar flow, becomes wavy and irregular in transitional flow, and mixes with water and disappears in turbulent flow.

In laminar flow the head loss due to friction varies as U, whereas in turbulent flow the head loss due to friction varies as U^2.

Most open channel flows like the flows in rivers and canals are turbulent. The Reynolds number of most open channel flows like those in rivers and canals is high, of the order of 10^6. For example, for river flow, U ≈ 1 m/s and R ≈ 5 m, so that Re ≈ 5×10^6. This indicates that the viscous forces are very weak relative to the inertial forces. Therefore, the Reynolds number does not play a significant role in determining the state of open channel flows.

Laminar flow occurs rarely. The overland flow and the groundwater flow with a free surface are two examples of laminar open channel flow.

(b) Effect of Gravity

The effect of the gravity forces relative to the inertial forces on open channel flow is determined by the *Froude number*, which may be written as

$$Fr = \sqrt{\frac{Inertial\ forces}{Gravity\ forces}} = \sqrt{\frac{Mass * inertial\ acceleration}{Mass * gravitational\ acceleration}}$$

$$= \sqrt{\frac{Inertial\ acceleration}{Gravitational\ acceleration}} = \sqrt{\frac{L/T^2}{g}} = \sqrt{\frac{(L/T)^2}{gl}} = \sqrt{\frac{U^2}{gl}} = \frac{U}{\sqrt{gL}} \quad (1.11)$$

where U is a characteristic velocity, taken as the mean velocity of flow, L is a characteristic length and g (= 9.81 m/s²) is the acceleration due to gravity. In open channel flow, the characteristic length commonly used is the hydraulic depth D (= A/B), so that

$$Fr = \frac{U}{\sqrt{gD}} \quad (1.12)$$

Then, if

i) $Fr = 1, U = \sqrt{gD}$, the flow is *critical*.

ii) $Fr < 1, U < \sqrt{gD}$, the flow is *subcritical*, and

iii) $Fr > 1, U > \sqrt{gD}$, the flow is *supercritical*.

When the flow is critical, the inertial and gravity forces are equal. When the flow is subcritical, the gravity forces are dominant and when the flow is supercritical, the inertial forces are dominant.

The flow in most rivers and canals and upstream of a sluice gate is subcritical. As an example, for river flow, U ≈ 1 m/s and D ≈ 5 m and hence Fr ≈ 0.14. Supercritical flow normally occurs downstream of a sluice gate and at the feet of drops and spillways. Flow upstream of a hydraulic jump is supercritical and downstream of a hydraulic jump is subcritical.

The Froude number of open channel flows varies over a wide range covering both subcritical and supercritical flows and the state of open channel flow is primarily governed by the gravity forces relative to the inertial forces. Therefore, the Froude number is the most important parameter to indicate the state of open channel flows.

Equation (1.12) states that at the critical state of flow (Fr = 1), the flow velocity is equal to $\sqrt{gD}$. It can be shown that $\sqrt{gD}$ is the velocity (or celerity) of an elementary or small-amplitude wave on the surface of still water (see Appendix 1.1 at the end of Chapter 1). The Froude number can then be defined as the ratio between the flow velocity U and the wave celerity c, i.e.

$$Fr = \frac{U}{c} \quad (1.13)$$

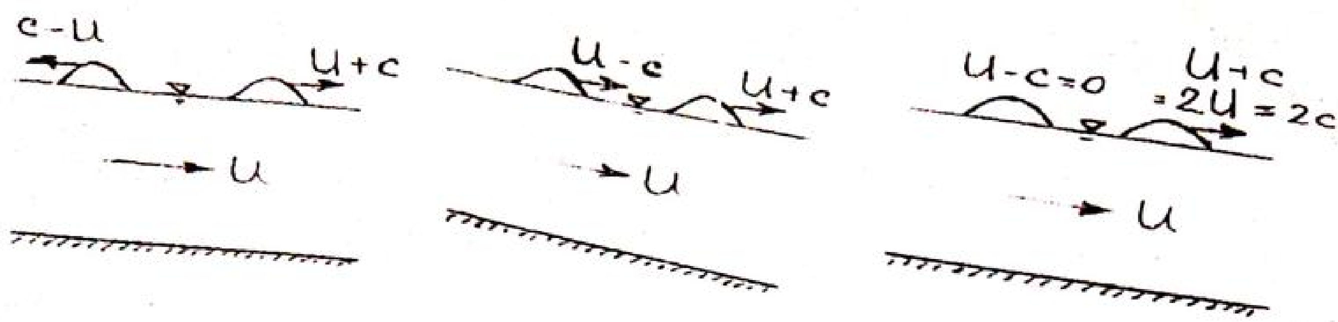
Obviously, then

i) for subcritical flow, Fr < 1 and U < c,

ii) for critical flow, Fr = 1 and U = c, and

iii) for supercritical flow, Fr > 1 and U > c.

A small-amplitude wave can be easily produced by gently throwing a small object in water. In subcritical flow, one wave front propagates upstream at a velocity (c-U) and the other wave front propagates downstream at a velocity (U + c) (Fig.1.7a). In supercritical flow, both the wave fronts propagate downstream at velocities (U - c) and (U + c), respectively (Fig. 1.7b). In critical flow, one wave front remains stationary and the other moves downstream at a velocity (U + c) (Fig. 1.7c). Thus, by observing an elementary wave whether it propagates upstream, remains stationary or propagates downstream can be used as a criterion for physically identifying subcritical, critical or supercritical flow, respectively.



Fr < 1, U < c
(a) Subcritical flow

Fr > 1, U > c
(b) Supercritical flow

Fr = 1, U = c
(c) Critical flow

Fig. 1.7 Propagation of wave in subcritical, supercritical and critical flows

The fact that a small disturbance such as an elementary wave can propagate upstream against the flow in subcritical flow, but not in supercritical flow, has important practical significance. In subcritical flow, conditions upstream are affected by downstream conditions. Therefore, subcritical flow is controlled from downstream. In supercritical flow, conditions upstream are not affected by downstream conditions, and hence, supercritical flow is controlled from upstream.

Combined Effect of Viscosity and Gravity: State of Flow

A combined effect of viscosity and gravity may produce any of the following four states of flow in an open channel based on the numerical values of the Froude and the Reynolds numbers:

i) Subcritical laminar	$Fr < 1$, $Re < 500$
ii) Supercritical laminar	$Fr > 1$, $Re < 500$
iii) Subcritical turbulent	$Fr < 1$, $Re > 12{,}500$
iv) Supercritical turbulent	$Fr > 1$, $Re > 12{,}500$

The first two flow states, subcritical laminar and supercritical laminar, are very rare. The flow in most rivers and canals is subcritical turbulent, and the flow at the feet of drops and spillways is supercritical turbulent.

Example 1.1

A trapezoidal channel has a bottom width of 6 m and side slopes of 2:1. Compute the discharge and determine the state of flow in this channel if the depth of flow is 1.5 m and the mean velocity of flow is 2.30 m/s. If elementary waves are created in this channel, determine the speed of the wave fronts upstream and downstream.

Solution Trapezoidal channel, $b = 6$ m, $s = 2$, $h = 1.5$ m, $U = 2.30$ m/s

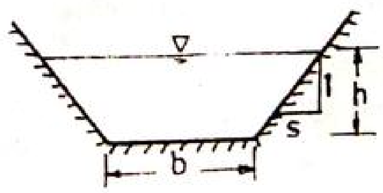
$$A = (b + sh)h = (6 + 2 \times 1.5) \times 1.5 = 13.5\ m^2$$

$$P = b + 2h\sqrt{1 + s^2} = 6 + 2 \times 1.5 \times \sqrt{1 + 2^2} = 12.71\ m$$

$$B = b + 2sh = 6 + 2 \times 2 \times 1.5 = 12\ m$$

$$R = A / P = 13.5 / 12.71 = 1.06\ m$$

$$D = A / B = 13.5 / 12 = 1.13\ m$$



$$\therefore Q = AU = 13.5 \times 2.30 = 31.05\ m^3/s$$

$$Re = \frac{UR}{\nu} = \frac{2.30 \times 1.06}{10^{-6}} = 2.44 \times 10^6 > 12{,}500$$

$$Fr = \frac{U}{\sqrt{gD}} = \frac{2.30}{\sqrt{9.81 \times 1.13}} = 0.69 < 1$$

Hence, the flow is subcritical turbulent.

Now, c = $\sqrt{gD} = \sqrt{9.81 \times 1.13} = 3.33\ m/s$

∴ Speed of the wave fronts upstream = c – U = 3.33 – 2.30 = 1.03 m/s

and, speed of the wave fronts downstream = c + U = 3.33 + 2.30 = 5.63 m/s

Example 1.2

A circular channel 2.75 m in diameter carries a discharge of 6.55 m³/s at a depth of 1.1 m. Determine the state of flow.

Solution Circular channel, d_0 = 2.75 m, Q = 6.55 m³/s, h = 1.1 m

$$\omega = 2\cos^{-1}(1 - 2h/d_0) = 2\cos^{-1}(1 - 2 \times 1.1/2.75) = 2.74\ rad$$

$$A = (\omega - \sin\omega)d_o^2/8 = (2.74 - \sin 2.74) \times 2.75^2/8 = 2.22\ m^2$$

$$P = \omega d_o/2 = 2.74 \times 2.75/2 = 3.77\ m$$

$$B = d_o \sin(\omega/2) = 2.75 \times \sin(2.74/2) = 2.69\ m$$

$$R = A/P = 2.22/3.77 = 0.59\ m$$

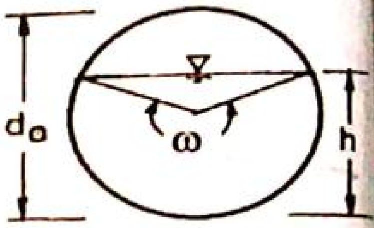
$$D = A/B = 2.22/2.69 = 0.82\ m$$

$$U = Q/A = 6.55/2.22 = 2.95\ m/s$$

$$Re = \frac{UR}{\nu} = \frac{2.95 \times 0.59}{10^{-6}} = 1.74 \times 10^6 > 12{,}500$$

$$Fr = \frac{U}{\sqrt{gD}} = \frac{2.95}{\sqrt{9.81 \times 0.82}} = 1.04 > 1$$

Hence, the flow is supercritical turbulent.



1.6 VELOCITY DISTRIBUTION

Velocity Distribution in a Channel Section

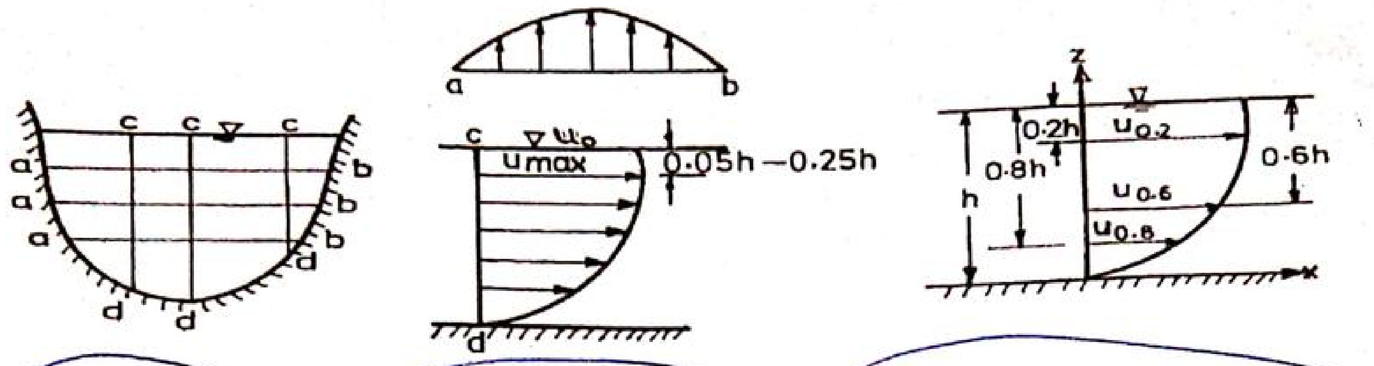
Owing to the presence of the free surface and the friction over the channel bed and banks, the velocities are not uniformly distributed in an open channel flow section. The velocity is zero at the solid boundary (no-slip condition) and gradually increases with distance from the boundary (Fig.1.8). The measured maximum velocity usually occurs below the free surface at a distance of 0.05 to 0.25 of the depth and is about 10% to 30% higher than the cross-sectional mean velocity.

In turbulent flow, the variation of velocity along a vertical can be approximated by a *logarithmic* or *power law*. In this case, the average of the velocities at 0.2 and 0.8 of the depth or the velocity at 0.6 of the depth below the water surface (Fig.1.8c) is approximately equal to the mean velocity in the vertical.

Measurement of Velocity and Discharge

The velocity of flow in an open channel can be measured with a current meter. It is the standard practice of the U.S. Geological Survey to determine the average velocity in a vertical $\overline{U}$ by measuring the velocities at 0.2 and 0.8 of the depth below the free surface when the depth is more than 0.61 m (2 ft), or at 0.6 of the depth when the depth is less than 0.61 m, i.e.

$$\overline{U} \approx \frac{u_{0.2} + u_{0.8}}{2} \approx u_{0.6} \qquad (1.14)$$



(a) Channel section (b) Velocity variation along a-b and c-d (c) Logarithmic velocity distribution along a vertical

Fig.1.8 Velocity distribution

The discharge is obtained from the measurement of velocity and area by the *area-velocity method*. With reference to Fig.1.5, the total discharge is computed as

$$Q = \sum_{i=1}^{N-1} \Delta Q_i \tag{1.15}$$

where ΔQ_i = discharge in the ith segment
= cross-sectional area of the ith segment × average velocity at the ith vertical
= $\Delta A_i . \overline{U}_i$

The mean velocity in the section is equal to the discharge divided by the area, i.e.

$$U = \frac{Q}{A} \tag{1.16}$$

The surface velocity u_0 is related to the average velocity in a vertical $\overline{U}$ by

$$\overline{U} = k u_0 \tag{1.17}$$

where k is a coefficient whose value ranges between 0.80 and 0.95 depending on the channel section. The surface velocity can be determined by float tracking or other surface velocity measuring devices.

Example 1.3

The data collected during the stream-gauging operation at a certain river section are given in Table 1.2. Compute the discharge and the mean velocity for the entire section.

Solution

The measurements made 2 m and 15 m from the left bank involve velocity at 0.6 depth which represents the mean velocity in the vertical. Other measurements involve velocities at 0.2 and 0.8 depths and the mean velocity is obtained by averaging the velocities at 0.2 and 0.8 depths. The width associated with each measurement extends halfway between the adjacent

Table 1.2 Computation of discharge from stream-gauging

Distance from left bank (m)	Total Depth (m)	Meter depth (m)	Velocity (m/s)	Width (m)	Area (m^2)	Mean velocity (m/s)	Discharge (m^3/s)
0	0						
2	1.00	0.60	0.54	2.00	2.00	0.54	1.08
4	3.50	2.80	0.98				
		0.70	1.62	2.00	7.00	1.30	9.10
6	5.20	4.16	1.35				
		1.04	1.60	2.50	13.00	1.48	19.18
9	6.30	5.04	1.36				
		1.26	1.81	2.50	15.75	1.59	24.96
11	4.40	3.52	1.51				
		0.88	1.72	2.00	8.80	1.62	14.21
13	2.20	1.32	1.16	2.00	4.40	1.16	5.10
15	0.80	0.48	0.64	2.00	1.60	0.64	1.02
17	0						
Total:					52.55		74.65

segments. So, for example, for the measurement taken 6 m from the left bank, the width is [(6-4)/2+(9-6)/2] = 2.50 m. The corresponding area ΔA is (5.20×2.50) = 13.00 m^2 and the corresponding discharge ΔQ is (13.00×1.48) = 19.18 m^3/s. The areas and discharges associated with other measurements are computed similarly and are shown in Table 1.2 and summed to give total cross-sectional area A = 52.55 m^2 and total discharge Q = 74.65 m^3/s. Therefore, the mean velocity for the entire section = 74.65/52.55 = 1.42 m/s.

1.7 VELOCITY DISTRIBUTION COEFFICIENTS

The flow in a straight prismatic channel is in fact three-dimensional, i.e. the flow properties like the velocity and the pressure vary in the longitudinal, lateral and vertical directions. However, the variations of the flow parameters in the lateral and vertical directions are usually small compared to those in the longitudinal direction. Consequently, a majority of open channel flow problems are analyzed by considering that the flow is one-dimensional, i.e. we consider the cross-sectional mean values of the flow parameters that vary from section to section only.

In the one-dimensional method of flow analysis, the discharge and the mean velocity of flow in a channel section are computed as follows. Let u be the velocity over an elementary area ΔA of the channel cross-section (Fig. 1.9). Then, the discharge passing the entire channel section is given by

$$Q = \int_0^A u\,dA \qquad (1.18)$$

and the mean velocity of flow U is given by

$$U = \frac{Q}{A} = \frac{1}{A}\int_0^A u\,dA \qquad (1.19)$$

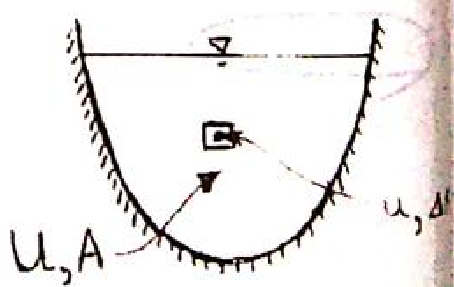


Fig.1.9 Definition sketch

Owing to non-uniform velocity distribution in a channel section, the kinetic energy and the momentum of flow computed from the cross-sectional mean velocity are generally less than their actual values. To get the actual kinetic energy of flow, the kinetic energy based on the mean velocity is multiplied by the coefficient α, known as the *kinetic energy coefficient*. Similarly, to get the actual momentum, the momentum based on the mean velocity is multiplied by the coefficient β, known as the *momentum coefficient*. The energy and momentum coefficients α and β together are known as the *velocity distribution coefficients*.

The kinetic energy of flow passing ΔA (Fig.1.9) per unit time is equal to

$$\frac{1}{2} \times \rho u \Delta A \times u^2 = \frac{\rho}{2} u^3 \Delta A$$

where ρ is the mass density of water. Therefore, the total kinetic energy of flow passing the channel section is equal to

$$\frac{\rho}{2}\int_0^A u^3 dA$$

where A is the total area of the cross-section. The total kinetic energy based on the mean velocity U and corrected for the non-uniform distribution of velocity is

$$\alpha \frac{\rho}{2} U^3 A$$

Equating the above two quantities and rearranging

$$\alpha = \frac{\int_0^A u^3 dA}{U^3 A} \approx \frac{\sum u^3 \Delta A}{U^3 A} \qquad (1.20)$$

The momentum of flow passing ΔA (Fig.1.9) per unit time is $\rho u \Delta A \times u = \rho u^2 \Delta A$. Therefore, the total momentum of flow passing the channel section per unit time is equal to

$$\rho\int_0^A u^2 dA$$

The total momentum based on the mean velocity U and corrected for the non-uniform distribution of velocity is $\beta \rho U^2 A$ so that

$$\beta = \frac{\int_0^A u^2 dA}{U^2 A} \approx \frac{\sum u^2 \Delta A}{U^2 A} \qquad (1.21)$$

The energy and momentum coefficients are always positive and never less than unity. For uniform velocity distribution in the channel section, α = β = 1. In all other cases, α > β > 1. The values of α and β are higher for laminar flow than for turbulent flow, since the effect of turbulence is to make the distribution of velocity more uniform over the channel section and reduce α and β.

Experimental results (Watts et al., 1967) suggest that, when the channel is straight and prismatic and the flow is turbulent and uniform or gradually varied, the two velocity distribution coefficients do not normally exceed 1.10 and 1.04, respectively, and one can assume α = β = 1. However, in channels of complex cross-section, upstream from weirs, in the vicinity of obstructions, near pronounced irregularities in alignment or when the flow is concentrated in one part of the section, values of α and β may even be greater than 2 and 1.35, respectively

Although the numerical values of α and β vary over a wide range depending on the velocity distribution, the ratio (α-1)/(β-1) tends to vary only slightly, in the range 2.8 to 3.0 (Henderson, 1966).

Example 1.4

In a wide channel the velocity varies along a vertical as u = 1 + 3z/h, where h is the depth of flow and u is the velocity at a distance z from the channel bottom. (i) Compute the discharge per unit width, (ii) determine the state of flow, and (iii) compute the velocity distribution coefficients α and β and the ratio (α-1)/(β-1), if h = 5 m.

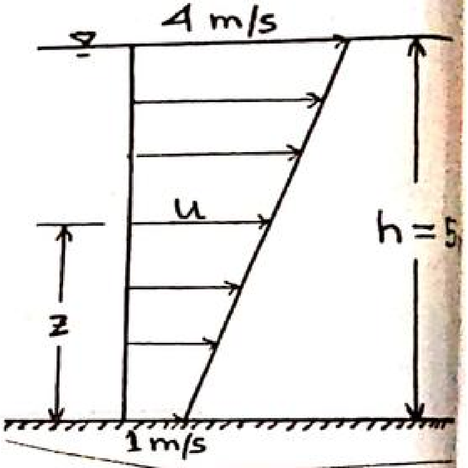
Solution

For a wide channel we can consider unit width of the channel and replace the area A in Eq.(1.19) by the flow depth h and dA by dz. Then, the cross-sectional mean velocity U becomes the depth-averaged velocity $\bar{U}$. Therefore,

$$\bar{U} = \frac{1}{h}\int_0^h u dz = \frac{1}{h}\int_0^h (1+\frac{3z}{h})dz = \frac{1}{h}[h+\frac{3h^2}{2h}] = 1+\frac{3}{2} = 2.50 \ m/s$$

Alternatively, since the velocity varies linearly from 1 m/s at the channel bottom (z/h = 0) to 4 m/s at the water surface (z/h = 1), the depth-averaged velocity is obtained as

$$\bar{U} = \frac{1+4}{2} = 2.50 \ m/s$$



(i) Then, the discharge per unit width is

$$q = \bar{U}h = 2.50 \times 5 = 12.50 \quad m^2/s$$

(ii) The Reynolds and Froude numbers Re and Fr are then

$$Re = \frac{\bar{U}R}{\nu} = \frac{\bar{U}h}{\nu} = \frac{2.50\times 5}{10^{-6}} = 12.5\times 10^6 > 12{,}500$$

(for a wide channel, R ≈ h)

$$Fr = \frac{\bar{U}}{\sqrt{gD}} = \frac{\bar{U}}{\sqrt{gh}} = \frac{2.50}{\sqrt{9.81\times 5}} = 0.36 < 1 \quad (\because D = h)$$

Hence, the flow is subcritical turbulent.

(iii) The kinetic energy and momentum coefficients α and β are then

$$\alpha = \frac{1}{\bar{U}^3 h}\int_0^h u^3 dz = \frac{1}{\bar{U}^3 h}\int_0^h \left(1+\frac{3z}{h}\right)^3 dz$$

$$= \frac{1}{\bar{U}^3 h}\int_0^h \left(1+\frac{9z}{h}+\frac{27z^2}{h^2}+\frac{27z^3}{h^3}\right)dz$$

$$= \frac{1}{\bar{U}^3 h}(h+4.5h+9h+6.75h) = \frac{21.25}{\bar{U}^3} = \frac{21.25}{2.5^3} = 1.36$$

$$\beta = \frac{1}{\bar{U}^2 h}\int_0^h u^2 dz = \frac{1}{\bar{U}^2 h}\int_0^h \left(1+\frac{3z}{h}\right)^2 dz$$

$$= \frac{1}{\bar{U}^2 h}\int_0^h \left(1+\frac{6z}{h}+\frac{9z^2}{h^2}\right)dz = \frac{1}{\bar{U}^2 h}(h+3h+3h) = \frac{7}{\bar{U}^2} = \frac{7}{2.5^2} = 1.12$$

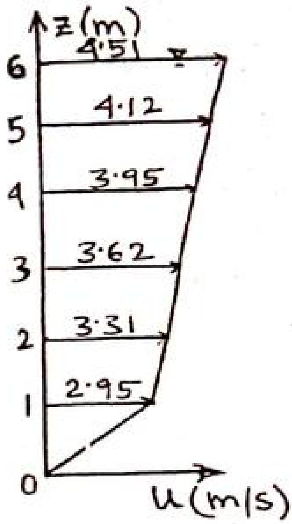
$$\therefore (\alpha-1)/(\beta-1) = 0.36/0.12 = 3$$

Note that when u is expressed as a function of z/h, the numerical values of $\bar{U}$, α and β become independent of the depth of flow.

Example 1.5

Using the trapezoidal rule of numerical integration, compute the discharge per unit width, the mean velocity and the numerical values of α and β for the following velocity measurements (u is the velocity at a distance z from the channel bottom) along a vertical in a wide channel, when the total depth is 6 m.

z (m)	0	1	2	3	4	5	6
u (m/s)	0	2.95	3.31	3.62	3.95	4.12	4.51



Solution $h = 6m \qquad \Delta z = 1\ m$

$$q = \int_0^h u dz = \sum u \Delta z = \Delta z \sum u$$

$$= 1 \times \left[\frac{0 + 4.51}{2} + 2.95 + 3.31 + 3.62 + 3.95 + 4.12\right] = 20.21\ m^2/s$$

$$\bar{U} = \frac{q}{h} = \frac{20.21}{6} = 3.57\ m/s$$

$$\alpha = \frac{\int_0^h u^3 dz}{\bar{U}^3 h} = \frac{\sum u^3 \Delta z}{\bar{U}^3 h} = \frac{\Delta z}{\bar{U}^3 h}\sum u^3$$

$$= \frac{1}{3.37^3 \times 6}\left[\frac{0^3 + 4.51^3}{2} + 2.95^3 + 3.31^3 + 3.62^3 + 3.95^3 + 4.12^3\right] = 1.25$$

$$\beta = \frac{\int_0^h u^2 dz}{\bar{U}^2 h} = \frac{\sum u^2 \Delta z}{\bar{U}^2 h} = \frac{\Delta z}{\bar{U}^2 h}\sum u^2$$

$$= \frac{1}{3.37^2 \times 6}\left[\frac{0^2 + 4.51^2}{2} + 2.95^2 + 3.31^2 + 3.62^2 + 3.95^2 + 4.12^2\right] = 1.11$$

1.8 PRESSURE DISTRIBUTION

Hydrostatic Pressure Distribution

Let us consider a vertical column of water of height h and cross-sectional area ΔA (Fig. 1.10). Let p be the intensity of pressure or unit pressure (force/unit area) at the bottom of the water column. Then,

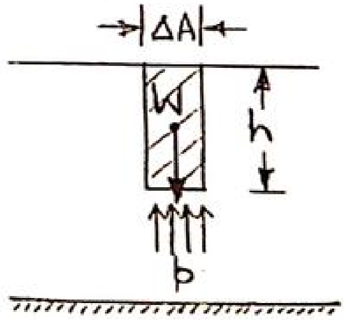


Fig.1.10 Definition sketch

$$p = \frac{Force}{Area} = \frac{Weight\ of\ water\ column}{Area} = \frac{W}{\Delta A} = \frac{\gamma h \Delta A}{\Delta A} = \gamma h = \rho g h \qquad (1.22)$$

Equation (1.22) indicates that the pressure at any point is directly proportional to the depth of the point below the free surface. This is known as the *hydrostatic distribution of pressure* and h is the *hydrostatic pressure head.*

The hydrostatic distribution of pressure is valid for parallel flow in a horizontal or small slope channel. When the curvature of the streamlines is small, the flow is known as *parallel flow* (Fig.1.11a). Uniform flow is practically parallel flow. Gradually varied flow may also be regarded as parallel flow since the curvature of the streamlines is small and negligible. Hence, the hydrostatic law of pressure distribution holds exactly for uniform flow and approximately for gradually varied flow in a horizontal or small slope channel.

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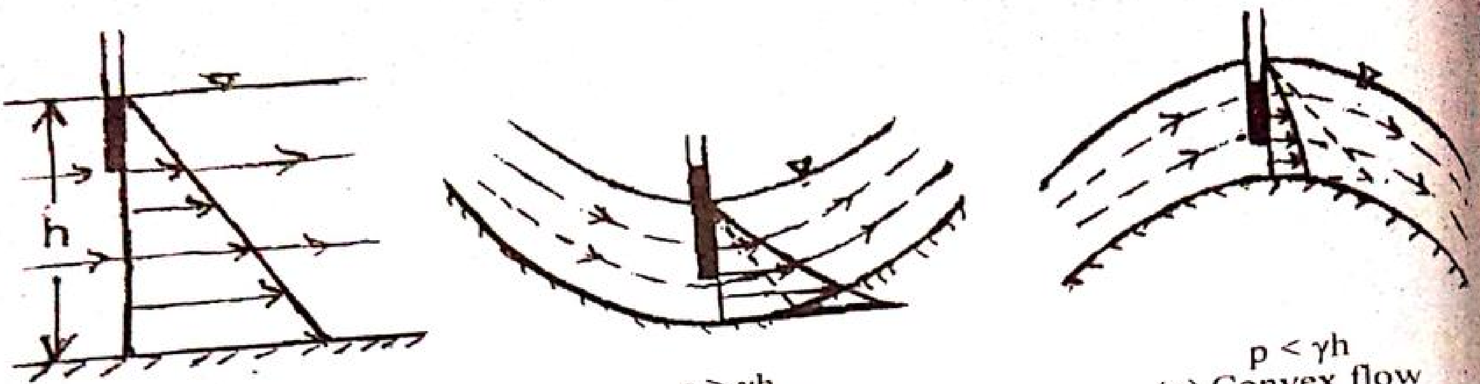


p = γh
(a) Parallel flow

p > γh
(b) Concave flow

p < γh
(c) Convex flow

Fig.1.11 Pressure distribution in parallel, concave and convex flows

Pressure Distribution in Curvilinear Flow

When the curvature of the streamlines is considerable, the flow is known as *curvilinear flow*. Such situations may occur when the bottom of the channel is curved, at sluice gates and at free overfalls. In such cases, the pressure distribution is not hydrostatic. Curvilinear flows may either be *concave* or *convex*. In concave flow (Fig.1.11b), the centrifugal forces resulting from streamline curvature combine with the gravity forces and the pressure is more than hydrostatic. In convex flow (Fig.1.11c), the centrifugal forces act against the gravity forces and the pressure is less than hydrostatic.

Let us consider the forces acting in the vertical direction on a water column of height h and cross-sectional area ΔA (Fig.1.12). Then,

Mass of the water column = $\rho h \Delta A$

If r is the radius of curvature of the streamline and u is the flow velocity at the point under consideration, then

$$\text{Centrifugal acceleration} = \frac{u^2}{r}$$

and

$$\text{Centrifugal force} = \rho h \Delta A \times \frac{u^2}{r}$$

The intensity of pressure as a result of the centrifugal force is

$$p_c = \frac{Force}{Area} = \frac{\rho h \Delta A}{\Delta A} \times \frac{u^2}{r} = \rho h \frac{u^2}{r} \qquad (1.23)$$

and the pressure head

$$h_c = \frac{p_c}{\gamma} = \frac{\rho h}{\rho g}\frac{u^2}{r} = \frac{hu^2}{gr} \qquad (1.24)$$

For practical purposes, the velocity u is replaced by the cross-sectional mean velocity U.

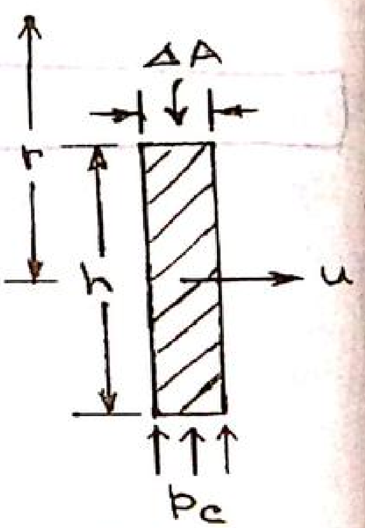


Fig.1.12 Definition sketch

The total pressure acting at the bottom of the water column is the algebraic sum of the hydrostatic pressure and the pressure due to the centrifugal action, i.e.

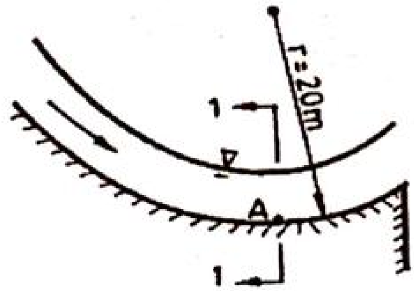
$$\text{Total pressure head} = h \pm h_c = h \pm \frac{hU^2}{gr} = h(1 \pm \frac{U^2}{gr}) \qquad (1.25)$$

and

$$\text{Total pressure} = \gamma(h \pm h_c) = \gamma h(1 \pm \frac{U^2}{gr}) \qquad (1.26)$$

where the plus and minus signs are used with concave and convex flows, respectively

Example 1.6 A spillway flip bucket has a radius of curvature of 20 m. If the flow depth at section 1-1 is 3m and the discharge per unit width is 66 m^2/s, compute the pressure at A.



Solution $r = 20$ m, $q = 66$ m^2/s, $h = 3$ m

$$U = \frac{q}{h} = \frac{66}{3} = 22\ m/s$$

$$p = \gamma h\left(1 + \frac{U^2}{gr}\right) = \rho g h\left(1 + \frac{U^2}{gr}\right)$$

$$= 1000 \times 9.81 \times 3 \times \left(1 + \frac{22^2}{9.81 \times 20}\right)$$

$$= 102030\ N/m^2$$

Effect of Slope on Pressure Distribution

Consider a water column of height h and cross-sectional area ΔA (Fig.1.13). The pressure at B in this case balances the component of the weight of the water column AB normal to the bed. Now, weight of the water column

$$W = \gamma d \Delta A$$

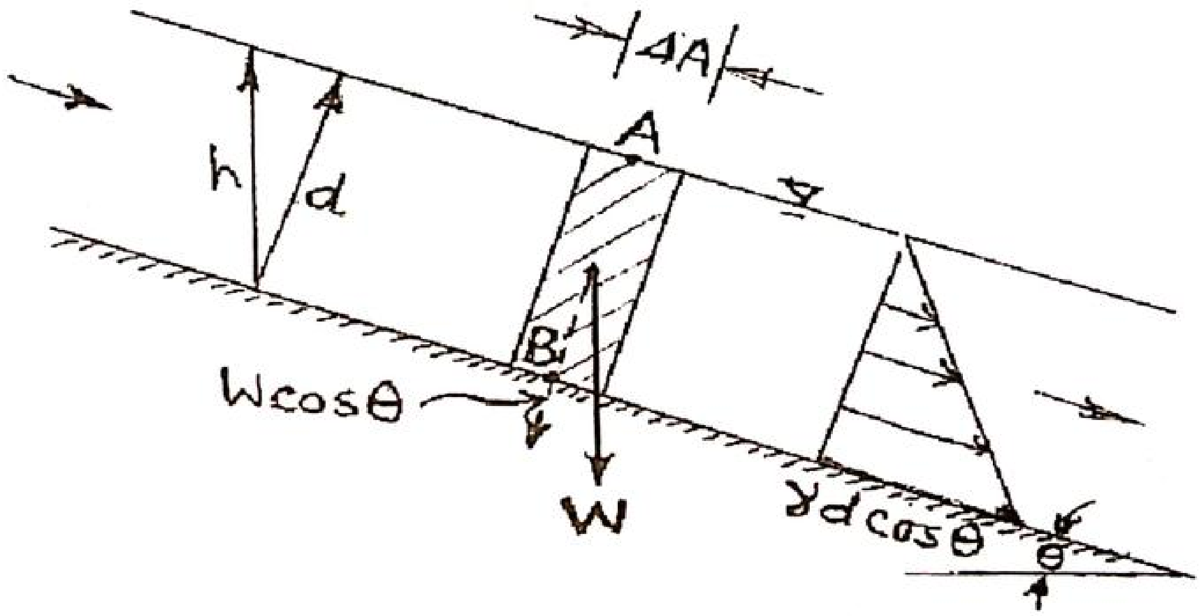


Fig.1.13 Pressure distribution in a channel of large slope

The component of W normal to the channel bed, i.e. along AB = Wcosθ = γdΔAcosθ. Hence, the intensity of pressure at the bottom of the water column

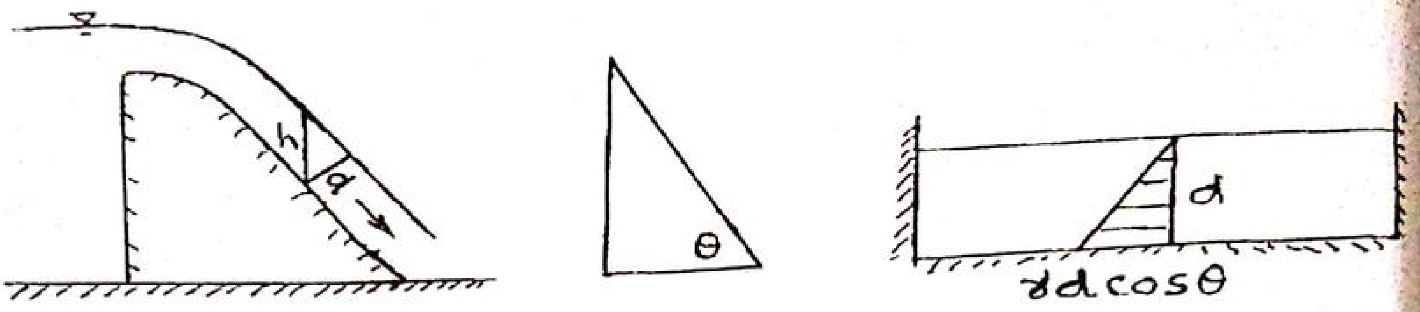
$$p = \frac{Force}{Area} = \frac{W\cos\theta}{\Delta A} = \frac{\gamma d \Delta A \cos\theta}{\Delta A} = \gamma d\cos\theta = \gamma h \cos^2\theta \quad (\because d = h\cos\theta) \quad (1.27)$$

$$\therefore \text{ Pressure head, } \frac{p}{\gamma} = d\cos\theta = h\cos^2\theta \quad (1.28)$$

Thus, for channels of large slope, the pressure head at any vertical depth is equal to the depth of flow h multiplied by a correction factor $\cos^2\theta$. For a small slope channel, $\theta \le 6^o$, the use of Eq. (1.22) instead of Eq. (1.28) involves an error less than about 1% and can safely be ignored. So, the effect of slope on pressure distribution is considered for a channel of large slope only

Example 1.7

Prove that the shear force and the overturning moment on the side walls of a steep rectangular chute are $(1/2)\gamma h^2 \cos^3\theta$ and $(1/6)\gamma h^3 \cos^4\theta$, respectively, where h is the depth of flow and θ is the angle of bottom slope of the chute.



Solution Pressure at the bed of the chute, $p = \gamma d \cos\theta$

Shear force on the side walls, $F = \frac{1}{2} \times p \times d = \frac{1}{2} \times \gamma d \cos\theta \times d$ (*per unit length*)

$$= \frac{1}{2}\gamma d^2 \cos\theta = \frac{1}{2}\gamma h^2 \cos^3\theta \quad (\because d = h\cos\theta)$$

Overturning moment (about point O) $= F \times arm = F \times d/3$

$$= \frac{1}{2}\gamma d^2 \cos\theta \times \frac{d}{3} = \frac{1}{6}\gamma d^3 \cos\theta = \frac{1}{6}\gamma h^3 \cos^4\theta$$

PROBLEMS AND EXERCISES

1.1(a) State whether the following open channel flows are (i) steady or unsteady, (ii) subcritical or supercritical, (iii) gradually varied, rapidly varied or spatially varied, and (iv) laminar or turbulent:

1. Flood flow in a river
2. Flow upstream of a hydraulic jump
3. Overland flow
4. Tidal flow in an estuary
5. Flow downstream of a sluice gate.

(b) State whether the following open channels are (i) natural or artificial, (ii) prismatic or non-prismatic, (iii) rigid or mobile boundary, and (iv) small or large slope channels.

1. Alluvial river
2. Irrigation canal (unlined)
3. Laboratory flume
4. Spillway chute

1.2 Define (i) open channel flow, (ii) prismatic channel, (iii) channel of large slope, (iv) stage, (v) wide channel, (vi) steady flow, (vii) unsteady flow, (viii) uniform flow, (ix) gradually varied flow, (x) rapidly varied flow, (xi) spatially varied flow, (xii) Reynolds number, (xiii) laminar flow, (xiv) turbulent flow, (xv) Froude number, (xvi) subcritical flow, (xvii) critical flow, (xviii) supercritical flow, (xix) parallel flow, and (xx) curvilinear flow.

1.3(a) Write three differences between pipe flow and open channel flow.

(b) State why the solution of an open channel flow problem becomes difficult when the depth of flow is unknown.

(c) State why uniform flow can be steady only.

(d) How can you physically identify whether the flow in an open channel is laminar or turbulent?

(e) How can you physically identify whether the flow in an open channel is subcritical, critical or supercritical?

(f) Explain why the Froude number is more important than the Reynolds number to determine the state of open channel flow.

(g) Explain why an elementary wave can move upstream in subcritical flow, but not in supercritical flow. What is the practical significance of this phenomenon?

1.4 State why the velocity distribution coefficients α and β are used in open channel flow. Deduce expressions for them. Why are the numerical values of α and β greater for laminar flows than for turbulent flows?

1.5 Deduce the expression for the pressure head correction for curvilinear flow (Eq.1.24).

1.6(a) The depth and mean velocity upstream and downstream of a vertical sluice gate in a horizontal rectangular channel are 4 m and 1 m and 2 m/s and 8 m/s, respectively. The width of the channel is 6 m. Determine the state of flow both upstream and downstream of the gate.

(b) Consider the following data for the Padma (Ganges) river at the Baruria station in Faridpur on the 2nd July, 1989: $A = 33{,}500\ m^2$, $Q = 56{,}200\ m^3/s$ and $B = 3820$ m. Compute the state of flow. Assume that the river is wide.

1.7 Water flows in an open channel at a depth of 1 m and a mean velocity of 3 m/s. Compute the discharge and determine the state of flow if the channel is

i) wide	ii) rectangular with $b = 6$ m
iii) trapezoidal with $b = 6$ m and $s = 2$	iv) triangular with $s = 2$
v) parabolic with $B = 4$ m	vi) circular whose diameter is 2.5 m.

If elementary waves are created in these channels, determine the speeds of the wave fronts upstream and/or downstream.

1.8(a) The average depth of water in a wide river connected to sea is 5 m. Determine the time taken by a tidal wave to travel from the river mouth to 30 km upstream (i) when there is no flow in the river, and (ii) when the average flow velocity in the river is 1 m/s.

(b) Waves of small amplitude are created at the center of a circular-shaped pond of radius 50 m. The waves are found to reach the edge of the pond in 10 s. Estimate the approximate volume of water in the pond assuming that the depth of water in the pond is same everywhere.

(c) Two islands A and B situated in an ocean are 1000 km apart. An earthquake in island A creates a single tidal wave (tsunami) which propagates across the ocean to island B. If the average water depth in the ocean is 1 km, estimate the time of arrival of the tsunami in island B.

1.9(a) In a wide river the velocity varies along a vertical as $u = 1 + 2z/h$, where h is the total depth and u is the velocity at a distance z from the channel bottom. The river is 5 m deep. (i) Compute the discharge per unit width, (ii) determine the state of flow, and (iii) compute the numerical values of the velocity distribution coefficients α and β and the ratio $(\alpha-1)/(\beta-1)$.

(b) Solve the above problem if $u = 1+2(z/h)^{1/2}$.

1.10(a) For laminar flow the velocity distribution along a vertical can be approximated by

$$u = u_o \sin \frac{\pi z}{2h}$$

where u is the velocity at a distance z from the channel bottom, h is the depth of flow and u_o is the velocity at the free surface. Compute the velocity distribution coefficients α and β and the ratio $(\alpha-1)/(\beta-1)$.

(b) For turbulent flow the velocity distribution along a vertical can be approximated by $u \propto z^n$, when $n = 1/7$ (Prandtl's one-seventh power law). Determine the velocity distribution coefficients α and β and the ratio $(\alpha-1)/(\beta-1)$ in terms of n and for $n = 1/7$. Compare the numerical values of α and β with those obtained for laminar flow in Problem 1.10(a).

1.11(a) Solve Example 1.5 using the Simpson's rules of numerical integration.

(b) Figure 1.14 shows the velocity distribution downstream of a sluice gate under submerged condition. Using the trapezoidal rule of numerical integration, compute the discharge per unit width, the mean velocity of flow and the numerical values of α and β.

1.12(a) A steep rectangular chute has a slope of 1H:3V. Compute the pressure at the bed of the chute if the vertical depth of water flowing over the chute is 1 m. Also, compute the force and the overturning moment on its side walls.

(b) While computing the shear force and the overturning moment on the side walls of a steep spillway chute having an inclination of 1H:3V and h = 1 m, an engineer assumed that the pressure distribution is hydrostatic. Are the computed results correct? If not, compute (i) the correct values of the shear force and the overturning moment, and (ii) the % error.

(c) A high-head overflow spillway is shown in Fig.1.15. The flip bucket at the toe of the spillway acts to change the direction of flow from the slope of the spillway face to the horizontal and to discharge the flow into the air. If $r_1 = r_2 = 20$ m, $h_1 = h_2 = h_3 = h_4 = 1$ m and the discharge over the spillway is 6.5 $m^3/s/m$, determine the intensities of pressure at points 1, 2, 3 and 4

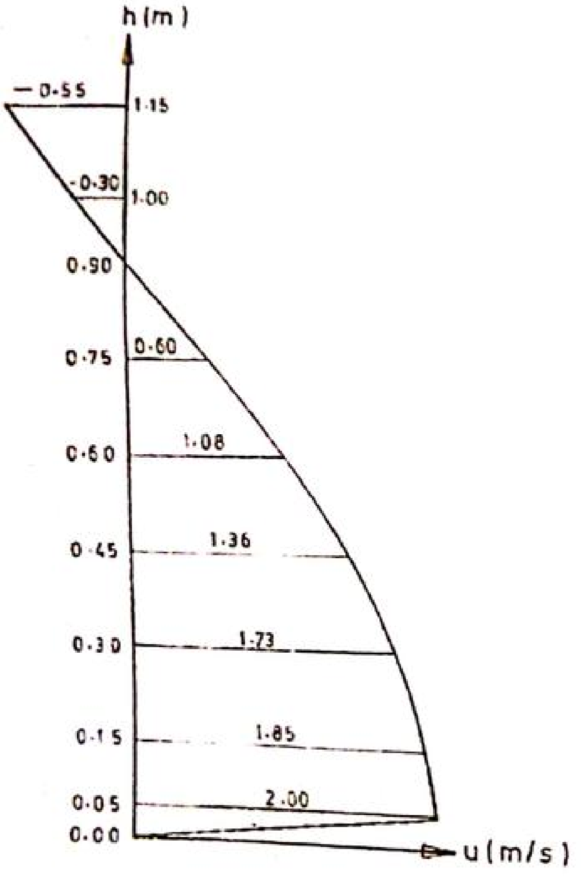


Fig. 1.14

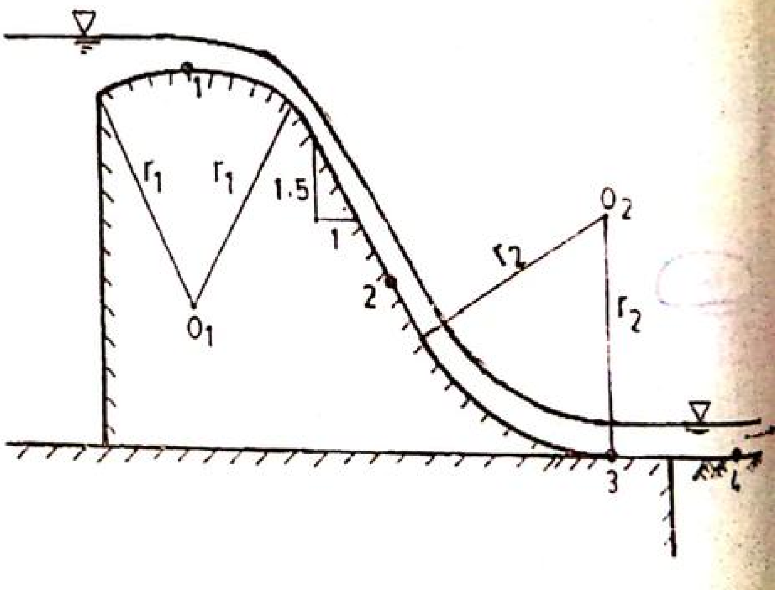


Fig. 1.15

Appendix 1.1

CELERITY OF A SMALL - AMPLITUDE WAVE

The celerity of a small-amplitude wave can be obtained considering the movement of a solitary wave of height Δh traveling to the right with celerity c in a channel as shown in Fig.1.16a. The cross-sectional areas of the channel corresponding to depths h and h + Δh are A and A + ΔA, respectively. The situation defined by Fig.1.16a is obviously unsteady and cannot be analyzed by elementary techniques. However, if the wave form does not change during its travel, the situation may be rendered into one of steady flow by applying a velocity of magnitude c in the direction opposite to that of wave travel as shown in Fig 1.16b.

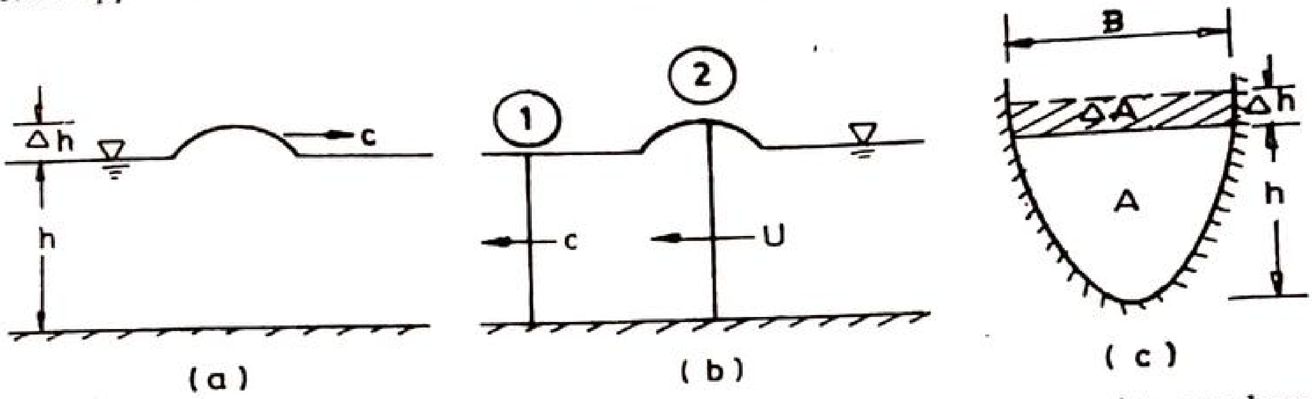


Fig.1.16 Propagation of a wave: (a) unsteady flow, (b) steady flow appeared to an observer moving with the wave crest, and (c) section through the wave crest

Now, application of the equation of continuity for steady flow (derived in Art.2.2) between sections 1 and 2 yields

$$cA = U(A + \Delta A)$$

which on simplification gives

$$U = c\left(1 - \frac{\Delta A}{A}\right) \tag{1.29}$$

Assuming a horizontal bed and neglecting friction, application of the energy equation (presented in Art.2.3) between sections 1 and 2 yields

$$h + \frac{c^2}{2g} = h + \Delta h + \frac{c^2}{2g}\left(1 - \frac{\Delta A}{A}\right)^2 \tag{1.30}$$

With reference to Fig.1.16c, the elementary water area ΔA near the free surface is equal to BΔh. Since the hydraulic depth D = A/B, solving Eq.(1.30) for c and simplifying, one obtains

$$c = \sqrt{gD} \tag{1.31}$$

For a rectangular channel, D = h and the celerity of a wave of small amplitude is given by

$$c = \sqrt{gh} \tag{1.32}$$

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Chapter 2

BASIC EQUATIONS FOR STEADY ONE-DIMENSIONAL FLOW

2.1 GENERAL

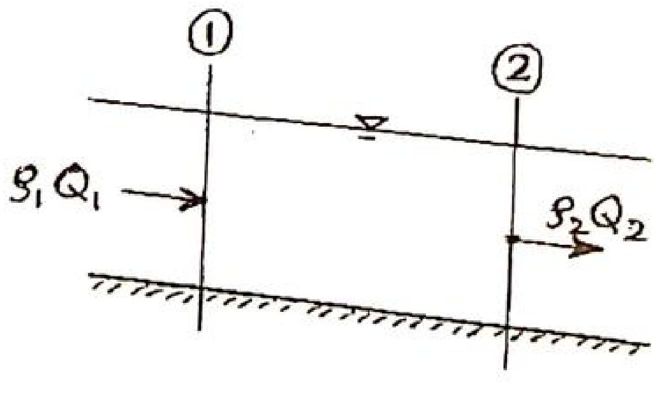
Water movement is the key process in open channel flow. The three basic equations to describe water movement are the continuity, the energy and the momentum equations derived from the three fundamental laws of physics – conservation of mass, conservation of energy and conservation of momentum. The conservation of mass says that mass can neither be created nor destroyed. The conservation of energy says that energy cannot be created or destroyed, but may be transformed from one form to another. The law of conservation of momentum says that a moving body cannot gain or lose momentum unless acted upon by an external force, which is a statement of Newton's second law of motion.

As stated earlier, the flow in an open channel is, in fact, three-dimensional and the flow properties like the velocity and the pressure vary in the longitudinal, lateral and vertical directions. Since the variations of the flow parameters in the lateral and vertical directions are small compared to those in the longitudinal direction, a majority of open channel flow problems are analyzed by considering that the flow is one-dimensional, i.e. the variations of the flow properties only in the longitudinal direction are taken into consideration. This analysis gives the cross-sectional mean values of the flow properties, which do not vary within the cross-section, but vary from section to section.

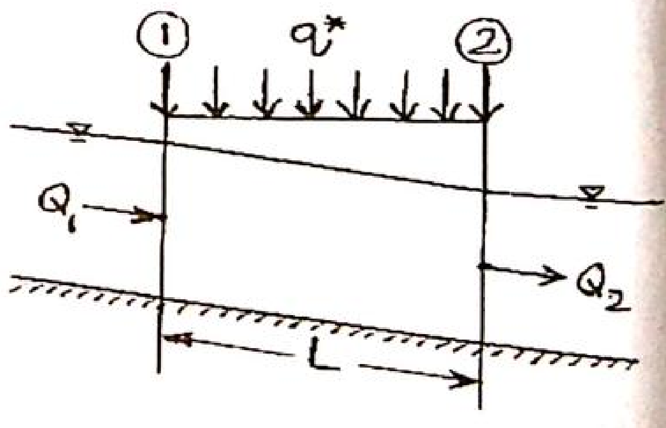
2.2 CONTINUITY EQUATION

The principle of conservation of mass implies that during a time interval the mass of water entering the control volume minus the mass of water leaving the control volume equals the change of mass within the control volume. If the flow is steady, there cannot be any change of mass within the control volume and the mass entering is equal to the mass leaving. Therefore, if there is no lateral addition or withdrawal of water (Fig.2.1a), the mass flow rate (mass per unit time) passing various flow sections must be the same, i.e.

$$\rho_1 Q_1 = \rho_2 Q_2 = \ldots\ldots \quad (2.1)$$



(a) Without lateral inflow or outflow



(b) With lateral inflow

Fig.2.1 Definition sketch for the continuity equation where ρ is the mass density of water and Q is the discharge (volume flow rate) and the subscripts designate different channel sections.

Since $Q = AU$ and water is practically incompressible ($\rho_1 = \rho_2$), Eq.(2.1) takes the form

$$Q_1 = Q_2 = \ldots \qquad or \qquad U_1 A_1 = U_2 A_2 = \ldots \tag{2.2}$$

where A is the cross-sectional area and U is the mean velocity of flow.

Equation (2.2) is the usual form of the continuity equation for steady one-dimensional open channel flow of an incompressible fluid without lateral inflow or outflow. It indicates that *in steady flow not only $\partial Q/\partial t = 0$, but also $\partial Q/\partial x = 0$, i.e. the discharge in the channel is constant.*

If there is a lateral addition or withdrawal of water at the rate of q^* per unit length (Fig.2.1b), flow is spatially varied (Art. 1.4), and the discharge Q_2 at section 2 at a distance L from section 1 where the discharge is Q_1 is given by

$$Q_2 = Q_1 \pm q^* L \tag{2.3}$$

where the + sign is to be used for inflow and the - sign is to be used for outflow.

2.3 ENERGY EQUATION

The total energy or head at a channel section is given by

$$H = z_b + h + \alpha \frac{U^2}{2g} \tag{2.4}$$

where z_b is the elevation or datum head, h is the depth of flow and $\alpha U^2/2g$ is the velocity or kinetic energy head based on the cross-sectional mean velocity U and corrected for the non-uniform distribution of velocity and α is the kinetic energy coefficient. For a channel of large slope, Eq. (2.4) becomes

$$H = z_b + h \cos^2\theta + \alpha \frac{U^2}{2g} \tag{2.5}$$

According to the principle of conservation of energy, the total energy at the upstream section 1 must be equal to the total energy at the downstream section 2 plus the frictional loss of energy h_f between the two sections (Fig. 2.2), i.e.

$$H_1 = H_2 + h_f$$

or

$$z_{b1} + h_1 + \alpha_1 \frac{U_1^2}{2g} = z_{b2} + h_2 + \alpha_2 \frac{U_2^2}{2g} + h_f \tag{2.6}$$

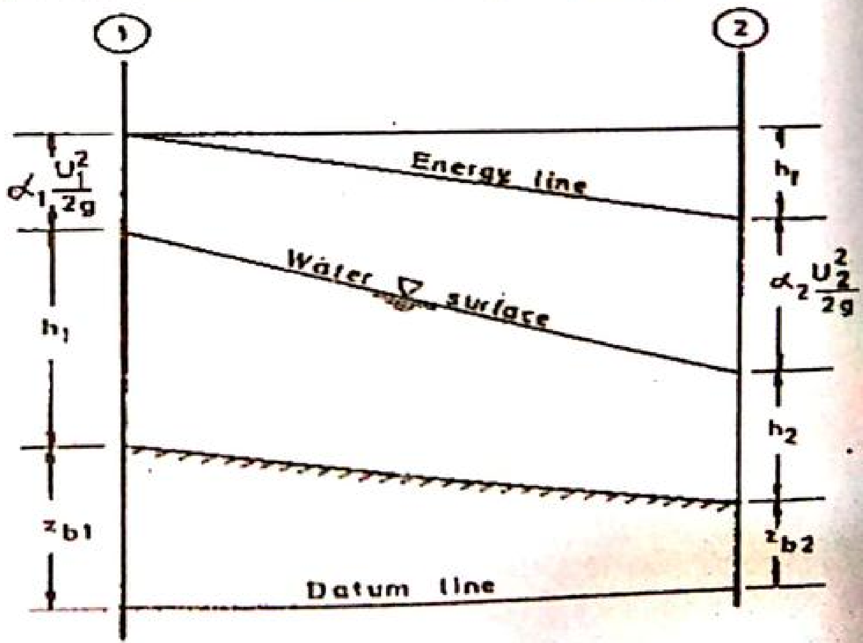


Fig. 2.2 Definition sketch for the energy equation

F-4

For a channel of large slope, Eq.(2.6) becomes

$$z_{b1} + h_1 \cos^2\theta + \alpha_1 \frac{U_1^2}{2g} = z_{b2} + h_2 \cos^2\theta + \alpha_2 \frac{U_2^2}{2g} + h_f \qquad (2.7)$$

Either of these two equations is known as the *energy equation.*

For an ideal fluid, $\alpha_1 = \alpha_2 = 1$ and $h_f = 0$. Hence, Eq.(2.6) becomes

$$z_{b1} + h_1 + \frac{U_1^2}{2g} = z_{b2} + h_2 + \frac{U_2^2}{2g} \qquad (2.8)$$

This is the well-known *Bernoulli equation.*

Each term in the energy equation has the dimension of length and represents energy per unit weight of water, i.e. in m-N/N or m-kg/kg or simply m. This is why the term 'head' is used for them. The expression of energy in this form is particularly convenient for dealing with problems in open channel flow.

For the flow of a real fluid like water, the total energy decreases in the downstream direction due to energy or head loss. As a result, in an open channel flow the energy grade line always slopes downward.

The frictional energy loss term h_f in the energy equation signifies the energy that is transformed from mechanical (potential plus kinetic) energy into heat energy. This energy slightly raises the temperature of water, the channel and the surroundings.

Normally, the total head loss h_L for flow in a non-prismatic channel can be considered to be made up of the frictional head loss h_f and the eddy loss h_e, i.e.

$$h_L = h_f + h_e \qquad (2.9)$$

The eddy loss h_e results from flow contractions and expansions and may be appreciable in non-prismatic channels. It is generally taken to be proportional to the absolute magnitude of the change in the velocity head between the two sections, or

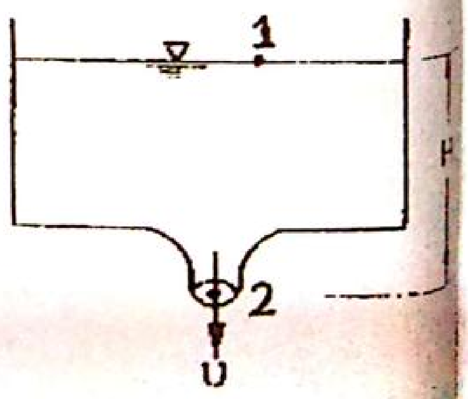
$$h_e = k\left|\alpha_1 \frac{U_1^2}{2g} - \alpha_2 \frac{U_2^2}{2g}\right| \qquad (2.10)$$

where k is a coefficient which is taken to be 0.1 for gradual contractions, 0.2 for gradual expansions and 0.5 for abrupt expansions or contractions. The eddy loss is usually neglected in prismatic channels.

The loss of energy may also be due to other reasons, like the presence of bends, flow past submerged bodies, etc. and has to be included in the energy equation when it is encountered.

Example 2.1 Derivation of the Law of Torricelli

If a constant level is maintained in a vessel with atmospheric pressure at the water surface and at the discharge point (Fig. 2.3), then application of the Bernoulli equation between points 1 and 2 yields



$$H + 0 + \frac{U_1^2}{2g} = 0 + 0 + \frac{U_2^2}{2g} \qquad (2.11)$$

Fig. 2.3

For a wide vessel, $U_1 << U_2$. Hence, from Eq.(2.11), writing U for U_2, it follows that

$$U = \sqrt{2gH} \qquad (2.12)$$

which is the Law of Torricelli.

2.4 MOMENTUM EQUATION

The momentum equation is based on Newton's second law of motion which states that the algebraic sum of all the external forces acting on a fluid mass in any particular direction is equal to the time rate of change of momentum in that direction.

Let us consider the control volume bounded by sections 1 and 2 (Fig. 2.4). The various forces acting on the control volume in the longitudinal direction are:

i) the resultant hydrostatic pressure forces F_{p1} and F_{p2} at the two end sections,

ii) the force due to gravity, Wsinθ, which is the component of the weight of water in the longitudinal direction, and

iii) the external frictional force F_f due to friction acting on the surface of contact between water and the channel.

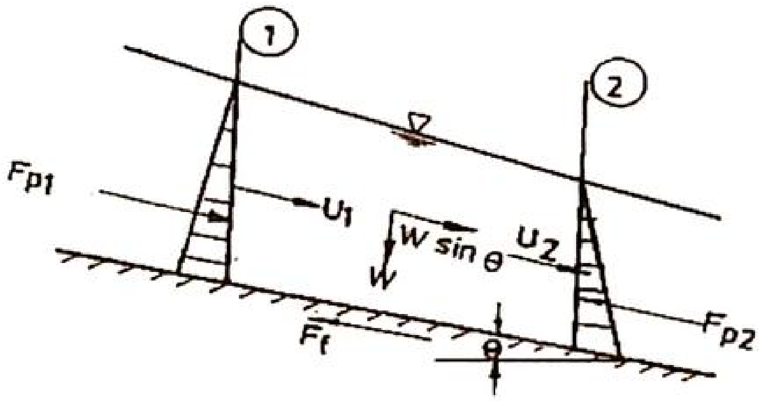


Fig.2.4 Definition sketch for the momentum equation

The momentum of flow passing a channel section per unit time based on the mean velocity U and corrected for the non-uniform distribution of velocity = βρQU, where β is the momentum coefficient. Then, applying the Newton's second law of motion, we can write

$$\rho Q(\beta_2 U_2 - \beta_1 U_1) = F_{p1} - F_{p2} + W \sin\theta - F_f \qquad (2.13)$$

which is the momentum equation for one-dimensional steady flow for the flow situation shown in Fig. 2.4.

Note that the terms in the momentum equation have the units of force, i.e. Newton or kg-m/s^2.

If any other external force is present, it has to be included in the momentum equation. The momentum equation is a particularly useful tool in analyzing rapidly varied expanding flow like a hydraulic jump where the energy losses are significant. The momentum equation is very useful in estimating the forces on different hydraulic structures (e.g. force on a sluice gate).

Example 2.2

Derive the expression for the normal force when a jet of water strikes a stationary flat plate.

Solution When a jet of water with a velocity U strikes a stationary flat plate normally (Fig. 2.5), the force on the plate is equal to the rate of change of momentum of the jet. The jet leaves the plate tangentially so that all its momentum in a direction normal to the plate is destroyed. Hence, the normal force on the plate is

$$F = \rho Q(U - 0) = \rho QU = \rho AU^2 \qquad (2.14)$$

where A is the cross-sectional area of the jet.

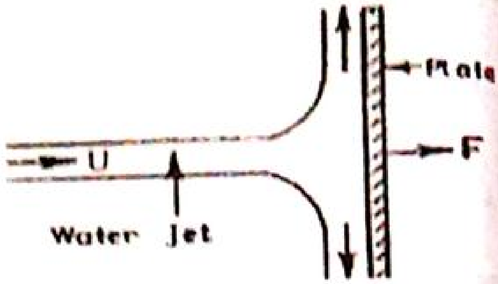


Fig. 2.5 Jet striking a flat plate

2.5 APPLICABILITY OF THE EQUATIONS

For steady one-dimensional open channel flow we have three basic equations that can be used to compute three unknown quantities. However, in many flow problems, usually it is required to compute two unknown quantities and we need only two equations. For example, it may be required to compute flow depth h and flow velocity U at a downstream section in a channel when the flow conditions at an upstream section are known. The equation of continuity is invariably used as it is the simplest of the three equations. Then, the choice remains whether we will use the energy or the momentum equation.

The energy equation contains a term h_L of internal energy losses. So this equation can be used initially only when this energy loss term h_L is small and negligible. On the other hand, the momentum equation contains an external friction force F_f. So this equation can be used initially only when F_f is small and negligible and if any other external force is not present.

As an example, let us consider the flow under a sluice gate as shown in Fig. 2.6. Flow under a sluice gate is an example of converging flow in which energy losses between sections 1 and 2 are usually small and negligible. The flow depth h_2 and the flow velocity U_2 at section 2 can be determined from the known flow conditions at section 1 and using the continuity and energy equations. Initially the momentum equation cannot be used for this situation because of the force on the sluice gate that is unknown and not negligible, although the external friction force F_f is small and negligible. However, once h_2 and U_2 are determined, the momentum equation can be used to compute the force on the sluice gate.

For the hydraulic jump downstream of the sluice gate, the energy equation cannot be initially used because of the significant energy loss h_L involved in the jump. However, the momentum equation can be used without difficulty since the jump takes place in a short distance and the friction force F_f is small and negligible and there is no other external force. Therefore, the continuity and momentum equations are used to compute the flow depth h_3 and the flow velocity U_3. The energy equation can then be used to compute the energy loss h_L.

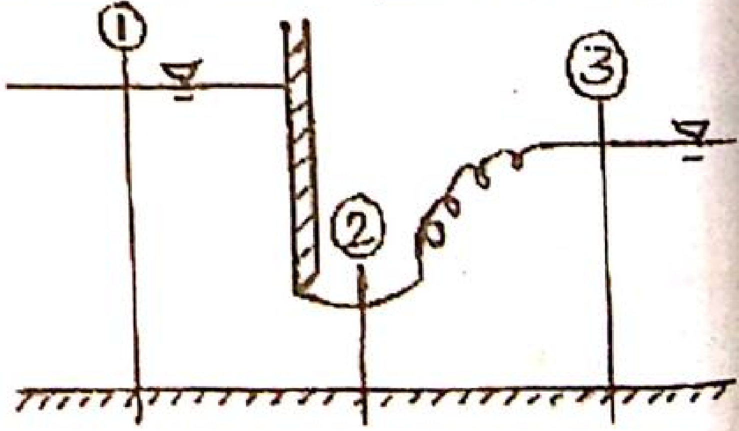


Fig.2.6 Flow beneath a sluice gate

For steady uniform and gradually varied flows, the energy and the momentum equations give similar results and we can use either of these two equations. However, the energy equation is preferred because it is easy to understand and use than the momentum equation, as energy is a scalar quantity and momentum is a vector quantity.

Example 2.3

Figure 2.7 shows a sharp-crested weir in a rectangular channel. If the discharge per unit width of the weir is 4 m^2/s, estimate the energy loss due to the weir and force on the weir plate for the submerged flow condition as shown.

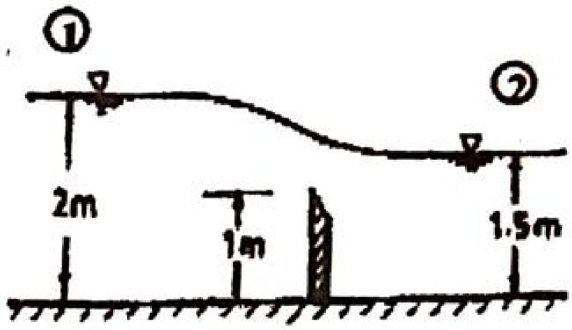


Fig. 2.7

Solution Let the force exerted by the weir plate on water is F. Then, assuming unit width

$$U_1 = \frac{q}{h_1} = \frac{4}{2} = 2\ m/s$$

$$U_2 = \frac{q}{h_2} = \frac{4}{1.5} = 2.67\ m/s$$

Applying the energy equation between sections 1 and 2, we obtain

$$h_1 + \frac{U_1^2}{2g} = h_2 + \frac{U_2^2}{2g} + h_f$$

$$\therefore h_f = (h_1 - h_2) + (\frac{U_1^2}{2g} - \frac{U_2^2}{2g}) = (2.00 - 1.50) + \frac{(2^2 - 2.67^2)}{2 \times 9.81}$$

$$= 0.50 - 0.16 = 0.34\ m\ of\ water$$

Applying the momentum equation between sections 1 and 2 and assuming unit width, we obtain

$$\rho q(U_2 - U_1) = \frac{1}{2}\gamma h_1^2 - \frac{1}{2}\gamma h_2^2 - F$$

$$\therefore F = \frac{1}{2}\gamma(h_1^2 - h_2^2) - \rho q(U_2 - U_1)$$

$$= \frac{1}{2} \times 1000 \times 9.81 \times (2^2 - 1.5^2) - 1000 \times 4 \times (2.67 - 2.0)$$

$$= 8583.75 - 2666.67 = 5917.08\ N$$

The force on the weir plate is equal and opposite to F.

PROBLEMS AND EXERCISES

2.1 What are the three basic equations to describe open channel flow? State the principles on which these are based.

2.2(a) What does the frictional loss term h_f signify?

(b) When does the eddy loss h_e occur?

(c) State the applicability of the energy equation for (i) contracting flow, and (ii) expanding flow.

(d) Which water will be warmer from the hydraulic point of view – water stored in a tank or water flowing out of a tap? Why?

2.3 Derive the momentum equation (Eq.2.13). State two uses of this equation.

2.4 When a Pitot tube is placed under water as shown in Fig. 2.8, the water rises in the tube to a height H. Show that the velocity of stream upstream of the tube is

$$u = \sqrt{2gH} \tag{2.15}$$

2.5 The inlet and exit angles of a ski-jump spillway (Fig.2.9) are 45° and the flow over it has a velocity of 25 m/s and a depth of 0.5 m. Neglecting all losses, estimate the maximum elevation of the outflow trajectory. Also, compute the horizontal and vertical forces on the spillway as a result of the change in the flow direction. Assume unit width.

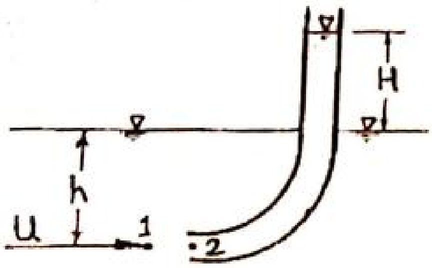


Fig. 2.8 (Problem 2.4)

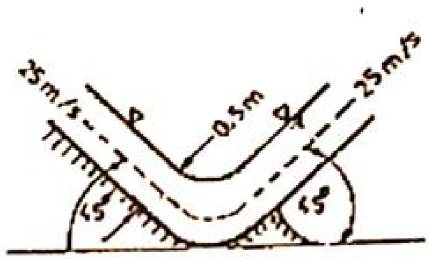


Fig. 2.9 (Problem 2.5)

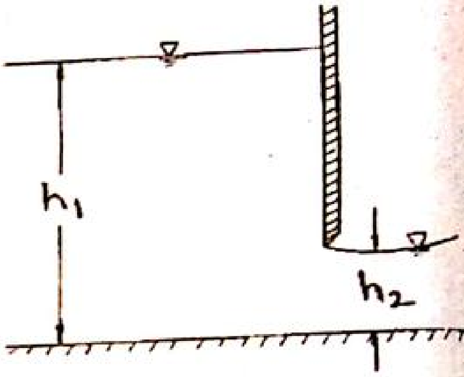


Fig. 2.10 (Problem 2.6)

2.6(a) Show that the force on a vertical sluice gate in a horizontal rectangular channel (Fig.2.10) is given by

$$F = \frac{1}{2}\gamma \frac{(h_1 - h_2)^3}{h_1 + h_2} \tag{2.16}$$

where γ is the specific weight of water.

(b) The depths of flow a short distance upstream and at the vena contract a downstream of a vertical sluice gate in a horizontal rectangular channel are 4 m and 1 m, respectively. The width of the channel is 6 m.

i) Neglecting energy losses and taking $\alpha_1 = \alpha_2 = 1$, compute the discharge under the gate.

ii) Compute the force on the sluice gate and compare it with that obtained by assuming hydrostatic distribution of pressure. Assume that the coefficient of contraction for the sluice gate, $C_c = 0.61$.

2.7 A bridge across a river has its piers placed symmetrically at the rate of 30 m center to center. Upstream of the bridge the water depth is 10 m and the velocity is 4 m/s. When the flow has gone far enough downstream to even out again after the disturbance caused by the piers, the water depth is 9 m. Compute the thrust on each pier. Neglect the bed slope and the bed friction.

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Chapter 3

SPECIFIC ENERGY AND CRITICAL FLOW

3.1 SPECIFIC ENERGY

Specific energy (E) at a channel section is the energy measured with respect to the channel bottom. From Eq. (2.4) with $z_b = 0$, the specific energy at a channel section becomes

$$E = h + \alpha \frac{U^2}{2g} \tag{3.1}$$

Equation (3.1) indicates that the specific energy is the sum of the depth of flow and the velocity head. Since U = Q/A, Eq. (3.1) may also be written with α = 1 as

$$E = h + \frac{Q^2}{2gA^2} \tag{3.2}$$

Equation (3.2) shows that the specific energy depends on the channel section, the depth of flow h and the discharge Q.

The concept of specific energy introduced by Bakhmeteff in 1912 is very useful in the analysis of many open channel flow problems.

Specific Energy Curve

The variation of specific energy with depth for a given section and a constant discharge using Eq. (3.2) is shown in Fig. 3.1. The resulting curve, which is known as the specific energy curve or E-h curve, has two limbs CA and CB. As $h \to 0$, $U^2/2g \to \infty$, $E \to \infty$ and the limb CA approaches the E axis asymptotically toward the right. As $h \to \infty$, $U^2/2g \to 0$, $E \to h$ and the limb CB approaches the line OP whose equation is E = h.

For all points on the specific energy curve except point C, there are two values of h for a given value of E, the lower depth h_1 and the higher depth h_2. These are known as the *alternate depths*. At point C on the specific energy curve, the specific energy is minimum. The state of flow represented by point C is obtained by taking the first derivative of E with respect to h from Eq.(3.2) keeping Q constant, i.e.

$$\frac{dE}{dh} = 1 + \frac{Q^2}{2g}\frac{d}{dh}(A^{-2}) = 1 + \frac{Q^2}{2g}(-2)A^{-3}\frac{dA}{dh} = 1 - \frac{Q^2}{gA^3}\frac{dA}{dh} = 1 - \frac{U^2}{gA}\frac{dA}{dh}$$

The elementary water area near the free surface is $dA = Bdh$, so that $dA/dh = B$. Since $D = A/B$ and $Fr = U/\sqrt{gD}$, we obtain

$$\frac{dE}{dh} = 1 - \frac{U^2 B}{gA} = 1 - \frac{U^2}{gA/B} = 1 - \frac{U^2}{gD} = 1 - Fr^2 \tag{3.3}$$

Now, for minimum specific energy, dE/dh = 0, and hence $1 - Fr^2 = 0$, i.e. Fr = 1, which is the criterion for critical flow. Thus, *at the critical state of flow, the specific energy is minimum for a given discharge*.

Obviously, when the depth of flow is greater than the critical depth, the velocity of flow is less than the critical velocity for the given discharge and the flow is subcritical. When the depth of flow is less than the critical depth, the velocity of flow is greater than the critical velocity and the flow is supercritical. Hence, h_1 is the depth of supercritical flow and h_2 is the depth of subcritical flow and the limbs CB and CA represent subcritical and supercritical flows, respectively.

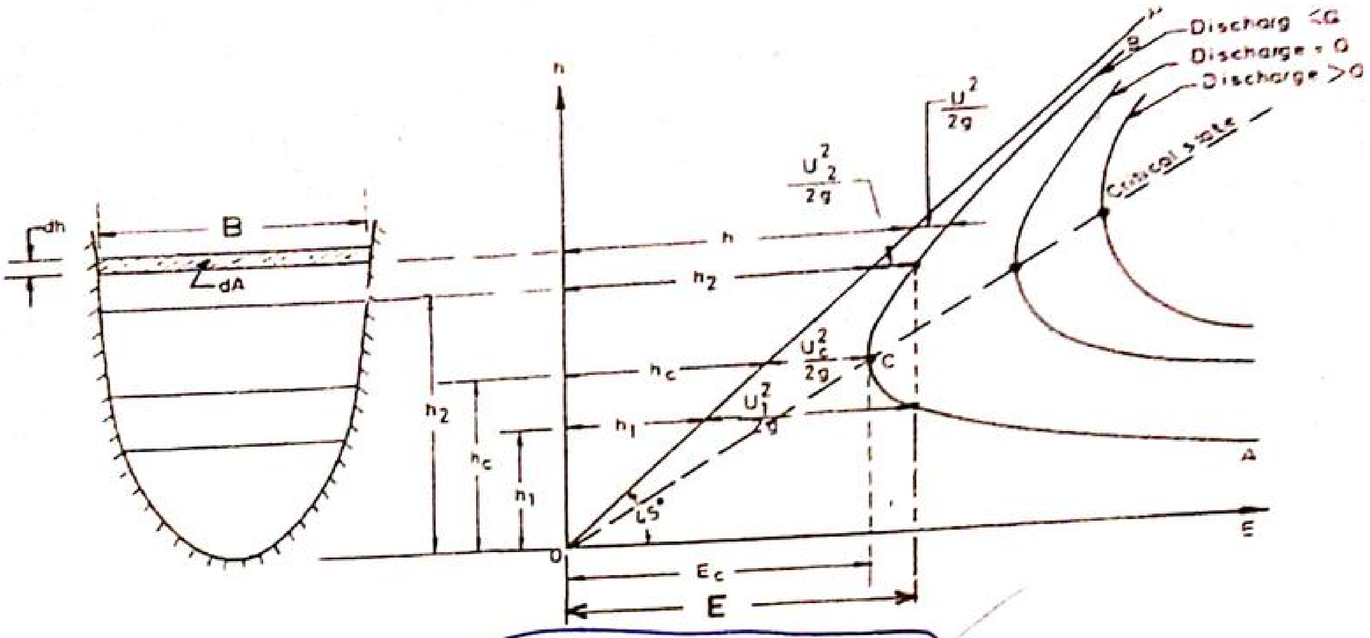


Fig. 3.1 Specific energy curve

When the discharge changes, the specific energy also changes. As shown in Fig. 3.1, the specific energy curve for the flow rate greater than Q lies to the right of the curve for Q indicating that the specific energy increases with an increase in discharge and vice versa.

In an open channel flow the energy grade line always slopes downward and the available energy is decreased. The specific energy, however, remains constant for uniform flow and can either increase or decrease along the channel in varied flow.

The E-h curve is almost vertical near the critical state (Fig. 3.1) and a small change in E results in a large change in h. As a result, flow at or near the critical condition is unstable and there will be a wavy water surface. Hence, it is undesirable to design channels at or near the critical state.

Discharge-Depth Curve

So far the variation of E with h for a given Q has been considered. It is also of practical interest to study the variation of Q with h for a given E. Equation (3.2) may be written as

$$Q^2 = 2gA^2(E-h) \qquad (3.4)$$

Clearly, the variation of Q with h will be of the general form shown in Fig. 3.2. When either h = 0 or h = E, Q = 0 and there will be a maximum value of Q for some value of h between 0 and E which may be obtained as follows. Differentiating Eq. (3.4) with respect to h keeping E constant and using Eq.(3.1), we obtain

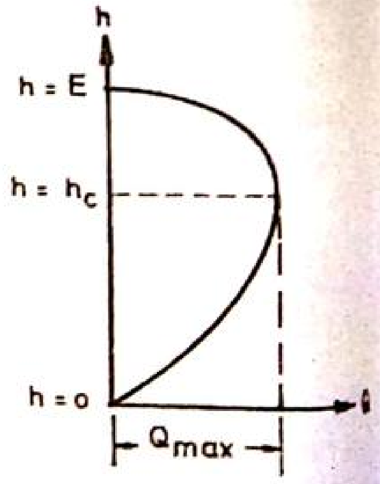


Fig.3.2 Discharge-depth curve

$$2Q\frac{dQ}{dh} = 2g[A^2(-1)+(E-h)\times 2A\frac{dA}{dh}] = 2g[-A^2+\frac{U^2}{2g}\times 2AB]$$

$$= -2gA^2[1-\frac{U^2}{gA/B}] = -2gA^2[1-\frac{U^2}{gD}] = -2gA^2(1-Fr^2) \qquad (3.5)$$

For maximum discharge, dQ/dh = 0, and we obtain Fr = 1, which is the criterion for critical flow. Thus, *at the critical state of flow, the discharge is maximum for a given specific energy.* When the specific energy increases, the discharge also increases and vice versa.

3.2 CRITICAL FLOW

Section Factor

The critical flow is the flow for which the Froude number is equal to unity. Using Eq.(1.12), it can be shown that at the critical state of flow

$$\frac{U_c^2}{2g} = \frac{D_c}{2} \quad (3.6)$$

and when $\alpha \neq 1$

$$\alpha\frac{U_c^2}{2g} = \frac{D_c}{2} \quad (3.7)$$

i.e., the velocity head is equal to one-half of the hydraulic depth.

The product of the flow area and the square root of the hydraulic depth is known as the section factor in connection with critical flow, denoted by Z, i.e.

$$Z = A\sqrt{D} \quad (3.8)$$

It can be computed when the channel section and the depth of flow h are given.

When the flow is critical, the product of the flow area and the square root of the hydraulic depth is known as the section factor for critical flow computation, denoted by Z_c, i.e.

$$Z_c = A_c\sqrt{D_c} \quad (3.9)$$

Since U = Q/A, Eqs. (3.7) and (3.9) give

$$\alpha\frac{Q^2}{2gA_c^2} = \frac{D_c}{2} \quad or, \; A_c^2 D_c = \frac{Q^2}{g/\alpha} \quad or, \; Z_c = \frac{Q}{\sqrt{g/\alpha}} \quad (3.10)$$

Thus, Z_c can be computed using Eq.(3.9) when the channel section and the critical depth h_c are given, or alternatively, using Eq.(3.10) when the discharge Q and the energy coefficient α are given. Equations (3.9) and (3.10) are very useful in the analysis of critical flow.

Using Eqs.(1.12), (3.8) and (3.10), we obtain

$$Fr^2 = \frac{\alpha U^2}{gD} = \frac{\alpha Q^2}{gA^2 D} = \frac{\alpha Q^2/g}{A^2 D} = \frac{Z_c^2}{Z^2} \quad (3.11)$$

Obviously, when the flow is critical

$$Fr = 1 \;\; \therefore \;\; 1 - Fr^2 = 0 \;\; or, \; 1 - \frac{Z_c^2}{Z^2} = 1 - \frac{\alpha Q^2}{gA^2 D} = 0 \quad (3.12)$$

Hydraulic Exponent for Critical Flow Computation

The section factor Z is a function of the depth of flow for a given channel section. It is convenient to express Z in the form

$$Z^2 = C_1 h^M \quad (3.13)$$

where C_1 is a coefficient and M is an exponent which is known as the hydraulic exponent for critical flow computation.

Taking logarithm of both sides of Eq.(3.13) and then differentiating with respect to h, we obtain

F-5

$$\frac{d(\ln Z)}{dh} = \frac{M}{2h} \qquad (3.14)$$

Also, substituting D = A/B in Eq. (3.8), taking logarithm of both sides of this equation and then differentiating with respect to h, we get

$$\frac{d(\ln Z)}{dh} = \frac{3}{2}\frac{1}{A}\frac{dA}{dh} - \frac{1}{2}\frac{1}{B}\frac{dB}{dh} = \frac{3B}{2A} - \frac{1}{2B}\frac{dB}{dh} = \frac{1}{2A}(3B - D\frac{dB}{dh}) \qquad (3.15)$$

Equating the right sides of Eqs. (3.14) and (3.15) and solving for M, we obtain

$$M = \frac{h}{A}(3B - D\frac{dB}{dh}) \qquad (3.16)$$

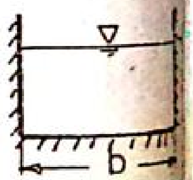
which is the general equation for the hydraulic exponent for critical flow computation M and indicates that M is a function of the channel section and the depth of flow. The values of M for different channel sections are given in Table 3.1.

Table 3.1 Values of M for different channel sections

Channel section	M
1. Rectangle and wide	3
2. Triangle	5
3. Parabola	4
4. Trapezoid	$\frac{3[1+2s(h/b)]^2 - 2s(h/b)[1+s(h/b)]}{[1+2s(h/b)][1+s(h/b)]}$
5. Circle	$(1-\cos\omega/2)\frac{12\sin\omega/2}{\omega-\sin\omega} - \frac{\cos\omega/2}{\sin^2\omega/2}$

Example 3.1

Determine the numerical value of the hydraulic exponent for critical flow computation M for a rectangular channel.

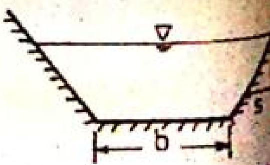


Solution For a rectangular channel, A = bh, B = b, D = A/B = h, dB/dh = 0

$$\therefore\ M = \frac{h}{A}(3B - D\frac{dB}{dh}) = \frac{h}{bh}(3b - h\times 0) = 3$$

Example 3.2

Compute the hydraulic exponent for critical flow computation M for a trapezoidal channel with b = 6.1 m, s = 2 and h = 2 m.



Solution h/b = 2/6.1 = 0.328

$$\therefore\ M = \frac{3[1+2s(h/b)]^2 - 2s(h/b)[1+s(h/b)]}{[1+2s(h/b)][1+s(h/b)]}$$

$$= \frac{3(1+2\times2\times0.328)^2 - 2\times2\times0.328\times(1+2\times0.328)}{(1+2\times2\times0.328)(1+2\times0.328)} = 3.62$$

Alternative solution $A = (6.1 + 2\times2)\times2 = 20.2\ m^2$, $B = 6.1 + 2\times2\times2 = 14.1\ m$, $D = A/B = 20.2/14.1 = 1.43\ m$, $dB/dh = 2s = 2\times2 = 4$

$$M = \frac{h}{A}\left(3B - D\frac{dB}{dh}\right) = \frac{2}{20.2}(3\times14.1 - 1.43\times4) = 3.62$$

3.3 COMPUTATION OF CRITICAL DEPTH

Analytical Method

The critical depth is an important parameter in the analysis of open channel flow. It may be computed when the channel section, the energy coefficient α and the discharge Q are given. For rectangular, triangular and parabolic channels, the formula for the critical depth can be easily obtained.

Rectangular channel

We know, when the flow is critical

$$Z_c = \frac{Q}{\sqrt{g/\alpha}} \quad \text{and} \quad Z_c = A_c\sqrt{D_c}$$

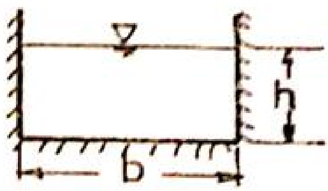
$$A_c\sqrt{D_c} = \frac{Q}{\sqrt{g/\alpha}}$$

or

$$A_c^2 D_c = \frac{\alpha Q^2}{g} \tag{3.17}$$

For a rectangular channel, $A_c = bh_c$ and $D_c = A_c/B_c = bh_c/b = h_c$. Hence

$$b^2 h_c^2 \cdot h_c = \frac{\alpha Q^2}{g}$$



or

$$h_c^3 = \frac{\alpha Q^2}{g b^2}$$

which gives

$$h_c = \sqrt[3]{\frac{\alpha Q^2}{g b^2}} \tag{3.18}$$

Wide Channel

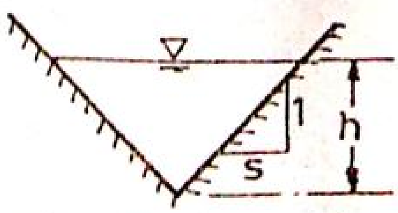
For a wide channel, $q = Q/b$. Therefore

$$h_c = \sqrt[3]{\frac{\alpha q^2}{g}} \tag{3.19}$$

Triangular channel

For a triangular channel, $A_c = sh_c^2$, $B_c = 2sh_c$ and $D_c = A_c/B_c = sh_c^2/2sh_c = h_c/2$. Hence, from Eq. (3.17)

$$s^2 h_c^4 \cdot \frac{h_c}{2} = \frac{\alpha Q^2}{g}$$



or

$$h_c^5 = \frac{2\alpha Q^2}{g s^2}$$

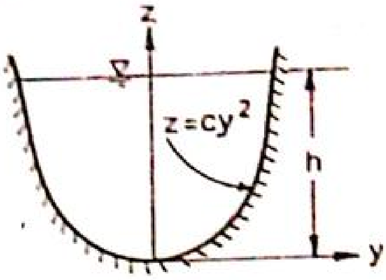
which gives

$$h_c = \sqrt[5]{\frac{2\alpha Q^2}{g s^2}} \qquad (3.20)$$

Parabolic channel $(z = cy^2)$

For a parabolic channel, $A = 4h_c^{3/2}/3\sqrt{c}$, $B = 2h_c^{1/2}/\sqrt{c}$ and $D_c = A_c/B_c = 2h_c/3$. Hence from Eq. (3.17)

$$\frac{16h_c^3}{9c} \cdot \frac{2h_c}{3} = \frac{\alpha Q^2}{g}$$



or

$$h_c^4 = \frac{27\alpha c Q^2}{32g}$$

which gives

$$h_c = \sqrt[4]{\frac{27\alpha c Q^2}{32g}} \qquad (3.21)$$

Example 3.3

Compute the critical depth and velocity in a (i) wide rectangular channel with $q = 3\ m^2/s$, (ii) rectangular channel with $b = 6$ m and $Q = 20\ m^3/s$, (iii) triangular channel with $s = 2$ and $Q = 10\ m^3/s$, and (iv) parabolic channel whose profile is given by $y^2 = 4z$ and $Q = 20\ m^3/s$. In all cases, assume $\alpha = 1.12$

Solution $\alpha = 1.12$

(i) Wide channel $q = 3\ m^2/s$

$$h_c = \sqrt[3]{\frac{\alpha q^2}{g}} = \sqrt[3]{\frac{1.12 \times 3^2}{9.81}} = 1.01\ m$$

$$U_c = \frac{q}{h_c} = \frac{3}{1.01} = 2.97\ m/s$$

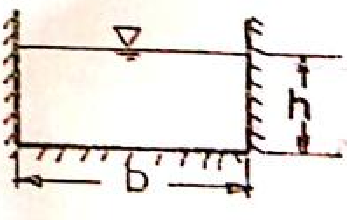
(ii) Rectangular channel, $b = 6$ m, $Q = 20\ m^3/s$

$$h_c = \sqrt[3]{\frac{\alpha Q^2}{g b^2}} = \sqrt[3]{\frac{1.12 \times 20^2}{9.81 \times 6^2}} = 1.08\ m$$

Then,

$$A_c = bh_c = 6 \times 1.08 = 6.50\ m^2$$

$$U_c = \frac{Q}{A_c} = \frac{20}{6.50} = 3.08\ m/s$$



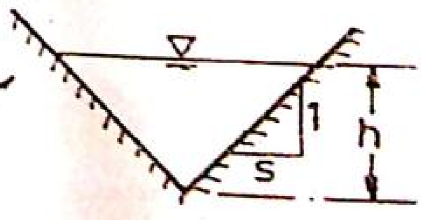
(iii) Triangular channel, $s = 2$, $Q = 10\ m^3/s$

$$h_c = \sqrt[5]{\frac{2\alpha Q^2}{g s^2}} = \sqrt[5]{\frac{2 \times 1.12 \times 10^2}{9.81 \times 2^2}} = 1.42\ m$$

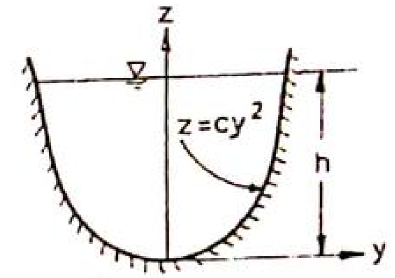
Then,

$$A_c = sh_c^2 = 2 \times 1.42^2 = 4.01\ m^2$$

$$U_c = \frac{Q}{A_c} = \frac{10}{4.01} = 2.49\ m/s$$



(iv) Parabolic channel, Q = 20 m³/s. Since $y^2 = 4z$, $z = 0.25y^2$ and hence $c = 0.25$



$$h_c = \sqrt[4]{\frac{27\alpha c Q^2}{32g}} = \sqrt[4]{\frac{27\times1.12\times0.25\times20^2}{32\times9.81}} = 1.76\ m$$

Then,

$$A_c = \frac{4h_c^{3/2}}{3\sqrt{c}} = \frac{4\times1.76^{3/2}}{3\times\sqrt{0.25}} = 6.24\ m^2$$

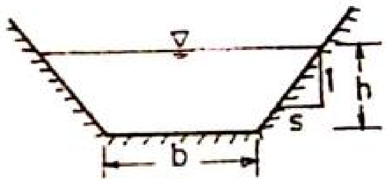
$$U_c = \frac{Q}{A_c} = \frac{20}{6.24} = 3.21\ m/s$$

Trial-and-Error Method

For other channel sections, like the trapezoidal and circular sections, the critical depth can be conveniently obtained by the trial-and-error solution of Eqs. (3.9) and (3.10). The actual value of the section factor Z_c is obtained using Eq.(3.10). Then we assume several values of h and compute the section factor Z using Eq.(3.9) until the computed value of Z is close to the actual value of Z_c.

Example 3.4

For a trapezoidal channel with b = 6 m and s = 2, compute the critical depth and velocity if Q = 50 m³/s. Take α = 1.



Solution Trapezoidal channel, b = 6 m, s = 2, Q = 50 m³/s, α = 1

$$Z_c = \frac{Q}{\sqrt{g/\alpha}} = \frac{50}{\sqrt{9.81/1}} = 15.964$$

Now, assume several values of h and compute the section factor Z until the computed value of Z is close to 15.964.

h (m)	A (m²)	B (m)	D (m)	Z=A√D	Remarks
1.00	8.000	10.00	0.800	7.155	h small
2.00	20.000	14.00	1.429	23.905	h large
1.60	14.720	12.40	1.187	16.038	h closest
1.59	14.596	12.36	1.181	15.862	

Hence, the critical depth, h_c = 1.60 m and the critical velocity

$$U_c = \frac{Q}{A_c} = \frac{50}{14.720} = 3.40\ m/s$$

Example 3.5

A circular channel 2 m in diameter carries a discharge of 4 m³/s. Compute the critical depth and velocity. Take α = 1.10.

Solution Circular channel, d_0 = 2 m, Q = 4 m³/s, α = 1.10

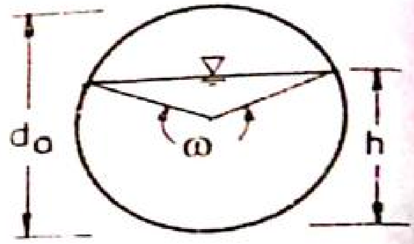
$$Z_c = \frac{Q}{\sqrt{g/\alpha}} = \frac{4}{\sqrt{9.81/1.10}} = 1.339$$

In case of a circular section, it is very convenient to compute the section factor Z using ω instead of h.

ω (rad)	A (m^2)	B (m)	D (m)	Z=A√D
1	0.079	0.959	0.083	0.023
2	0.545	1.683	0.324	0.310
3	1.429	1.995	0.716	1.210
4	2.378	1.819	1.308	2.719
3.10	1.529	2.000	0.765	1.337
3.11	1.539	2.000	0.770	1.350

Hence, ω_c = 3.10 radians and the critical depth

$$h_c = \frac{d_0}{2}(1-\cos\frac{\omega_c}{2}) = \frac{2}{2}(1-\cos\frac{3.10}{2}) = 0.98\ m$$



and, the critical velocity

$$V_c = \frac{Q}{A_c} = \frac{4}{1.529} = 2.62\ m/s$$

Numerical Methods

A number of numerical methods are available for solving non-linear algebraic equations involving a single variable, e.g. the method of bisection, the method of iteration, the method of false position, the secant method, the Newton-Raphson method, etc. (Churchhouse, 1981; Chapra and Canale, 1988). These methods can be conveniently used to compute the critical depth in trapezoidal and circular channels. The application of the method of bisection and the Newton-Raphson method for computing critical depth is considered here

Bisection method: The bisection method is very convenient to solve an algebraic equation f(x) = 0 which contains only one root. Suppose, we want to compute the critical depth in a channel for a given section, discharge Q and energy coefficient α. We choose two depths h_{min} and h_{max} such that the function

$$f(h) = 1 - Fr^2 = 1 - \frac{\alpha Q^2 B}{g A^3} \qquad (3.22)$$

is negative for h = h_{min} (lower limit of h) and positive for h = h_{max} (upper limit of h), and the root of the equation f(h) = 0, which is the critical depth, lies between h_{min} and h_{max}. We bisect the interval, i.e. the depth is taken equal to (h_{min} + h_{max})/2 and f(h) is determined. If f(h) is positive, then the root is less than (h_{min} + h_{max})/2 and the upper limit is taken as (h_{min} + h_{max})/2. On the other hand, if f(h) is negative, then the lower limit is taken as (h_{min} + h_{max})/2. The procedure is repeated till the desired accuracy is attained.

Example 3.6

For a trapezoidal channel with b = 6 m and s = 2, compute the critical depth by the method of bisection if Q = 14 m^3/s and α = 1.

Solution A = (6+2h)h, B = 6+4h, $\alpha Q^2/g$ = 1 × 14^2/9.81 = 19.98

$$f(h) = 1 - \frac{\alpha Q^2 B}{g A^3} = 1 - \frac{19.98(6+4h)}{[(6+2h)h]^3}$$

Initially the values of h_{min} and h_{max} are taken as 0 and 10 m, respectively. The computation is carried out as follows

h_{min}	h_{max}	$h=(h_{min}+h_{max})/2$	f(h)	Root lies between
0	10	5	0.999	0 and 5
0	5	2.5	0.984	0 and 2.5
0	2.5	1.25	0.8168	0 and 1.25
0	1.25	0.625	- 0.8254	0.625 and 1.25
0.625	1.25	0.9375	0.5159	0.625 and 0.9375
0.625	0.9375	0.7813	0.1160	0.625 and 0.7813
0.625	0.7813	0.7031	- 0.2468	0.7031 and 0.7813
0.7031	0.7813	0.7422	- 0.0455	0.7422 and 0.78
0.7422	0.7813	0.7617	0.0396	0.7422 and 0.7
0.7422	0.7617	0.7519	- 0.0018	0.7520 and 0.76
0.7520	0.7617	0.7568	0.0192	0.7520 and 0.756
0.7520	0.7568	0.7544	0.0088	0.7520 and 0.75

Hence, the critical depth, h_c = 0.75 m

In general, 12 to 15 iterations, depending on the values of h_{min} and h_{max} taken, reduce to interval within which the root is correct up to 0.01 m.

Newton - Raphson method: The Newton-Raphson method is particularly convenient for solving an algebraic equation which is easily differentiable and when the value of the desired root is known approximately. Let f(x) = 0 be the equation, with $x = x_n$ an approximation of the root. Then, a better approximation x_{n+1} to the root is obtained using the equation

$$x_{n+1} = x_n + \Delta x \tag{3.23}$$

where

$$\Delta x = -\frac{f(x_n)}{f'(x_n)} \tag{3.24}$$

Suppose we want to compute the critical depth in a channel for a given section, Q and α. Obviously, when $h = h_c$

$$1 - Fr^2 = 0 \qquad or, \qquad 1 - \frac{\alpha Q^2 B}{gA^3} = 0 \qquad or, \qquad A^3 - \frac{\alpha Q^2 B}{g} = 0$$

If we now assume

$$f(h) = A^3 - \frac{\alpha Q^2 B}{g} \tag{3.25}$$

then

$$f'(h) = 3A^2 B - \frac{\alpha Q^2}{g}\frac{dB}{dh} \tag{3.26}$$

For a given section, $f(h)$ and $f'(h)$ depend on the depth of flow only and can be easily calculated.

Example 3.7

For a trapezoidal channel with b = 6 m and s = 2, compute the critical depth by the Newton-Raphson method if Q = 14 m^3/s and α = 1.

Solution $A = (b + sh)h = (6 + 2h)h$, $B = b + 2sh = 6 + 4h$, $dB/dh = 2s = 4$

$$f(h) = A^3 - \frac{\alpha Q^2 B}{g} = [(6+2h)h]^3 - \frac{1\times 14^2 \times (6+4h)}{9.81}$$

$$= [(6+2h)h]^3 - 19.98(6+4h)$$

and

$$f'(h) = 3A^2 B - \frac{\alpha Q^2}{g}\frac{dB}{dh} = 3[(6+2h)h]^2 (6+4h) - \frac{1\times 14^2 \times 4}{9.81}$$

$$= 24(3+2h)[(3+h)h]^2 - 79.92$$

The initial value of h is taken as 1 m. The computation of critical depth is carried out as follows.

h	$f(h)$	$f'(h)$	$\Delta h = -\frac{f(h)}{f'(h)}$	$h = h + \Delta h$
1.000	312.204	1840.082	-0.170	0.830
0.830	70.776	1050.264	-0.067	0.773
0.773	8.488	815.536	-0.010	0.753
0.753	0.496	783.752	-0.001	0.752

Hence, the critical depth, $h_c = 0.75$ m

The Newton-Raphson method is particularly suitable for computing the critical depth in an open channel. Normally, it is necessary to repeat the procedure 3 to 4 times to obtain a value of the root correct up to 0.01 m.

3.4 CONTROL, TRANSITION AND FLOW MEASUREMENT

Control

Any feature which produces a direct relationship between the depth (or the stage) and the discharge is a control. From this definition, it follows that for any feature which acts as a control the discharge can be computed once the depth of flow (or the stage) is known and vice versa.

The location of a control in a channel is governed by the state of flow in the channel. It has been shown in Art. 1.5 that a small disturbance (e.g. an elementary wave) can travel upstream in subcritical flow and can only travel downstream in supercritical flow. That is to say, subcritical flow is affected by conditions downstream and supercritical flow is affected by conditions upstream. Accordingly, *subcritical flow is subjected to downstream control and supercritical flow is subjected to upstream control*. In other words, *the flow upstream of a control must be subcritical and that downstream of a control must be supercritical*.

Three control sections are illustrated in Fig 3.3. In Fig. 3.3(a), a sluice gate in a horizontal channel changes the flow from subcritical to supercritical and provides control for both upstream and downstream from it. In Fig. 3.3(b), a change in channel slope from mild to steep causes the flow to change from subcritical to supercritical and a control section exists at or near the break in slope. In Fig. 3.3(c), the free overfall at the end of a horizontal channel causes the flow to change from subcritical to supercritical and a control section occurs behind the brink.

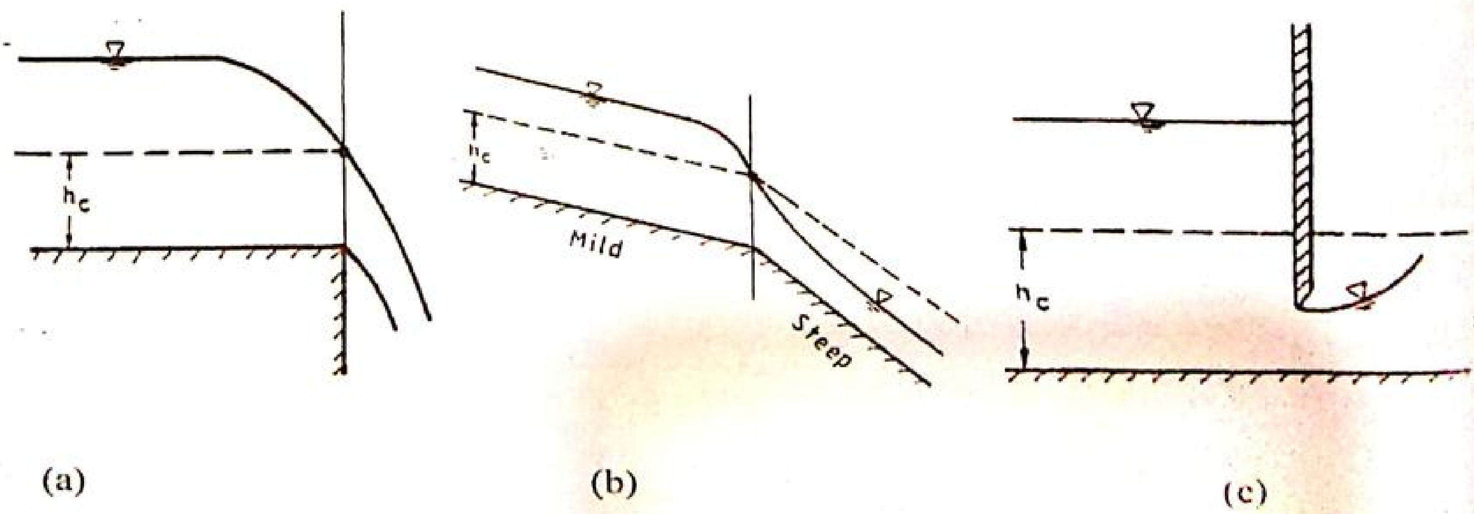


Fig. 3.3 Control sections

A control can be conveniently used for flow measurement since the discharge is readily obtained once the depth of flow (or the stage) is known or measured.

A critical flow section is a control section, because it establishes a direct relationship between the depth and the discharge, represented by Eq.(3.10). As a result, a critical flow section can be conveniently used for flow measurement. The control sections in Fig.3.3 are actually the critical flow sections.

Transition

A transition may be defined as a change either in the direction or slope or cross-section (i.e. change in bed level and/or width) of the channel. A transition may produce critical flow and a change in the state of flow in the channel. In this section, two simple transitions, viz. change in the channel bed level and change in the channel width are considered. The following two examples show how these two transitions produce critical flow in a channel.

Example 3.8

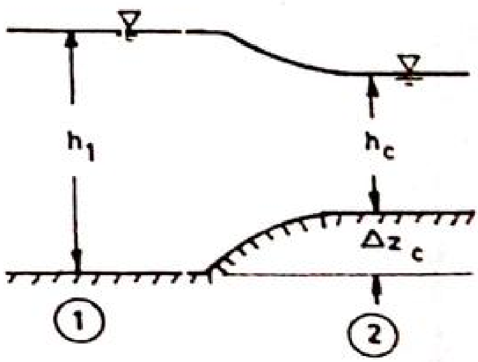
Water flows at a velocity of 1 m/s and a depth of 1.50 m in a long rectangular channel 3 m wide. Compute the height of a smooth upward step in the channel bed to produce critical flow and the change in water level produced by the step. Neglect energy losses and take $\alpha = 1$.

Solution Taking section 1 upstream of the step and section 2 over the step where the flow is critical, we obtain

$$Q = A_1U_1 = bh_1U_1 = 3\times 1.50\times 1 = 4.5\ m^3/s$$

$$h_c = \sqrt[3]{\frac{\alpha Q^2}{gb^2}} = \sqrt[3]{\frac{1\times 4.5^2}{9.81\times 3^2}} = 0.61\,m$$

$$U_c = \frac{Q}{A_c} = \frac{Q}{bh_c} = \frac{4.5}{3\times 0.61} = 2.45\ m/s$$



Applying the energy equation between sections 1 and 2 and taking the original channel bed as datum, we get

$$0 + h_1 + \frac{U_1^2}{2g} = \Delta z_c + h_c + \frac{U_c^2}{2g}$$

$$\therefore \Delta z_c = h_1 + \frac{U_1^2}{2g} - h_c - \frac{U_c^2}{2g} = 1.50 + \frac{1^2}{2\times 9.81} - 0.61 - \frac{2.45^2}{2\times 9.81} = 0.63\ m$$

Hence, the height of the step required to produce critical flow, $\Delta z_c = 0.63$ m, and drop in water level $= h_1 - h_c - \Delta z_c = 1.50 - 0.61 - 0.63 = 0.26\,m$

This example shows that a critical state of flow can be produced by raising the channel bed by the amount $\Delta z = \Delta z_c$. It can be shown that when $\Delta z > \Delta z_c$, then also flow over the step remains critical. So, the amount by which the channel bed is to be raised to produce critical flow is given by $\Delta z \geq \Delta z_c$. In a broad-crested weir (Fig. 3.4), the critical flow is produced by raising the channel bed.

Example 3.9

Water flows at a velocity of 1 m/s and a depth of 1.50 m in a long rectangular channel 3 m wide. Compute the contraction in width of the channel for producing critical flow and the change in water level produced by this contraction. Neglect energy losses and take $\alpha = 1$

Solution Taking section 1 upstream of the contraction and section 2 over the contraction where the flow is critical, we obtain $Q = 4.5\ m^3/s$ as in Example 3.8. Applying the energy equation between sections 1 and 2 and taking the channel bed as datum, we get

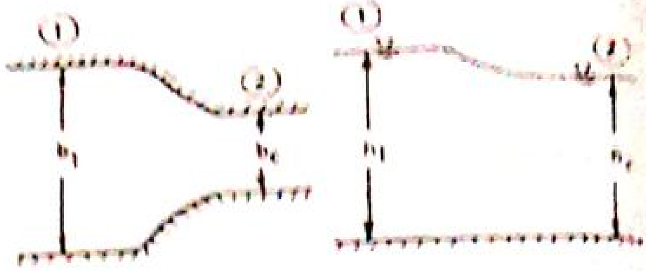
$$0 + h_1 + \frac{U_1^2}{2g} = 0 + h_c + \frac{U_c^2}{2g}$$

or

$$h_1 + \frac{U_1^2}{2g} = h_c + \frac{U_c^2}{2g} = h_c + \frac{D_c}{2} = h_c + \frac{h_c}{2} = \frac{3}{2}h_c$$

$$h_c = \frac{2}{3}\left(h_1 + \frac{U_1^2}{2g}\right) = \frac{2}{3}\left(1.50 + \frac{1^2}{2\times 9.81}\right)$$

$$= \frac{2}{3}\times 1.55 = 1.03\ m$$



and

$$U_c = \sqrt{gD_c/\alpha} = \sqrt{gh_c/1} = \sqrt{9.81\times 1.03/1} = 3.19\ m$$

Since, $Q = A_c U_c = b_c h_c U_c$ we get

$$b_c = \frac{Q}{h_c U_c} = \frac{4.5}{1.03\times 3.19} = 1.37\ m$$

Hence, the width required to produce critical flow, b_c = 1.37 m and drop in water level = $h_1 - h_c$ = 1.50 − 1.03 = 0.47 m

This example shows that a critical state of flow can be produced by reducing the channel width, the width to be provided is given by $b_2 = b_c$. It can be shown that when $b_2 < b_c$, then also flow over the contraction remains critical. So, the width to be provided to produce critical flow in the channel is given by $b_2 \le b_c$. In a Venturi flume (Fig. 3.5), the critical flow is produced by reducing the channel width.

Flow Measurement

A definite relationship between depth and discharge exists at a control section which offers a theoretical basis for the measurement of discharge in open channels. Since a critical flow section is a control section, it can be used for flow measurement. Based on the principle of critical flow, various devices for flow measurement have been developed. In such devices, the critical depth is produced either by raising the channel bed, as in a broad-crested weir, or by reducing the channel width, as in a critical flow flume.

Broad-Crested Weir

If the channel bed is raised by an amount Δz such that $\Delta z \ge \Delta z_c$ over a length sufficient to develop parallel flow over the hump, the flow over the hump will be critical. Such a device is called a broad-crested weir and provides an excellent means of measuring the discharge in open channels.

Consider a rectangular broad-crested weir as shown in Fig. 3.4. Using Eq.(3.18), the discharge over the weir with α = 1 is given by

$$Q = \sqrt{g}\, b h_c^{1.5} = 3.13 b h_c^{1.5} \qquad (3.27)$$

where b is the width of the channel.

The usual difficulty in using Eq.(3.27) for computing discharge in an open channel lies in locating the critical flow section and measuring the critical depth accurately. This difficulty is however, avoided by measuring the depth of flow upstream of the weir where the flow is not affected by the presence of the weir. With reference to Fig.3.4, neglecting the velocity of approach and the frictional losses and applying the energy equation between the upstream section and the critical flow section, one obtains

$$h_1 = h_c + \frac{U_c^2}{2g} = h_c + \frac{D_c}{2} = h_c + \frac{h_c}{2} = \frac{3}{2}h_c \quad (\because \text{for a rectangular channel, } D = h) \qquad (3.28)$$

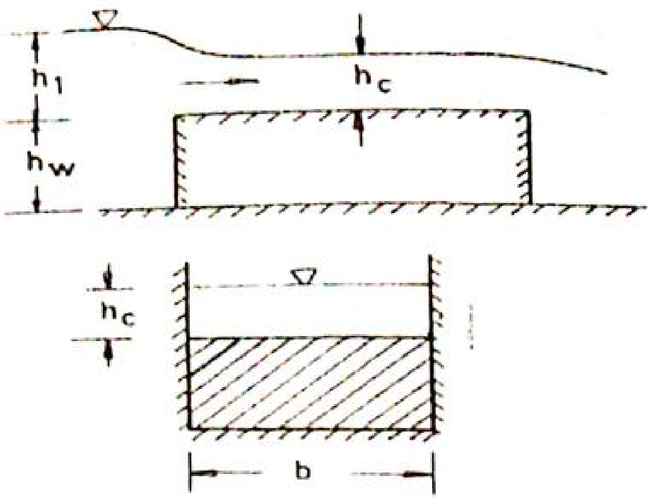


Fig. 3.4 Flow over a rectangular broad-crested weir

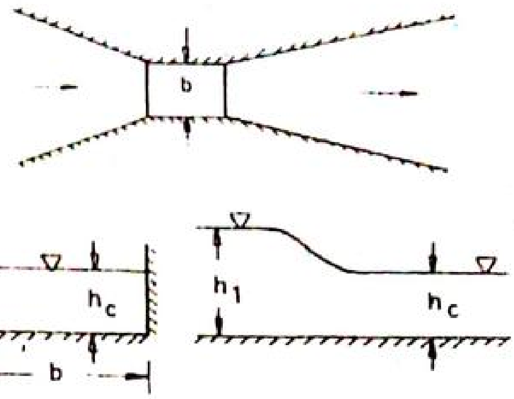


Fig. 3.5 Flow over a rectangular Venturi flume

Using Eq.(3.28), Eq.(3.27) becomes

$$Q = (2/3)^{1.5}\, b\sqrt{g}\, h_1^{1.5} = 1.705 b h_1^{1.5} \qquad (3.29)$$

If it is desired to include the velocity of approach, then Eq.(3.28) becomes

$$h_1 + \frac{U_1^2}{2g} = h_c + \frac{U_c^2}{2g} = h_c + \frac{h_c}{2} = \frac{3}{2}h_c \qquad (3.30)$$

and the discharge equation, Eq.(3.29), becomes

$$Q = 1.705 b\left(h_1 + \frac{U_1^2}{2g}\right)^{1.5} \qquad (3.31)$$

Example 3.10

A broad-crested weir is built in a rectangular channel of width 2 m. The height of the weir crest above the channel bed is 1.20 m and the head over the weir is 0.80 m. Calculate the discharge.

Solution Rectangular channel, b = 2 m, h_w = 1.20 m, h_1 = 0.80 m and $A_1 = b(h_w+h_1) = 2\times(1.20+0.80) = 4.00\ m^2$. Initially neglect the velocity of approach. Then, the discharge is obtained using Eq.(3.29) as

$$Q = 1.705 b h_1^{1.5} = 1.705\times 2\times 0.80^{1.5} = 2.440\ m^3/s$$

A more accurate discharge can be obtained by using Eq.(3.31) as follows.

Assumed Q	A_1	U_1	$U_1^2/2g$	$h_1+U_1^2/2g$	Computed Q
2.440	4.00	0.6100	0.0190	0.8190	2.527
2.527	4.00	0.6318	0.0203	0.8203	2.534
2.534	4.00	0.6334	0.0204	0.8204	2.534

Hence, the discharge, Q = 2.534 m^3/s

Critical Flow Flume

A broad-crested weir has the disadvantage of having a dead water region upstream in which silt and debris can accumulate. This difficulty can be overcome by the use of a critical flow flume in which the occurrence of critical flow is forced by a contraction in channel width, followed by a short length of supercritical flow and a hydraulic jump. Obviously, the width at the contracted section or throat (Fig. 3.5) must be equal to or less than the width required for producing critical flow, i.e. $b \le b_c$

The critical flow flume, also known as the *Venturi flume*, has been designed in various forms. The discharge through a rectangular Venturi flume is given by Eq. (3.27) or Eq. (3.29) or (3.31), where b is now the throat width and h_c is the critical depth at the throat (Fig. 3.5).

PROBLEMS AND EXERCISES

3.1 Define (i) specific energy, (ii) alternate depths, (iii) control, and (iv) transition.

3.2 Prove that at the critical state of flow, (i) the specific energy is minimum for a given discharge, and (ii) the discharge is maximum for a given specific energy.

3.3 State why it is undesirable to design channels at or near the critical state.

3.4 Derive the general expression for the hydraulic exponent for critical flow computation M, and then determine the numerical value of M for a (i) rectangular channel, (ii) triangular channel and (iii) parabolic channel.

3.5 Derive the expression for critical depth in a (i) rectangular channel, (ii) triangular channel and (iii) parabolic channel.

3.6 State why a critical flow section is used for flow measurement. How can you produce critical flow in a channel? State how a broad-crested weir and a Venturi flume act as a control.

3.7 A rectangular channel has a bottom width of 6 m. (a) Construct the specific energy curve for a discharge of 15 m^3/s and determine the critical depth and the minimum value of the specific energy. (b) Construct the discharge-depth curve for a specific energy of 3 m and determine the critical depth and the maximum value of the discharge.

3.8 Compute the numerical values of the hydraulic exponent for critical flow computation M for a depth of 1 m in a trapezoidal channel with b = 6 m, s = 2 and Q = 20 m^3/s.

3.9 Prove the following equations when the flow is critical in a rectangular channel:

i) $h_c = \sqrt[3]{\dfrac{\alpha Q^2}{gb^2}} = \sqrt[3]{\dfrac{\alpha q^2}{g}}$ ii) $U_c = \sqrt{\dfrac{gh_c}{\alpha}}$

iii) $E_c = 1.5 h_c$ iv) $Q = \sqrt{g/\alpha}\; b h_c^{1.5}$

3.10 Compute the critical depth and velocity in a (i) wide channel with q = 4 m^2/s, (ii) rectangular channel with b = 6 m and Q = 35 m^3/s, (iii) triangular channel with s = 1 and Q = [illegible] m^3/s, and (iv) parabolic channel whose profile is given by $y^2 = 5z$ with Q = 25 m^3/s. In all cases assume α = 1.12.

3.11 (a) Compute the critical depth and velocity in a trapezoidal channel with b = 6 m, s = 2, α = 1.12 and Q = 30 m^3/s by (i) the trial-and-error method, (ii) the bisection method, and (iii) the Newton-Raphson method.

(b) Compute the critical depth and velocity in a circular channel with d_0 = 3 m and Q = [illegible] m^3/s by the trial-and-error method, if (i) α = 1, and (ii) α = 1.12.

3.12 Show that the relation between the alternate depths h_1 and h_2 for a rectangular channel is given by

$$\frac{2h_1^2 h_2^2}{h_1 + h_2} = h_c^3$$

where h_c is the critical depth.

3.13 The depth upstream of a vertical sluice gate in a rectangular channel is 2 m and the discharge under the gate is 30.67 m^3/s. The channel is 6 m wide. Compute the downstream depth.

3.14 Water is flowing at a velocity of 2 m/s and a depth of 2.5 m in a long rectangular channel 6 m wide. Compute the height of a smooth upward step in the channel bed to produce critical flow. Also, compute the change in water level produced by the step. Neglect energy losses and take α = 1.

3.15 Water is flowing at a velocity of 2 m/s and a depth of 2.5 m in a long rectangular channel 6 m wide. Compute the contraction in width of the channel for producing critical flow. Also, compute the change in water level produced by the contraction. Neglect energy losses and take α = 1.

3.16 A bridge is to be constructed across a 10 km wide river carrying a discharge of 1,00,000 m^3/s at a depth of 10 m. If it is intended to provide the minimum length of the bridge by reducing the river width, what would be the minimum river width without affecting the upstream flow? Neglect energy losses and assume α = 1.

3.17(a) A broad-crested weir is built in a rectangular channel of width 1 m. The height of the weir crest above the channel bed is 0.60 m and the head over the weir is 0.40 m. Calculate the discharge.

(b) Compute the discharge through a Venturi flume having a throat width of 0.30 m when the upstream depth is 0.50 m.

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Chapter 4

UNIFORM FLOW

1ST class

4.1 INTRODUCTION

When uniform flow occurs in a channel, (i) the discharge, the mean velocity and the depth of flow remain constant along the length of the channel, and (ii) the total energy line, the water surface and the channel bottom are parallel, i.e. $S_f = S_w = S_0$. In uniform flow, water is neither accelerated nor retarded and the net external force on water is zero.

The flow in a long straight prismatic channel under normal condition, i.e. when there is no inflow or outflow or no transition or control structures like sluice gates, weirs, dams etc., tends to be uniform.

Uniform flow is considered to be steady only, since unsteady uniform flow is not practically possible. True uniform flow does not normally occur in natural channels, because changes in the cross-section along the length of the channel induce non-uniform flow conditions. Still, the concept of uniform flow is central to the understanding and solution of most problems in open channel hydraulics.

4.2 ESTABLISHMENT OF UNIFORM FLOW

The condition for the establishment of uniform flow in an open channel can be determined considering the momentum equation, Eq.(2.13). For uniform flow, $U_1 = U_2$, $F_{p1} = F_{p2}$ and $\beta_1 = \beta_2$. Hence, Eq.(2.13) reduces to

$$W \sin\theta = F_f \qquad (4.1)$$

which indicates that ***when uniform flow occurs in a channel, the component of the gravity force causing the flow is equal to the force of friction or resistance.***

The above condition implies that (i) flow cannot be uniform in a horizontal channel for which $\theta = 0$, and for uniform flow to occur, the channel must have a slope in the downstream direction, (ii) flow cannot be uniform in an adverse slope channel in which both $W\sin\theta$ and F_f act in the same direction, which is opposite to the direction of flow, (iii) flow cannot be uniform in a frictionless channel for which $F_f = 0$, and (iv) uniform flow of an ideal fluid is impossible, since an ideal fluid has no friction.

The condition for the establishment of uniform flow, $W\sin\theta = F_f$, can be used to explain why the flow in a long straight prismatic channel under normal condition tends to be uniform. Suppose that at some location of a channel $W\sin\theta > F_f$ and flow is non-uniform. As the flow proceeds downstream, the flow is accelerated since $W\sin\theta > F_f$ and the flow velocity increases. Since $F_f \propto U^2$, the friction or resistance force also increases and a balance between $W\sin\theta$ and F_f tends to reach and the flow tends to be uniform. On the other hand, if $W\sin\theta < F_f$ at some location of a channel, the flow is retarded and the flow velocity decreases. Hence, the friction or resistance force also decreases and a balance between $W\sin\theta$ and F_f tends to reach and the flow tends to be uniform as the flow proceeds downstream. Thus, uniform flow seems to be self adjusting and any departure from the condition $W\sin\theta = F_f$ tends to reestablish this condition.

4.3 VELOCITY DISTRIBUTION

Shear Stress

When water flows in a channel, the pull of water produces a force that acts on the channel bed in the direction of flow. This force is known as the shear or tractive or drag force and is equal to the friction or resistance force F_f. If the average value of this force per unit wetted area, which is known as the shear stress, is denoted by τ_0, then

$$F_f = \tau_0 P L \tag{4.2}$$

where P is the wetted perimeter and L is the length of the channel. Therefore, from Eq.(4.1)

$$W \sin\theta = F_f = \tau_0 PL \tag{4.3}$$

When θ is small and in radian, sinθ ≈ tanθ ≈ θ. Also, tanθ = S_0. Therefc omponent of the gravity force = Wsinθ = γALsinθ ≈ γALtanθ = γALS_0, where γ is the sp ic weight of water. A is the cross-sectional area and S_0 is the channel bottom slope. Theref e,

$$\gamma ALS_0 = \tau_0 PL \tag{4.4}$$

or

$$\tau_0 = \gamma \frac{A}{P} S_0 = \gamma RS_0 = \rho gRS_0 \tag{4.5}$$

For a wide channel, R ≈ h, hence Eq. (4.5) becomes

$$\tau_0 = \gamma hS_0 = \rho ghS_0 \tag{4.6}$$

Friction Velocity

The quantity $\sqrt{\tau_0 / \rho}$ has the dimensions of velocity. So, a velocity u* is introduced such that

$$u* = \sqrt{\tau_0 / \rho} \tag{4.7}$$

where u* is known as the shear or friction or drag velocity. It does not represent a velocity which is physically real. However, it is used as the velocity scale in the study of velocity distribution in open channels.

Using Eqs.(4.5) and (4.7) it can be shown that

$$u* = \sqrt{gRS_0} \tag{4.8}$$

and when the channel is wide

$$u* = \sqrt{ghS_0} \tag{4.9}$$

Laminar or Viscous Sublayer

Even in a turbulent flow, there is a very thin layer near the boundary in which the flow is laminar. This layer is known as the laminar or viscous sublayer. The thickness of this layer is given by

$$\delta_s = \frac{11.6\nu}{u*} \tag{4.10}$$

Smooth and Rough Boundaries

As stated earlier, the velocity distribution across a channel section is not uniform owing to the presence of boundary or surface roughness. The effect of boundary roughness on the velocity distribution in turbulent flow was first investigated by Nikuradse who introduced the concept of *equivalent sand grain roughness* (k_s) as standard for all other types of roughness elements. The ratio k_s/R of the roughness height to the hydraulic radius is known as the *relative roughness*. Table 4.1 gives the values of equivalent sand grain roughness for different materials.

Table 4.1 Equivalent sand grain roughness (k_s) for various materials

Sl. No.	Material	k_s(mm)
1.	Glass	0.0003
2.	Wrought iron, steel	0.046
3.	Asphalted cast iron	0.12
4.	Galvanized iron	0.15
5.	Cast iron	0.26
6.	Concrete	0.30 – 3.0
7.	Riveted steel	0.90 – 9.0

The boundary surfaces are classified based on the following criteria:

1. Hydraulically smooth boundary

A boundary is said to be hydraulically smooth when

$$\frac{u_* k_s}{\nu} \leq 5 \quad \text{and} \quad k_s < \delta_s \qquad (4.11)$$

The roughness elements are well-covered by the viscous sublayer and do not affect the velocity distribution outside the sublayer. The velocity distribution depends on the viscosity of water.

2. Hydraulically rough boundary

A boundary is said to be hydraulically rough when

$$\frac{u_* k_s}{\nu} \geq 70 \quad \text{and} \quad k_s > \delta_s \qquad (4.12)$$

The roughness elements project through the viscous sublayer and the velocity distribution outside the sublayer is affected by the surface roughness. The viscosity of water has no effect on the velocity distribution.

3. Transition boundary

For a transition boundary

$$5 < \frac{u_* k_s}{\nu} < 70 \qquad (4.13)$$

The velocity distribution is affected both by the viscosity of water and the surface roughness.

Velocity Distribution in Turbulent Flow

Along a Vertical

The velocity distribution along a vertical in a wide channel in turbulent flow is given by

$$\frac{u_z}{u*} = \frac{1}{\kappa} \ln \frac{z}{z_0} \tag{4.14}$$

where u_z is the velocity at a distance z from the channel bottom, κ (= 0.4) is the *von Karman constant* and z_0 is the zero velocity level, i.e. u = 0 at z = z_0. Equation (4.14) is commonly known as the *Prandtl-von Karman universal logarithmic velocity distribution law*.

Experimental evidence suggests that the logarithmic velocity profile is a good approximation for the full depth of the flow. The values of z_0 for different boundaries are as follows:

1. Hydraulically smooth boundary ($u^*k_s/\nu \leq 5$)

$$z_o = 0.11\frac{\nu}{u*} \tag{4.15}$$

2. Hydraulically rough boundary ($u^*k_s/\nu \geq 70$)

$$z_o = 0.033k_s \tag{4.16}$$

3. Transition boundary ($5 < u^*k_s/\nu < 70$)

$$z_0 = 0.11\frac{\nu}{u*} + 0.033k_s \tag{4.17}$$

Depth-Averaged Velocity

For logarithmic velocity distribution, Eq.(4.14), Vanoni (1941) showed that the flow velocity, measured at 0.632h from the free surface, is equal to the depth-averaged velocity in the vertical. Also, it can be shown that the velocity at 0.6h depth from the free surface or the average of the velocities at 0.2h and 0.8h depths from the free surface, when h is the total depth of flow, is approximately equal to the average velocity in the vertical.

Cross-Sectional Mean Velocity

The cross-sectional mean velocity for turbulent flow in open channels are given by the following equations:

1. Hydraulically smooth boundary ($u^*k_s/\nu \leq 5$)

$$\frac{U}{u*} = 5.75\log\left(\frac{3.64u*R}{\nu}\right) \tag{4.18}$$

2. Hydraulically rough boundary ($u^*k_s/\nu \geq 70$)

$$\frac{U}{u*} = 5.75\log\left(\frac{12.2R}{k_s}\right) \tag{4.19}$$

3. Transition boundary ($5 < u^*k_s/\nu < 70$)

$$\frac{U}{u*} = 5.75\log\left(\frac{12.2R}{k_s + 3.35\nu/u*}\right) \tag{4.20}$$

F-7

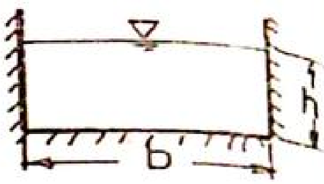
Example 4.1

A rectangular channel is 6 m wide and laid on a slope of 0.25%. The channel is made of concrete (k_s = 2 mm) and carries water at a depth of 0.50 m. Compute the mean velocity of flow.

Solution k_s = 2 mm = 0.002 m S_0 = 0.25/100 = 0.0025

A = bh = 6 × 0.50 = 3.0 m^2 P = b + 2h = 6 + 2 × 0.50 = 7.0 m

R = A/P = 3.0/7.0 = 0.43 m



$$u^* = \sqrt{gRS_0} = \sqrt{9.81 \times 0.43 \times 0.0025} = 0.1025\ m/s$$

$$\frac{k_s u^*}{\nu} = \frac{0.002 \times 0.1025}{10^{-6}} = 205 > 70$$

Hence, the boundary is hydraulically rough and the mean velocity of flow is obtained by Eq.(4.19), i.e.

$$\frac{U}{u^*} = 5.75 \log \frac{12.2R}{k_s} = 5.75 \log \frac{12.2 \times 0.43}{0.002} = 19.65$$

$$\therefore\ U = 19.65 \times 0.1025 = 2.014\ m/s$$

4.4 UNIFORM FLOW FORMULAS

Chezy Formula

The Chezy formula can be found mathematically from two assumptions. The first assumption states that, in steady uniform flow the component of the gravity force causing the flow must be equal to the force of friction or resistance, as indicated by Eq.(4.1). When the channel slope is small, the component of the gravity force = Wsinθ = γALsinθ ≈ γALtanθ = $\gamma ALS_0 = \gamma ALS_f$.

The second assumption states that, in turbulent flow the resistance force per unit wetted area varies as the square of the mean velocity. The total wetted area is the product of the wetted perimeter P and the length of the channel L. Hence, the total force of resistance is given by

$$F_f = kPLU^2 \tag{4.21}$$

where k is a constant of proportionality. Hence, Eq.(4.1) gives

$$\gamma ALS_f = kPLU^2 \tag{4.22}$$

Equation (4.22) can be rearranged to yield

$$U = CR^{1/2} S_f^{1/2} \tag{4.23}$$

where $\sqrt{\gamma / k}$ is written as one constant C.

Equation (4.23) is probably the first steady uniform flow formula developed by the French engineer Antoine Chezy in 1769 and is applicable for steady uniform and nearly uniform flows in open channels. It can be used in any systems of units. In this formula, C is the resistance factor, specifically known as *Chezy's C*. Its SI units are $m^{1/2}$/s. The numerical value of Chezy's C varies with the systems of units. For the alluvial rivers of Bangladesh, the Chezy's C varies from 30 $m^{1/2}$/s to 80 $m^{1/2}$/s.

Darcy-Weisbach Formula

The Darcy-Weisbach formula, first presented by Julius Weisbach in 1845 and primarily developed for pipe flow, is given by

$$h_f = f\frac{L}{d_0}\frac{U^2}{2g} \tag{4.24}$$

where h_f is the frictional loss, f is the friction factor, L is the length of the pipe, d_0 is the diameter of the pipe, U is the mean velocity of flow and g is the acceleration due to gravity. Since $d_0 = 4R$ and the energy gradient $S_f = h_f/L$, the above formula may be written as

$$U = \sqrt{\frac{8g}{f}}R^{1/2}S_f^{1/2} \tag{4.25}$$

This formula is same in all the systems of units and may be applied to uniform and nearly uniform flows in open channels. The friction factor f is dimensionless and its numerical value remains same in all the systems of units.

Manning Formula 2nd class

In 1889 the Irish engineer Robert Manning presented an empirical formula for steady uniform flow in open channels. In SI and English units this formula is given by

$$U = \frac{1}{n}R^{2/3}S_f^{1/2} \tag{4.26}$$

and

$$U = \frac{1.486}{n}R^{2/3}S_f^{1/2} \tag{4.27}$$

respectively, where U is the mean velocity, n ($s/m^{1/3}$ or $sec/ft^{1/3}$) is the Manning's roughness coefficient, specifically known as *Manning's n*, R is the hydraulic radius and S_f is the slope of the energy line.

The Manning's roughness coefficient n has the dimensions of $TL^{-1/3}$. But its numerical value is kept same in all the systems of units. To keep the numerical value of n same in all the systems of units, the Manning formula becomes different in different systems of units. For example, the Manning formula needs to be multiplied by $(3.28)^{1/3} = 1.486$ to convert it from SI units (Eq. 4.26) to English units (Eq.4.27), as shown below.

$$n \text{ in } s/m^{1/3} = \frac{n}{3.28^{1/3}} \text{ in } sec/ft^{1/3} \quad \text{(since 1 m = 3.28 ft)}$$

$$U = \frac{1}{n}R^{2/3}S_f^{1/2} \quad \text{in SI units}$$

$$= \frac{1}{n/3.28^{1/3}}R^{2/3}S_f^{1/2} \quad \text{in English units}$$

$$= \frac{3.28^{1/3}}{n}R^{2/3}S_f^{1/2} \quad \text{in English units}$$

$$= \frac{1.486}{n}R^{2/3}S_f^{1/2} \quad \text{in English units}$$

The Manning formula has been verified by many laboratory and field measurements and found to give satisfactory results. It is valid for fully rough turbulent flow for values of R/k_s up to about 1500. Therefore, it has been the most widely used of all the uniform flow formulas for open channel flow computation. The values of Manning's n for some surfaces are given in Table 4.2.

Table 4.2 Values of Manning's roughness coefficient n for some surfaces

Surface	Value of n	Surface	Value of n
Glass	0.010	Plastic	0.010
Cement	0.011	Concrete	0.013
Wood	0.015	Earth canals	0.025
Rivers	0.025	Flood plains	0.040

Relationship Between Chezy's C, Darcy-Weichbach Friction Factor f and Manning's n

Using Eqs.(4.23), (4.25) and (4.26), we obtain the following relationships between C and n, C and f, and n and f in SI units:

$$C = \frac{1}{n} R^{\frac{1}{6}} \tag{4.28}$$

$$\frac{C}{\sqrt{g}} = \sqrt{\frac{8}{f}} \tag{4.29}$$

$$n = R^{\frac{1}{6}} \sqrt{\frac{f}{8g}} \tag{4.30}$$

Strickler Formula for Estimating Manning's n

The most simple and the best-known of the methods used for estimating Manning's n is the empirical formula presented by the Swiss engineer Strickler in 1923 based on data from streams with beds consisting of coarse material and free from bed undulations. The formula originally proposed by Strickler is

$$n = \frac{d_{50}^{1/6}}{21.1} = 0.047 d_{50}^{1/6} \tag{4.31}$$

where d_{50} is the median diameter or the diameter of the bed material in meters such that 50 percent of the material by weight is smaller.

The Strickler formula has two major advantages: (i) it relates n to the size of the grains which can be measured easily, and (ii) since d_{50} is raised to 1/6th power, an error in estimating its value has a corresponding less effect on the computed value of n.

Factors Affecting Manning's n

The value of n is highly variable and depends on a number of factors, which are to some extent interdependent. The factors that exert the greatest influence upon Manning's n in both natural and artificial channels are briefly described below.

i) **Roughness of the surface**: The value of Manning's n depends on the roughness of the surface which in turn depends on the size and shape of the grains of the material forming the channel perimeter. In general, fine-grained soils (e.g. clay, silt and sand) result in a low value of n and coarse-grained soils (e.g. gravels and boulders) result in a high value of n.

ii) **Vegetation**: The presence of vegetation in a channel retards the flow and increases n depending on the height, density, distribution and type of vegetation. Owing to the seasonal growth of aquatic plants, the value of n may increase in the growing season and diminish in the dormant season.

iii) **Channel irregularity**: Channel irregularities include sand bars, depressions, holes, humps, etc. and increase the value of n.

iv) **Channel alignment**: The value of n is low for straight channels and high for curved channels and increases with the curvature of the channel.

v) **Silting and scouring**: In general, silting converts an irregular channel into a regular one and decreases n, whereas scouring does the reverse and increases n.

vi) **Obstruction**: The presence of obstructions like logs, bridge piers, boats, ships, launches, steamers, etc. tends to increase n depending on the size, shape, number and distribution of the obstructions.

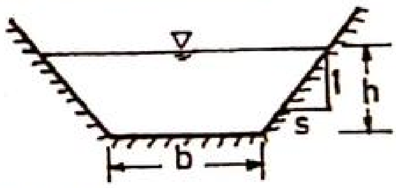
vii) **Stage and discharge**: The value of n generally decreases with inc[illegible] stage and discharge. However, the value of n may be high when the flood plains in a river a[illegible] submerged at high stages.

viii) **Suspended material and bed load**: The suspended material and bed load cause an increase in Manning's n because additional energy is required to move the sediment.

Example 4.2

An open channel lined with concrete (d_{50} = 1.5 mm) is laid on a slope of 0.1%. The channel is trapezoidal with b = 6 m and s = 2. Compute the uniform flow discharge in the channel if the depth of flow is 2 m. Also, compute the numerical values of Chezy's C and friction factor f.

Solution $S_f = S_0 = 0.1\% = 0.1/100 = 0.001$, $d_{50} = 1.5\text{ mm} = 1.5/1000\text{ m} = 0.0015\text{ m}$



$$n = 0.047 d_{50}^{1/6} = 0.047 \times 0.0015^{1/6} = 0.016$$

$$A = (b + sh)h = (6+2\times 2)\times 2 = 20\ m^2$$

$$P = b + 2h\sqrt{1+s^2} = 6 + 2\times 2\times\sqrt{1+2^2} = 14.94\,m$$

$$R = A/P = 20/14.94 = 1.34\ m$$

$$Q = \frac{1}{n} A R^{2/3} S_0^{1/2} = \frac{1}{0.016}\times 20\times 1.34^{2/3}\times 0.001^{1/2} = 48.29\ m^3/s$$

$$C = \frac{1}{n} R^{1/6} = \frac{1}{0.016}\times 1.34^{1/6} = 66.01\ m^{1/2}/s$$

$$f = \frac{8g}{C^2} = \frac{8\times 9.81}{66.01^2} = 0.018$$

4.5 NORMAL DEPTH, SECTION FACTOR, CONVEYANCE AND HYDRAULIC EXPONENT

Normal Depth

The depth of uniform flow is known as normal depth and is designated by h_n. Similarly, the discharge of uniform flow is known as the normal discharge designated by Q_n, the velocity of uniform flow is known as the normal velocity designated by U_n and so on. The Chezy and the Manning formulas for the discharge can then be written respectively as

$$Q_n = C A_n R_n^{1/2} S_n^{1/2} \tag{4.32}$$

and

$$Q_n = \frac{1}{n} A_n R_n^{2/3} S_n^{1/2} \tag{4.33}$$

Section Factor

The product of flow area and two-thirds power of hydraulic radius, i.e. $AR^{2/3}$, is known as the section factor in connection with the Manning formula. It can be computed if the channel section and the depth of flow h are known.

When the flow is uniform, the product of flow area and two-thirds power of hydraulic radius, i.e. $A_nR_n^{2/3}$, is known as the *section factor for uniform flow computation* in connection with the Manning formula. Using Eq.(4.33), it can be shown that

$$A_n R_n^{2/3} = \frac{nQ_n}{\sqrt{S_n}} \tag{4.34}$$

Thus, $A_nR_n^{2/3}$ can be computed either if the channel section and the normal depth h_n are given, or alternatively, using Eq.(4.34) from given values of the Manning's n, the discharge and the slope.

When the Chezy formula is used, then $AR^{1/2}$ is the section factor and $A_nR_n^{1/2}$ is the section factor for uniform flow computation. Using Eq.(4.32), it can be shown that

$$A_n R_n^{1/2} = \frac{Q_n}{C\sqrt{S_n}} \tag{4.35}$$

The section factor is an important parameter in the computation of uniform flow.

Conveyance

The conveyance for a channel section in terms of the Manning formula is given by

$$K = \frac{1}{n} AR^{2/3} \tag{4.36}$$

which can be computed when the section, the roughness coefficient n and the depth h are given.

When the flow is uniform, then using the Manning formula (Eq.4.33), it can be shown that

$$Q_n = K_n\sqrt{S_n} \tag{4.37}$$

where

$$K_n = \frac{1}{n} A_n R_n^{2/3} \tag{4.38}$$

is the *conveyance for uniform flow*. It can be computed using Eq.(4.37) when the discharge and the slope are given or using Eq.(4.38) from the given section, n and the normal depth h_n.

In terms of the Chezy formula, the conveyance is given by

$$K = CAR^{1/2} \tag{4.39}$$

and using Eq.(4.32), we obtain

$$Q_n = K_n\sqrt{S_n} \tag{4.40}$$

where

$$K_n = CA_nR_n^{1/2} \tag{4.41}$$

is the conveyance for uniform flow.

Hydraulic Exponent for Uniform Flow Computation

The conveyance K is a function of the depth of flow for a given channel section and roughness and it is convenient to express K in the form

$$K^2 = C_2 h^N \tag{4.42}$$

where C_2 is a coefficient and N is an exponent which is known as the hydraulic exponent for uniform flow computation.

Taking logarithm of both sides of Eq.(4.42) and then differentiating with respect to h, we obtain

$$\frac{d(\ln K)}{dh} = \frac{N}{2h} \tag{4.43}$$

Also, using R = A/P in Eq.(4.36), taking logarithm of both sides of the resulting equation, then differentiating it with respect to h and using dA/dh = B, we get

$$\frac{d(\ln K)}{dh} = \frac{5}{3}\frac{1}{A}\frac{dA}{dh} - \frac{2}{3}\frac{1}{P}\frac{dP}{dh} = \frac{5B}{3A} - \frac{2}{3P}\frac{dP}{dh} = \frac{1}{3A}\left(5B - 2R\frac{dP}{dh}\right) \tag{4.44}$$

Equating the right sides of Eqs. (4.43) and (4.44) and solving for N, we obtain

$$N = \frac{2h}{3A}\left(5B - 2R\frac{dP}{dh}\right) \tag{4.45}$$

which is the general equation for the hydraulic exponent N when the conveyance is expressed in terms of the Manning formula.

When the conveyance is expressed in terms of the Chezy formula, Eq.(4.39), it can be shown similarly that

$$N = \frac{h}{A}\left(3B - R\frac{dP}{dh}\right) \tag{4.46}$$

Equations (4.45) and (4.46) indicate that the numerical value N depends on the channel shape and the depth of flow. It also depends on whether the conveyance is expressed in terms of the Manning or the Chezy formula. The values of N for different channel sections are given in Table 4.3.

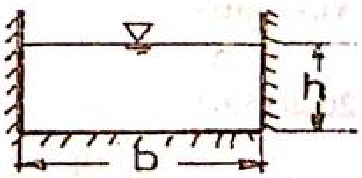
Example 4.3

Derive the expression for the hydraulic exponent for uniform flow computation N for a rectangular channel based on the Manning formula. Then, compute the numerical values of N for a wide channel.

Solution For a rectangular channel, A = bh, B = b, P = b + 2h, R = A/P, dP/dh = 2

$$N = \frac{2h}{3A}\left(5B - 2R\frac{dP}{dh}\right) = \frac{2h}{3bh}\left(5b - 2\times\frac{bh}{b+2h}\times 2\right)$$

$$= \frac{2}{3}\left(5 - \frac{4h}{b+2h}\right) = \frac{2}{3}\left[5 - \frac{4(h/b)}{1+2(h/b)}\right]$$



This is the expression for N for a rectangular channel based on the Manning formula.

For a wide channel, $h/b \approx 0$. Hence,

$$N = \frac{2}{3}\left[5 - \frac{4(h/b)}{1+2(h/b)}\right] = \frac{2}{3}\left[5 - \frac{4\times 0}{1+2\times 0}\right] = \frac{10}{3} = 3.33$$

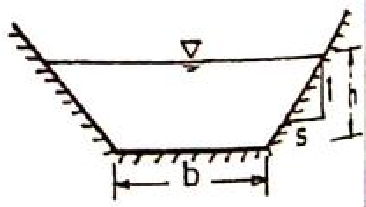
Table 4.3 Values of N for different channel sections

Channel section	Value of N when conveyance is expressed by	
	Manning formula	Chezy formula
1. Wide (b>>h)	3.33	3.00
2. Triangle	5.33	5.00
3. Rectangle	$\frac{2}{3}\left[5 - \frac{4(h/b)}{1+2(h/b)}\right]$	$3 - \frac{2(h/b)}{1+2(h/b)}$
4. Trapezoid	$\frac{2}{3}\left[5\frac{1+2s(h/b)}{1+s(h/b)} - 4\frac{\sqrt{1+s^2}(h/b)}{1+2\sqrt{1+s^2}(h/b)}\right]$	$3\frac{1+2s(h/b)}{1+s(h/b)} - \frac{2\sqrt{1+s^2}(h/b)}{1+2\sqrt{1+s^2}(h/b)}$
5. Circle	$\frac{16h}{3d_o}\left(\frac{5\sin\omega/2}{\omega-\sin\omega} - \frac{1}{\omega\sin\omega/2}\right)$	$\frac{4h}{d_o}\left(\frac{6\sin\omega/2}{\omega-\sin\omega} - \frac{1}{\omega\sin\omega/2}\right)$
6. Parabola (perimeter equation $z = cy^2$)	$3 + \frac{4}{3+2ch}$	$3 + \frac{3}{3+2ch}$

Example 4.4

Compute the hydraulic exponent for uniform flow computation N of a trapezoidal channel with b = 6.1 m, s = 2 and h = 2 m based on the Manning formula.

Solution We have, b = 6.1 m, s = 2, h = 2 m and h/b = 2/6.1 = 0.328



$$\because N = \frac{2}{3}\times 5\times\frac{1+2s(h/b)}{1+s(h/b)} - \frac{2}{3}\times 4\times\frac{\sqrt{1+s^2}\,(h/b)}{1+2\sqrt{1+s^2}\,(h/b)}$$

$$= \frac{2}{3}\times 5\times\frac{1+2\times 2\times 0.328}{1+2\times 0.328} - \frac{2}{3}\times 4\times\frac{\sqrt{1+2^2}\times 0.328}{1+2\sqrt{1+2^2}\times 0.328}$$

$$= 4.65 - 0.79 = 3.86$$

Alternative solution

$A = (6.1+2\times 2)\times 2 = 20.2\ m^2$, $P = 6.1 + 2\times 2\times\sqrt{1+2^2} = 15.04$ m, R = A/P = 20.2/15.04 = 1.34 m, B = 6.1 + 2×2×2 = 14.1 m, $dP/dh = 2\sqrt{1+s^2} = 2\sqrt{5}$

$$\therefore\ N = \frac{2h}{3A}\left(5B - 2R\frac{dP}{dh}\right) = \frac{2\times 2}{3\times 20.2}(5\times 14.1 - 2\times 1.34\times 2\sqrt{5}) = 3.86$$

4.6 COMPUTATION OF NORMAL DEPTH

Analytical Method

The normal depth is an important parameter in the analysis of open channel flow. It may be computed using the Manning or the Chezy formula when the channel section, the discharge Q, the bottom slope S_0 and the Manning's n or the Chezy's C are given. For wide and triangular channels, the formula for the normal depth can be easily obtained.

A. Using the Manning formula

Wide channel

The Manning formula, Eq.(4.33), can be written for a given discharge Q and a given bottom slope S_0 as

$$A_n R_n^{2/3} = \frac{nQ}{\sqrt{S_0}} \tag{4.47}$$

since for uniform flow, $S_0 = S_f = S_w = S_n$. For a wide channel, A = h, R = h and Q is replaced by q. Therefore,

$$h_n . h_n^{2/3} = \frac{nq}{\sqrt{S_0}}$$

or

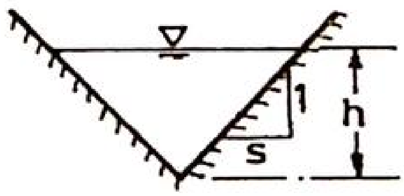
$$h_n^{5/3} = \frac{nq}{\sqrt{S_0}}$$

$$\therefore \; h_n = \left(\frac{nq}{\sqrt{S_o}} \right)^{3/5} \tag{4.48}$$

Triangular channel

For a triangular channel, $A = sh^2, P = 2h\sqrt{1+s^2}$ and $R = \frac{A}{P} = \frac{sh}{2\sqrt{1+s^2}}$. Hence, from Eq.(4.47), we obtain

$$sh_n^2 . \left(\frac{sh_n}{2\sqrt{1+s^2}} \right)^{2/3} = \frac{nQ}{\sqrt{S_0}}$$



or

$$\frac{s^{5/3} h_n^{8/3}}{2^{2/3}(1+s^2)^{1/3}} = \frac{nQ}{\sqrt{S_0}}$$

or

$$h_n^{8/3} = \frac{2^{2/3}(1+s^2)^{1/3}}{s^{5/3}} \frac{nQ}{\sqrt{S_0}}$$

$$\therefore \; h_n = \frac{2^{1/4}(1+s^2)^{1/8}}{s^{5/8}} \left(\frac{nQ}{\sqrt{S_o}} \right)^{3/8} \tag{4.49}$$

B. Using the Chezy formula

Using the Chezy formula, Eq. (4.32), it can be shown similarly that, for a wide channel

$$h_n = \left(\frac{q}{C\sqrt{S_o}}\right)^{2/3} \tag{4.50}$$

and, for a triangular channel

$$h_n = \frac{2^{1/5}(1+s^2)^{1/10}}{s^{3/5}}\left(\frac{Q}{C\sqrt{S_o}}\right)^{2/5} \tag{4.51}$$

Example 4.5

A wide channel with $S_0 = 0.0025$ carries a discharge of 3 m^2/s. Compute the normal depth and velocity (i) using the Manning formula when n = 0.020, and (ii) using the Chezy formula when $C = 45\ m^{1/2}/s$.

Solution (i) Using the Manning formula

$$h_n = \left(\frac{nq}{\sqrt{S_o}}\right)^{3/5} = \left(\frac{0.020 \times 3}{\sqrt{0.0025}}\right)^{3/5} = 1.12\ m$$

$$U_n = \frac{q}{h_n} = \frac{3}{1.12} = 2.69\ m/s$$

(ii) Using the Chezy formula

$$h_n = \left(\frac{q}{C\sqrt{S_0}}\right)^{2/3} = \left(\frac{3}{45 \times \sqrt{0.0025}}\right)^{2/3} = 1.21\ m$$

$$U_n = \frac{q}{h_n} = \frac{3}{1.21} = 2.48\ m/s$$

Example 4.6

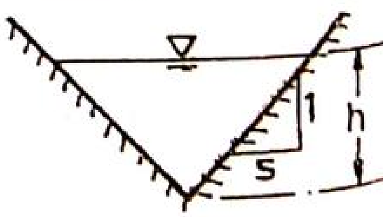
For a triangular channel with s = 2, $S_0 = 0.0016$ and n = 0.015, determine the normal depth and velocity if $Q = 10\ m^3/s$.

Solution Triangular channel, s = 2, $S_0 = 0.0016$, n = 0.015, $Q = 10\ m^3/s$

$$\therefore\ h_n = \frac{2^{1/4}(1+s^2)^{1/8}}{s^{5/8}}\left(\frac{nQ}{\sqrt{S_o}}\right)^{3/8} = \frac{2^{1/4}(1+2^2)^{1/8}}{2^{5/8}}\left(\frac{0.015 \times 10}{\sqrt{0.0016}}\right)^{3/8} = 1.55\ m$$

$$A_n = sh_n^2 = 2 \times 1.55^2 = 4.79\ m^2$$

$$\therefore\ U_n = \frac{Q}{A_n} = \frac{10}{4.79} = 2.09\ m/s$$



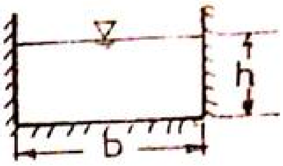
Trial-and-Error Method

For other channel sections, like the rectangular, trapezoidal, circular and parabolic sections, the normal depth can be conveniently obtained by the trial-and-error solution of Eq. (4.34) or (4.35).

Example 4.7

For a rectangular channel with b = 6 m, n = 0.025 and $S_0 = 0.0025$, compute the normal depth and velocity when $Q = 20\ m^3/s$.

Solution Rectangular channel, b = 6 m, n = 0.025, S_0 = 0.0025, Q = 20 m^3/s



$$A_n R_n^{2/3} = \frac{nQ}{\sqrt{S_o}} = \frac{0.025 \times 20}{\sqrt{0.0025}} = 10.000$$

Now, assume several values of h and compute the section factor $AR^{2/3}$ until the computed value of $AR^{2/3}$ is close to 10.000.

h(m)	A(m^2)	P(m)	R(m)	$AR^{2/3}$	Remarks
1.00	6.000	8.000	0.750	4.952	h too small
2.00	12.000	10.000	1.200	13.551	h too large
1.60	9.600	9.200	1.043	9.876	
1.62	9.720	9.240	1.052	10.054	
1.61	9.660	9.220	1.048	9.965	h closest

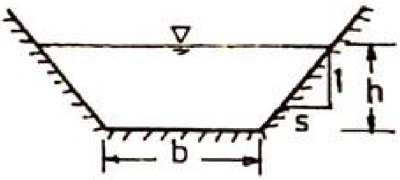
Hence, the normal depth, h_n = 1.61 m and the normal velocity

$$U_n = \frac{Q}{A_n} = \frac{20}{9.66} = 2.07 \ m/s$$

Example 4.8

For a trapezoidal channel with b = 6 m, s = 2, n = 0.025 and S_0 = 0.001, compute the normal depth and velocity when Q = 14 m^3/s.

Solution Trapezoidal channel, b = 6 m, s = 2, n = 0.025, S_0 = 0.001, Q = 14 m^3/s



$$A_n R_n^{2/3} = \frac{nQ}{\sqrt{S_o}} = \frac{0.025 \times 14}{\sqrt{0.001}} = 11.068$$

h(m)	A(m^2)	P(m)	R(m)	$AR^{2/3}$
1.00	8.000	10.472	0.764	6.684
2.00	20.000	14.944	1.338	24.288
1.30	11.180	11.814	0.946	10.776
1.31	11.292	11.858	0.952	10.929
1.32	11.405	11.903	0.958	11.084

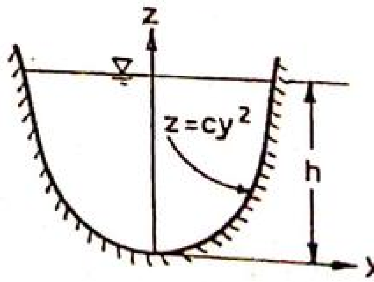
Hence, the normal depth, h_n = 1.32 m and the normal velocity

$$U_n = \frac{Q}{A_n} = \frac{14}{11.405} = 1.23 \ m/s$$

Example 4.9

Compute the normal depth and velocity in a parabolic channel with Q = 20 m^3/s, n = 0.025 and S_0 = 0.0025 when the profile of the channel is given by $y^2 = 4z$.

Solution Parabolic channel, Q = 20 m^3/s, n = 0.025, S_0 = 0.0025



$$A_n R_n^{2/3} = \frac{nQ}{\sqrt{S_o}} = \frac{0.025 \times 20}{\sqrt{0.0025}} = 10.000$$

Since $y^2 = 4z$, we have $z = 0.25y^2$ so that c = 0.25. Also, $x = 4h/B = 2h^{1/2}\sqrt{c} = 2h^{1/2}\sqrt{0.25} = h^{1/2}$.

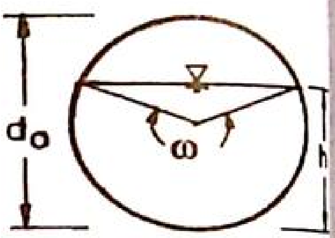
h(m)	B(m)	A(m²)	P(m)	R(m)	$AR^{2/3}$
1.00	4.000	2.667	4.591	0.581	1.856
2.00	5.657	7.542	7.192	1.049	7.785
3.00	6.928	13.856	9.562	1.449	17.743
2.26	6.013	9.060	7.822	1.158	9.993
2.27	6.027	9.120	7.846	1.162	10.008

Hence, the normal depth, h_n = 2.26 m and the normal velocity

$$U_n = \frac{Q}{A_n} = \frac{20}{9.060} = 2.21\ m/s$$

Example 4.10

A circular channel 2 m in diameter is laid on a slope of 0.001 and carries a discharge of 4 m^3/s. Compute the normal depth and velocity when n = 0.013.



Solution Circular channel, d_0 = 2 m, S_0 = 0.001, Q = 4 m^3/s, n = 0.013

$$A_n R_n^{2/3} = \frac{nQ}{\sqrt{S_o}} = \frac{0.013 \times 4}{\sqrt{0.001}} = 1.644$$

$\omega(rad)$	A(m²)	P(m)	R(m)	$AR^{2/3}$
1	0.079	1	0.079	0.015
2	0.545	2	0.273	0.229
3	1.429	3	0.476	0.872
4	2.378	4	0.595	1.682
3.94	2.328	3.94	0.591	1.639
3.95	2.337	3.95	0.592	1.647

Hence, ω_n = 3.95 radians and the normal depth

$$h_n = \frac{d_o}{2}\left(1 - \cos\frac{\omega_n}{2}\right) = \frac{2}{2}\left(1 - \cos\frac{3.95}{2}\right) = 1.39\ m$$

and, the normal velocity

$$U_n = \frac{Q}{A_n} = \frac{4}{2.337} = 1.71\ m/s$$

Numerical Methods

The numerical methods which have been used in Art. 3.3 for computing the critical depth in a channel, i.e. the method of bisection and the Newton-Raphson method, can also be used to compute the normal depth in rectangular, trapezoidal, circular and parabolic channels.

Bisection method: Suppose that we want to compute the normal depth in a channel for a given section, discharge Q, roughness coefficient n and bottom slope S_0. We choose two depths h_{min} and h_{max} such that the function

$$f(h) = AR^{2/3} - A_n R_n^{2/3} = AR^{2/3} - \frac{nQ}{\sqrt{S_0}} \qquad (4.52)$$

is negative for h = h_{min} (lower limit of h) and positive for h = h_{max} (upper limit of h), and the root of the equation f(h) = 0, which is the normal depth, lies between h_{min} and h_{max}. We bisect the interval, i.e. the depth is taken equal to $(h_{min} + h_{max})/2$ and f(h) is determined. If f(h) is positive, then the root is less than $(h_{min} + h_{max})/2$ and the upper limit is taken as $(h_{min} + h_{max})/2$. On the other hand, if f(h) is negative, then the lower limit is taken as $(h_{min} + h_{max})/2$. The procedure is repeated till the desired accuracy is attained.

Example 4.11

For a trapezoidal channel with b = 6 m, s = 2, n = 0.025 and S_0 = 0.001, compute the normal depth by the method of bisection if Q = 14 m^3/s.

Solution $A = (6 + 2h)h \qquad P = 6 + 4.472h \qquad R = A/P$

$$f(h) = AR^{2/3} - \frac{nQ}{\sqrt{S_0}} = \frac{A^{5/3}}{P^{2/3}} - \frac{0.025 \times 14}{\sqrt{0.001}} = \frac{[(6+2h)h]^{5/3}}{(6+4.472h)^{2/3}} - 11.068$$

Initially the values of h_{min} and h_{max} are taken as 0 and 10 m, respectively. The computation is carried out as follows.

h_{min}	h_{max}	$h=(h_{min}+h_{max})/2$	f(h)	Root lies between
0	10	5	148.647	0 and 5
0	5	2.5	26.562	0 and 2.5
0	2.5	1.25	-1.041	1.25 and 2.5
1.25	2.50	1.875	10.381	1.25 and 1.875
1.25	1.875	1.5625	4.105	1.25 and 1.5625
1.25	1.5625	1.4063	1.394	1.25 and 1.4063
1.25	1.4063	1.3281	0.143	1.25 and 1.3281
1.25	1.3281	1.2891	- 0.458	1.2891 and 1.3281
1.2891	1.3281	1.3086	- 9.357	1.3086 and 1.3281
1.3086	1.3281	1.3184	- 0.009	1.3184 and 1.3281
1.3184	1.3281	1.3233	0.067	1.3184 and 1.3233

Hence, the normal depth, h_n = 1.32 m

Newton-Raphson method: Suppose, we want to compute the normal depth in a channel for given section, Q, n and S_0. Obviously, when h = h_n

$$AR^{2/3} - A_n R_n^{2/3} = AR^{2/3} - \frac{nQ}{\sqrt{S_0}} = 0 \qquad or, \quad A^{5/3} - \frac{nQ}{\sqrt{S_0}} P^{2/3} = 0 \qquad \text{(since R = A/P)}$$

If we now assume

$$f(h) = A^{5/3} - \frac{nQ}{\sqrt{S_0}} P^{2/3} \tag{4.53}$$

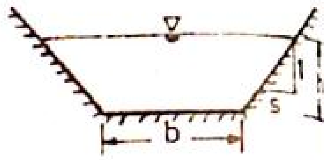
then

$$f'(h) = \frac{5}{3} A^{2/3} \frac{dA}{dh} - \frac{nQ}{\sqrt{S_0}} \times \frac{2}{3} P^{-1/3} \frac{dP}{dh} = \frac{5}{3} A^{2/3} B - \frac{2nQ}{3\sqrt{S_0}} P^{-1/3} \frac{dP}{dh} \quad (\because \frac{dA}{dh} = B) \tag{4.54}$$

For a given channel section, $f(h)$ and $f'(h)$ depend on the depth of flow only and can be easily calculated.

Example 4.12

For a trapezoidal channel with b = 6 m, s = 2, n = 0.025 and S_0 = 0.001, compute the normal depth by the Newton-Raphson method if Q = 14 m^3/s.



Solution $A=(6+2h)h, P=6+2\sqrt{5}h, B=6+4h, dP/dh=2\sqrt{5}$

$$\frac{nQ}{\sqrt{S_0}}=\frac{0.025\times 14}{\sqrt{0.001}}=11.068$$

Using Eqs.(4.53) and (4.54), we obtain

$$f(h)=[(6+2h)h]^{5/3}-11.068(6+2\sqrt{5}h)^{2/3}=3.175[(3+h)h]^{5/3}-17.569(3+\sqrt{5}h)^{2/3}$$

and

$$f'(h)=5.291(3+2h)[(3+h)h]^{2/3}-26.191(3+\sqrt{5}h)^{-1/3}$$

The computation of normal depth is carried out as follows.

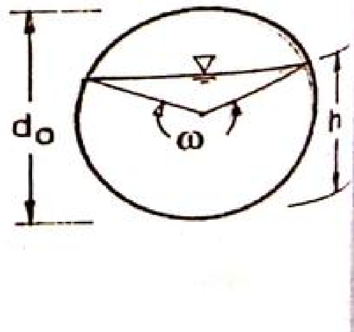
h	$f(h)$	$f'(h)$	$\Delta h=-\frac{f(h)}{f'(h)}$	$h=h+\Delta h$
1.000	-20.979	51.579	0.407	1.407
1.407	7.493	89.555	- 0.083	1.324
1.324	0.406	81.215	- 0.005	1.319
1.319	0.003	79.094	- 0.000	1.319

Hence, the normal depth, h_n = 1.32 m

Normal Depth for a Conduit with Gradually Closing Top

In some channel sections the top width either remains constant or increases with flow depth. The section factor $AR^{2/3}$ for these channel sections increases with an increase in the depth of flow and there is only one normal depth for a given discharge. The rectangular, triangular, trapezoidal and parabolic sections fall in this category. However, for channel sections having a gradually closing top, the section factor at first increases with depth and decreases with depth when the full depth is approached. Consequently, it is possible to have two normal depths for the same value of the section factor and, therefore, for the same discharge. The circular section falls in this later category.

In a circular section when the depth is greater than about $0.82d_0$, it is possible to have two different normal depths for the same discharge and a small disturbance in the water surface may cause the water surface to seek alternate normal depths, thus contributing to the instability of the water surface. Therefore, in the design of a circular channel section, it is desirable to restrict the depth to a value less than or equal to $0.80d_0$.



4.7 COMPUTATION OF NORMAL AND CRITICAL SLOPES

The normal slope (S_n) is the longitudinal slope of the channel that is required to maintain uniform flow in the channel. When the Manning formula is used

$$S_n = \frac{n^2 U_n^{\ 2}}{R_n^{\ 4/3}} = \frac{n^2 Q_n^{\ 2}}{A_n^{\ 2} R_n^{\ 4/3}} \tag{4.55}$$

or, when the Chezy formula is used

$$S_n = \frac{U_n^{\ 2}}{C^2 R_n} = \frac{Q_n^{\ 2}}{C^2 A_n^{\ 2} R_n} \tag{4.56}$$

So, when the channel section, Q, n or C and h_n are given, the normal slope can be obtained using Eq.(4.55) or (4.56).

The critical slope (S_c) is the longitudinal slope of the channel for which the flow in the channel is both uniform and critical, i.e. uniform flow occurs in a critical state i.e. $S_n = S_c$, $U_n = U_c$ and $h_n = h_c$. When the channel section, n or C and h or Q are given, the critical slope can be determined using the Manning formula as

$$S_c = \frac{n^2 U^2}{R^{4/3}} = \frac{n^2 Q^2}{A^2 R^{4/3}} \tag{4.57}$$

or, using the Chezy formula as

$$S_c = \frac{U^2}{C^2 R} = \frac{Q^2}{C^2 A^2 R} \tag{4.58}$$

The condition for critical flow, Fr = 1, produces a direct relationship between Q and h_c. So, when Q is given, the critical depth h_c (= h_n) is first computed using the critical condition and then the critical slope is computed using Eq.(4.57) or (4.58). On the other hand, when h_n (= h_c) is given, the mean velocity U or the discharge Q is first determined using the critical flow condition and then the critical slope is computed using Eq.(4.57) or (4.58).

Example 4.13

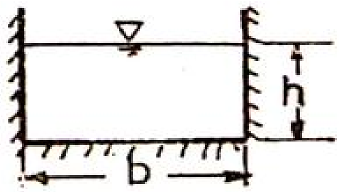
A rectangular channel has a bottom width of 6 m, $\alpha = 1.12$ and n = 0.020. (i) For $h_n = 1$ m and Q = 11 m^3/s, determine the normal slope. (ii) Determine the critical slope for Q = 11 m^3/s. (iii) Determine the critical slope for $h_n = 1$ m.

Solution Rectangular channel. b = 6 m, $\alpha = 1.12$, n = 0.020

(i) $h_n = 1$ m   Q = 11 m^3/s

$A = bh = 6 \times 1 = 6\ m^2$, $P = b + 2h = 6 + 2\times 1 = 8$ m, $R = A/P = 0.75$ m

$$\therefore S_n = \left(\frac{nQ}{AR^{2/3}}\right)^2 = \left(\frac{0.020\times 11}{6\times 0.75^{2/3}}\right)^2 = 0.0020$$



(ii) Q = 11 m^3/s

$$h_c = \sqrt[3]{\frac{\alpha Q^2}{gb^2}} = \sqrt[3]{\frac{1.12\times 11^2}{9.81\times 6^2}} = 0.73\ m$$

$\therefore h_n = h_c = 0.73\ m$

$A = bh = 6\times 0.73 = 4.36\ m^2$, $P = b + 2h = 6 + 2\times 0.73 = 7.45$ m, $R = A/P = 0.58$ m

$$\therefore S_c = \left(\frac{nQ}{AR^{2/3}}\right)^2 = \left(\frac{0.020\times 11}{4.36\times 0.58^{2/3}}\right)^2 = 0.0053$$

(iii) $h_c = h_n = 1$ m

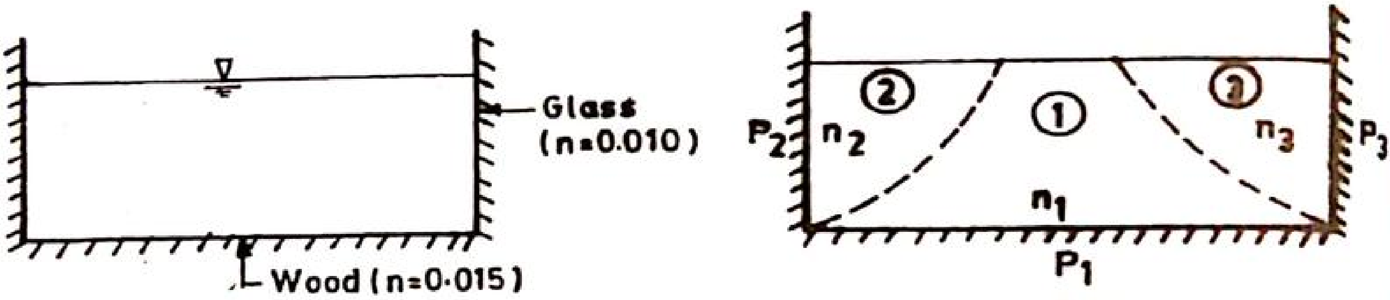
$A = bh = 6 \times 1 = 6\ m^2$, $P = b + 2h = 6 + 2 \times 1 = 8$ m, $R = A/P = 0.75$ m

$$Q = \sqrt{g/\alpha}\, b h_c^{1.5} = \sqrt{9.81/1.12} \times 6 \times 1^{1.5} = 17.76\ m^3/s$$

$$\therefore S_c = \left(\frac{nQ}{AR^{2/3}}\right)^2 = \left(\frac{0.020 \times 17.76}{6 \times 0.75^{2/3}}\right)^2 = 0.0051$$

4.8 CHANNEL SECTION WITH COMPOSITE ROUGHNESS

A channel section in which the roughness is different for different parts of the perimeter is known as a channel section with composite roughness. A good example of such a section is provided by a rectangular flume built with a wooden bottom and glass walls having different n-values for wood and glass (Fig. 4.1a).



(a) (b)

Fig. 4.1 Channel section with composite roughness

In applying the Manning (or the Chezy) formula to compute flow in such a section, it is necessary to compute an equivalent roughness coefficient n (or Chezy's C) for the entire perimeter. Consider a channel in which the flow area is divided into 3 parts, as shown in Fig.4.1(b), of which the wetted perimeters P_1, P_2 and P_3 and the corresponding coefficients of roughness n_1, n_2 and n_3 are known. Following Horton (1933), it is assumed that *each part of the area has the same mean velocity that is also equal to the mean velocity of the whole section*, i.e.

$$U_1 = U_2 = U_3 = U \tag{4.59}$$

Using Eq. (4.26), we can write Eq. (4.59) as

$$\frac{1}{n_1} R_1^{2/3} S_0^{1/2} = \frac{1}{n_2} R_2^{2/3} S_0^{1/2} = \frac{1}{n_3} R_3^{2/3} S_0^{1/2} = \frac{1}{n} R^{2/3} S_0^{1/2} \tag{4.60}$$

Since the bottom slope S_0 is same for all the three parts and R = A/P, we obtain

$$\frac{1}{n_1}\left(\frac{A_1}{P_1}\right)^{2/3} = \frac{1}{n_2}\left(\frac{A_2}{P_2}\right)^{2/3} = \frac{1}{n_3}\left(\frac{A_3}{P_3}\right)^{2/3} = \frac{1}{n}\left(\frac{A}{P}\right)^{2/3} \tag{4.61}$$

so that

$$A_1 = \left(\frac{n_1}{n}\right)^{3/2}\left(\frac{P_1}{P}\right)A, \quad A_2 = \left(\frac{n_2}{n}\right)^{3/2}\left(\frac{P_2}{P}\right)A \text{ and } A_3 = \left(\frac{n_3}{n}\right)^{3/2}\left(\frac{P_3}{P}\right)A \tag{4.62}$$

Since $A = A_1 + A_2 + A_3$, we obtain

$$A = \left(\frac{n_1}{n}\right)^{3/2}\left(\frac{P_1}{P}\right)A + \left(\frac{n_2}{n}\right)^{3/2}\left(\frac{P_2}{P}\right)A + \left(\frac{n_3}{n}\right)^{3/2}\left(\frac{P_3}{P}\right)A$$

or

$$Pn^{3/2} = P_1 n_1^{3/2} + P_2 n_2^{3/2} + P_3 n_3^{3/2}$$

or

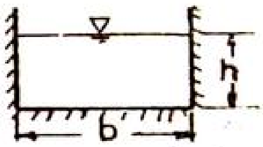
$$n = \left(\frac{P_1 n_1^{3/2} + P_2 n_2^{3/2} + P_3 n_3^{3/2}}{P} \right)^{2/3} \qquad (4.63)$$

which gives the equivalent n for the entire section.

Example 4.14

The sides of a laboratory flume are made of glass ($n = 0.010$) and the bottom is made of wood ($n = 0.015$). The flume is rectangular with $b = 1$ m and is laid on a slope of 0.001. Compute the discharge in the flume if $h_n = 0.4$ m.

Solution Designating the perimeter of the bottom as P_1 and the perimeter of one side as P_2, we have $P_1 = 1$ m, $P_2 = 0.4$ m, $P = P_1 + 2P_2 = 1 + 2\times 0.4 = 1.8$ m, $n_1 = 0.015$, $n_2 = 0.010$. Then,



$$A = 1\times 0.4 = 0.4 \text{ m}^2, \quad R = A/P = 0.4/1.8 = 0.22 \text{ m}$$

$$n = \left(\frac{P_1 n_1^{3/2} + 2P_2 n_2^{3/2}}{P} \right)^{2/3} = \left(\frac{1\times 0.015^{3/2} + 2\times 0.4\times 0.010^{3/2}}{1.8} \right)^{2/3} = 0.013$$

$$Q = \frac{1}{n} AR^{2/3} S_0^{1/2} = \frac{1}{0.013} \times 0.4 \times 0.22^{2/3} \times 0.001^{1/2} = 0.36 \; m^3/s$$

4.9 COMPOUND CROSS-SECTION

The cross-section of a channel which consists of several distinct subsections (Fig.4.2) is known as a compound cross-section. For example, an alluvial river subjected to seasonal floods generally consists of a main channel and two side channels. The side channels are usually shallower and rougher than the main channel. So, the mean velocity in the main channel is greater than the mean velocity in the side channels.

In dealing with a compound cross-section, the subsections (the main and the side channels) are first separated by drawing vertical lines (shown as dotted lines in Fig.4.2). It is assumed that (i) the water surface and the energy line are horizontal across the cross-section, and (ii) the vertical separation lines do not contribute to the wetted perimeters of the subsections.

The discharges in the subsections are computed by applying the Manning formula separately to each subsection, i.e.

$$Q_1 = \frac{1}{n_1} A_1 R_1^{2/3} S_0^{1/2} \qquad (4.64a)$$

$$Q_2 = \frac{1}{n_2} A_2 R_2^{2/3} S_0^{1/2} \qquad (4.64b)$$

$$Q_3 = \frac{1}{n_3} A_3 R_3^{2/3} S_0^{1/2} \qquad (4.64c)$$

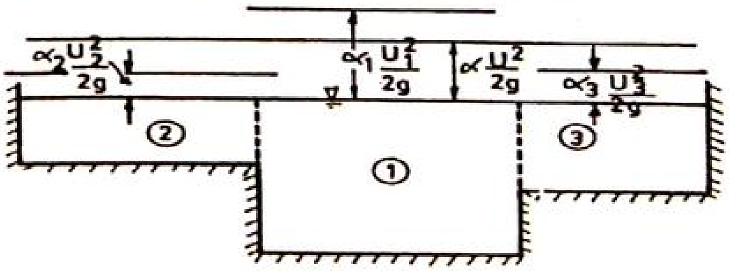


Fig. 4.2 Compound cross-section

The total discharge for the entire section is equal to the sum of these discharges, i.e

$$Q = Q_1 + Q_2 + Q_3 \tag{4.65}$$

The mean velocity for the entire section is equal to the total discharge divided by the total area, i.e.

$$U = \frac{Q}{A} = \frac{Q_1 + Q_2 + Q_3}{A_1 + A_2 + A_3} \tag{4.66}$$

where the total area A is given by

$$A = A_1 + A_2 + A_3 \tag{4.67}$$

The equivalent n-value for the entire section can be computed using the Manning formula for the entire section as

$$n = AR^{2/3}S_0^{1/2}/Q \tag{4.68}$$

The energy and momentum coefficients for the entire section are obtained using the equations

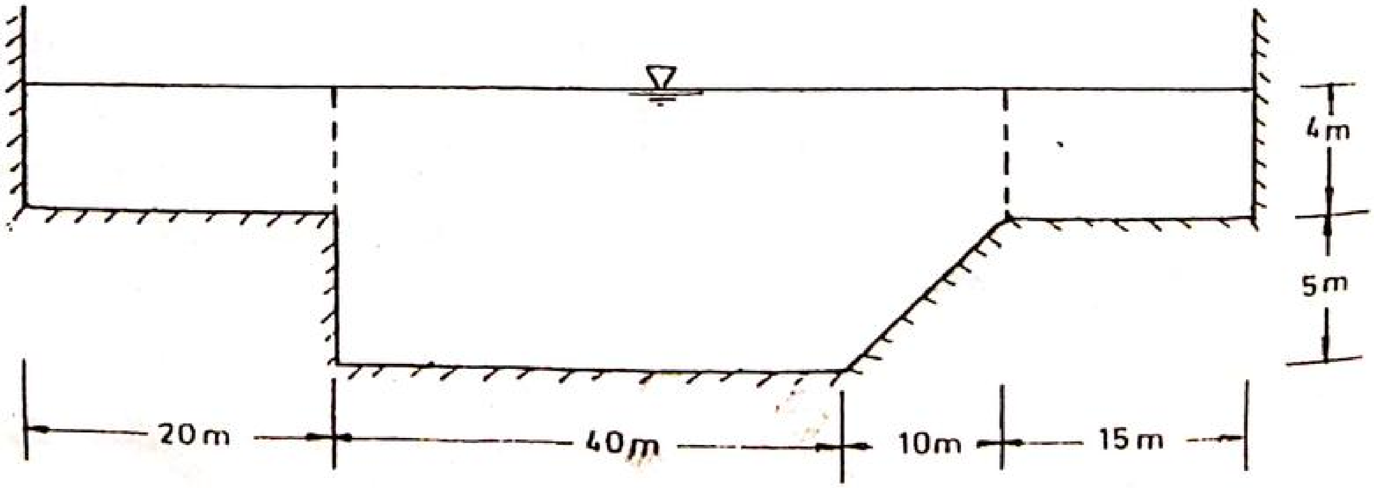
$$\alpha = \frac{\alpha_1 K_1^3/A_1^2 + \alpha_2 K_2^3/A_2^2 + \alpha_3 K_3^3/A_3^2}{K^3/A^2} \tag{4.69}$$

$$\beta = \frac{\beta_1 K_1^2/A_1 + \beta_2 K_2^2/A_2 + \beta_3 K_3^2/A_3}{K^2/A} \tag{4.70}$$

where α_1, α_2 and α_3 are the energy coefficients, β_1, β_2 and β_3 are the momentum coefficients and K_1, K_2 and K_3 are the conveyances of the individual subsections, respectively, and K $(= K_1 + K_2 + K_3)$ is the conveyance for the entire section.

Example 4.15

A channel consists of a main section and two side sections as shown in the following figure. Compute the total discharge and the mean velocity of flow for the entire section when n = 0.025 for the main section, n = 0.035 for the side sections and S_0 = 0.0001. Also, compute the numerical values of n, α and β for the entire section assuming that α = 1.12 and β = 1.04 for the main and side sections.



Solution

The main and the two side (left and right) sections are separated by drawing vertical (dotted) lines as shown. The computed values of A, P, R, Q and K for the three subsections and A, P, Q and K for the entire section are shown below.

Section	A	P	R	Q	K
Main	425.0	56.18	7.565	655.12	65512
Left	80.0	24.00	3.333	51.00	5100
Right	60.0	19.00	3.158	36.90	3690
Σ	565.0	99.18		743.02	74302

Hence, the total discharge, Q = 743.02 m^3/s

The mean velocity U, roughness coefficient n, energy coefficient α and momentum coefficient β for the entire section are obtained as follows

$$U = \frac{Q}{A} = \frac{743.02}{565.0} = 1.315\ m/s$$

$$n = AR^{2/3}S_0^{1/2}/Q = 565.0 \times (565.0/99.18)^{2/3} \times 0.0001^{1/2}/743.02 = 0.024$$

$$\alpha = \frac{\alpha_1 K_1^3/A_1^2 + \alpha_2 K_2^3/A_2^2 + \alpha_3 K_3^3/A_3^2}{K^3/A^2}$$

$$= \frac{1.12 \times 65512^3/425^2 + 1.12 \times 5100^3/80^2 + 1.12 \times 3690^3/60^2}{74302^3/565^2} = 1.236$$

$$\beta = \frac{\beta_1 K_1^2/A_1 + \beta_2 K_2^2/A_2 + \beta_3 K_3^2/A_3}{K^2/A}$$

$$= \frac{1.04 \times 65512^2/425 + 1.04 \times 5100^2/80 + 1.04 \times 3690^2/60}{74302^2/565} = 1.090$$

4.10 COMPUTATION OF FLOOD DISCHARGE BY SLOPE-AREA METHOD

The slope-area method is an indirect method of estimating the flood discharge in a river from past records of stages at different sections using the Manning or the Chezy formula. Flood flows in natural channels are usually unsteady and varied. The use of a steady uniform flow formula for computing flood discharge is acceptable only when the changes in flood stage and discharge are sufficiently gradual and the data available are not sufficient to justify the use of a more sophisticated method.

The selection of a suitable channel reach is probably the most important aspect of the slope-area method. The following points must be considered in selecting the channel reach.

i) Reliable and good quality highwater marks must be available in the selected reach.

ii) A straight and uniform reach is preferred and a gradually contracting reach should be chosen over an expanding reach if there is a choice. The change in conveyance in the reach should be less than 30%.

iii) The length of the reach L should be at least 75 times the average depth of flow, the fall of water surface F should be equal to or greater than the velocity head and the fall should at least be equal to 0.15 m.

With reference to Fig. 4.3, applying the energy equation between sections 1 and 2, we get

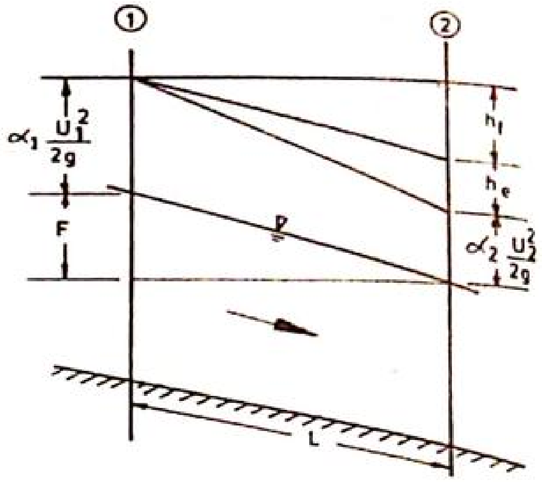
$$F + \alpha_1 \frac{U_1^2}{2g} = \alpha_2 \frac{U_2^2}{2g} + h_f + h_e \qquad (4.71)$$

or

$$h_f = F + (\alpha_1 \frac{U_1^2}{2g} - \alpha_2 \frac{U_2^2}{2g}) - h_e \qquad (4.72)$$

where h_e is the eddy loss, given by

$$h_e = k\left|\alpha_1 \frac{U_1^2}{2g} - \alpha_2 \frac{U_2^2}{2g}\right| \qquad (4.73)$$



and the coefficient k is assumed to range between 0 and 0.1 for gradual contractions, between 0 and 0.2 for gradual expansions and to have a value of 0.5 for abrupt expansions or contractions.

Fig. 4.3 Definition sketch

The following data are required for the slope-area method: (i) the cross-sectional areas A_1 and A_2 of the upstream and downstream sections of the selected reach, (ii) the wetted perimeters P_1 and P_2, (iii) the Manning roughness coefficients n_1 and n_2, (iv) the energy coefficients α_1 and α_2, (v) the length of the reach L, and (vi) the fall of the water surface F between the two sections. In addition, the eddy-loss coefficient k is to be given if the eddy loss is to be included.

The computation of flood discharge using this method involves the following steps:

i) Compute the conveyances K_1 and K_2 for the two end sections.

ii) Compute the geometric mean conveyance for the reach, i.e.

$$K = \sqrt{K_1 K_2} \qquad (4.74)$$

iii) Since the discharge Q is not known initially, as a first approximation assume that $h_f = F$ and hence

$$S_f = \frac{h_f}{L} = \frac{F}{L} \qquad (4.75)$$

iv) The first approximation of the discharge (which is also the uniform flow discharge) is then computed using the equation

$$Q = K\sqrt{S_f} \qquad (4.76)$$

v) A more accurate value of the energy slope is now obtained using the equation

$$S_f = \frac{h_f}{L} \qquad (4.77)$$

where h_f is given by Eq.(4.72). The corresponding discharge is then computed by Eq.(4.76) using the revised slope given by Eq.(4.77).

vi) Repeat step (v) until the assumed and the computed discharges agree.

Example 4.16

Compute the flood discharge through a river reach of 850 m using the following data:

$A_1 = 10350\ m^2,\ P_1 = 2035\ m,\ n_1 = 0.030,\ \alpha_1 = 1.15$

$A_2 = 9275\ m^2,\ P_2 = 1965\ m,\ n_2 = 0.030,\ \alpha_2 = 1.18$

The fall of water surface in the reach is 0.76 m. Neglect eddy loss.

Solution $L = 850$ m, $F = 0.76$ m, $h_e = 0$

$$K_1 = \frac{1}{n_1} A_1 R_1^{2/3} = \frac{1}{0.030} \times 10350 \times (10350/2035)^{2/3} = 1020320$$

$$K_2 = \frac{1}{n_2} A_2 R_2^{2/3} = \frac{1}{0.030} \times 9275 \times (9275/1965)^{2/3} = 869950$$

$$K = \sqrt{K_1 K_2} = \sqrt{1020320 \times 869950} = 942140$$

Approxi-mation	Assumed $Q(m^3/s)$	F (m)	$\alpha_1 \frac{U_1^2}{2g}$ (m)	$\alpha_2 \frac{U_2^2}{2g}$ (m)	h_f (m)	S_f ($\times 10^{-4}$)	Computed $Q(m^3/s)$
1st	-	0.76	-	-	0.7600	8.94	28170
2nd	28170	"	0.4343	0.5548	0.6395	7.52	25840
3rd	25840	"	0.3653	0.4668	0.6585	7.75	26220
4th	26220	"	0.3762	0.4806	0.6556	7.73	26170
5th	26170	"	0.3747	0.4788	0.6559	7.72	26170

The flood discharge, $Q = 26,170\ m^3/s$

PROBLEMS AND EXERCISES

4.1 Define (i) shear stress, (ii) friction velocity, (iii) laminar sublayer, (iv) relative roughness, (v) hydraulically smooth boundary, (v) hydraulically rough boundary, (vi) normal slope, (vii) critical slope, (viii) channel section with composite roughness, and (ix) compound cross-section

4.2(a) State the condition for the establishment of uniform flow in an open channel.

(b) Explain why flow cannot be uniform in (i) a horizontal channel, (ii) an adverse slope channel, and (iii) a frictionless channel.

(c) "Uniform flow of an ideal fluid is impossible" – justify.

4.3 State the two assumptions which are to be made for the development of the Chezy formula.

4.4 Why is the Manning formula different in different systems of units? Convert the Manning formula from SI units to English or FPS units. Describe the factors which affect the Manning's roughness coefficient n.

4.5 Write the SI units of Chezy's C, Darcy-Weisbach friction factor f and Manning's n. State whether the numerical values of C, f and n are same or different in different systems of units.

4.6 (a) Write two major advantages of the Strickler formula.

(b) In the design of a circular channel section, it is desirable to restrict the depth to a value less than or equal to $0.80d_0$. State why.

4.7(a) Derive the general expression for the hydraulic exponent for uniform flow computation N based on the Manning formula, and then determine the numerical value of N for a (i) wide channel, and (ii) triangular channel.

(b) Derive the general expression for the hydraulic exponent for uniform flow computation N based on the Chezy formula, and then determine the numerical value of N for a (i) wide channel, and (ii) triangular channel.

4.8(a) Derive the expression for normal depth in a (i) wide channel, and (ii) triangular channel, based on the Manning formula.

(b) Derive the expression for normal depth in a (i) wide channel, and (ii) triangular channel, based on the Chezy formula.

4.9 Derive the expression for the equivalent roughness coefficient n for a composite cross-section (Eq. 4.63).

4.10 State the three points which are to be considered in selecting the channel reach for computing flood discharge by the slope-area method.

4.11 Assuming that the velocity distribution along a vertical in an open channel is logarithmic, compute the position of the mean velocity below the free surface. Also show that (i) the velocity at 0.6 depth, and (ii) the average of the velocities at 0.2 and 0.8 depths are approximately equal to the mean velocity in a vertical.

4.12(a) A trapezoidal channel has a bottom width of 6.0 m, side slopes of 1.5H:1V, a depth of flow of 2.0 m, n = 0.025 and $S_0 = 0.0001$. Assuming that the flow is uniform, (i) compute Q, (ii) compute C, f, τ_0 and u*, and (iii) compute k_s, determine whether the channel boundary is smooth or rough and state if the Manning formula is applicable for computing flow in this channel. Assume that the velocity distribution is logarithmic.

(b) Consider the following data for the Padma (Ganges) river at the Baruria station in Faridpur on the 2nd July, 1989: $A = 33{,}500\ m^2$, $Q = 56{,}200\ m^3/s$ and $B = 3820$ m. Assuming that the flow is uniform, (i) compute n, C, f, u_* and τ_0, and (ii) determine whether the channel boundary is smooth or rough taking the velocity distribution as logarithmic. Assume that the river is wide. Longitudinal slope of the river is 4 cm/km.

4.13(a) Using the Manning formula and taking h = 1 m, compute the hydraulic exponent for uniform flow computation N for a (i) rectangular channel with b = 6 m, and (ii) trapezoidal channel with b = 6 m and s = 2.

(b) Solve Problem 4.13(a) using the Chezy formula.

4.14(a) A wide channel with n = 0.025 and $S_0 = 0.0025$ carries a discharge o . Compute the normal depth and velocity.

(b) A wide channel with $S_0 = 0.006$ and $C = 50\ m^{1/2}/s$ carries a disch of $4\ m^2/s$. Compute the normal depth and velocity.

4.15(a) A triangular channel with side slopes 1:1 is laid on a slope of 0.001. If n = 0.015 and h_n = 1 m, compute the discharge.

(b) A triangular channel with s = 1, n = 0.025 and $S_0 = 0.0025$ carries a discharge of $5\ m^3/s$. Compute the normal depth and velocity.

4.16 Uniform flow occurs in an open channel with $h_n = 1$ m, $S_0 = 0.0001$ and n = 0.015. Compute the discharge if the channel is (i) rectangular with b = 6 m, (ii) trapezoidal with b = 6 m and s = 1, (iii) triangular with s = 1.5, (iv) parabolic whose profile is given by $y^2 = 4z$, and (v) circular whose diameter is 1.5 m.

4.17 Compute the normal depth and velocity in a (i) rectangular channel with b = 8 m and Q = $22\ m^3/s$, (ii) trapezoidal channel with b = 6 m, s = 2 and Q = $30\ m^3/s$, (iii) parabolic channel whose profile is given by $y^2 = 5z$ and Q = $15\ m^3/s$, and (iv) circular channel whose diameter is 2 m and Q = $3\ m^3/s$. In all cases, take n = 0.025 and $S_0 = 0.0025$.

4.18(a) A rectangular channel having n = 0.025 and $S_0 = 0.0001$ carries a discharge of $6\ m^3/s$ at a normal depth of 1.5 m. Compute the bottom width.

(b) A trapezoidal channel having side slopes of 1.5H:1V, n = 0.020 and $S_0 = 0.0002$ carries a discharge of $25\ m^3/s$ at a normal depth of 2 m. Compute the bottom width.

4.19 A trapezoidal channel has a bottom width of 6 m, side slopes of 1.5:1, $\alpha = 1.12$ and n = 0.025. (i) Determine the normal slope at a normal depth of 1 m when the discharge is $20\ m^3/s$. (ii) Determine the critical slope when the discharge is $20\ m^3/s$. (iii) Determine the critical slope when the normal depth is 1 m.

4.20 An unlined irrigation canal is trapezoidal in shape with b = 6 m, s = 1, n = 0.025, h = 2 m and $S_0 = 0.0005$. (a) Compute the discharge carried by the canal under uniform flow condition. (b) It is proposed to line the canal with concrete having n = 0.013. Compute the discharge that would be carried by the canal when (i) only the sides are lined, (ii) only the bottom is lined, and (iii) both the bottom and the sides are lined.

4.21 A rectangular testing channel is 0.60 m wide and is laid on a slope of 0.1%. When the channel bed and walls were made smooth by neat cement, the measured normal depth of flow was 0.40 m for a discharge of $0.23\ m^3/s$. The same channel was then roughened by cemented sand grains and the measured normal depth was 0.35 m for a discharge of $0.12\ m^3/s$. Determine the discharge for a normal depth of 0.45 m if the bed is roughened and the walls are made smooth.

4.22 A channel consists of a main section and two side sections as shown in Fig.4.4. Compute the total discharge, the mean velocity of flow and the Manning's n for the entire section when n = 0.025 for the main channel, n = 0.045 for the side channels and S_0 = 0.0002. Also, compute the numerical values α and β for the entire section assuming that α = β = 1.00 for the main and the side sections.

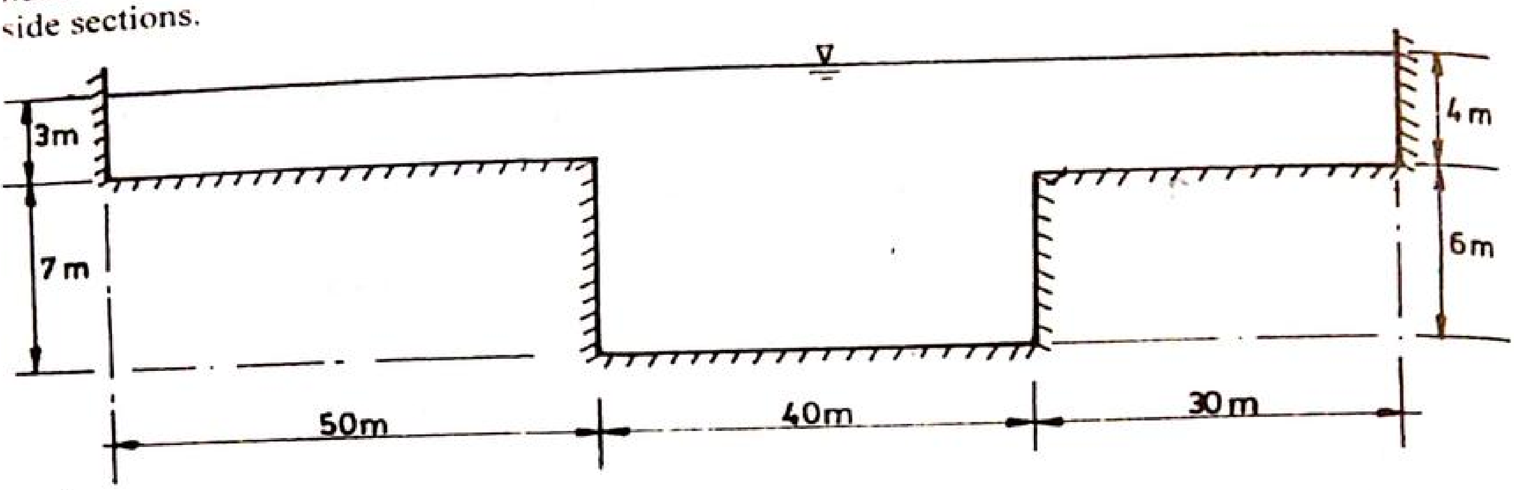


Fig. 4.4 (Prob. 4.22)

4.23 Compute the flood discharge through a river reach 1000 m long having a fall in water surface of 0.85 m. Neglect the eddy loss. Use the following data:

Section	A (m^2)	P (m)	n	α
Upstream	12,000	2,150	0.030	1.15
Downstream	10,500	2,050	0.030	1.18

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Chapter 5

DESIGN OF CHANNELS

5.1 INTRODUCTION

The hydraulic design of a channel, like an irrigation canal or a drainage channel, involves the determination of its alignment, shape, size and bottom slope to convey the required discharge. Since a channel may be lined or unlined and may or may not carry sediment, we consider the design of three types of channels:(i) rigid-boundary or non-erodible channels carrying clear water with little or no sediment, (ii) mobile-boundary or erodible channels carrying clear water which scour but do not silt, and (iii) alluvial or mobile boundary channels carrying sediment which both scour and silt.

The channel alignment is selected so that the channel length is as short as possible. The longitudinal slope of the channel generally depends on the topography of the land. The type of materials forming the channel body determines the roughness characteristics and the side slope of the channel.

The shape of the cross-section is generally decided by the discharge and the engineering properties of the material forming the channel body. Normally, a trapezoidal section is used when the discharge is large. For small discharges, triangular sections are used. Rectangular cross-sections are also used when the discharge is small or in special situations, such as rock cuts, steep chutes and cross-drainage works.

In the design of a channel, it is desirable to maintain subcritical flow in the channel having a Froude number range of 0.3 to 0.4. When the Froude number is high and the flow approaches the critical state, the water surface becomes unstable and wavy and large disturbances are expected at bends and obstructions.

The *minimum permissible velocity* is the lowest mean velocity of flow that will prevent sedimentation and vegetative growth. In general, a velocity of 2 to 3 ft/sec (0.61 to 0.91 m/s) will prevent sedimentation when the silt load of the flow is low (Chow, 1959). A velocity of 2.5 ft/sec (0.75 m/s) is usually sufficient to prevent the growth of vegetation.

The *maximum permissible velocity* is the greatest mean velocity that will not cause erosion of the channel body. This velocity is generally taken as 2 m/s (Ranga Raju, 1993).

The *freeboard* is the vertical distance between the top of the channel and the water surface at the design condition. It is provided to prevent the water level from overtopping the sides of the channel due to its fluctuation caused by wind, tide, superelevation at bends, hydraulic jump, etc. Freeboards varying from 5% to 30% of the depth of flow are commonly used in design. As a rough estimate of freeboard, the United States Bureau of Reclamation (USBR) suggested the formula

$$F_b = \sqrt{ch} \tag{5.1}$$

where F_b is the freeboard in ft, h is the depth of flow in ft and c is a factor varying from 1.5 for Q < 20 ft^3/sec (0.57 m^3/s) to 2.5 for Q > 300 ft^3/sec (8.52 m^3/s). In case of lined channels, the top of the lining is generally located half the total freeboard above the water surface.

5.2 RIGID-BOUNDARY OR NON-ERODIBLE CHANNELS CARRYING NO SEDIMENT

Most of the lined channels and the built-up unlined channels fall into this category. A uniform flow formula, like the Manning or the Chezy formula, is used to compute the section dimensions of the channel, by maintaining a velocity which will prevent sedimentation and growth of vegetation. The maximum permissible velocity is not usually the criterion in the design of non-erodible channels. However, when the flow velocity is very high, the rapidly flowing water tends to lift the lining blocks and push them out of position. So, the mean velocity of flow is restricted to 2 m/s to avoid any danger to the lining materials.

The materials used for lining include concrete, stone or brick masonry, steel, cast iron, timber, glass, plastic, geotextile, etc. The choice of a material depends mainly on the availability and cost of the material and the purpose and the method of construction. The provision of lining in a channel (i) permits the water to flow at high velocities, (ii) decreases seepage and percolation losses, (iii) reduces the costs of operation and maintenance, and (iv) ensures the stability of the channel section. In some cases, in particular when water is very scarce, a lined channel may be more practicable and economical than an unlined channel in spite of the initial cost of lining.

Best Hydraulic Section

A channel section that conveys the maximum discharge for a given area is known as the best hydraulic section. Since $Q \propto AR^{2/3}$ and $R = A/P$, the best hydraulic section gives minimum wetted perimeter P and maximum hydraulic radius R for a given area A. It is to be noted that a minimum wetted perimeter also gives minimum amount of lining.

Among all possible open channel cross-sections, the best hydraulic section is a semicircle ($h = d_0/2$). Also, for any channel section of a given geometric shape, there is a relationship between the various geometric elements to form the best hydraulic section such that a semicircle can be inscribed in it. Thus, the best hydraulic rectangular section is one-half of a square ($B = 2h$), the best hydraulic triangular section is one-half of a square ($s = 1$), the best hydraulic trapezoidal section is one-half of a regular hexagon ($s = 1/\sqrt{3}$) and for the best hydraulic parabolic section the top width is equal to $2\sqrt{2}$ times the depth of flow ($B = 2\sqrt{2}h$). Table 5.1 lists the geometric elements of some best hydraulic sections.

Table 5.1 Geometric elements of some best hydraulic sections

Cross-section	A	P	R	B	D
Rectangle (half of a square)	$2h^2$	$4h$	$h/2$	$2h$	h
Triangle (half of a square)	h^2	$2\sqrt{2}h$	$\sqrt{2}h/4$	$2h$	$h/2$
Trapezoid (half of a hexagon)	$\sqrt{3}h^2$	$2\sqrt{3}h$	$h/2$	$4\sqrt{3}h/3$	$3h/4$
Circle (semi-circle)	$\pi h^2/2$	πh	$h/2$	$2h$	$\pi h/4$
Parabola($B=2\sqrt{2}h$)	$4\sqrt{2}h^2/3$	$8\sqrt{2}h/3$	$h/2$	$2\sqrt{2}h$	$2h/3$

Example 5.1

Show that the best hydraulic rectangular section is one-half of a square. Also, determine the geometric elements of best hydraulic rectangular section.

Solution For a rectangular section, $A = bh \quad \therefore b = A/h$

$$\therefore P = b + 2h = \frac{A}{h} + 2h$$

Considering A to be constant and differentiating P with respect to h, we get

$$\frac{dP}{dh} = -\frac{A}{h^2} + 2$$

For P to be minimum, dP/dh = 0. Hence, we get

$$-\frac{A}{h^2} + 2 = 0 \quad or, \frac{bh}{h^2} = 2$$

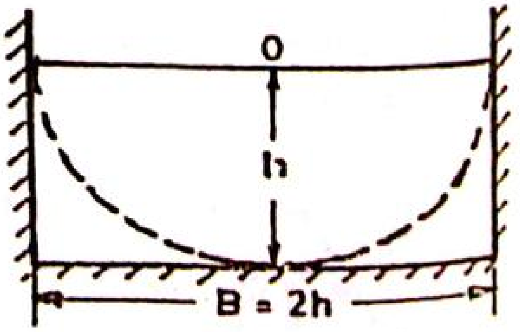


Fig.5.1 Best hydraulic rectangular section

or $$b = 2h$$

This means that the best hydraulic rectangular section is one-half of a square, as shown in Fig. 5.1. A semi-circle with O as center and h as radius can be inscribed in the best hydraulic rectangular section. Also, for the best hydraulic rectangular section, we get

$A = bh = 2h^2$ $\quad P = b + 2h = 4h$ $\quad R = A/P = 2h^2/4h = h/2$

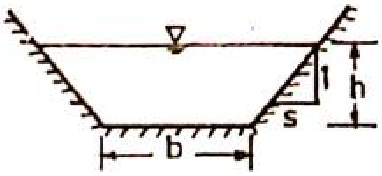
$B = b = 2h$ $\quad D = A/B = 2h^2/2h = h$

as given in Table 5.1.

Example 5.2

Show that the best hydraulic trapezoidal section is one-half of a regular hexagon.

Solution For a trapezoidal section, $A = (b + sh)h$ (i)



or, $b = \dfrac{A}{h} - sh$

$\therefore P = b + 2h\sqrt{1+s^2}$

$= \dfrac{A}{h} - sh + 2h\sqrt{1+s^2}$ (ii)

or, $P = \dfrac{A}{h} + (2\sqrt{1+s^2} - s)h$ (iii)

Considering A and s to be constant and differentiating P with respect to h, we get

$$\frac{dP}{dh} = -\frac{A}{h^2} + 2\sqrt{1+s^2} - s$$

For the minimum value of P, dP/dh = 0. Hence, we get

$A = (2\sqrt{1+s^2} - s)h^2$ (iv)

Combining (i) and (iv), we get

$$(b + sh)h = (2\sqrt{1+s^2} - s)h^2$$

or, $b = 2(\sqrt{1+s^2} - s)h$ (v)

Substituting (v) into (ii), we get

$P = 2(\sqrt{1+s^2} - s)h + 2\sqrt{1+s^2}h = 2h(2\sqrt{1+s^2} - s)$ (vi)

Dividing (iv) by (vi), we get

$R = A/P = h/2$ (vii)

i.e. for the best hydraulic trapezoidal section, the hydraulic radius R is one-half of the depth of flow h, irrespective of the side slope.

From (vi), we have

$P^2 = 4h^2(2\sqrt{1+s^2} - s)^2$ (viii)

and from (iv), we have

$h^2 = \dfrac{A}{2\sqrt{1+s^2} - s}$ (ix)

Using (ix) in (viii), we get

$P^2 = \dfrac{4A(2\sqrt{1+s^2} - s)^2}{2\sqrt{1+s^2} - s} = 4A(2\sqrt{1+s^2} - s)$ (x)

Considering A to be constant and differentiating P with respect to s, we get

$$2P\frac{dP}{ds}=4A\left(\frac{2\times\frac{1}{2}\times 2s}{\sqrt{1+s^2}}-1\right)=4A\left(\frac{2s}{\sqrt{1+s^2}}-1\right)$$

For minimum P, dP/ds = 0. Hence

$$\frac{2s}{\sqrt{1+s^2}}-1=0 \quad or,\ 2s=\sqrt{1+s^2} \quad or,\ 4s^2=1+s^2 \quad or,\ 3s^2=1$$

$$\therefore\ s=\frac{1}{\sqrt{3}} \qquad \text{(xi)}$$

i.e. $\tan\phi=\frac{1}{s}=\sqrt{3}=\tan 60^0$

$$\therefore\ \phi=60^0 \qquad \text{(xii)}$$

This means that the section is one-half of a regular hexagon, as shown in Fig. 5.2.

Now, from Fig. 5.2

$$OA=OB=OP\sin\phi=\frac{1}{2}PQ\sin\phi$$

$$=\frac{PQ}{2\sqrt{1+s^2}}=(b+2sh)\times\frac{1}{2\sqrt{1+s^2}}$$

$$=2\sqrt{1+s^2}\times h\times\frac{1}{2\sqrt{1+s^2}} \quad \text{(using (v))}$$

$$=h$$

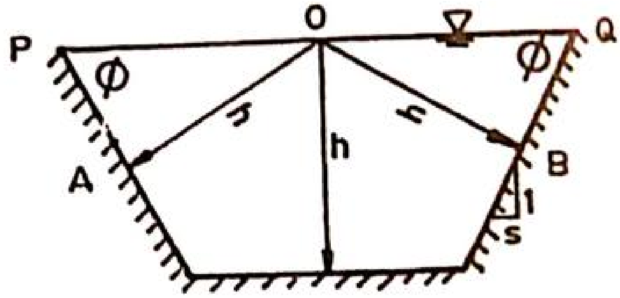


Fig.5.2 Best hydraulic trapezoidal section

Thus, a semi-circle with O as center and h as radius can be inscribed in the best hydraulic trapezoidal section.

Procedure for Designing a Channel with Best Hydraulic Section

When the design discharge Q, the Manning's roughness coefficient n and the bottom slope S_0 are known or determined, the design of rigid boundary or non-erodible channels based on the concept of best hydraulic section involves the following steps:

1. Compute the section factor for uniform flow computation $AR^{2/3}$ using

$$AR^{2/3}=\frac{nQ}{\sqrt{S_0}}$$

2. Substitute the expressions for A and R from Table 5.1 in the above equation and solve directly for the depth.
3. Check for the minimum permissible velocity for siltation and/or for vegetation.
4. Check for the Froude number of the flow.
5. Add a proper freeboard to the depth of the channel section.

Example 5.3

A trapezoidal channel carrying 20 m^3/s is built with non-erodible bed having a slope of 1 in 1000 and n = 0.025. Design the channel by the concept of best hydraulic section.

Solution Trapezoidal channel, Q = 20 m^3/s, S_0 = 1 in 1000 = 0.001, n = 0.025

$$AR^{2/3} = \frac{nQ}{\sqrt{S_0}} = \frac{0.025 \times 20}{\sqrt{0.001}} = 15.81$$

From Table 5.1, $A = \sqrt{3}h^2$ and $R = 0.5h$. Hence,

$$\sqrt{3}h^2 \times (0.5h)^{2/3} = 15.81$$

which gives

$$h^{8/3} = 14.49 \qquad \therefore\ h = 14.49^{3/8} = 2.73 \text{ m}$$

The side slope of the best hydraulic trapezoidal section is given by $s =$ [illegible]$\sqrt{3} = 0.577$ so that $b = 2h(\sqrt{1+s^2} - s) = 4.12$ m, $A = \sqrt{3}h^2 = 12.86$ m^2, $P = 2\sqrt{3}h = 9.44$ m. [illegible] $3h/3 = 6.30$ m, $R = 0.5h = 1.36$ m and $D = 3h/4 = 2.05$ m. Also, $U = Q/A = 1.56$ m/s, which is greater than the minimum permissible velocity. The Froude number, $Fr = U/\sqrt{gD} = 0.$[illegible] which seems satisfactory. Since h = 8.95 ft and Q = 705.75 ft^3/sec, c = 2.5 and Eq.(5.1) gives a freeboard of 4.73 ft or 1.44 m.

Practical Rigid-Boundary Channel Sections

From the practical point of view, the best hydraulic section is not necessarily the most economic section. This is because (i) the area to be excavated to achieve the area for the best hydraulic section may be significantly larger, (ii) the type of material in the channel body may not permit the adoption of the slope required by the best hydraulic section, (iii) the cost of excavation depends not only on the amount of material which is removed, but also on the ease of access of the site and the cost of depositing the material removed, and (iv) the sharp corners in a cross-section, which are virtually the zones of stagnation, may lead to deposition if the water carries silt.

In view of the above factors, the best hydraulic sections need to be modified in practice. The side slopes of a channel depend mainly on the kind of material (Table 5.2). Usually the slopes are steeper in cutting than in filling. For lined channels, the side slopes usually vary from 1:1 to 2:1 and roughly correspond to the angle of repose of the natural soil (Fig. 5.6). The angle of repose is the angle made by a heap of soil with the horizontal under natural condition.

Table 5.2 Side slopes for channels built in various kinds of materials (Chow, 1959)

Material	Side slope
Rock	Nearly vertical
Muck and peat soils	0.25:1
Stiff clay or earth with concrete lining	0.5:1 to 1:1
Earth with stone lining or earth for large channels	1:1
Firm clay or earth for small ditches	1.5:1
Loose sandy earth	2:1
Sandy loam or porous clay	3:1

Also, the triangular and trapezoidal sections are provided with rounded corners instead of sharp corners, as shown in Fig. 5.3 (Ranga Raju, 1993). For the triangular section

$$A = h^2(\phi + \cot\phi) \tag{5.2}$$

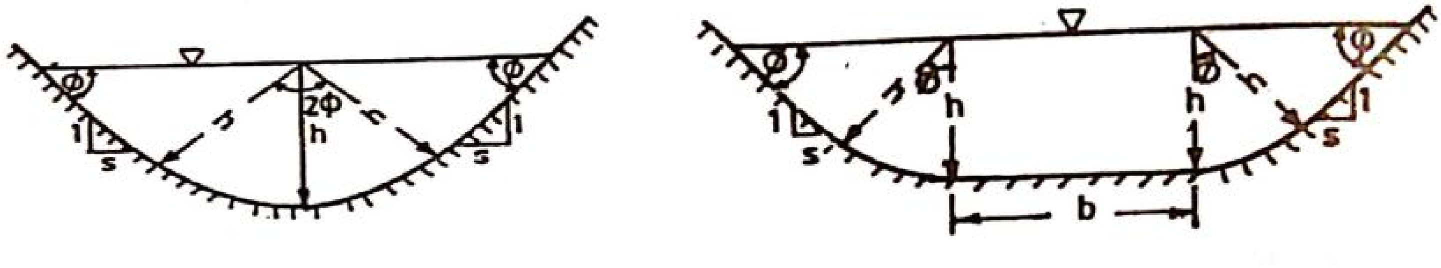
$$P = 2h(\phi + \cot\phi) \tag{5.3}$$

and for the trapezoidal section (5.4)

$$A = bh + h^2(\phi + \cot\phi) \quad (5.5)$$

$$P = b + 2h(\phi + \cot\phi)$$

For given values of Q, n, s and S_0, the depth of flow in a triangular channel can be determined directly using the Manning formula, and the determination of the depth of flow and the bottom width of a trapezoidal channel is based on the maximum permissible velocity.



(a) (b)

Fig. 5.3 Lined channel sections for (a) Q < 55 m^3/s and (b) Q > 55 m^3/s (Ranga Raju, 1993)

Example 5.4

A channel lined with concrete is to be laid on a slope of 1 in 3600. The side slope of the channel is to be maintained at 1:1 and the lining is expected to give n = 0.013. Determine the section dimensions if (a) Q = 35 m^3/s, and (b) Q = 100 m^3/s and the maximum permissible velocity is 2 m/s.

Solution (a) Since Q < 55 m^3/s, use a triangular section with rounded corner as shown in Fig. 5.3(a). Here, $s = \cot\phi = 1$ and $\phi = 45^0 = \pi/4$ radian = 0.785 radian. Then,

$$A = h^2(\phi + \cot\phi) = 1.785h^2$$

$$P = 2h(\phi + \cot\phi) = 3.571h$$

$$R = A/P = 0.5h$$

Now, using the Manning formula, we get

$$35 = (1/0.013) \times 1.785h^2 \times (0.5h)^{2/3} \times (1/3600)^{1/2}$$

from which we obtain

$$h^{8/3} = 24.28 \qquad \therefore h = 24.28^{3/8} = 3.31 \text{ m}$$

(b) Since Q > 55 m^3/s, use a trapezoidal section with rounded corners as shown in Fig. 5.3(b). Here, $s = \cot\phi = 1$ and $\phi = 0.785$ radian. As the maximum permissible velocity is 2 m/s, so

$$A = 100/2 = 50.0 \text{ m}^2$$

Again, from the Manning formula

$$100 = (1/0.013) \times 50 \times R^{2/3} \times (1/3600)^{1/2}$$

which gives

$$R = 1.95 \text{ m}$$

Since R = A/P, we have

$$P = A/R = 50.0/1.95 = 25.66 \ m$$

So, we have the equations

$$A = bh + h^2(\phi + \cot\phi) = bh + 1.785h^2 = 50.00$$

$$P = b + 2h(\phi + \cot\phi) = b + 3.571h = 25.66$$

Eliminating b between the above two equations, we get the quadratic equation

$$h^2 - 14.37h + 28.00 = 0$$

which gives h = 2.32 m and 12.04 m. Using these two values of h, we get b = 17.36 m and -17.34 m. Since b cannot be negative, we accept

h = 2.32 m and b = 17.36 m.

5.3 ERODIBLE CHANNELS WHICH SCOUR BUT DO NOT SILT: LANE'S SHEAR FORCE MOTHOD

The erodible channels with coarse non-cohesive materials on the bou[illegible] and carrying either clear water or water with fine sediment in suspension which will not [illegible] are designed by the shear or tractive force method developed by Lane (1955) of USBR. Th[e m]ethod is based on the *threshold* or *incipient or impending motion* condition of the soil partic[les] which denotes the limiting condition at which the soil particles just begin to move. In this method it is assumed that a channel scours when the shear stress developed on the channel boundary exceeds the critical shear stress value. The average shear stress on the boundary of an open channel at which the soil particles just begin to move is called the *critical shear stress* τ_c. Since the threshold or incipient or impending motion condition of the particles on the channel boundary (or over a part of the boundary which is more susceptible to scour) is considered, the design is economical.

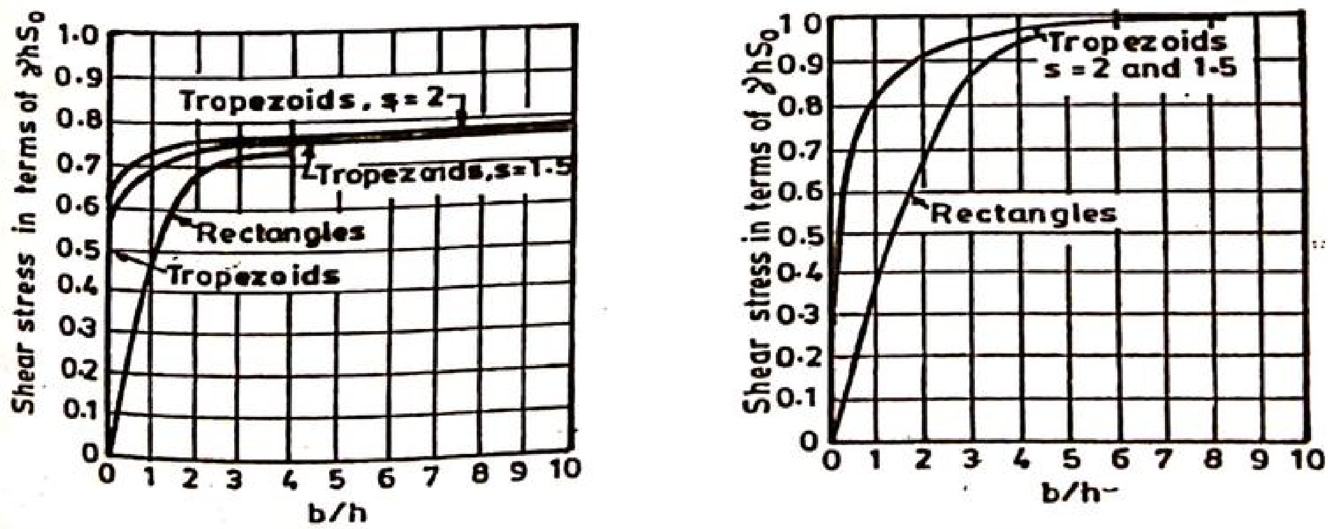
As stated in Art. 4.3, in uniform flow the average shear stress on the channel boundary is given by

$$\tau_0 = \gamma R S_0 \qquad (5.6)$$

and when the channel is wide

$$\tau_0 = \gamma h S_0 \qquad (5.7)$$

The shear stress, however, is not uniformly distributed along the wetted perimeter when the channel is not wide. For a trapezoidal section, the maximum shear stress on the bottom is close to $\gamma h S_0$ and on the sides close to $0.76\gamma h S_0$, as shown in Fig.5.4 (Lane, 1955).



(a) On sides (b) On bottom

Fig. 5.4 Maximum shear stresses on sides and bottom of trapezoidal channels

Shear Stress Ratio

On a soil particle resting on the sloping side of a trapezoidal channel in which water is flowing (Fig. 5.5), two forces, viz. the shear force $a\tau_s$ and the gravity force component $W_s \sin\varphi$, tend to move the soil particle. Since the two forces act on the soil particle at right angles to each other, the resultant force on the soil particle is

$$\sqrt{W_s^2 \sin^2 \phi + a^2 \tau_s^2}$$

where a is the effective area of the soil particle, τ_s is the shear stress on the sloping side and W_s is the submerged weight of the particle.

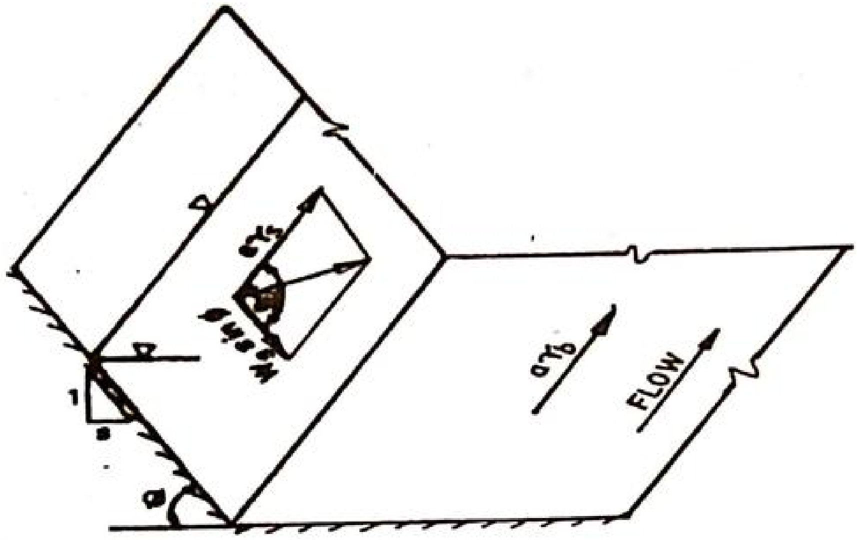


Fig.5.5 Forces acting on a soil particle resting on side and bottom of a trapezoidal channel

The force resisting the movement of the soil particle is equal to $W_s \cos\varphi \tan\psi$, where ψ is the *angle of repose* and tanψ is the *coefficient of friction*. For the incipient motion condition of the soil particle

$$W_s \cos\phi \tan\psi = \sqrt{W_s^2 \sin^2 \phi + a^2 \tau_s^2} \tag{5.8}$$

which gives

$$\tau_s = \frac{W_s}{a} \cos\phi \tan\psi \sqrt{1 - \frac{\tan^2 \phi}{\tan^2 \psi}} \tag{5.9}$$

Similarly, for the incipient motion condition of a soil particle on the level or horizontal bottom, Eq.(5.9) gives with $\phi = 0$

$$\tau_b = \frac{W_s}{a} \tan\psi \tag{5.10}$$

where τ_b is the shear stress or unit tractive force on the level bottom.

From Eqs.(5.9) and (5.10), the shear stress ratio K, which is the ratio between τ_s and τ_b, is given by

$$K = \frac{\tau_s}{\tau_b} = \cos\phi \sqrt{1 - \frac{\tan^2 \phi}{\tan^2 \psi}} = \sqrt{\cos^2 \phi - \frac{\sin^2 \phi}{\tan^2 \psi}} = \sqrt{1 - \sin^2 \phi - \frac{\sin^2 \phi}{\tan^2 \psi}}$$

$$= \sqrt{1 - \sin^2 \phi \left(1 + \frac{1}{\tan^2 \psi}\right)} = \sqrt{1 - \sin^2 \phi \left(\frac{\sin^2 \psi + \cos^2 \psi}{\sin^2 \psi}\right)}$$

or

$$K = \sqrt{1 - \frac{\sin^2 \phi}{\sin^2 \psi}} \tag{5.11}$$

For cohesive and fine non-cohesive materials, the gravity force component causing the particle to roll down the side slope is much smaller than the cohesive forces and can safely be neglected. Therefore, the effect of the angle of repose is to be considered only for coarse non-cohesive materials. In general, the angle of repose increases with both size and angularity of the material. Figure 5.6 shows the curves prepared by USBR for the angle of repose for non-cohesive materials larger than 0.2 inch (5 mm) in diameter. In this figure, d_{75} is the diameter of a particle than which 25% (by weight) of the material is larger.

The USBR recommends a value of permissible shear stress in pounds per square foot on level bottom for coarse non-cohesive materials equal to $0.40 d_{75}$, where d_{75} is in inches.

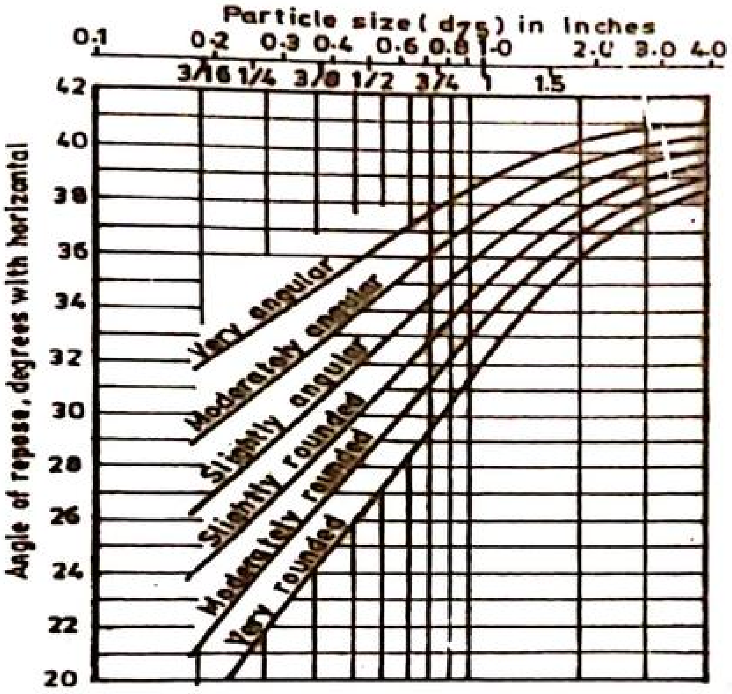


Fig.5.6 Angle of repose of non-cohesive material (Lane, 1955)

Threshold Condition

The threshold or incipient or impending motion condition denotes the limiting condition at which the sediment particles just begin to move. Shields in 1936 gave a semi-empirical approach to define the threshold of movement. His results can be stated in terms of two dimensionless parameters

$$Re^* = \frac{u^* d}{\nu} \tag{5.12}$$

and

$$\tau_c^* = \frac{\tau_c}{\gamma(s_s - 1)d} = \frac{u^{*2}}{g(s_s - 1)d} \tag{5.13}$$

when s_s is the specific gravity of the sediment and d is the size of the sediment particle. The parameter Re^* is known as the *particle Reynolds number*.

The Shields curve, plotted in Fig.5.7 (Henderson, 1966), delineates the threshold of movement, i.e. the regime above the curve represents a moving bed and that below the curve represents a rigid bed. The first part of the curve represents laminar flow, the middle part represents a transition region and the last part represents a region of fully developed turbulence. The boundary is rough at large values of Re^* and when Re^* exceeds a value of about 400, τ_c^* remains constant at 0.056.

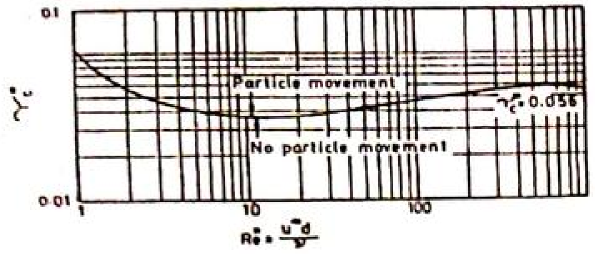


Fig.5.7 Shields curve for incipient motion condition (Henderson,1966)

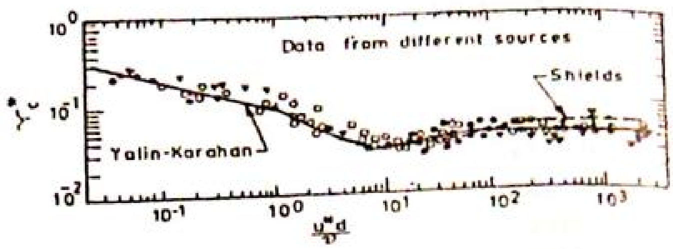


Fig.5.8 Yalin-Karahan curve for incipient motion condition (Ranga Raju, 1993)

The Shields curve is based on a limited amount of field data. The relationship proposed by Yalin and Karahan (1979) based on considerably more data is shown in Fig. 5.8 and is more reliable. As a result, it is recommended that the relationship of Yalin and Karahan be used to define τ_c. In the Yalin-Karahan relationship, when Re^* exceeds a value of about 70, τ_c^* remains constant at 0.045. However, the computation of τ_c using Fig. 5.8 requires a trial procedure. Ranga Raju (1993) presented a modified form of the Yalin-Karahan curve as shown in Fig.5.9 which gives the value of τ_c directly without involving any trial.

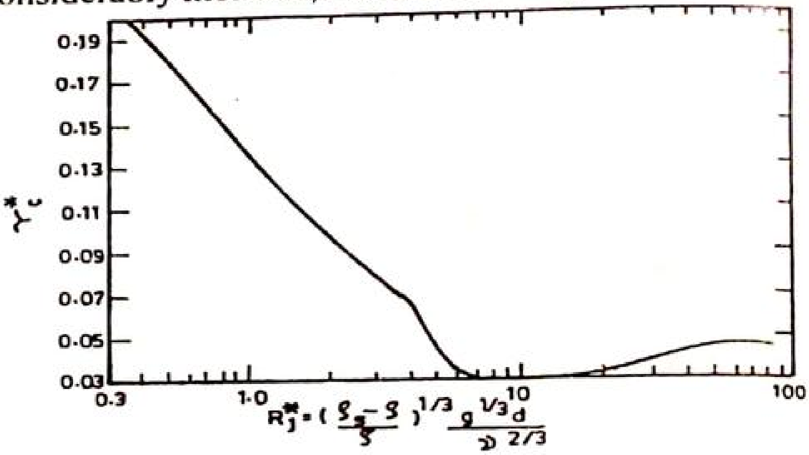


Fig. 5.9 Modified form of Yalin-Karahan curve (Ranga Raju, 1993)

Since sinuous channels are apt to scour more easily than straight channels, the permissible shear stresses for sinuous channels are reduced approximately, as suggested by Lane (1955), by 10% for slightly sinuous channels (sinuosity 1.5-2.0), 25% for moderately sinuous channels (sinuosity 2.0-2.5) and 40% for very sinuous channels (sinuosity > 2.5). The sinuosity is the ratio between the channel (curved) length and the valley (straight) length of a curved channel.

Procedure for Designing a Trapezoidal Channel in Coarse Non-Cohesive Material

For a trapezoidal channel in coarse non-cohesive material, the shear stress on the sloping sides is usually less than that on the bottom. Hence, the side force is generally the limiting factor in the channel design. The section dimensions should therefore be determined for the side force and checked for the bottom force.

When the design discharge Q, the bottom slope S_0, the relevant engineering properties of the soil and the Manning's n are known or determined, the design of a trapezoidal channel section in coarse non-cohesive materials involves the following steps:

1. Estimate the angle of repose ψ for the perimeter material (Fig.5.6).
2. Assume s and b/h_n. Compute ϕ (= $\cot^{-1}s$).
3. Determine the maximum shear stress developed on sides in terms of h_n (Fig.5.4(a))
4. Calculate K (Eq.(5.11)).
5. Determine τ_b, the permissible shear stress on bottom ($\tau_b = 0.40 d_{75}$ when τ_b is in pounds per square foot and d_{75} is in inches, or $\tau_b = \tau_c$ obtained from the modified Yalin-Karahan curve, Fig.5.9) and correct for sinuosity, if any.
6. Compute τ_s, the permissible shear stress on sides, using $\tau_s = K\tau_b$.
7. Compute the normal depth h_n by equating the computed value of τ_s in step 6 to the

shear stress obtained in step 3.

8. Compute b.

9. Compute Q and compare this with the design Q.

10. Repeat steps 2 to 9 until the computed Q is close to the design Q.

11. Compare the actual shear stress on bottom τ_b (Fig.5.4(b)) with the permissible value obtained in step 5.

Example 5.5

A trapezoidal channel is to be laid on a slope of 1 in 1000 and carry a discharge of 20 m^3/s. It is to be excavated in earth containing moderately rounded coarse non-cohesive particles with d_{50} = 2 cm, d_{75} = 2.5 cm and n = 0.025. Determine the section dimensions of the channel.

Solution $S_0 = 1/1000 = 0.001$

i) Design by the method of Lane

$d_{75} = 2.5/2.54 = 0.98$ in

Then, from Fig.5.6, $\psi = 33^0$

Assume s = 2 and $b/h_n = 4$. Then, $\phi = \cot^{-1} 2 = \tan^{-1} \frac{1}{2} = 26.56^0$

Then, from Fig.5.4(a), the maximum shear stress on sides is $0.75\gamma h_n S_0 = 0.75 \times 9810 \times h_n \times 0.001 = 7.36 h_n$ N/m².

$$K = \frac{\tau_s}{\tau_b} = \sqrt{1 - \frac{\sin^2\phi}{\sin^2\psi}} = \sqrt{1 - \frac{\sin^2 26.56^0}{\sin^2 33^0}} = 0.57$$

The permissible shear stress on bottom is

$\tau_b = 0.4 \times 0.98 = 0.39$ lb/ft² = 18.77 N/m² ($\because$ 1 lb/ft² = 47.86 N/m²)

Then, the permissible shear stress on sides is

$\tau_s = K\tau_b = 0.57 \times 18.77 = 10.72$ N/m²

For a state of incipient motion of the particles on sides

$7.36 h_n = 10.72$ $\therefore h_n = 1.47$ m

$\therefore b = 1.47 \times 4 = 5.83$ m

For this trapezoidal section, A = 12.73 m², P = 12.34 m and R = 1.03 m. Hence

$$Q = (1/0.025) \times 12.73 \times 1.03^{2/3} \times 0.001^{1/2} = 16.44\ m^3/s$$

which is less than the design discharge of 20 m³/s.

Further computations are done in tabular form as shown below for different values of b/h_n.

b/h_n	h_n (m)	b (m)	A (m²)	P (m)	R (m)	Q (m³/s)
4.5	1.47	6.62	14.05	13.19	1.065	18.53
5.0	1.47	7.35	15.13	13.92	1.086	20.22
4.9	1.47	7.20	14.91	13.78	1.082	19.88

Then, for s = 2 and $b/h_n = 5.0$, the section dimensions are h_n = 1.47 m, b = 7.35 m, A = 15.13 m², P = 13.92 m and R = 1.086 m, and Q = 20.22 m³/s which is very close to 20 m³/s.

With s = 2 and $b/h_n = 5.0$, the shear stress on bottom (Fig.5.4(b)) is $0.97\gamma h_n S_0 = 0.97 \times 9810 \times 1.47 \times 0.001 = 13.98$ N/m² which is less than 18.77 N/m², the permissible shear stress on bottom. Hence the design is acceptable.

Obviously, alternative section dimensions may be obtained by taking other values of s.

ii) Design by using modified Yalin-Karahan curve

The permissible shear stress on bottom τ_b taken to be equal to τ_c obtained from Fig. 5.9. The median size d_{50} is taken to be equal to the size of the sediment particle. Then,

$$R_t* = \left(\frac{\rho_s - \rho}{\rho}\right)^{1/3} \times \frac{g^{1/3} d_{50}}{\nu^{2/3}} = \left(\frac{2650-1000}{1000}\right)^{1/3} \times \frac{9.81^{1/3} \times 0.02}{(10^{-6})^{2/3}} = 505.92$$

The corresponding value of τ_c* from Fig. 5.9 is 0.045.

$$\therefore \tau_b = \tau_c = \tau_c * \gamma(s_s - 1)d_{50} = 0.045 \times 9810 \times (2.65-1) \times 0.02 = 14.57\ N/m^2$$

$\tau_s = K\tau_b = 0.57 \times 14.57 = 8.30\ N/m^2$

$\therefore\ 7.36h_n = 8.30$ $\quad\quad\quad \therefore h_n = 7.36/8.30 = 1.13$ m

$\therefore\ b = 1.13 \times 4 = 4.52$ m

For this trapezoidal section, A = 7.66 m², P = 9.57 m and R = 0.80 m. Hence

$$Q = (1/0.025) \times 7.66 \times 0.80^{2/3} \times 0.001^{1/2} = 8.35\ m^3/s$$

which is far less than the design discharge of 20 m³/s.

Further computations are done in tabular form as shown below for different values of b/h_n.

b/h_n	h_n (m)	b (m)	A (m²)	P (m)	R (m)	Q (m³/s)
8.0	1.13	9.04	12.77	14.03	0.906	15.12
10.0	1.13	11.30	15.32	16.35	0.096	18.56
11.0	1.13	12.43	16.60	17.43	0.949	20.23
10.9	1.13	12.32	16.47	17.37	0.948	20.10

Then, for s = 2 and b/h_n = 10.9, h_n = 1.13 m, b = 12.32 m, A = 16.47 m², P = 17.37 m, R = 0.948 m and Q = 20.10 m³/s which is very close to 20 m³/s.

With s = 2 and b/h_n = 10.9, the maximum unit tractive force on bottom (Fig.5.4(b)) is $\gamma h_n S_0 = 9810 \times 1.13 \times 0.001 = 11.08\ N/m^2$ which is less than 14.57 N/m², the permissible tractive force on bottom. Hence, the design is acceptable.

5.4 ALLUVIAL CHANNELS: REGIME APPROACH

An alluvial channel is a channel transporting the same type of material as that comprising the channel perimeter. Such a channel can be stable only when sediment inflow into channel is equal to sediment outflow, i.e. the channel cross-section and bottom slope do not change due to erosion and deposition. A channel is said to be in a regime when it has adjusted its shape and slope to an equilibrium condition.

The two commonly adopted methods for the design of stable alluvial channels are the shear stress approach, considered earlier, and the regime approach. The shear stress approach is more rational, as it makes use of the laws governing sediment transport and resistance laws. The regime theory is purely empirical and has been developed using data of stable canals in India and Pakistan carrying sediment load generally less than 500 ppm by weight.

The first regime formula developed by Kennedy (1895) is given by

$$U_0 = 0.546h^{0.64} \quad (5.14)$$

where U_0 is the non-silting and non-scouring velocity and h is the depth of flow. The main limitation of the Kennedy equation is that it does not specify a definite width, thereby making an infinite number of width-to-depth ratios possible. However, experience shows that stability is possible only if the width does not vary over a wide range. Lindley (1919) recognized this fact and introduced a relation between non-silting and non-scouring velocity and the bed width. Later on, Lacey (1930,1946) carried out extensive investigations on the design of stable channels in alluvium using data of stable canals in the Indo-Gangetic plains and put forward his new theory. He differentiated between two regime conditions: (i) initial regime, and (ii) final regime. A channel under *initial or false regime* is not a channel in regime, although it appears to be in regime as there is no silting or scouring, and the regime theory is not applicable to them. According to Lacey, an artificially constructed channel having a certain fixed section and certain fixed slope can be in *true or final regime* if the discharge is constant, flow is uniform, the silt grade and the silt charge are constant and the channel flows through incoherent alluvium of the same type as is transported without changing its cross-section and slope. Lacey's regime theory is applicable only to channels which are in final regime.

The various equations proposed by Lacey for the design of stable channels in alluvium are

$$P = 4.75\sqrt{Q} \quad .15)$$

$$R = 0.47(Q/f_s)^{1/3} \quad (5.16)$$

and

$$S_0 = \frac{f_s^{5/3}}{3340Q^{1/6}} \quad (5.17)$$

with

$$f_s = 1.76\sqrt{d} \quad (5.18)$$

where P is the wetted perimeter in m, R is the hydraulic radius in m, Q is the discharge in m^3/s, d is the average particle size in mm and f_s is the *silt factor* which takes into account the effect of grain size of the material forming the channel.

The alluvial channels are usually provided with trapezoidal sections having side slopes equal to or less than the angle of repose of the perimeter material. But due to deposition of fine sediments, the final side slopes attained by the channels are much steeper. Hence, it is customary to assume a side slope of 0.5:1 (i.e. s = 0.5) for the design of alluvial channels.

Example 5.6

Design a stable alluvial channel using the Lacey method. The channel is to carry 10 m^3/s through 1 mm sand.

Solution $Q = 10\ m^3/s \quad d = 1\ mm$

$$f_s = 1.76\sqrt{d} = 1.76\sqrt{1} = 1.76$$

$$S_0 = \frac{f_s^{5/3}}{3340Q^{1/6}} = \frac{1.76^{5/3}}{3340\times 10^{1/6}} = 5.23\times 10^{-4}$$

$$R = 0.47(Q/f_s)^{1/3} = 0.47\times(10/1.76)^{1/3} = 0.84\ m$$

$$P = 4.75\sqrt{Q} = 4.75\sqrt{10} = 15.02\ m$$

so that

$$A = PR = 15.02\times 0.84 = 12.60\ m^2$$

Assuming that the side slope is 1/2H:1V so that s = 0.5, we obtain

$$P = 15.02 = b + 2h\sqrt{1+0.5^2} = b + 2.236h$$

$$A = 12.60 = (b+0.5h)h = bh + 0.50h^2$$

Eliminating b between the above two equations, we get the quadratic equation

$$h^2 - 8.652h + 7.258 = 0$$

which gives h = 0.94 m and 7.71 m. Using these two values of h, we get b = 15.02 − 2.236h = 12.92 m and − 2.22 m. Since b cannot be negative, we accept

h = 0.94 m and b = 12.92 m.

PROBLEMS AND EXERCISES

5.1 Define (i) minimum permissible velocity, (ii) maximum permissible velocity, (iii) freeboard, (iv) best hydraulic section, (v) angle of repose, (vi) threshold or incipient or impending motion condition, (vii) critical shear stress, and (viii) shear stress ratio.

5.2 What are the advantages of lining a canal? Write the names of the materials commonly used for lining.

5.3 Show that (i) the best hydraulic circular section is a semi-circle, (ii) the best hydraulic rectangular section is one-half of a square, (iii) the best hydraulic triangular section is one-half of a square, (iv) the best hydraulic trapezoidal section is one-half of a regular hexagon, and (v) for the best hydraulic parabolic section the top width is equal to $2\sqrt{2}$ times the depth of flow.

5.4 "The best hydraulic section is not necessarily the most economic section" why?

5.5 Deduce the expression for the shear stress ratio K (Eq. 5.11).

5.6 What is the main limitation of the Kennedy method? What is the improvement of the Lacey method over the Kennedy method? What is a regime channel according to Lacey?

5.7 The cross-sectional area of a channel is 40 m^2. Calculate the wetted perimeter and the hydraulic radius of the best hydraulic section if the channel is (i) rectangular, (ii) triangular, (iii) trapezoidal, (iv) circular, and (v) parabolic. Which section has the minimum wetted perimeter?

5.8 Design the best hydraulic trapezoidal section to carry a discharge of 20 m^3/s on a slope of 1 in 2500 if s = 1 and n = 0.012.

5.9 A lined channel (n = 0.015) is to be laid on a slope of 1 in 2000. The side slope of the channel is to be maintained at 1.5:1. (i) Determine the depth of flow of a triangular section with rounded corner to carry a discharge of 40 m^3/s. (ii) Determine the dimensions of a trapezoidal section with rounded corners to carry a discharge of 80 m^3/s when the maximum permissible velocity is 2 m/s.

5.10 An irrigation canal has to carry a discharge of 30 m^3/s through a coarse non-cohesive material having d_{50} = 2.5 cm, d_{75} = 3 cm and n = 0.025. The angle of repose of the perimeter material is 32^0. The canal is to be trapezoidal in shape having s = 2 and laid on a slope of 1 in 1000. Compute the bottom width and the depth of flow (i) using the method of Lane, and (ii) using the modified Yalin-Karahan diagram.

5.11 Design a stable alluvial channel using the Lacey method. The channel has to carry a discharge of 25 m^3/s through 1.5 mm sand.

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GRADUALLY VARIED FLOW

6.1 INTRODUCTION AND BASIC ASSUMPTION

In gradually varied flow, the depth of flow and the mean velocity vary gradually along the length of the channel ($\partial h/\partial x \approx 0$, $\partial U/\partial x \approx 0$). The streamlines are practically parallel and the pressure distribution is approximately hydrostatic. Since in gradually varied flow the depth of flow changes gradually, to produce a significant change in depth, long channel lengths are usually involved in the analysis of gradually varied flow. Consequently, the frictiona[illegible] [illegible]es, which are proportional to the channel length, play a dominating role in determining th[illegible] [illegible]aracteristics and must be included.

The analysis of gradually varied flow involves the assumption that *th[illegible] [illegible]ction losses in gradually varied flow are not significantly different from those in uniform flow*. By virtue of this assumption, the friction slope in gradually varied flow is computed using a uniform flow formula, i.e

$$S_f = \frac{Q^2}{C^2 A^2 R} \qquad (6.1)$$

when the Chezy formula is used, and

$$S_f = \frac{n^2 Q^2}{A^2 R^{4/3}} \qquad (6.2)$$

when the Manning formula is used.

6.2 GOVERNING EQUATIONS

Consider a channel in which the depth and mean velocity change gradually but the discharge remains constant along the length of the channel. It is also assumed that the channel slope is small. Then, at any channel section, the total energy is given by (Fig. 6.1)

$$H = z_b + h + \alpha \frac{U^2}{2g} = z_b + E \qquad (6.3)$$

where z_b is the elevation of the channel bottom above a horizontal datum, h is the depth of flow, U is the mean velocity of flow, α is the energy coefficient and E is the specific energy.

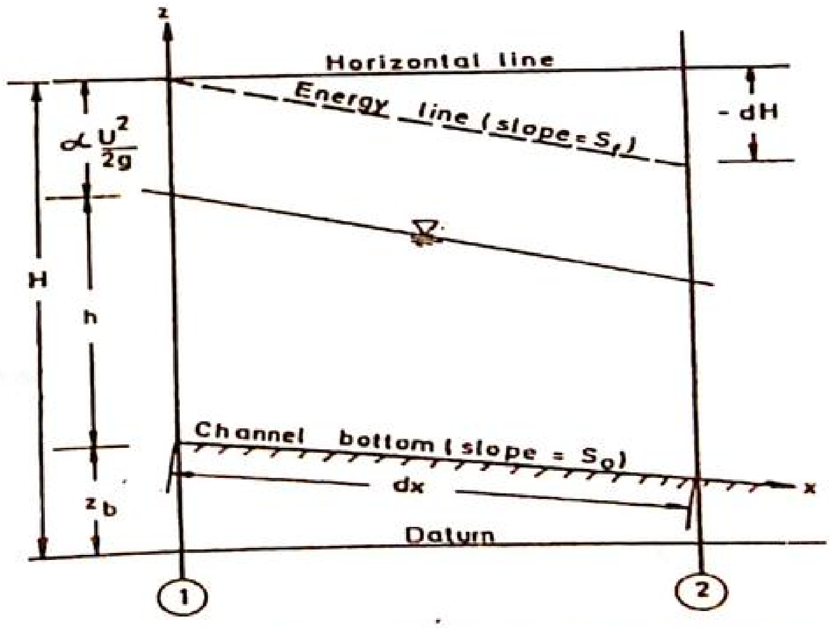


Fig. 6.1 Derivation of the gradually varied flow equation

Differentiating Eq.(6.3) with respect to x gives

$$\frac{dH}{dx} = \frac{dz_b}{dx} + \frac{dE}{dx} \qquad (6.4)$$

The term dH/dx represents the slope of the energy line. It is usual to consider the slope of the energy line which descends in the direction of flow as positive. Since the total energy of flow decreases (as x increases) in the direction of flow, it follows that

$$\frac{dH}{dx} = -S_f \quad (6.5)$$

The term dz_b/dx represents the bottom slope of the channel. It is usual to consider the bottom slope that falls in the direction of flow as positive. Since the change in the bottom elevation dz_b is negative when the channel bottom descends in the direction of flow, we have

$$\frac{dz_b}{dx} = -S_0 \quad (6.6)$$

Using Eqs.(6.5) and (6.6), Eq.(6.4) becomes

$$\frac{dE}{dx} = S_0 - S_f \quad (6.7)$$

Using Eq. (3.3), we can write

$$\frac{dE}{dx} = \frac{dE}{dh}\frac{dh}{dx} = (1 - Fr^2)\frac{dh}{dx} \quad (6.8)$$

Combining Eqs.(6.7) and (6.8), we obtain

$$\frac{dh}{dx} = \frac{S_0 - S_f}{1 - Fr^2} \quad (6.9)$$

Equation (6.9) is the basic differential equation of steady gradually varied flow and is known as the *dynamic equation of (steady) gradually varied flow*. It represents the slope of the water surface with respect to the channel bottom and gives the variation of h in the x direction.

The water surface curve or profile represents a *backwater curve* when the depth of flow increases in the direction of flow ($dh/dx > 0$) and a *drawdown curve* when the depth of flow decreases in the direction of flow ($dh/dx < 0$). When $dh/dx = 0$, the water surface is parallel to the channel bottom and the flow is uniform.

Other useful forms of Eq. (6.9) can also be obtained. The section factor Z and the section factor for critical flow computation Z_c can be expressed, respectively, as (Art. 3.2)

$$Z = A\sqrt{D} \quad (6.10)$$

and

$$Z_c = \frac{Q}{\sqrt{g/\alpha}} \quad (6.11)$$

so that

$$\frac{Z_c^2}{Z^2} = \frac{\alpha Q^2}{gA^2 D} = \frac{U^2}{gD/\alpha} = Fr^2 \quad (6.12)$$

where Z represents simply the numerical value of $A\sqrt{D}$ to be computed for Q at the actual depth h of the gradually varied flow and Z_c is the section factor to be computed for Q as if the flow in the channel is critical.

The slope of the channel bottom S_0 and the slope of the energy line S_f can be expressed, respectively, as (Art. 4.5)

$$S_0 = \frac{Q^2}{K_n^{\ 2}} \tag{6.13}$$

and

$$S_f = \frac{Q^2}{K^2} \tag{6.14}$$

so that

$$\frac{S_f}{S_0} = \frac{K_n^{\ 2}}{K^2} \tag{6.15}$$

where K is the numerical value of the conveyance to be computed for Q at the actual depth h of the gradually varied flow and K_n is the conveyance to be computed for Q as if the flow in the channel is uniform.

Substitution of Eqs. (6.12) and (6.15) into Eq.(6.9) gives

$$\frac{dh}{dx} = S_0 \frac{1-(K_n/K)^2}{1-(Z_c/Z)^2} \tag{6.16}$$

Now, for a given channel section, the section factor and the conveyance are functions of the depth of flow, i.e.

$$Z^2 = C_1 h^M \qquad Z_c^{\ 2} = C_1 h_c^{\ M} \tag{6.17}$$

$$K^2 = C_2 h^N \qquad K_n^{\ 2} = C_2 h_n^{\ N} \tag{6.18}$$

where C_1 and C_2 are the coefficients and M and N are the hydraulic exponents for critical flow computation and uniform flow computation, respectively. With the above expressions, Eq.(6.16) becomes

$$\frac{dh}{dx} = S_0 \frac{1-(h_n/h)^N}{1-(h_c/h)^M} \tag{6.19}$$

For a wide channel, M = 3, and when the Chezy formula is used, N = 3, and hence

$$\frac{dh}{dx} = S_0 \frac{1-(h_n/h)^3}{1-(h_c/h)^3} \tag{6.20}$$

When the Manning formula is used, N = 10/3, and therefore

$$\frac{dh}{dx} = S_0 \frac{1-(h_n/h)^{10/3}}{1-(h_c/h)^3} \tag{6.21}$$

Equations (6.20) and (6.21) can be used to compute gradually varied flow in wide channels.

6.3 CHARACTERISTICS AND CLASSIFICATION OF FLOW PROFILES

Types of Bottom Slopes

The channel bottom slopes are conveniently classified as *sustaining* or *positive slope* ($S_o > 0$) and *non-sustaining slope* ($S_o \le 0$). A positive slope is the slope for which the channel bottom falls in the direction of flow. It may be

i) *mild* ($S_o < S_c$, $h_n > h_c$)
ii) *critical* ($S_o = S_c$, $h_n = h_c$), and
iii) *steep* ($S_o > S_c$, $h_n < h_c$).

Uniform flow can occur in positive slope channels only. In a mild slope channel the uniform flow is subcritical, in a critical slope channel the uniform flow is critical and in a steep slope channel the uniform flow is supercritical.

A non-sustaining slope is the slope for which the channel bottom does not fall in the direction of flow. It may be

i) *horizontal* ($S_o = 0$), and
ii) *adverse* ($S_o < 0$)

A horizontal slope is a zero slope. An adverse slope is a negative slope for which the channel bottom rises in the direction of flow.

The symbols M, C, S, H and A are respectively used to designate the Mild, the Critical, the Steep, the Horizontal and the Adverse slopes.

Types of Flow Profiles

For the given discharge and channel conditions, the normal depth line (NDL) and the critical depth line (CDL) divide the space above the channel into three zones:

Zone 1 Space above upper line ($h > h_n$, $h > h_c$)
Zone 2 Space between two lines ($h_n > h > h_c$ or $h_c > h > h_n$)
Zone 3 Space between channel bed and lower line ($h < h_n$, $h < h_c$)

The three zones are designated by 1 to 3 starting from the top. The flow profiles are classified according to the channel slope and the zone in which the flow profile lies. The name of a flow profile contains the symbol of the channel slope followed by the zone number. Thus, the name of the flow profile which lies in Zone 1 of a Mild slope channel is M1. It will be shown later that the horizontal and the adverse slope channels have only two zones and the flow profiles H1 and A1 are physically not possible. Thus, ***the flow profiles may be classified into thirteen different types designated as M1, M2, M3, C1, C2, C3, S1, S2, S3, H2, H3, A2 and A3. Of the thirteen flow profiles, twelve are for gradually varied flow and one, C2, is for uniform flow.*** The general characteristics of these flow profiles are given in Table 6.1 and their shapes are shown in Fig. 6.2. Some examples of flow profiles are given in Fig. 6.3.

Behavior of Flow Profiles at Specific Depths

Let us now consider the theoretical behavior of the flow profiles at some specific depths.

i) When $h \rightarrow h_n$, Eq.(6.19) shows that $dh/dx \rightarrow 0$, i.e. the flow profile approaches the normal depth line tangentially.

ii) When $h \rightarrow h_c$, Eq.(6.19) shows that $dh/dx \rightarrow \infty$, i.e. the flow profile becomes vertical in crossing the critical depth line. This indicates a hydraulic jump if the depth changes suddenly from a lower value to a higher value or a hydraulic drop if the depth changes abruptly from a higher value to a lower value. In both cases the flow becomes rapidly varied and the theory of gradually varied flow does not apply.

iii) When $h \rightarrow \infty$, Eq.(6.19) shows that $dh/dx \rightarrow S_o$, i.e. the flow profile tends to be horizontal. Note that the slope of the channel bottom S_0 is determined with respect to the horizontal line, whereas the slope of the flow profile dh/dx is determined with respect to the channel bottom. Therefore, the flow profile, whose slope is $dh/dx = S_0$, must be horizontal.

iv) As $h \rightarrow 0$, Eq.(6.19) shows that $dh/dx \rightarrow \infty/\infty$. However, using Eqs.(6.20) and (6.21), it can be shown that when $h = 0$ and the channel is wide

$$\frac{dh}{dx} = S_0\left(\frac{h_n}{h_c}\right)^3 \quad (6.22)$$

when the Chezy formula is used, and

$$\frac{dh}{dx} = \infty \quad (6.23)$$

when the Manning formula is used. Thus, the theoretical behavior of the flow profile at or near h = 0 depends on the type of uniform flow formula used in the computation. However, this result is not of much practical importance since zero depth does never occur.

Table 6.1 Types of flow profiles in prismatic channels

Channel slope	Zone	Designation	Relation of h to h_n and h_c	Type of curve	Type of flow
Horizontal $S_0 = 0$	1	-	-	-	-
	2	H2	$h > h_c$	Drawdown	Subcritical
	3	H3	$h_c > h$	Backwater	Supercritical
Mild $0 < S_0 < S_c$ $h_n > h_c$	1	M1	$h > h_n > h_c$	Backwater	Subcritical
	2	M2	$h_n > h > h_c$	Drawdown	Subcritical
	3	M3	$h_n > h_c > h$	Backwater	Supercritical
Critical $S_0 = S_c > 0$ $h_n = h_c$	1	C1	$h > h_c = h_n$	Backwater	Subcritical
	2	C2	$h_c = h = h_n$	Parallel to channel bottom	Uniform critical
	3	C3	$h_c = h_n > h$	Backwater	Supercritical
Steep $S_0 > S_c > 0$ $h_n < h_c$	1	S1	$h > h_c > h_n$	Backwater	Subcritical
	2	S2	$h_c > h > h_n$	Drawdown	Supercritical
	3	S3	$h_c > h_n > h$	Backwater	Supercritical
Adverse $S_0 < 0$	1	-	-	-	-
	2	A2	$h > h_c$	Drawdown	Subcritical
	3	A3	$h_c > h$	Backwater	Supercritical

General Procedure for Sketching Qualitative Flow Profiles

The general procedure for sketching the qualitative flow profiles in a channel is as follows:

1. Draw the profile of the channel. Plot the critical depth line (CDL) and normal depth line (NDL), if any.

2. For the zone in which the profile lies, determine the relation of the depth h to h_c and h_n, if any. For example, for Zone 1 of a mild slope channel, $h > h_n > h_c$.

3. Name the profile considering the channel slope and the zone in which it lies. For example, the name of the profile which lies in Zone 2 of a steep slope channel is S2.

4. Determine the sign of dh/dx from the signs of the numerator and denominator of the right hand side of Eq.(6.19). The numerator is positive if $h > h_n$ and negative if $h < h_n$. Similarly, the denominator is positive if $h > h_c$ and negative if $h < h_c$.

5. Determine whether the profile is a backwater curve or a drawdown curve. The profile is a backwater curve if $dh/dx > 0$ and a drawdown curve if $dh/dx < 0$.

6. Consider the conditions of the profile at its upstream and downstream ends which will help to determine the actual shape of the profile, i.e. whether the profile is concave or convex.

7. Sketch the qualitative flow profile.

8. Determine whether the flow is subcritical or supercritical. Flow is subcritical if $h > h_c$ and supercritical if $h < h_c$.

Flow Profiles in Mild Slope Channels ($S_o > 0$ and $h_n > h_c$)

Using Eq. (6.19), the sign of dh/dx in each zone can be determined as follows:

$$i)\ Zone\ 1:\ h > h_n > h_c,\ \frac{dh}{dx} = +\frac{+}{+} = +,\ i.e.\ \frac{dh}{dx} > 0$$

$$ii)\ Zone\ 2:\ h_n > h > h_c,\ \frac{dh}{dx} = +\frac{-}{+} = -,\ i.e.\ \frac{dh}{dx} < 0$$

$$iii)\ Zone\ 3:\ h_n > h_c > h,\ \frac{dh}{dx} = +\frac{-}{-} = +,\ i.e.\ \frac{dh}{dx} > 0$$

The water surface profile in Zone 1, designated as M1, is a backwater curve and represents subcritical flow. At the upstream boundary ($h \rightarrow h_n$, $dh/dx \rightarrow 0$), the profile is tangential to the normal depth line and at the downstream boundary ($h \rightarrow \infty$, $dh/dx \rightarrow S_o$), the profile tends to be horizontal. The M1 profile occurs behind a dam and upstream of a weir or sluice gate in a mild slope channel. It may be very long compared to other flow profiles. It represents the most common flow profile and it is the most important flow profile from the practical point of view.

The M2 drawdown curve in Zone 2 is tangential to the normal depth line at its upstream boundary ($h \rightarrow h_n$, $dh/dx \rightarrow 0$) and normal to the critical depth line ($h \rightarrow h_c$, $dh/dx \rightarrow \infty$), indicating a hydraulic drop, at its downstream boundary. This type of profile can occur at a free overfall and when a mild slope is followed by a steeper mild or critical or steep slope.

The M3 backwater profile in Zone 3 starts theoretically from the channel bottom at its upstream end and terminates in a hydraulic jump at its downstream boundary ($h \rightarrow h_c$, $dh/dx \rightarrow \infty$). The M3 profile occurs downstream of a sluice gate in a mild slope channel and when a supercritical flow enters a mild slope channel. The M3 profiles are relatively shorter than M1 and M2 profiles.

Flow Profiles in Steep Slope Channels ($S_o > 0$ and $h_n < h_c$)

The S1 backwater profile begins with a hydraulic jump at the upstream boundary and tends to be horizontal at the downstream boundary. The S1 profile occurs behind a dam or upstream of a weir built in a steep channel.

The S2 drawdown curve starts from the critical depth line with a vertical slope at its upstream end and is tangential to the normal depth line at its downstream end. It is usually very short and acts like a transition between a hydraulic drop and uniform flow. This type of profile can occur downstream of a transition of slope from mild to steep or steep to steeper.

The S3 backwater profile starts from the channel bottom and approaches the normal depth line tangentially at its downstream end. It may occur below a sluice gate on a steep slope or at a transition between steep slope and milder steep slope.

Flow Profiles in Critical Slope Channels ($S_o > 0$ and $h_n = h_c$)

In a critical slope channel the NDL and the CDL coincide since $h_n = h_c$. Therefore, Zone 2 and the C2 profile which satisfy the condition $h_n = h = h_c$ also coincide with the NDL and the CDL. The C2 profile thus represents uniform critical flow and may occur in a long critical slope channel. The C1 backwater profile in Zone 1 starts from the $h_n = h_c$ line and tends to be horizontal downstream. The C3 backwater profile in Zone 3 starts from the channel bottom and meets the $h_n = h_c$ line at its downstream end. Using the condition $h_n = h_c$, Eq.(6.20) gives

$$\frac{dh}{dx} = S_0$$

which indicates that the C1 and C3 profiles in a wide channel are exactly horizontal when the Chezy formula is used. Since $M \approx N$ generally, using the condition $h_c = h_n$ in Eq.(6.19) or (6.21), we obtain

$$\frac{dh}{dx} \approx S_0$$

which indicate that the C1 and C3 profiles are approximately horizontal. The C1 profile may occur upstream of a sluice gate on a critical slope or when a critical slope is followed by a mild or horizontal or adverse slope. The C3 profile may occur downstream of a sluice gate in a critical slope channel or at the transition between steep and critical slopes. The critical slope profiles are very rare.

Flow Profiles in Horizontal Channels ($S_o = 0$)

For a horizontal slope ($S_o = 0$), Eq.(6.13) gives $K_n = \infty$ or $h_n = \infty$, and, therefore, the condition $h > h_n > h_c$ for Zone 1 cannot exist and an H1 profile is not physically possible. With $S_o = 0$, combination of Eqs. (6.9), (6.12), (6.14) and (6.17) gives

$$\frac{dh}{dx} = \frac{-S_f}{1 - Fr^2} = \frac{-(Q/K)^2}{1-(h_c/h)^M} \qquad (6.24)$$

so that the sign of dh/dx is obtained as follows:

$$i)\ Zone\ 2: h > h_c, \quad \frac{dh}{dx} = \frac{-}{+} = -, \ i.e. \ \frac{dh}{dx} < 0$$

$$ii)\ Zone\ 3: h < h_c, \quad \frac{dh}{dx} = \frac{-}{-} = +, \ i.e. \ \frac{dh}{dx} > 0$$

The H2 drawdown profile has a horizontal asymptote at its upstream end ($h \to \infty$) and ends in a hydraulic drop at its downstream end ($h \to h_c$, $dh/dx \to \infty$). It may occur on a horizontal slope upstream of a free overfall. The H3 backwater profile, which is similar to the M3 profile, is obtained downstream of sluice gates and spillways on a horizontal slope.

Flow Profiles in Adverse Slope Channels ($S_o < 0$)

For an adverse slope ($S_o < 0$), Eq. (6.13) indicates that K_n^2 is negative and h_n is imaginary. Therefore, the condition $h > h_n > h_c$ for Zone 1 cannot exist and an A1 profile is not physically possible. Combining Eqs. (6.9), (6.12) and (6.17), we obtain

$$\frac{dh}{dx} = \frac{S_0 - S_f}{1-(h_c/h)^M} \qquad (6.25)$$

Since S_0 is negative and S_f is positive, the numerator of the right hand side of Eq.(6.25) is negative and, hence, the sign of dh/dx is obtained as follows:

$$i)\ Zone\ 2: h > h_c, \quad \frac{dh}{dx} = \frac{-}{+} = -, \ i.e. \ \frac{dh}{dx} < 0$$

$$ii)\ Zone\ 3: h < h_c, \quad \frac{dh}{dx} = \frac{-}{-} = +, \ i.e. \ \frac{dh}{dx} > 0$$

The A2 and A3 profiles are similar to H2 and H3 profiles and are very rare. Only short lengths of adverse slope profiles may be expected to occur in practice.

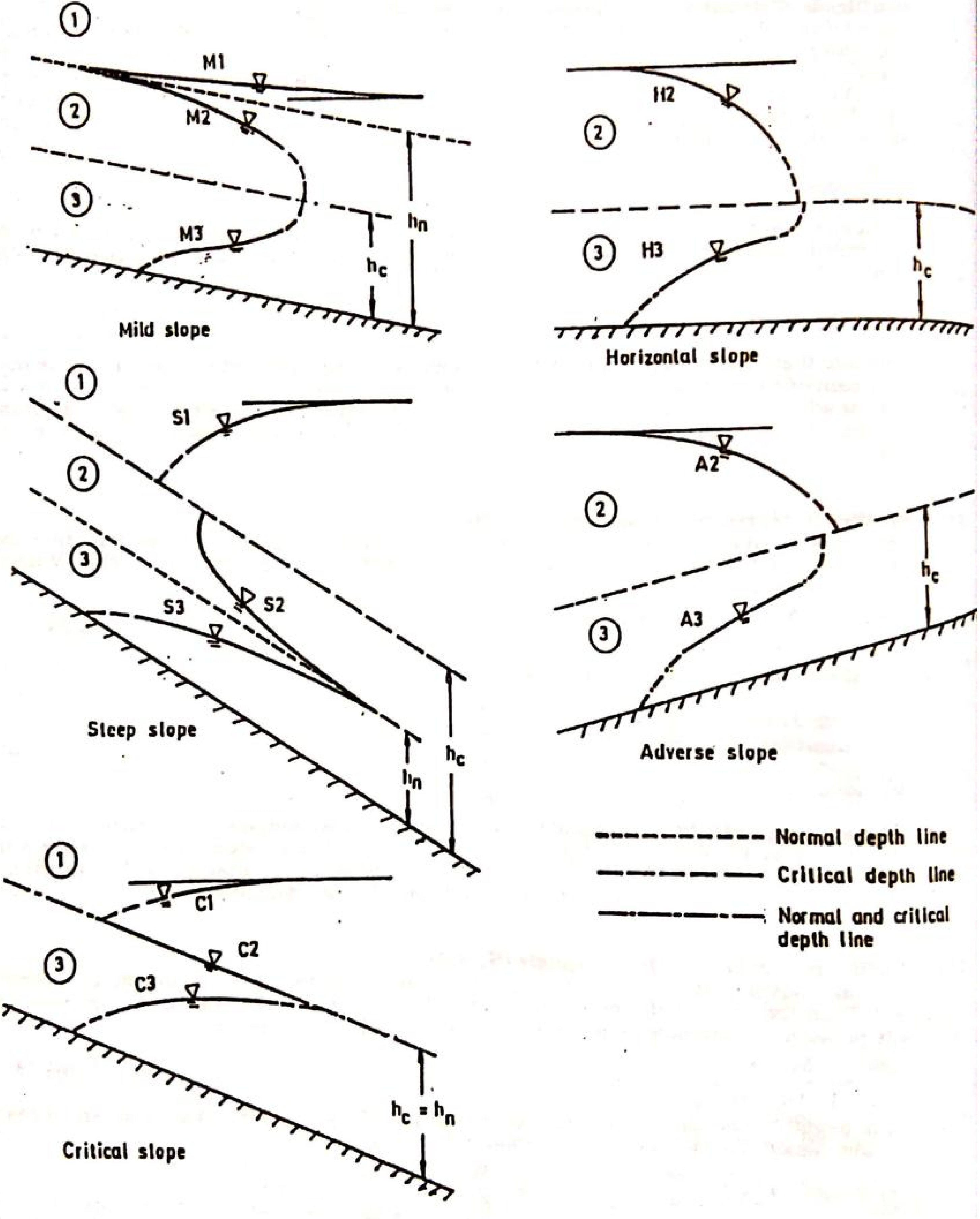


Fig.6.2 Classification of flow profiles of gradually varied flow

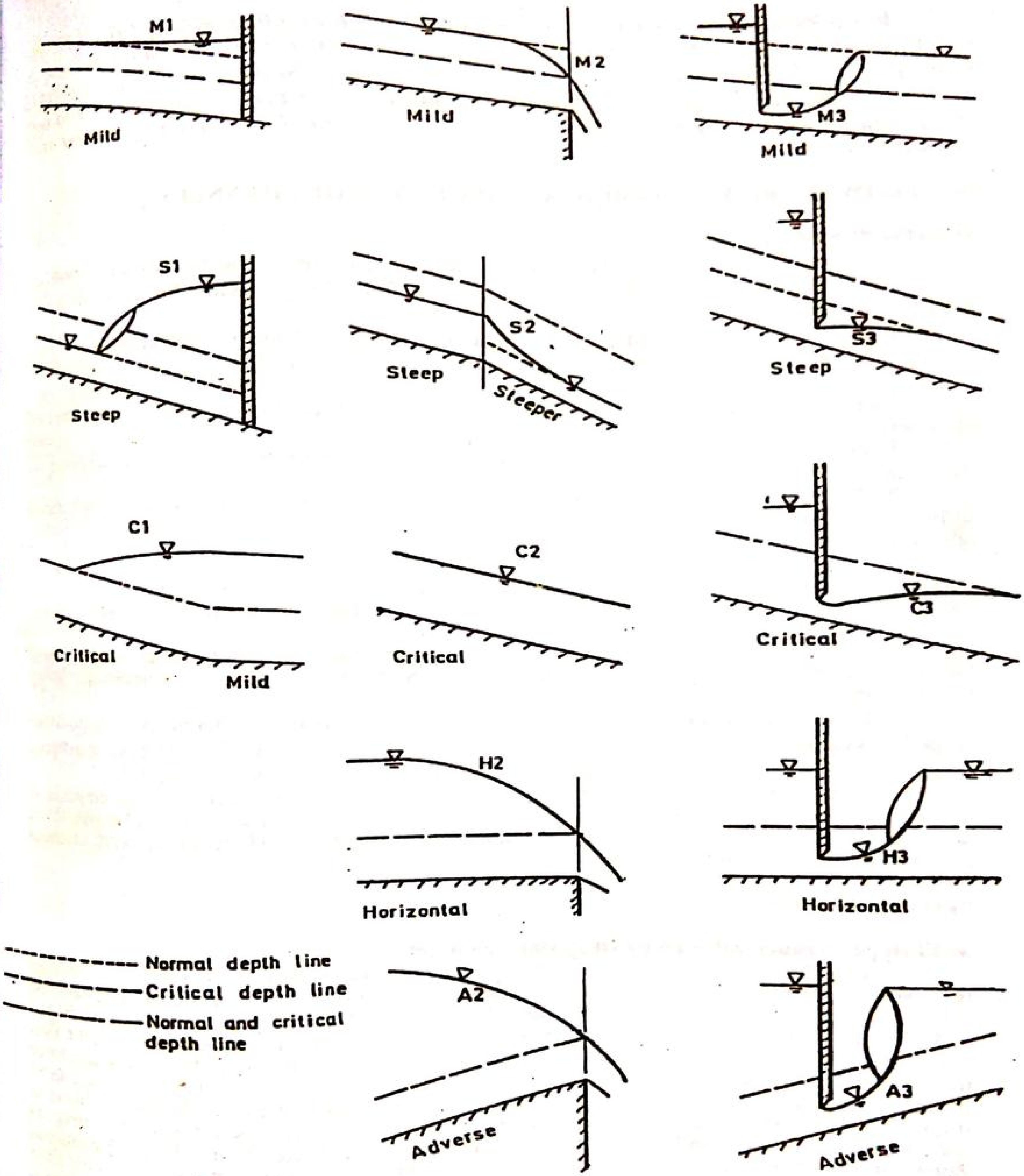


Fig. 6.3 Examples of flow profiles

It is evident from Table 6.1 and Figs. 6.2 and 6.3 that ***the profiles in Zone 1 (i.e. M1, S1 and C1) and Zone 3 (i.e. M3, S3, C3, H3 and A3) are backwater curves and those in Zone 2 (i.e. M2, S2, H2 and A2) excepting C2 are drawdown curves. The profiles in Zone 1 and the profiles M2, H2 and A2 of Zone 2 represent subcritical flow. The profiles in Zone 3 and the S2 profile of Zone 2 represent supercritical flow. The profile C2 represents uniform and critical flow.***

6.4 FLOW PROFILES IN SERIAL ARRANGEMENT OF CHANNELS

General Procedure

When two or more prismatic channels of the same cross-section but with different bottom slopes are combined and carry the same discharge, the following procedure for the analysis of flow profile is to be adopted:

1. Draw the channel profile. Plot the CDL and the NDL, if any, in each channel.
2. Locate all possible control sections at which the depth is known.
3. Starting from the known depth, draw the possible flow profiles in the channels.

The following points must be noted in connection with the flow profiles in a number of channels:

1. The critical depth h_c will be the same for all the channels, since it does not depend on the channel bottom slope S_0.
2. The normal depth h_n will be different in different channels. Since h_n is inversely related to the channel bottom slope S_0, the normal depth h_n for a channel will be higher if S_0 is lower and vice versa.
3. Flow upstream of a control must be subcritical and downstream of a control must be supercritical. The control itself locates the subcritical flow profile upstream of it and the supercritical flow profile downstream of it. In fact, the gradually varied flow profile(s) are the results of interaction between the flow and the control(s).
4. When the flow changes from subcritical to supercritical, a hydraulic drop usually forms. On the other hand, when the flow changes from supercritical to subcritical, a hydraulic jump usually forms.
5. Under normal condition, the flow in a long straight prismatic channel having positive slope is taken to be uniform. Therefore, the flow beyond the influence of a control or transition in a mild, critical or steep slope channel will be uniform, i.e. at the normal depth h_n.
6. Under normal condition, the flow in a long horizontal or adverse slope channel is subcritical. Therefore, the flow beyond the influence of a control or transition in a horizontal or adverse slope channel will be in Zone 2, i.e. the flow profile will be H2 in a horizontal channel and A2 in an adverse slope channel.

Specific Examples

Mild slope channel followed by steep slope channel

As an illustration, let us sketch the qualitative flow profiles in a mild slope channel followed by a steep slope channel (Fig. 6.4). Obviously, the flow is uniform both upstream and downstream of the transition point as shown. The five possible flow profiles, numbered from 1 to 5, over the transition in slope are shown in the figure. For the flow profiles marked 1, the flow is uniform in the entire mild slope channel and the flow passes from the uniform flow condition in the upstream channel to the uniform flow condition in the downstream channel through an S1 profile in Zone 1 and an S2 profile in Zone 2 of the steep slope channel. Obviously, this is an impossible case because an S1 profile must be a backwater curve, not a drawdown curve, as shown in Fig. 6.4. In a similar way, the flow profiles marked 5, consisting of an M2 profile in Zone 2 and an M3 profile in Zone 3 of the upstream mild slope channel, are also impossible since the profile M3 must be a backwater curve. Proceeding in this way we come to the conclusion that the flow profiles marked 3, consisting of an M2 drawdown profile in Zone 2 of the upstream mild slope channel and an S2 drawdown profile in Zone 2 of the downstream steep slope channel, are the only acceptable flow profiles. Obviously, the flow changes from subcritical in the upstream mild slope channel to supercritical in the downstream steep slope channel which is possible only through a hydraulic drop.

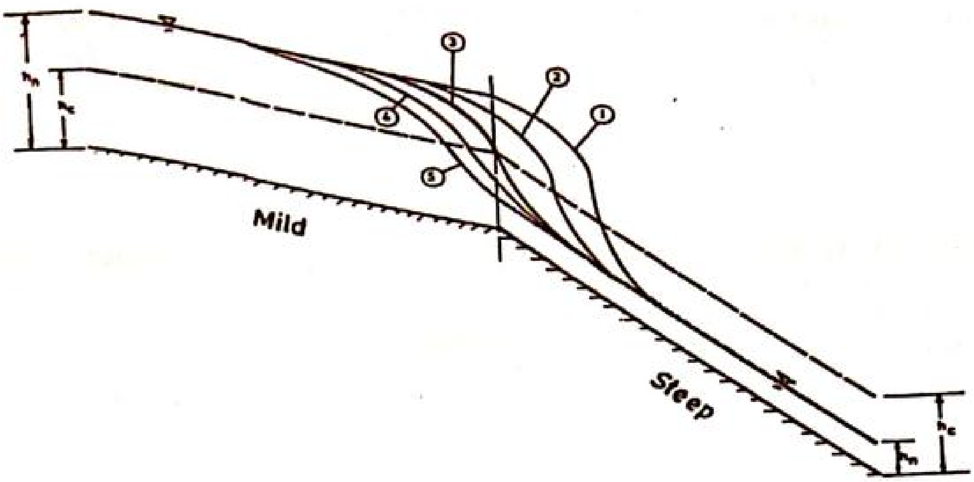


Fig. 6.4 Flow profiles in a mild slope channel followed by a steep slope channel

Steep slope channel followed by mild slope channel

In Fig. 6.5, the flow profiles in a steep slope channel followed by a mild slope channel are shown. The uniform flow is supercritical in the upstream channel and subcritical in the downstream channel. The change in the flow state from supercritical to subcritical can only occur through a hydraulic jump. The location of the jump depends on the relative magnitudes of the two slopes. If the magnitude of the two slopes are such that the jump is formed in the steep slope channel, an S1 profile is formed in the steep channel and the flow is uniform just from the beginning of the mild slope channel. Now, if the slope of the downstream mild slope channel is gradually increased, the normal depth line for the mild slope channel lowers and the jump gradually moves downstream and finally the jump forms on the mild slope channel. In this situation, the flow is uniform in the entire upstream channel, an M3 profile is formed in the downstream channel and the flow eventually returns to the uniform state after the hydraulic jump in the mild slope channel.

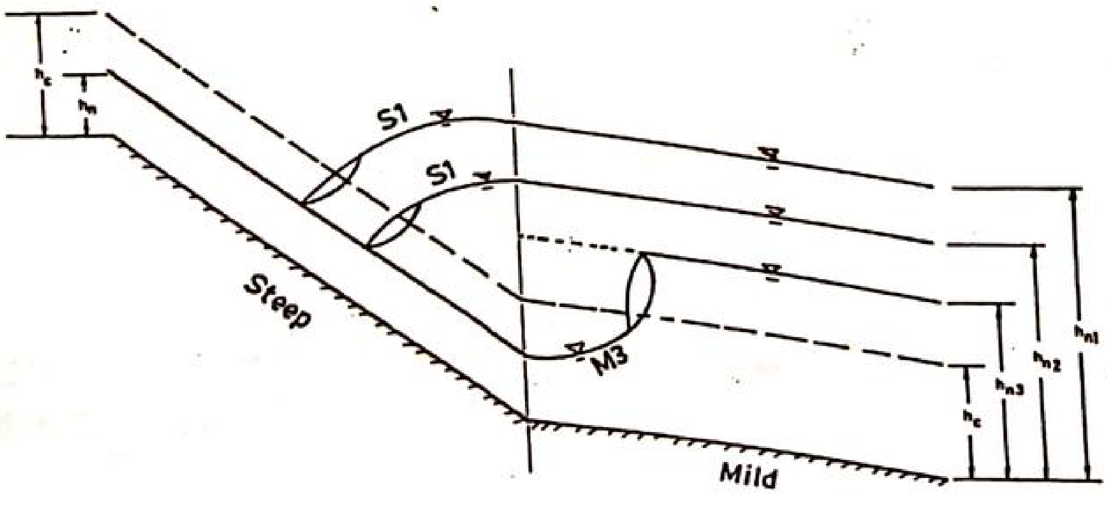


Fig. 6.5 Flow profile in a steep slope channel followed by a mild slope channel

It is to be noted that a hydraulic jump usually occurs when a steep slope channel is followed by either a mild or a horizontal or an adverse slope channel. The jump forms either in the upstream channel or in the downstream channel, but such a situation does never occur that a part of the jump forms in the upstream channel and the rest of the jump forms in the downstream channel.

Free overfall at the end of a mild slope channel

Suppose there is a free overfall at the end of a mild slope channel (Fig. 6.6). The flow is uniform in the channel far upstream of the free overfall. The water surface falls as a result of the free overfall, an M2 profile develops immediately upstream of the free overfall and the critical depth occurs just upstream of the brink.

Different water levels downstream of a mild slope channel

Three positions of the water level downstream of a mild slope channel are as shown in Fig. 6.7. Obviously,

i) when $h_d > h_n$, an M1 profile is formed,
ii) when $h_n > h_d > h_c$, an M2 profile is formed, and
iii) when $h_n > h_c > h_d$, the situation is similar to a free overfall and an M2 profile is formed.

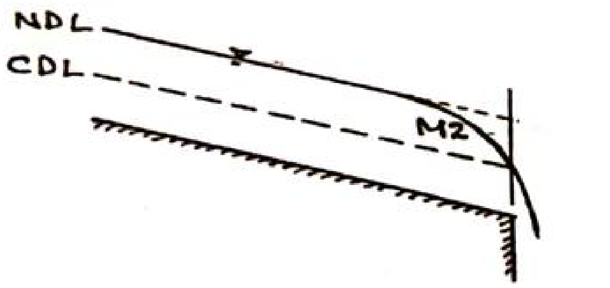


Fig. 6.6 Flow profile due to a free overfall at the end of a mild slope channel

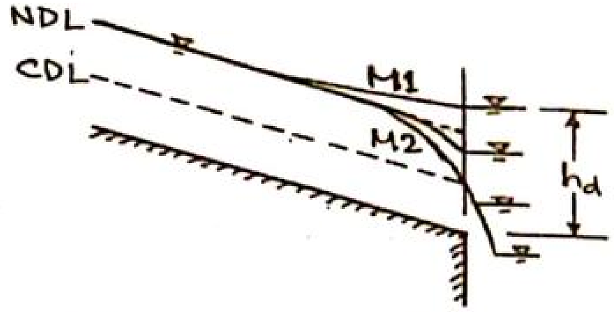
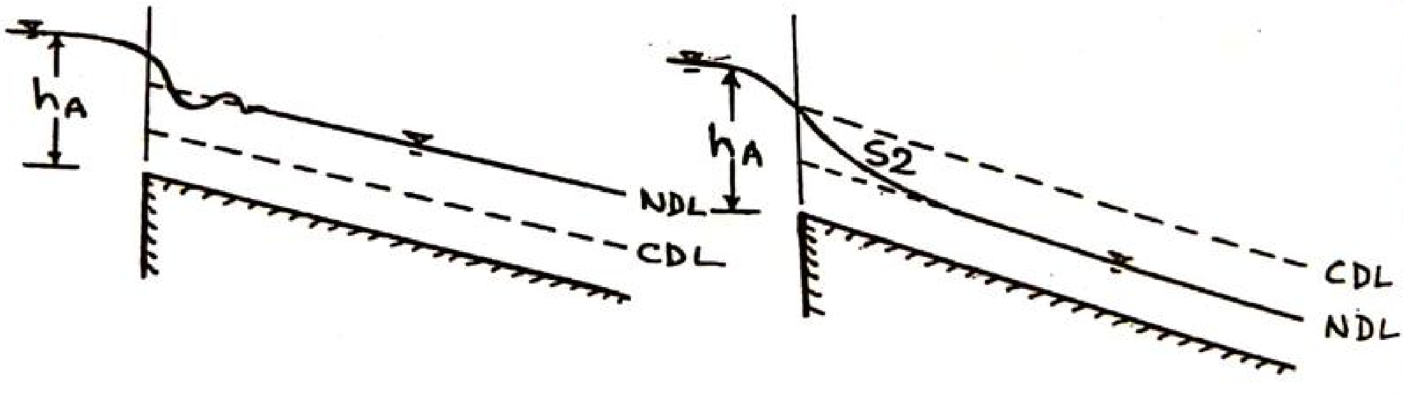


Fig. 6.7 Flow profiles for different positions of water level downstream of a mild slope channel

Flow from a reservoir into a channel

It is assumed that the water level in the reservoir h_A is more than h_c and h_n (if any) in the channel. The flow from the reservoir into the canal is subcritical and remains subcritical if the channel slope is mild, changes to critical if the channel slope is critical and changes to supercritical if the channel slope is steep. In case of a mild slope channel (Fig. 6.8a), the water surface from the reservoir joins the NDL of the channel through a transition and no profile is formed. In case of a steep slope channel (Fig. 6.8b), the depth at the entrance to the channel is critical and the water surface joins the NDL through an S2 profile.



(a) (b)

Fig. 6.8 Flow from a reservoir into a (a) mild slope channel, and (b) steep slope channel

Sluice gate in a steep slope channel

There is a vertical sluice gate in a steep slope channel (Fig. 6.9). The flow in the channel far upstream from of the sluice gate is uniform and supercritical. The presence of the sluice gate, which is a control, changes the flow to subcritical with the formation of a hydraulic jump and an S1 profile, as shown in Fig. 6.9. An S3 profile representing supercritical flow is formed downstream of the sluice gate through which the water surface joins the NDL downstream.

Overflow weir in a mild slope channel

The flow in the channel far upstream of the weir (Fig. 6.10) is uniform and subcritical. The depth above the weir is approximately equal to the critical depth, i.e. critical section occurs just upstream of the weir. Therefore, an M1 profile is formed upstream of the weir.

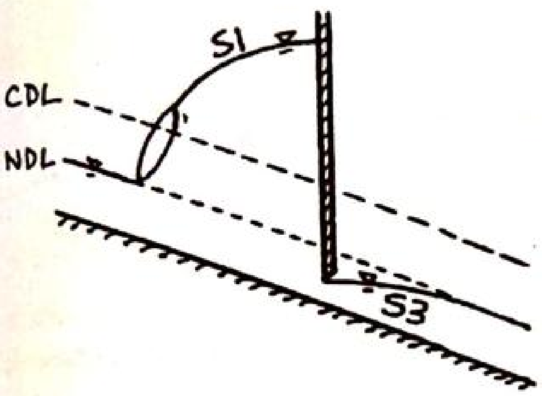


Fig. 6.9 Flow profiles upstream and downstream of a sluice gate in a steep slope channel

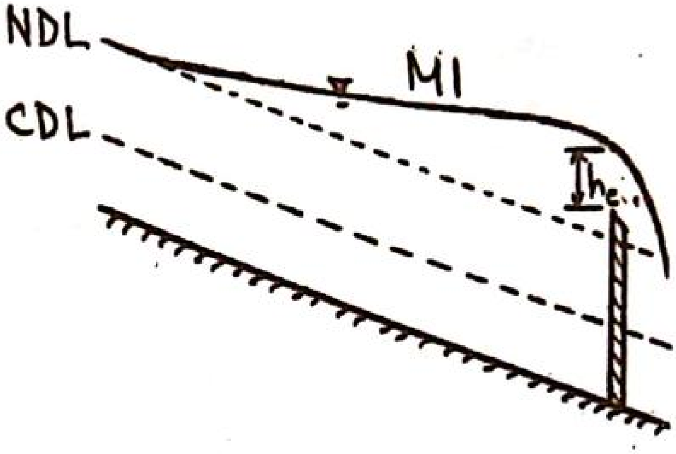
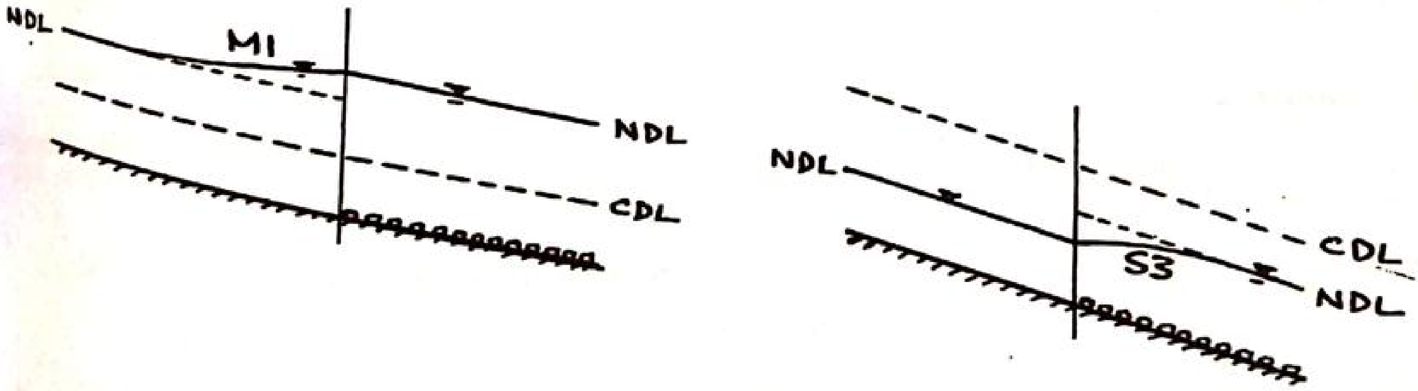


Fig. 6.10 Flow profile upstream of an overflow weir in a mild slope channel

Increase of surface roughness

An increase in surface roughness increases the normal depth h_n. As a result, the normal depth in the rough channel is higher than the normal depth in the smooth channel. When the channel is mild, an M1 profile is formed in the smooth channel through which the uniform flow in the smooth channel changes to uniform flow in the rough channel (Fig. 6.11a). In the case of a steep slope channel, an S3 profile is developed in the rough channel (Fig. 6.11b).



(a) (b)

Fig. 6.11 Flow profile as a result of increase in surface roughness in a (a) mild slope channel, and (b) steep slope channel

Change in channel width

An increase in channel width reduces the discharge per unit width q and the normal depth h_n. A decrease in width does the reverse. The flow profiles for a change in width in a mild slope channel is shown in Fig. 6.12.

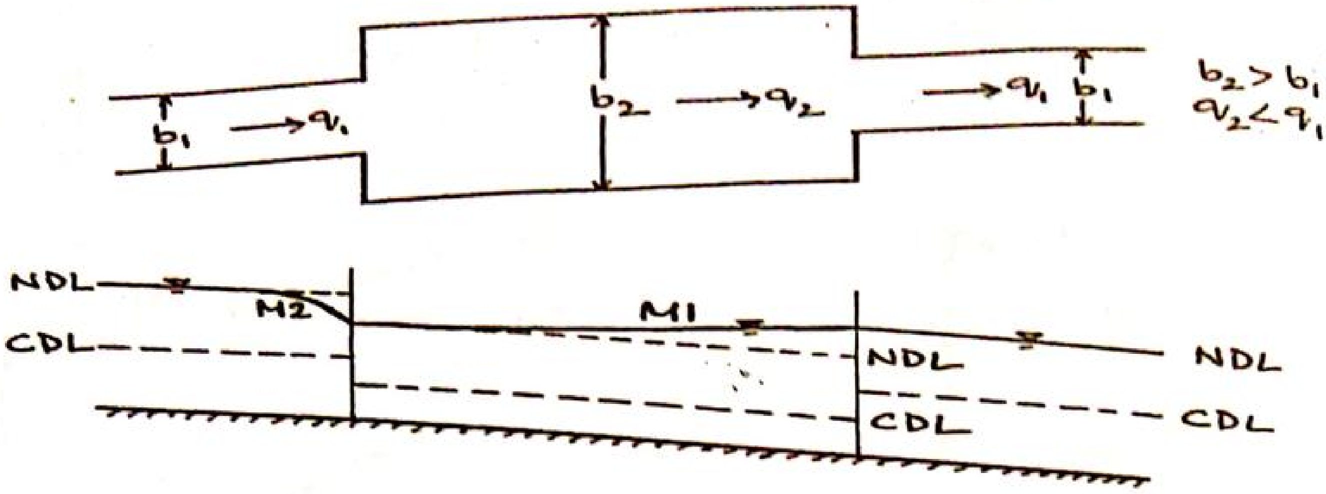


Fig. 6.12 Flow profile as a result of change in channel width in a mild slope channel

Example 6.1

A trapezoidal channel with b = 6 m, n = 0.025, s = 2 and S_0 = 0.001 carries a discharge of 28 m^3/s. At a certain section A of the channel the depth of flow is 1.30 m. (i) Determine the type of channel slope. (ii) Determine the type of flow profile. (iii) If at another section B, the depth of flow is 1.50 m, state whether section B is located upstream or downstream of section A.

Solution For the given section, n, S_0 and Q, the critical and normal depths are found to be

h_c = 1.14 m and h_n = 1.91 m

The actual depth of flow at section A , h_A = 1.30 m.

(i) Since $h_n > h_c$, the channel slope is mild.

(ii) Since $h_n > h_A > h_c$, flow occurs in Zone 2 and the profile is M2.

(iii) Since the M2 profile is a drawdown profile and the depth at section B (= 1.50 m) is more than the depth at section A (= 1.30 m), section B is located upstream of section A.

Example 6.2

A rectangular channel with b = 10 m, α = 1.10 and n = 0.025 has three reaches arranged serially. The bottom slopes of these reaches are 0.0040, 0.0065 and 0.0090, respectively. For a discharge of 35 m^3/s in this channel, sketch the resulting flow profiles.

Solution The critical depth for the given conditions is obtained as

$$h_c = \sqrt[3]{\frac{\alpha Q^2}{g b^2}} = \sqrt[3]{\frac{1.10 \times 35^2}{9.81 \times 10^2}} = 1.11 \ m$$

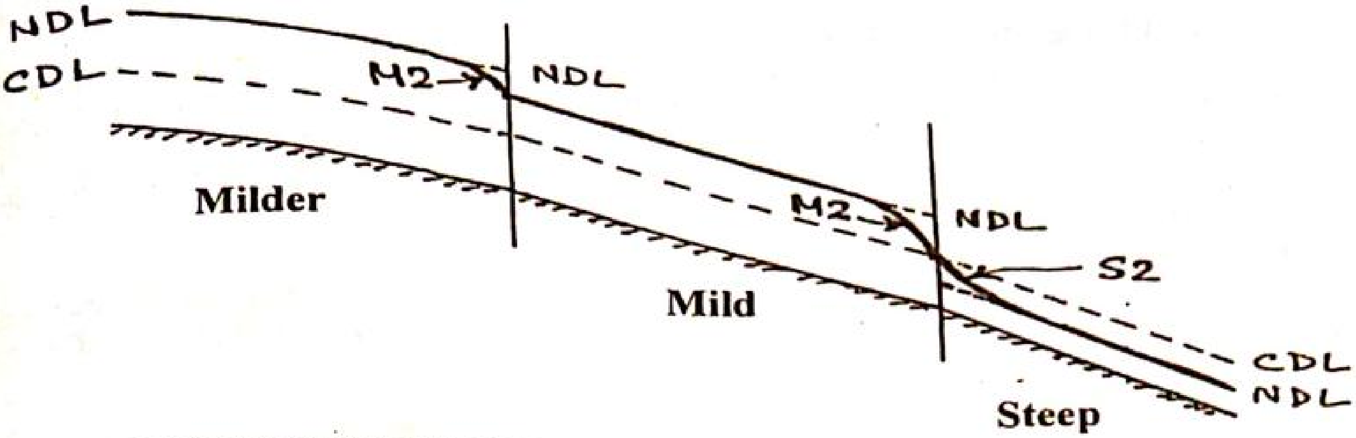
Since critical slope is the slope for which flow in the channel is both uniform and critical, hence

$h_n = h_c$ = 1.11 m

Therefore, A = 10× 1.11 = 11.12 m^2, P = 10 + 2× 1.11 = 12.22 m and R = A/P = 0.91 m, and

$$S_c = \left(\frac{nQ}{AR^{2/3}}\right)^2 = \left(\frac{0.025 \times 35}{11.12 \times 0.91^{2/3}}\right)^2 = 0.0070$$

Thus, the bottom slopes of the three reaches are milder, mild and steep, respectively. The resulting flow profiles are M2, M2 and S2, as shown in the following figure.



6.5 COMPUTATION OF FLOW PROFILES

6.5.1 Introduction

The computation of the gradually varied flow basically involves the integration of the dynamic equation of gradually varied flow. This equation is a non-linear ordinary differential equation of the first order and its solution requires one boundary condition for depth, i. e. the depth at the section where the computation begins must be given. This equation can be easily integrated (i) for a wide channel, and (ii) for a horizontal channel. For other channels, the integration of the gradually varied flow equation has to be performed either graphically or numerically.

The computation of gradually varied flow profile must begin with the known depth of flow at a control and proceed in the direction in which the control operates. Thus, ***the computation of the subcritical flow profiles must start from the downstream end of the channel reach and proceed upstream, and the computation of supercritical flow profiles must start from the upstream end of the channel reach and proceed downstream.***

The profiles M1, M2, S2 and S3 approach the normal depth line asymptotically, i.e. theoretically these profiles extent indefinitely before merging with the normal depth line. Such a situation presents difficulties from the computational point of view. As a result, the computation of these flow profiles is usually terminated at a section where the depth of flow is about 5% greater or less than the normal depth.

In computing a flow profile, the following data or information are generally required:

1. The discharge Q for which the flow profile is desired.
2. The depth of flow or stage at the section where the computation begins.
3. The channel sections.
4. The bottom slope S_0 of the channel.
5. The energy coefficient α.
6. The Manning's n or Chezy's C.

There are many methods for computing gradually varied flow profiles. However, these methods can be broadly classified into the following three categories:

i) Methods used for computing flow profiles in prismatic channels
ii) Methods used for computing flow profiles in non-prismatic channels
iii) Numerical methods.

The methods used for computing flow profiles in prismatic or regular or uniform channels compute a longitudinal distance x for a given h directly without involving any trial. The direct step method and the direct integration method fall in this category. The methods used for computing flow profiles in non-prismatic or irregular or non-uniform channels compute h from a given x. In this case, a trial-and-error procedure is necessary. The standard step method falls in this second category. The numerical methods compute h from given x without involving any trial. The Runge –Kutta methods fall in this category.

6.5.2 Direct Integration Method

Direct integration of the gradually varied flow equation for computing flow profiles in a wide channel and in a horizontal channel is simple and considered here. The integration of the gradually varied flow equation for other cases is presented by Chow (1959) and Gill (1976).

The main advantage of the direct integration method over other methods is that the total length of the flow profile may be computed accurately using a single step.

(i) Flow Profile in a Wide Channel: Bresse Method

Equation (6.19) can be integrated exactly for a wide rectangular channel with the conveyance expressed in terms of the Chezy formula. For this case, M = N = 3 and we have

$$\frac{dh}{dx}=S_0\frac{1-(h_n/h)^3}{1-(h_c/h)^3} \qquad (6.26)$$

Putting $u = h/h_n$, so that $du = dh/h_n$ in Eq.(6.26) and rearranging yields

$$dx=\frac{h_n}{S_0}\left[1-\left(1-\frac{h_c^{\,3}}{h_n^{\,3}}\right)\frac{1}{1-u^3}\right]du \qquad (6.27)$$

which on integration gives

$$x=\frac{h_n}{S_0}\left[u-\left(1-\frac{h_c^{\,3}}{h_n^{\,3}}\right)\phi\right]+C_1 \qquad (6.28)$$

where ϕ is the *Bresse function* given by

$$\phi=\int_0^u\frac{du}{1-u^3}=\frac{1}{6}\ln\frac{u^2+u+1}{(u-1)^2}-\frac{1}{\sqrt{3}}\tan^{-1}\frac{\sqrt{3}}{2u+1} \qquad (6.29)$$

and C_1 is a constant of integration. This integration was first performed by J. A. Ch. Bresse in 1860. A determination of the flow profile by this solution is widely known as the *Bresse method.*

The length of the flow profile between two consecutive sections of depths h_1 and h_2 is obtained from Eq.(6.28) as

$$L=x_2-x_1=\frac{h_n}{S_0}\left[(u_2-u_1)-\left(1-\frac{h_c^{\,3}}{h_n^{\,3}}\right)(\phi_2-\phi_1)\right] \qquad (6.30)$$

where ϕ_1 and ϕ_2 are the values of ϕ corresponding to $u_1 = h_1/h_n$ and $u_2 = h_2/h_n$, respectively.

Using the Bresse method, the flow profile in a wide positive (mild, critical and steep) slope channels can be computed.

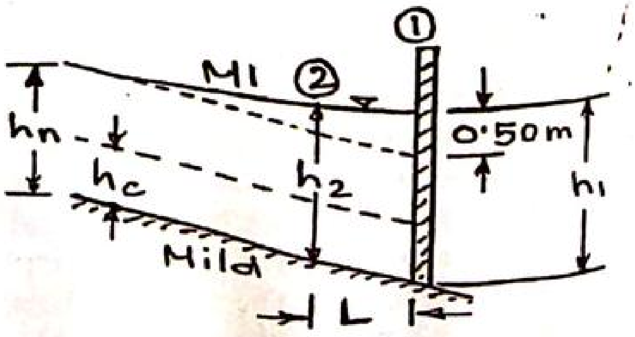
Example 6.3

A wide rectangular channel with Chezy's C = 47 $m^{1/2}/s$ and S_0 = 0.0001 carries a discharge of 2 m^2/s. A dam raises the water level by 0.50 m above the normal depth at the dam site. Compute the length of the resulting flow profile between the dam site and the location where the depth is 2.90 m.

Solution Since the channel is wide,

$$h_c=\sqrt[3]{\frac{q^2}{g}}=\sqrt[3]{\frac{2^2}{9.81}}=0.742\ m$$

$$h_n=\sqrt[3]{\frac{q^2}{C^2S_0}}=\sqrt[3]{\frac{2^2}{47.0^2\times0.0001}}=2.626\ m$$



Since $h_n > h_c$, the channel slope is mild.

Now, $h_1 = 2.626 + 0.50 = 3.126\ m$, $u_1 = h_1 / h_n = 1.190$,

$h_2 = 2.90\ m$, $u_2 = h_2 / h_n = 1.104$.

Since h_1 or $h_2 > h_n > h_c$, the profile is M1.

$$\phi_1 = \frac{1}{6}\ln\frac{u_1^2 + u_1 + 1}{(u_1 - 1)^2} - \frac{1}{\sqrt{3}}\tan^{-1}\frac{\sqrt{3}}{2u_1 + 1}$$

$$= \frac{1}{6}\ln\frac{1.190^2 + 1.190 + 1}{(1.190 - 1)^2} - \frac{1}{\sqrt{3}}\tan^{-1}\frac{\sqrt{3}}{2\times 1.190 + 1} = 0.7667 - 0.2734 = 0.4933$$

$$\phi_2 = \frac{1}{6}\ln\frac{u_2^2 + u_2 + 1}{(u_2 - 1)^2} - \frac{1}{\sqrt{3}}\tan^{-1}\frac{\sqrt{3}}{2u_2 + 1}$$

$$= \frac{1}{6}\ln\frac{1.104^2 + 1.104 + 1}{(1.104 - 1)^2} - \frac{1}{\sqrt{3}}\tan^{-1}\frac{\sqrt{3}}{2\times 1.104 + 1} = 0.9545 - 0.2858 = 0.6687$$

Hence, the length of the profile is obtained using Eq.(6.30) as

$$L = \frac{h_n}{S_0}\left[(u_2 - u_1) - \left(1 - \frac{h_c^3}{h_n^3}\right)(\phi_2 - \phi_1)\right]$$

$$= \frac{2.626}{0.0001}\left[(1.104 - 1.190) - \left(1 - \frac{0.742^3}{2.626^3}\right)(0.6687 - 0.4933)\right] = -6760.06\ m$$

(ii) Flow Profile in a Horizontal Channel

For a horizontal channel, $S_0 = 0$ and Eq. (6.9) becomes

$$\frac{dh}{dx} = \frac{-S_f}{1 - Fr^2} = \frac{-(Q/K)^2}{1 - (h_c/h)^M} \qquad (6.31)$$

where $Q = K\sqrt{S_f}$. Since the critical slope S_c is the slope that will produce a discharge Q at a normal depth equal to the critical depth h_c, $Q = K_c\sqrt{S_c}$ and hence Eq.(6.31) becomes

$$\frac{dh}{dx} = S_c\frac{-(K_c/K)^2}{1 - (h_c/h)^M} \qquad (6.32)$$

Since $K^2 = C_2 h^N$ and $K_c^2 = C_2 h_c^N$, so that $(K_c/K)^2 = (h_c/h)^N$, Eq.(6.32) becomes

$$\frac{dh}{dx} = S_c\frac{-(h_c/h)^N}{1 - (h_c/h)^M} \qquad (6.33)$$

Putting $p = h/h_c$ in Eq.(6.33) so that $dp = dh/h_c$ and rearranging, we obtain

$$h_c\frac{dp}{dx} = S_c\frac{-(1/p)^N}{1 - (1/p)^M} = S_c\frac{p^M}{p^N(1 - p^M)} = S_c\frac{p^{M-N}}{1 - p^M} \qquad (6.34)$$

or

$$dx = \frac{h_c}{S_c}(p^{N-M} - p^N)dp \qquad (6.35)$$

Integrating the above equation, we get

$$x = \frac{h_c}{S_c}\left(\frac{p^{N-M+1}}{N - M + 1} - \frac{p^{N+1}}{N + 1}\right) + C_1 \qquad (6.36)$$

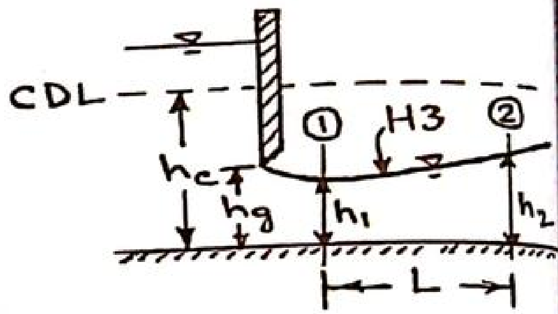
where C_1 is a constant of integration. The length of the flow profile between two consecutive sections of depths h_1 and h_2 is given by

$$L = x_2 - x_1 = \frac{h_c}{S_c}\left[\frac{1}{N-M+1}(p_2^{N-M+1} - p_1^{N-M+1}) - \frac{1}{N+1}(p_2^{N+1} - p_1^{N+1})\right] \quad (6.37)$$

When M and N are not constant, then they are to be computed for the average depth $\bar{h} = (h_1 + h_2)/2$ and taken to be constant for the reach.

Example 6.4

A vertical sluice gate having a coefficient of contraction, $C_c = 0.61$ and a gate opening, $h_g = 1.00$ m, discharges 25 m^3/s into a horizontal rectangular channel 5 m wide. Compute the length of the flow profile between the vena contracta and the location where the depth is 0.75 m. Take n = 0.015 and α = 1.12.



Solution Depth at the vena contracta,

$h_1 = C_c \times h_g = 0.61 \times 1.00 = 0.61$ m

$$h_c = \sqrt[3]{\frac{\alpha Q^2}{g b^2}} = \sqrt[3]{\frac{1.12 \times 25^2}{9.81 \times 5^2}} = 1.419\ m$$

Since $h_1 < h_c$, an H3 backwater profile is created.

The critical slope is obtained using the Manning formula with $h_n = h_c = 1.419$ m. Then, $A = 5 \times 1.419 = 7.09\ m^2$, $P = 5 + 2 \times 1.419 = 7.84$ m, $R = 7.09/7.84 = 0.91$ m so that

$$S_c = \left(\frac{nQ}{AR^{2/3}}\right)^2 = \left(\frac{0.015 \times 25}{7.09 \times 0.91^{2/3}}\right)^2 = 0.0032$$

The channel is rectangular. Hence, M = 3 and the value of N is computed for the average depth, $\bar{h} = (h_1 + h_2)/2 = (0.61 + 0.75)/2 = 0.68$ m so that $\bar{h}/b = 0.68/5 = 0.136$.

$$\therefore N = \frac{2}{3}\left[5 - \frac{4(\bar{h}/b)}{1 + 2(\bar{h}/b)}\right] = \frac{2}{3}\left[5 - \frac{4 \times 0.136}{1 + 2 \times 0.136}\right] = 3.048$$

Therefore, $N - M + 1 = 1.048$ and $N + 1 = 4.048$. Also, $p_1 = h_1/h_c = 0.61/1.419 = 0.4297$ and $p_2 = h_2/h_c = 0.75/1.419 = 0.5284$. Hence, the length of the flow profile is obtained using Eq. (6.37) as

$$L = \frac{h_c}{S_c}\left[\frac{1}{N-M+1}(p_2^{N-M+1} - p_1^{N-M+1}) - \frac{1}{N+1}(p_2^{N+1} - p_1^{N+1})\right]$$

$$= \frac{1.419}{0.0032}\left[\left(\frac{0.5284^{1.048} - 0.4297^{1.048}}{1.048}\right) - \left(\frac{0.5284^{4.048} - 0.4297^{4.048}}{4.048}\right)\right]$$

$$= 37.63 \text{ m}$$

6.5.3 Direct Step Method

In general, a step method is characterized by dividing the channel into short reaches (Fig. 6.13) and carrying the computation step by step from one end of the reach to the other. The direct step method is applicable to prismatic channels and predicts a longitudinal distance x for a given depth h explicitly without involving any trial.

In this method the equation

$$\frac{dE}{dx}=S_0-S_f \quad (6.38)$$

is used. In finite difference form this equation can be written as

$$\frac{\Delta E}{\Delta x}=S_0-S_f \quad (6.39)$$

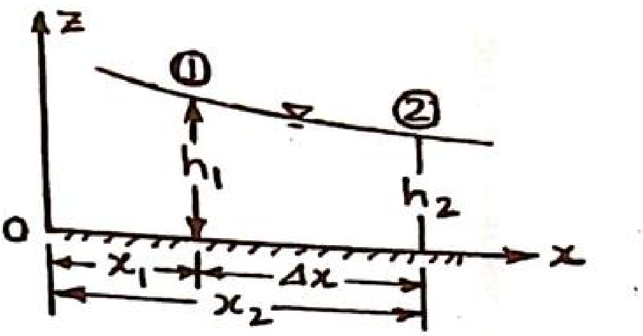


Fig. 6.13 Definition sketch for direct step method

In this equation, all variables with the exception of Δx, are functions of the depth of flow h. So, by selecting the value of h, Eq. (6.39) can be solved for Δx as

$$\Delta x=\frac{\Delta E}{S_0-\bar{S}_f}=\frac{E_2-E_1}{S_0-\bar{S}_f} \quad (6.40)$$

where

$$\bar{S}_f=(S_{f1}+S_{f2})/2 \quad (6.41)$$

is the mean value of the friction slope over the space interval Δx, and S_{f1} and S_{f2} are the friction slopes at sections 1 and 2, respectively. Since $\Delta x = x_2 - x_1$ and the distance x_1 of section 1 from the origin is known, the distance x_2 from the origin can be obtained from

$$x_2=x_1+\Delta x \quad (6.42)$$

Example 6.5

A trapezoidal channel with b = 6 m and s = 2 is laid on a slope of 0.0025 and carries a discharge of 30 m^3/s. The depth produced by a dam immediately upstream of it is 2.50 m. Compute the resulting flow profile. Take α = 1.12 and n = 0.025.

Solution

For the given data, the critical and normal depths are found to be h_c = 1.23 m and h_n = 1.55 m. Since $h_n > h_c$, the channel slope is mild, and since $h > h_n > h_c$, the profile is M1.

The channel bottom at the section where h = 2.5 m is taken as the origin and the distances measured downstream are taken to be positive. The computation begins at x = 0 and proceeds upstream step by step until the depth is 1.60 m, which is about 3% higher than the normal depth. The computation of Δx using Eq.(6.40) for various values of h are given in Table 6.2 which is almost self-explanatory.

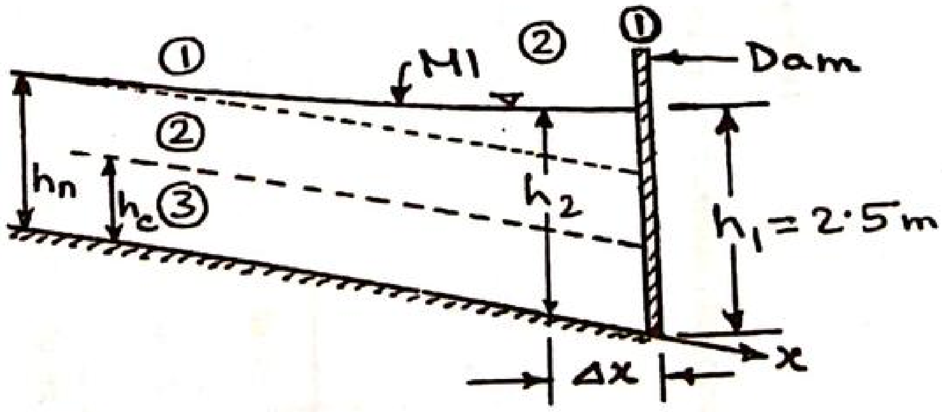


Table 6.2 Computation of flow profile for Example 6.5 by direct step method

Trapezoidal channel, b = 6 m, s = 2, S_0 = 0.0025, Q = 30 m^3/s, α = 1.12, n = 0.025, h_c = 1.23 m, h_n = 1.55 m, Mild slope, M1 profile

h	A	P	R	U	$\alpha U^2/2g$	E	ΔE	S_f	$\bar{S}_f$	$S_0 - \bar{S}_f$	Δx	x
2.50	27.50	17.18	1.601	1.091	0.0679	2.5679	-	0.000387	-	-	-	0.00
2.20	22.88	15.84	1.445	1.311	0.0981	2.2981	-0.2698	0.000658	0.000523	0.001978	-136.43	-136.43
2.00	20.00	14.94	1.338	1.500	0.1284	2.1284	-0.1697	0.000954	0.000806	0.001694	-100.18	-236.61
1.80	17.28	14.05	1.230	1.736	0.1721	1.9721	-0.1533	0.001429	0.001916	0.001308	-119.46	-356.07
1.70	15.98	13.60	1.175	1.887	0.2012	1.9012	-0.0709	0.001795	0.001612	0.000888	-79.84	-435.91
1.60	14.72	13.16	1.119	2.038	0.2371	1.8371	-0.0641	0.002235	0.002015	0.000485	-132.10	-568.01

6.5.4 Computation of Flow Profiles in Non-prismatic Channels

Standard Step Method

The direct integration method and the direct step method, considered above, compute a longitudinal distance x for a given depth h in a prismatic channel. These methods cannot be employed to compute flow profiles in non-prismatic channels where it is necessary to compute the depth of flow h or the stage z_w for a given longitudinal distance x.

In irregular or non-prismatic channels like the natural rivers, the channel properties are usually measured only at certain fixed sections, known as *stations*, and it is necessary to calculate the depth h or the stage z_w from the chosen value of x. Using the standard step method, the depth of flow h or the stage z_w for a given x can be computed. Therefore, this method is best suited to the computation of flow profiles in natural channels.

Since the depth or stage is unknown, we must use some sort of trial process. In the computation of flow profiles in non-prismatic channels like rivers, the *stage* z_w is most commonly used instead of the depth of flow h.

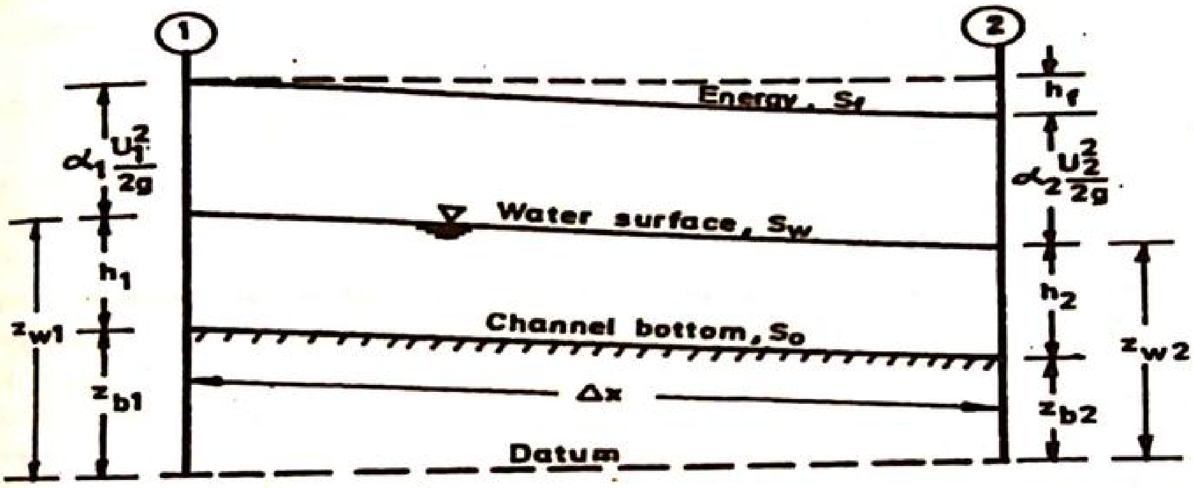


Fig. 6.14 Channel reach definition for standard step method

The application of the energy equation between the two stations shown in Fig.6.14 yields

$$z_{b1} + h_1 + \alpha_1 \frac{U_1^2}{2g} = z_{b2} + h_2 + \alpha_2 \frac{U_2^2}{2g} + h_f + h_e \tag{6.43}$$

or,

$$z_{w1} + \alpha_1 \frac{U_1^2}{2g} = z_{w2} + \alpha_2 \frac{U_2^2}{2g} + h_f + h_e \tag{6.44}$$

where

$$z_{w1} = z_{b1} + h_1 \tag{6.45}$$

and

$$z_{w2} = z_{b2} + h_2 \tag{6.46}$$

are the stages and z_{b1} and z_{b2} are the elevations of the channel bottom at sections 1 and 2, respectively, h_f is the friction loss in the reach and h_e is the eddy loss occurring in the reach.

The friction loss is given by

$$h_f = \bar{S}_f \Delta x = \frac{1}{2}(S_{f1} + S_{f2})\Delta x \tag{6.47}$$

Where S_{f1} and S_{f2} are the friction slopes at sections 1 and 2, respectively.

The eddy losses, which may be appreciable in non-prismatic channels, are generally taken to be proportional to the absolute magnitude of the change in the velocity head in the reach or

$$h_e = k\left|\alpha_1 \frac{U_1^2}{2g} - \alpha_2 \frac{U_2^2}{2g}\right| \tag{6.48}$$

where k is a coefficient which is assumed to range between 0 and 0.1 for gradually converging reaches, between 0 and 0.2 for gradually diverging reaches and to have a value of 0.5 for abrupt expansions or contractions. The eddy loss h_e is assumed to be zero for prismatic channels.

The total heads at sections 1 and 2 are given by

$$H_1 = z_{b1} + h_1 + \alpha_1 \frac{U_1^2}{2g} \tag{6.49}$$

and

$$H_2 = z_{b2} + h_2 + \alpha_2 \frac{U_2^2}{2g} \tag{6.50}$$

Using Eqs. (6.49) and (6.50), Eq.(6.43) or (6.44) becomes

$$H_2 = H_1 - h_f - h_e \tag{6.51}$$

Equation (6.51) is solved by trial-and-error, i.e. for a given space interval Δx, a value of h_2 (or z_{w2}) is assumed which allows the computation of H_2 by Eq. (6.50), h_f and h_e are then computed and H_2 is estimated by Eq. (6.51). If the two computed values of H_2 agree, then the assumed depth (or stage) at station 2 is correct. If not, the calculations are repeated with an improved trial value of h_2 (or z_{w2}). If the value of h_2 (or z_{w2}) computed by Eq.(6.50) is more than that computed by Eq.(6.51), then h_2 (or z_{w2}) has to be reduced for the next trial and vice versa.

The important step in this analysis is the selection of an improved trial value of h_2 or z_{w2}. On the basis of ith trial, the (i+1)th trial value of the depth h_2 (and hence the stage z_{w2}) can be found by the *Newton-Raphson method*. Let $F(h_2)$ be a function such that

$$F(h_2) = H_2 - (H_1 - h_f - h_e) \tag{6.52}$$

Then using Eqs.(6.46),(6.47) and (6.50), one obtains

$$F(h_2) = z_{b2} + h_2 + \alpha_2 \frac{U_2^2}{2g} - H_1 + \frac{1}{2}S_{f1}\Delta x + \frac{1}{2}S_{f2}\Delta x + h_e$$

Then

$$\frac{dF(h_2)}{dh_2} = \frac{d}{dh_2}\left(z_{b2} + h_2 + \alpha_2 \frac{U_2^2}{2g} - H_1 + \frac{1}{2}S_{f1}\Delta x + \frac{1}{2}S_{f2}\Delta x + h_e\right) \tag{6.53}$$

Since z_{b2}, H_1 and S_{f1} are already known, their derivatives with respect to h_2 are equal to zero. The derivative dh_e/dh_2 is neglected, since the variation of h_e with respect to h_2 is small (if desired, it can be included). Further, $d(\alpha U^2/2g)/dh = -Fr^2$, where Fr is the Froude number, and using Eqs.(6.14) and (6.18), it can be proved that $dS_f/dh = -NS_f/h$, where N is the hydraulic exponent for uniform flow computation. Therefore, Eq.(6.53) reduces to

$$\frac{dF(h_2)}{dh_2} = 1 - Fr_2^2 - \frac{N_2 S_{f2}\Delta x}{2h_2} \tag{6.54}$$

Then, according to the Newton-Raphson method, the amount by which the depth h_2 (or stage z_{w2}) must be adjusted is given by

$$\Delta h_2 \ (or\ \Delta z_{w2}) = -\frac{F(h_2)}{dF(h_2)/dh_2} = -\frac{F(h_2)}{1 - Fr_2^2 - N_2 S_{f2}\Delta x/2h_2} \qquad (6.55)$$

Equation (6.55) reduces the number of trial values, usually to 3 or 4, needed to compute the depth of flow or stage correctly.

For a wide channel, N = 3 or 3.33 depending on whether the conveyance is expressed by the Chezy or the Manning formula and R ≈ h. Hence, for a river, which can be considered wide, one can use the equation

$$\Delta h_2 \ (or\ \Delta z_{w2}) = -\frac{F(h_2)}{dF(h_2)/dh_2} = -\frac{F(h_2)}{1 - Fr_2^2 - 3S_{f2}\Delta x/2R_2} \qquad (6.56)$$

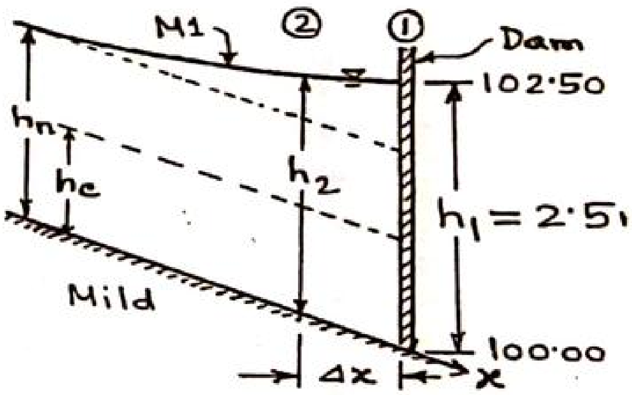
to obtain the next trial value of Δh_2 or Δz_{w2}.

Example 6.6

Considering the channel described in Example 6.5, compute the depths (or stages) at distances of 100 m, 200 m and 300 m upstream from the dam site by the standard step method. The elevation of the channel bottom at the dam site as 100.00 m. Take $h_e = 0$.

Solution

The channel bottom at the dam site is taken as the origin (x = 0) and distances measured downstream are taken to be positive. The computation starts at the dam and proceeds upstream step by step. The computation of depths (or stages) at the three upstream sections are given in Table 6.3.



At section 2 which is situated at a distance of 100 m upstream from the dam site (Δx = -100 m), the trial value of h_2 is taken as 2.50 m. The elevation of the channel bottom at section 2 is (100.00+100×0.0025) m = 100.25 m. The stage at this section is (100.25 + 2.50) m = 102.75 m. The value of H_2 determined by Eq.(6.50) is 102.8179 m and the value of H_2 determined by Eq.(6.51) is 102.6076 m. These two values of H_2 do not agree and hence it is necessary to revise the trial value of h_2 (and z_{w2}). The next trial value of h_2 is obtained as follows:

$F(h_2)$ = 102.8179 - 102.6076 = 0.2103

$B_2 = b + 2sh_2 = 6 + 2\times 2\times 2.5 = 16$ m

$D_2 = A_2/B_2 = 27.50/16 = 1.7188$ m

$Fr_2^2 = \alpha U_2^2/(gD_2) = 1.12\times 1.091^2/(9.81\times 1.7188) = 0.0791$

$$N_2 = \frac{2h_2}{3A_2}(5B_2 - 2R_2\frac{dP}{dh}) = \frac{2\times 2.50}{3\times 27.50}(5\times 16 - 2\times 1.601\times 2\sqrt{5}) = 3.981$$

$$\Delta h_2 = -\frac{F(h_2)}{1 - Fr_2^2 - N_2 S_{f2}\Delta x/2h_2} = -\frac{0.2103}{1 - 0.0791 - 3.981\times 0.000397\times(-100)/(2\times 2.50)}$$

$$= -\frac{0.2103}{0.9528} = -0.221$$

Hence, the next trial value of h_2 = (2.50 - 0.221) m = 2.279 m

Note that if the channel is assumed to be wide, then taking N = 3, we obtain Δh_2 = - 0.2103 /0.9447 = - 0.223 m and the next trial value of h_2 is 2.277 m, which is practically the same as that obtained when N = 3.981.

Table 6.3 Computation of flow profile for Example 6.5 by standard step method

Trapezoidal channel, b = 6 m, s = 2, S_0 = 0.0025, Q = 30 m^3/s, α = 1.12, n = 0.025, h_c = 1.23 m, h_n = 1.55 m, Mild slope, M1 profile

x	h	z_w	A	P	R	$R^{2/3}$	U	$\frac{\alpha U^2}{2g}$	H	S_f	$\bar{S}_f$	Δx	h_f	H
0	2.500	102.500	27.500	17.180	1.601	1.368	1.091	0.0679	102.5679	0.0003972	-	-	-	102.5679
-100	2.500 2.279 2.278	102.750 102.529 102.528	27.500 24.066 24.041	17.180 16.193 16.186	1.601 1.486 1.485	1.368 1.302 1.302	1.091 1.246 1.248	0.0679 0.0887 0.0889	102.8179 102.6180 102.6165	0.0003972 0.0005727 0.0005743	0.0003972 0.0004850 0.0004858	-100 -100 -100	-0.0397 -0.0485 -0.0486	102.6076 102.6164 102.6165
-200	2.278 2.072 2.070	102.778 102.572 102.570	24.041 21.022 20.995	16.186 15.267 15.259	1.485 1.377 1.376	1.302 1.238 1.237	1.248 1.427 1.429	0.0889 0.1163 0.1166	102.8665 102.6885 102.6869	0.0005743 0.0008309 0.0008339	0.0005743 0.0007026 0.0007041	-100 -100 -100	-0.0574 -0.0703 -0.0704	102.6739 102.6868 102.6869
-300	2.070 1.889 1.887	102.820 102.639 102.637	20.995 18.473 18.447	15.259 14.449 14.440	1.376 1.279 1.277	1.237 1.178 1.177	1.429 1.624 1.626	0.1166 0.1505 0.1510	102.9369 102.7897 102.7882	0.0008339 0.0011878 0.0011925	0.0008339 0.0010110 0.0010132	-100 -100 -100	-0.0834 -0.1011 -0.1013	102.7703 102.7880 102.7882

6.3.5 Computation of Flow Profiles by Numerical Methods

As stated earlier, the dynamic equation of gradually varied flow is a non-linear first-order ordinary differential equation and requires one boundary condition (i.e. the depth at the starting section) for its solution. There are a group of numerical methods, known as the *Runge-Kutta methods*, which are particularly suitable for solving this type of equation (Churchouse, 1981; Chapra and Canale, 1988). In using these methods, the channel is divided into short reaches of known space interval Δx. Starting from the known depth at one end of the channel, the depth at the end of Δx is systematically calculated till the other end of the channel is reached. The Runge-Kutta methods of various orders exist. Two of these methods, namely, the Euler method and the classical fourth-order Runge-Kutta method are considered here.

The dynamic equation of gradually varied flow can be expressed as

$$\frac{dh}{dx} = f(x,h) \tag{6.57}$$

where

$$f(x,h) = \frac{S_0 - S_f}{1 - Fr^2} = \frac{S_0 - S_f}{1 - \dfrac{\alpha Q^2 B}{gA^3}} = S_0 \frac{1-(h_n/h)^N}{1-(h_c/h)^M} \tag{6.58}$$

and suppose that the water depth h_j at point x_j is known, i.e.

$$h_j(x_j) = h_j \tag{6.59}$$

The Euler Method or the First-Order Runge-Kutta Method

In this method, the water depth h_{j+1} at the end of a space step Δx is obtained in a single step, i.e.

$$h_{j+1} = h_j + \Delta x\left(\frac{dh}{dx}\right)_j = h_j + \Delta x\, f(x_j, h_j) \tag{6.60}$$

This method is of first-order accuracy, i.e. accuracy ≈ 0(Δx). It is very slow and to obtain reasonable accuracy, we need to take a smaller value of Δx.

The Fourth-Order Runge-Kutta Method

In this method, the depth at the end of a space interval Δx is obtained using the equation

$$h_{j+1} = h_j + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4) \tag{6.61}$$

where

$$K_1 = \Delta x\, f(x_j, h_j)$$
$$K_2 = \Delta x\, f(x_j + \frac{1}{2}\Delta x, h_j + \frac{1}{2}K_1)$$
$$K_3 = \Delta x\, f(x_j + \frac{1}{2}\Delta x, h_j + \frac{1}{2}K_2)$$
$$K_4 = \Delta x\, f(x_j + \Delta x, h_j + K_3) \tag{6.62}$$

In this method, for a known depth h_j, the coefficients K_1,K_2, K_3 and K_4 are determined by repeated calculations and then by substitution in Eq.(6.61), h_{j+1} is found. This method is of fourth-order accuracy, i.e. accuracy ≈ 0(Δx^4).

Example 6.7

Determine the depth of flow 100 m upstream of the dam of Example 6.5 using the Euler and the fourth-order Runge-Kutta methods.

Solution Trapezoidal channel, b = 6 m, s = 2, $S_0 = 0.0025$, Q = 30 m^3/s, $\alpha = 1.12$, n = 0.025

$$n^2Q^2 = (0.025 \times 30)^2 = 0.5625$$

Euler method $h_1 = 2.50$ m, $A_1 = 27.50$ m^2, $P_1 = 17.18$ m, $R_1 = 1.601$ m, $R_1^{2/3} = 1.368$, $S_n = 0.000397$, $B_1 = 16$ m and $\Delta x = -100$ m

$$\left(\frac{dh}{dx}\right)_1 = \frac{S_0 - S_{f1}}{1 - \frac{\alpha Q^2 B_1}{g A_1^3}} = \frac{0.0025 - 0.000379}{1 - \frac{1.12 \times 30^2 \times 16}{9.81 \times 27.50^3}} = 2.283 \times 10^{-3}$$

$$h_2 = h_1 + \Delta x \left(\frac{dh}{dx}\right)_1 = 2.50 + (-100) \times 2.283 \times 10^{-3} = 2.2717\, m$$

Fourth-order Runge-Kutta method

From the Euler method, $(dh/dx)_1 = f(x_1,h_1) = 2.283 \times 10^{-3}$

$$\therefore K_1 = \Delta x \left(\frac{dh}{dx}\right)_1 = \Delta x\, f(x_1,h_1) = -100 \times 2.283 \times 10^{-3} = -0.2283$$

$$\therefore h_1 + \frac{1}{2}K_1 = 2.5 - 0.5 \times 0.2283 = 2.3858$$

Then, A = 25.699 m^2, P = 16.67 m, R = 1.542 m, $R^{2/3} = 1.3335$, B = 15.543 m, $S_f = 0.000478$ and $f(x_1+0.5\Delta x, h_1+K_1/2) = 2.2318 \times 10^{-3}$.

$$\therefore K_2 = \Delta x\, f(x_1 + 0.5\Delta x, h_1 + K_1/2) = -100 \times 2.2318 \times 10^{-3} = -0.22318$$

$$\therefore h_1 + \frac{1}{2}K_2 = 2.5 - 0.5 \times 0.22318 = 2.3884$$

Then, A = 23.739 m^2, P = 16.681 m, R = 1.543 m, $R^{2/3} = 1.335$, B = 15.553 m, $S_f = 0.000746$ and $f(x_1+0.5\Delta x, h_1+K_2/2) = 2.2331 \times 10^{-3}$.

$$\therefore K_3 = \Delta x\, f(x_1 + 0.5\Delta x, h_1 + K_2/2) = -100 \times 2.2331 \times 10^{-3} = -0.22331$$

$$\therefore h_1 + K_3 = 2.5 - 0.22331 = 2.2767$$

Then, A = 24.027 m^2, P = 16.182 m, R = 1.485, $R^{2/3} = 1.302$, B = 15.107 m, $S_f = 0.000575$ and $f(x_1+0.5\Delta x, h_1+K_3) = 2.1673 \times 10^{-3}$.

$$\therefore K_4 = \Delta x\, f(x_1 + 0.5\Delta x, h_1 + K_3) = -100 \times 2.1673 \times 10^{-3} = -0.21673$$

$$\therefore h_2 = h_1 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

$$= 2.50 + \frac{1}{6}(-0.22833 - 2 \times 0.22318 - 2 \times 0.22331 - 0.21673) = 2.2770\, m$$

PROBLEMS AND EXERCISES

6.1 Define (i) backwater curve, and (ii) drawdown curve.

6.2(a) Explain why the frictional losses must be included in the analysis of gradually varied flow.

(b) State the assumption regarding the frictional losses in gradually varied flow. By virtue of this assumption, how do you compute the friction slopes and the friction losses in gradually varied flow?

6.3 Show that the gradually varied flow equation for flow in a rectangular channel of variable width b may be expressed as

$$\frac{dh}{dx} = \frac{S_0 - S_f + (\alpha Q^2 h / gA^3)(db/dx)}{1 - \alpha Q^2 b / gA^3}$$

6.4 Show that the specific energy of the M1, S1, S2 and C1 profiles increases and of the M2, M3, S3, C3, H2, H3, A2 and A3 profiles decreases in the downstream direction.

6.5 Describe the different types of bottom slopes of open channels.

6.6(a) Explain the behavior of flow profiles when (i) $h \to h_n$, (ii) $h \to h_c$, (iii) $h \to 0$, and (iv) $h \to \infty$.

(b) State why a flow profile cannot show the behavior $h \to 0$ at its downstream end.

6.7(a) Show that the C1 and C3 profiles are horizontal or approximately horizontal.

(b) Explain why H1 and A1 profiles are not practically possible.

(c) Write the names of 2 flow profiles which are tangential to the NDL at their upstream end, and write the names of 2 flow profiles which are tangential to the NDL at their downstream end.

6.8(a) State the rule regarding the direction of computation of flow profiles.

(b) What data or information are generally needed for computing a flow profile?

6.9(a) Derive the expression for the length of a flow profile between two sections in a wide channel by the Bresse method (Eq.6.30).

(b) Derive the expression for the length of a flow profile between two sections in a horizontal channel (Eq. 6.37).

(c) Derive Eq.(6.55) for the improved trial value of h_2 in the computation of flow profile by the standard step method.

6.10 Sketch the possible flow profiles produced on the upstream and downstream of a sluice gate in a (i) mild, (ii) critical, (iii) steep, (iv) horizontal and (v) adverse slope channels.

6.11(a) Draw the possible flow profiles when there is a free overfall at the end of a (i) mild, (ii) critical, (iii) steep, (iv) horizontal and (v) adverse slope channels.

(b) There is a free overfall at the end of mild slope channel. Draw all the possible flow profiles for different water levels downstream of the channel.

(c) Same as Prob. 6.11(b), but now the channel is steep.

6.12 Sketch the possible flow profiles in the following combination of slopes:

a) mild-horizontal
b) horizontal-mild
c) steep-horizontal
d) critical-horizontal
e) horizontal-adverse
f) mild-critical
g) adverse-mild
h) critical-steep
i) mild-steeper mild
j) horizontal-critical
k) steep-critical
l) adverse-steep

6.13 Sketch the possible flow profiles in the following serial arrangement of channels. The flow is from left to right.

a) mild-milder-steep
b) critical-steep-mild
c) steep-mild-milder

d) horizontal-mild-critical
e) critical-adverse-horizontal
f) horizontal-adverse-steep-free overfall
g) mild-adverse-horizontal-free overfall
h) mild-critical-steep (there is a sluice gate on the critical slope channel)

6.14(a) Determine the flow profile developed as a result of an increase in surface roughness in a (i) mild slope, and (ii) steep slope channel.

(b) Determine the flow profile developed as a result of a decrease in surface roughness in a (i) mild slope, and (ii) steep slope channel.

6.15 A rectangular channel with b = 6.0 m and n = 0.020 carries a discharge of 24 m^3/s. Identify the flow profiles produced in the channel for the following changes in the bottom slope:

i) $S_0 = 0.0040$ to $S_0 = 0.0090$
ii) $S_0 = 0.0030$ to $S_0 = 0.0050$
iii) $S_0 = 0.0085$ to $S_0 = 0.0000$
iv) $S_0 = 0.0095$ to $S_0 = 0.0075$
v) $S_0 = 0.0000$ to $S_0 = 0.0045$

6.16 A rectangular channel 6 m wide and having n = 0.025 has four reaches arranged serially. The bottom slopes of the three reaches are 0.0016, 0.0150, 0.0096 and 0.0064, respectively. For a discharge of 20 m^3/s through this channel, sketch the resulting flow profiles.

6.17 A wide rectangular channel with $C = 45\ m^{1/2}/s$ and $S_0 = 0.0001$ carries a discharge of 1.8 m^2/s. A weir causes the water level to be raised by 0.50 m above the normal depth. Compute the length of the resulting flow profile between the weir site and the location where the depth is 2.80 m by the Bresse method.

6.18 A vertical sluice gate having $C_c = 0.61$ and gate opening = 0.60 m discharges 27 m^3/s into a horizontal rectangular channel 6 m wide. Compute the length of the flow profile between the vena contracta and the location where the depth is 0.50 m. Take n = 0.013.

6.19 A rectangular channel with b = 6 m, n = 0.025 and $S_0 = 0.0025$ carries a discharge of 40 m^3/s. At a section A of this channel the depth of flow is 2 m. (a) How far upstream or downstream from this section will the depth be 2.25 m? Use the direct step method. (b) What will be the depth at a distance of 50 m upstream of section A? Assume that the elevation of the channel bed at section A is 100.00 m. Use the standard step method. (c) Determine the depth at a distance of 50 m upstream of section A by (i) the Euler method, and (ii) the 4th-order Runge-Kutta method.

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HYDRAULIC JUMP

7.1 INTRODUCTION

In open channels when a supercritical flow is made to change abruptly to subcritical flow, the result is usually an abrupt rise of the water surface. This feature is known as the hydraulic jump (Fig. 7.1). A hydraulic jump may occur at the foot of a spillway or behind a sluice gate in a mild or horizontal or adverse slope channel, or when a steep slope channel is followed by a mild or horizontal or adverse slope channel.

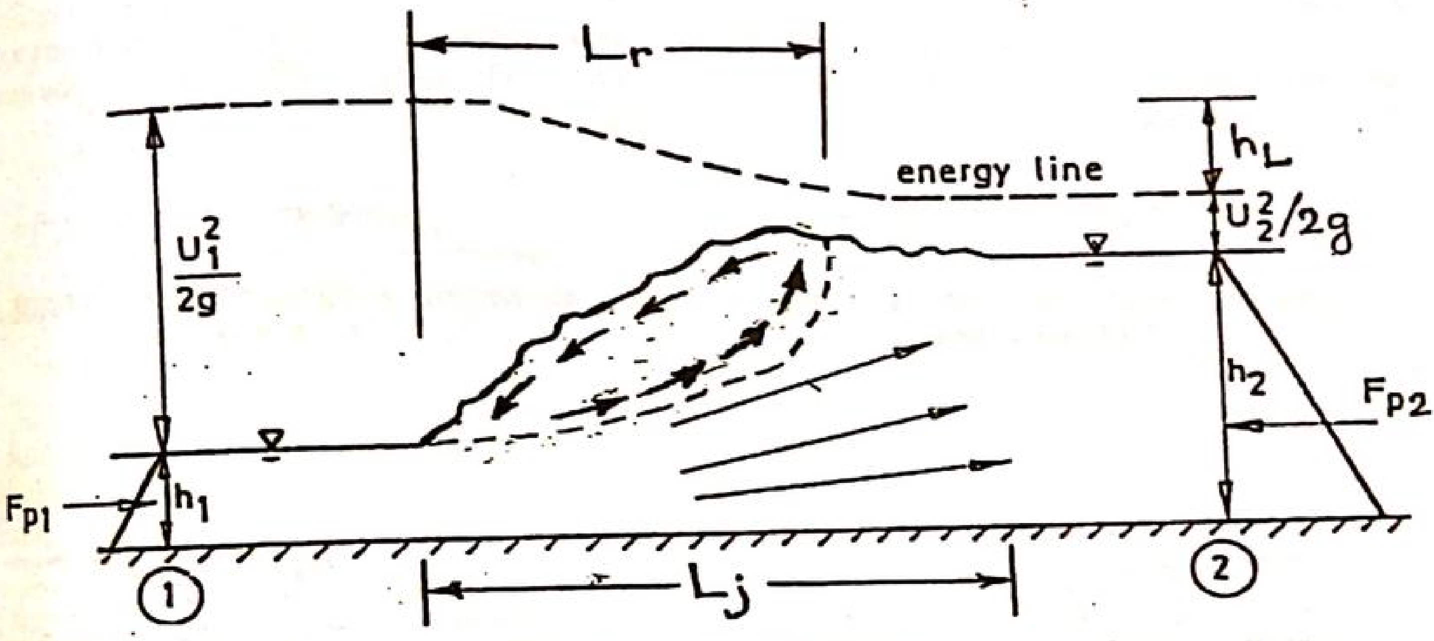


Fig. 7.1 Hydraulic jump

The depth of supercritical flow before the jump (h_1) is known as the *initial depth* and the depth of subcritical flow after the jump (h_2) is known as the *sequent or conjugate depth*. The strength of a hydraulic jump is determined by the upstream Froude number Fr_1. The principal parameter affecting the performance of the jump is also the upstream Froude number Fr_1.

Practical applications of the hydraulic jump are (i) the dissipation of kinetic energy in high-velocity flows over weirs, spillways, gates and other hydraulic structures to prevent scouring downstream, (ii) the increase of depth of water in channels for irrigation and water distribution purposes, (iii) the increase of the discharge of a sluice by repelling the tailwater so that it works under free-flow condition, (iv) the reduction of uplift pressure under a structure by increasing weight on its apron, (v) the mixing of chemicals for water purification or wastewater treatment, and (vi) the aeration of flows for city water supplies and the removal of air pockets from water supply lines.

7.2 JUMPS IN HORIZONTAL RECTANGULAR CHANNELS

Types of Jumps

The hydraulic jumps in horizontal rectangular channels have been most extensively studied and are known as the *classical jumps*. The United States Bureau of Reclamation (USBR) classified the hydraulic jumps in horizontal rectangular channels into the following five types (Fig.7.2) according to the Froude number Fr_1 of the incoming flow:

1. For $1 < Fr_1 < 1.7$, the water surface shows undulations and the change from initial to sequent depth is small and gradual. This jump is called an *undular jump*.

2. For $1.7 < Fr_1 < 2.5$, a series of small rollers appear on the jump surface, but the downstream water surface remains smooth. This type of jump is known as a *weak jump*.

3. For $2.5 < Fr_1 < 4.5$, the incoming jet oscillates between the bed and the bottom of the surface roller. Each oscillation produces a large surface wave of irregular period which may persist for a considerable distance downstream causing unlimited damage to earth banks and riprap. This jump is known as an *oscillating jump*.

4. For $4.5 < Fr_1 < 9.0$, a steady jump with appreciable energy dissipation and fairly smooth water surface downstream is formed. The action and position of the jump is least sensitive to the tailwater fluctuation. This type of jump is known as a *steady jump*.

5. For $Fr_1 > 9.0$, the jump surface and the water surface downstream become very rough and the high-velocity jet generates waves downstream. The jump is effective since the energy dissipation is high. This jump is known as a *strong jump*.

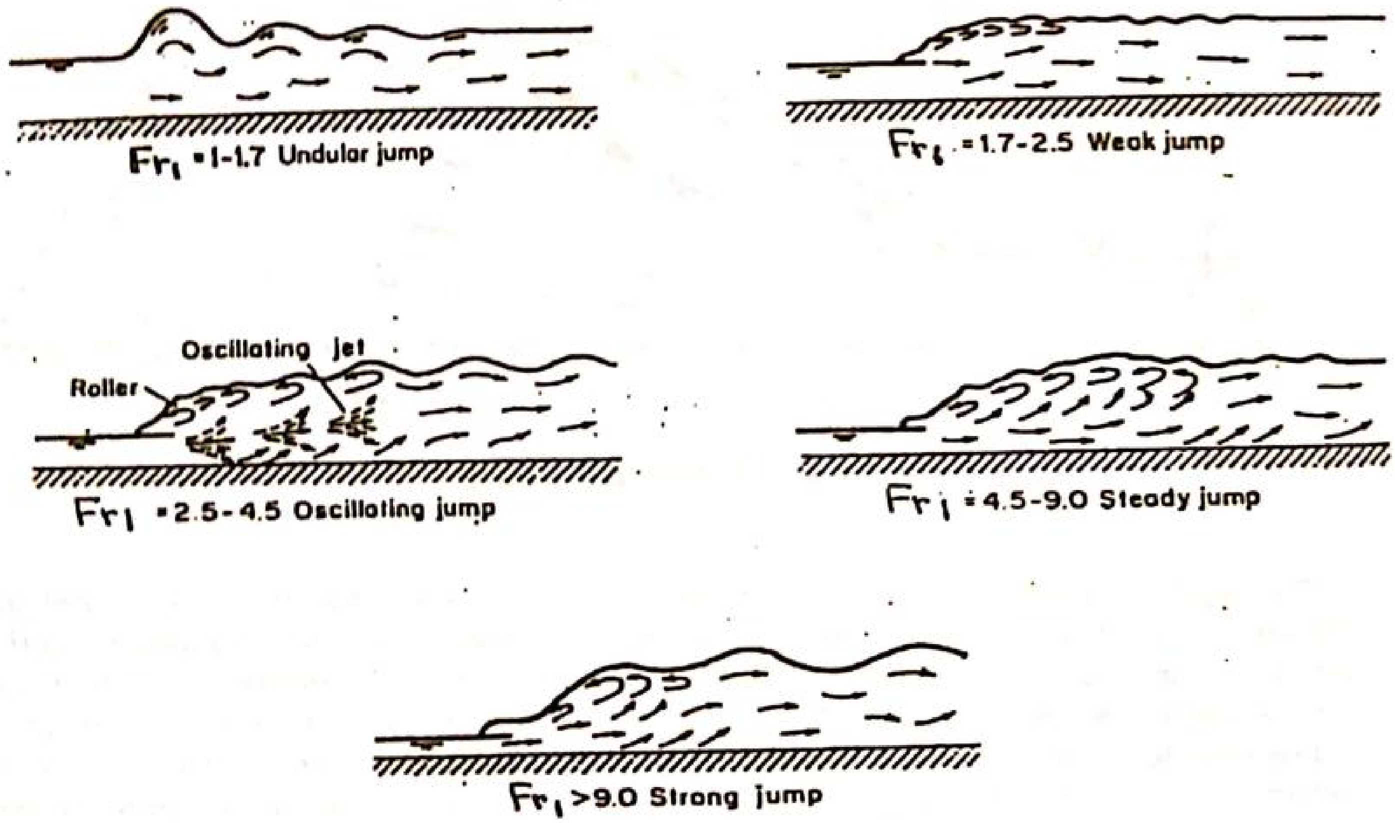


Fig. 7.2 Hydraulic jumps in horizontal rectangular channels

Sequent Depth

Since a hydraulic jump is accompanied by considerable energy loss, the energy principle cannot be used initially for its analysis. However, as the jump takes place in a short distance, the external friction force F_f can generally be neglected and the computation of hydraulic jumps always begins with the momentum equation for steady one-dimensional flow.

$$\rho Q(\beta_2 U_2 - \beta_1 U_1) = F_{p1} - F_{p2} + W \sin\theta - F_f \tag{7.1}$$

Consider a hydraulic jump occurring in a horizontal ($\theta = 0$) rectangular channel (Fig. 7.1). It is assumed that the velocity distribution is uniform and the pressure distribution is hydrostatic at the two end sections of the jump. Hence, $\beta_1 = \beta_2 = 1$ and the hydrostatic forces F_{P1} and F_{P2} may be expressed by

$$F_{p1} = \gamma \bar{z}_1 A_1 \tag{7.2}$$

and

$$F_{p2} = \gamma \bar{z}_2 A_2 \tag{7.3}$$

where $\bar{z}_1$ and $\bar{z}_2$ are the vertical distances of the centroids of the respective water areas A_1 and A_2 from the free surface. Since $U_1 = Q/A_1$ and $U_2 = Q/A_2$, the momentum equation, Eq.(7.1), may be expressed as

$$\frac{Q^2}{gA_1} + \bar{z}_1 A_1 = \frac{Q^2}{gA_2} + \bar{z}_2 A_2 \tag{7.4}$$

or

$$F_1 = F_2 \tag{7.5}$$

where

$$F = \frac{Q^2}{gA} + \bar{z}A \tag{7.6}$$

is known as the *specific force* and represents the force per unit weight of water. Equation (7.5) indicates that the specific forces before and after a hydraulic jump in a horizontal channel are equal.

Since for a rectangular channel $A_1 = bh_1$, $A_2 = bh_2$, $\bar{z}_1 = h_1/2$ and $\bar{z}_2 = h_2/2$, Eq.(7.4) gives

$$\frac{Q^2}{gb^2} = \frac{h_1 h_2}{2}(h_1 + h_2) \tag{7.7}$$

Since $Q = A_1U_1 = bh_1U_1$, Eq. (7.7) may be expressed as

$$\frac{U_1^2}{gh_1} = Fr_1^2 = \frac{1}{2}\frac{h_2}{h_1}\left(\frac{h_2}{h_1} + 1\right) \tag{7.8}$$

and since $Q = A_2U_2 = bh_2U_2$, Eq. (7.7) may also be expressed as

$$\frac{U_2^2}{gh_2} = Fr_2^2 = \frac{1}{2}\frac{h_1}{h_2}\left(\frac{h_1}{h_2} + 1\right) \tag{7.9}$$

where Fr_1 $(= U_1/\sqrt{gh_1})$ and Fr_2 $(= U_2/\sqrt{gh_2})$ are the Froude numbers of the flow before and after the jump. Equations (7.8) and (7.9) may be solved to obtain

$$\frac{h_2}{h_1} = \frac{1}{2}(\sqrt{1 + 8Fr_1^2} - 1) \tag{7.10}$$

and

$$\frac{h_1}{h_2} = \frac{1}{2}(\sqrt{1 + 8Fr_2^2} - 1) \tag{7.11}$$

respectively.

Equations (7.10) and (7.11) each contains three independent variables and two of them must be known before the third may be computed. Normally, the upstream conditions, i.e. h_1 and Fr_1, are known and the downstream depth h_2 can be determined using Eq. (7.10). It must, however, be realized that the downstream depth h_2 is the result of a control acting further downstream. If the downstream control produces the required depth h_2, then a jump will form.

The depth produced by a downstream control is called the *tailwater depth* h_t. If the tailwater depth is increased, the jump moves upstream and if the tailwater depth is decreased, the jump moves downstream.

Length of Jump

The length of a hydraulic jump L_j (Fig. 7.1) is the horizontal distance from the front face or toe of the jump to a point immediately downstream from the roller. It is an important design parameter, but it cannot be determined theoretically. While the beginning or toe of the jump is clearly defined, the downstream end of the jump cannot be located precisely. For this reason, the experimental data on the length of a jump show some scatter. For classical jumps occurring in horizontal rectangular channels, the length of the jump is preferably estimated from the Bradley and Paterka (1957) curve which is a plot of Fr_1 vs. L_j/h_2 (Fig.7.3). This curve has a fairly horizontal portion where $L_j/h_2 \approx 6.0$ to 6.1 in the range of the Froude numbers ($Fr_1 \approx 5$ to 13) yielding the best performance. Silvester (1964) demonstrated that for free hydraulic jumps in horizontal rectangular channels

$$\frac{L_j}{h_1} = 9.75(Fr_1 - 1)^{1.01} \qquad (7.12)$$

This functional relationship is probably the best because the resulting curve can best be defined by the experimental data. However, it was apparently not widely used because of the convenience of Fig. 7.3.

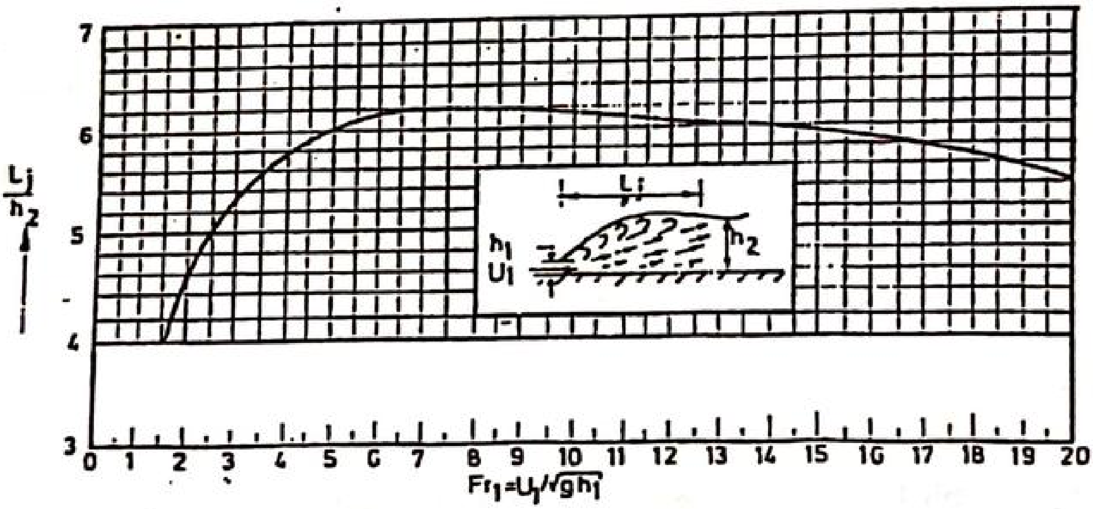


Fig.7.3 Length of hydraulic jumps in horizontal rectangular channels

Basic Characteristics

Energy loss: The energy loss involved in a hydraulic jump is the difference between the total energies immediately before and after the jump. It can be determined by applying the energy equation before and after the jump, i.e.

$$h_L = H_1 - H_2 = (z_{b1} + h_1 + \alpha_1 \frac{U_1^2}{2g}) - (z_{b2} + h_2 + \alpha_2 \frac{U_2^2}{2g})$$

For a jump on a horizontal channel ($z_{b1} = z_{b2}$), the energy loss h_L is given by (assuming $\alpha_1 = \alpha_2 = 1$)

$$h_L = H_1 - H_2 = (h_1 + \frac{U_1^2}{2g}) - (h_2 + \frac{U_2^2}{2g}) = E_1 - E_2 \qquad (7.13)$$

where E_1 and E_2 are the specific energies before and after the jump. Since $U_1 = Q/A_1$, $U_2 = Q/A_2$, $A_1 = bh_1$ and $A_2 = bh_2$, Eq.(7.13) may be expressed as

$$h_L = (h_1 - h_2) + \frac{Q^2}{2gb^2}\frac{(h_1+h_2)(h_2-h_1)}{h_1^2 h_2^2} \quad (7.14)$$

Then, combining Eqs.(7.7) and (7.14), it can be shown that for a hydraulic jump occurring in a horizontal rectangular channel

$$h_L = \frac{(h_2-h_1)^3}{4h_1h_2} \quad (7.15)$$

The ratio h_L/E_1 (= 1 - E_2/E_1) is known as the *relative loss.*

Efficiency: The ratio of the specific energy after the jump to that before the jump, E_2/E_1, is known as the efficiency of the jump. It can be shown that

$$\frac{E_1}{h_1} = \frac{h_1 + U_1^2/2g}{h_1} = \frac{1}{2}(2 + Fr_1^2) \quad (7.16)$$

and writing $U_2 = U_1h_1/h_2$ and using Eq.(7.10), it can be shown that

$$\frac{E_2}{h_1} = \frac{h_2 + U_2^2/2g}{h_1} = \frac{1}{2}\left[\frac{(1+8Fr_1^2)^{3/2} - 4Fr_1^2 + 1}{8Fr_1^2}\right] \quad (7.17)$$

Now, combining Eqs.(7.16) and (7.17), the efficiency E_2/E_1 is given by

$$\frac{E_2}{E_1} = \frac{(1+8Fr_1^2)^{3/2} - 4Fr_1^2 + 1}{8Fr_1^2(2+Fr_1^2)} \quad (7.18)$$

Height of jump: The height of the jump h_j is the difference between the sequent depth and the initial depth, i.e. $h_j = h_2 - h_1$. The ratio of the height of the jump to the specific energy before the jump, h_j/E_1, is known as the *relative height* of the jump. It can be shown that

$$\frac{h_j}{h_1} = \frac{1}{2}(\sqrt{1+8Fr_1^2} - 3) \quad (7.19)$$

Then, combination of Eqs.(7.16) and (7.19) gives

$$\frac{h_j}{E_1} = \frac{\sqrt{1+8Fr_1^2} - 3}{2+Fr_1^2} \quad (7.20)$$

Example 7.1

Water flows in a horizontal rectangular channel 6 m wide at a depth of 0.52 m and a velocity of 15.2 m/s. If a hydraulic jump forms in this channel, determine (i) the type of jump, (ii) the downstream depth needed to form the jump, (iii) the downstream Froude number, (iv) the efficiency of the jump, (v) the relative height of the jump, (vi) the length of the jump, (vii) the relative energy loss or energy dissipation in the jump, and (viii) the horse-power dissipation in the jump.

Solution Horizontal rectangular channel, b = 6 m, h_1 = 0.52 m, U_1 = 15.2 m/s

$$Q = A_1U_1 = bh_1U_1 = 6\times 0.52\times 15.2 = 47.42 \text{ m}^3/\text{s}$$

(i) $$Fr_1 = \frac{U_1}{\sqrt{gh_1}} = \frac{15.2}{\sqrt{9.81\times 0.52}} = 6.73$$

Hence, the jump is a steady jump.

(ii) $$\frac{h_2}{h_1} = \frac{1}{2}(\sqrt{1+8Fr_1^2} - 1) = \frac{1}{2}(\sqrt{1+8\times 6.73^2} - 1) = 9.03$$

$$\therefore h_2 = 9.03 \times 0.52 = 4.70\ m$$

(iii) $$U_2 = \frac{Q}{bh_2} = \frac{47.42}{6 \times 4.70} = 1.68\ m/s$$

$$\therefore Fr_2 = \frac{U_2}{\sqrt{gh_2}} = \frac{1.68}{\sqrt{9.81 \times 4.70}} = 0.25 < 1$$

As expected, the flow downstream of the jump is subcritical.

(iv) $$\textit{Efficiency},\ \frac{E_2}{E_1} = \frac{(1+8Fr_1^2)^{3/2} - 4Fr_1^2 + 1}{8Fr_1^2(2+Fr_1^2)}$$

$$= \frac{(1+8\times 6.73^2)^{3/2} - 4\times 6.73^2 + 1}{8\times 6.73^2 \times (2+6.73^2)} = 0.3937 = 39.37\%$$

or, alternatively

$$E_1 = h_1 + \frac{U_1^2}{2g} = 0.52 + \frac{15.2^2}{2\times 9.81} = 12.30\,m$$

$$E_2 = h_2 + \frac{U_2^2}{2g} = 4.70 + \frac{1.68^2}{2\times 9.81} = 4.84\,m$$

$$\therefore \textit{Efficiency} = \frac{E_2}{E_1} = \frac{4.84}{12.30} = 0.3937 = 39.37\%$$

(v) $$\text{Relative height of jump},\ \frac{h_j}{E_1} = \frac{\sqrt{1+8Fr_1^2} - 3}{2+Fr_1^2} = \frac{\sqrt{1+8\times 6.73^2} - 3}{2+6.73^2} = 0.3396 = 33.96\%$$

or, alternatively

$$\textit{Height of jump},\ h_j = h_2 - h_1 = 4.70 - 0.52 = 4.18\,m$$

$$\therefore \text{Relative height of jump} = \frac{h_j}{E_1} = \frac{4.14}{12.30} = 0.3396 = 33.96\%$$

(vi) Length of jump (using Fig. 7.3)

$$L_j = 6.1h_2 = 6.1 \times 4.70 = 28.67\ \text{m}$$

or by the Silvester formula (Eq. 7.12)

$$L_j = 9.75h_1(Fr_1 - 1)^{1.01} = 9.75 \times 0.52 \times (6.73-1)^{1.01} = 29.56\ \text{m}$$

(vii) The energy loss or energy dissipation in the jump

$$h_L = \frac{(h_2 - h_1)^3}{4h_1h_2} = \frac{(4.70-0.52)^3}{4\times 4.70 \times 0.52} = 7.47\ m \text{ of water}$$

Hence, the relative loss is given by

$$\frac{h_L}{E_1} = \frac{7.47}{12.30} = 0.6075 = 60.75\%$$

(viii) $$\textit{Power dissipation} = \gamma Q h_L = \rho g Q h_L = 1000 \times 9.81 \times 47.42 \times 7.47 = 3{,}474{,}970\ W$$

We know, 1 horse power = 745.7 W

$$\therefore \textit{Horse power dissipation} = \frac{3{,}474{,}970}{745.7} = 4660.01$$

7.3 JUMPS IN HORIZONTAL NON-RECTANGULAR CHANNELS

Sequent Depth

For hydraulic jumps occurring in horizontal non-rectangular channels, there are no equations similar to Eqs.(7.10) and (7.11) to compute the sequent depth. For these jumps the sequent depth can be determined from a trial-and- error solution of Eq.(7.5) or by applying the numerical methods to Eq. (7.5).

The trial-and-error or numerical solution of Eq. (7.5) requires the determination of $\bar{z}$ of the flow section. The expression for $\bar{z}$ of different channel sections derived from the basic principles are given in Table 7.1.

Table 7.1 Expression for $\bar{z}$ of different channel sections

Section	$\bar{z}$
1. Rectangle	$h/2$
2. Triangle	$h/3$
3. Trapezoid	$\frac{h}{6}\left(\frac{3b+2sh}{b+sh}\right)$
4. Parabola	$2h/5$
5. Circle	$\frac{2(d_0h-h^2)^{3/2}}{3A}-\frac{d_0}{2}+h$

Trial-and-error method

The trial-and-error solution of Eq.(7.5) is illustrated by the following example.

Example 7.2

A horizontal trapezoidal channel with b = 6 m and s = 2 carries a discharge of 120 m³/s. If the upstream depth of flow is 1 m, compute the downstream depth that will create a hydraulic jump.

Solution Trapezoidal channel, b = 6 m, s = 2, Q = 120 m³/s, h_1 = 1 m

At the upstream section

$$A_1 = (b + sh_1)h_1 = (6 + 2\times1)\times 1 = 8\ m^2$$

$$B_1 = b + 2sh_1 = 6 + 2\times2\times1 = 10\ m$$

$$D_1 = A_1/B_1 = 8/10 = 0.8\ m$$

$$U_1 = Q/A_1 = 120/8 = 15\ m/s$$

$$Fr_1 = U_1/\sqrt{gD_1} = 15/\sqrt{9.81\times0.8} = 5.35$$

$$\bar{z}_1 = \frac{h_1}{6}\left(\frac{3b+2sh_1}{b+sh_1}\right) = \frac{1}{6}\left(\frac{3\times6+2\times2\times1}{6+2\times1}\right) = 0.458\ m$$

$$F_1 = \frac{Q^2}{gA_1} + \bar{z}_1A_1 = \frac{120^2}{9.81\times8} + 0.458\times8 = 187.15$$

At the downstream section

$$F_2 = \frac{Q^2}{gA_2} + \bar{z}_2A_2$$

The condition which must be satisfied to cause a hydraulic jump between sections 1 and 2 is

$F_1 = F_2 = 187.15$

The value of h_2 which satisfies the above condition is determined by trial-and-error as follows.

h_2	A_2	$\bar{z}_2$	F_2
3.00	36.00	1.250	85.77
4.00	56.00	1.619	116.88
5.00	80.00	1.979	176.68
6.00	108.00	2.333	265.59
5.10	82.62	2.015	184.23
5.20	85.28	2.050	192.07
5.13	83.41	2.025	186.54
5.14	83.68	2.029	187.33

Hence, the downstream depth required to produce a hydraulic jump, $h_2 = 5.14$ m.

Energy Loss

For computing energy loss involved in hydraulic jumps in horizontal non-rectangular channels, there is no equation similar to Eq.(7.15). In such cases, the general energy equation, Eq.(7.13), is to be solved on a case-by-case basis.

Example 7.3

Compute the relative energy loss that will occur if there is a hydraulic jump in the channel considered in Example 7.2.

Solution From Example 7.2, $Q = 120\ m^3/s$, $h_1 = 1$ m, $U_1 = 15$ m/s, $h_2 = 5.14$ m, $A_2 = 83.68\ m^2$

$\therefore\ U_2 = Q/A_2 = 120/83.68 = 1.434$ m/s

$$E_1 = h_1 + \frac{U_1^2}{2g} = 1 + \frac{15^2}{2\times 9.81} = 12.47\ m$$

$$E_2 = h_2 + \frac{U_2^2}{2g} = 5.14 + \frac{1.434^2}{2\times 9.81} = 5.24\ m$$

$$\therefore \text{Re}lelative\ energy\ loss, \frac{h_L}{E_1} = \frac{E_1 - E_2}{E_1} = \frac{12.47 - 5.24}{12.47} = 0.5798 = 57.98\%$$

7.4 JUMPS IN SLOPING CHANNELS

In the analysis of hydraulic jumps in sloping channels, it is essential to consider the weight of water in the jump. In horizontal channels the effect of this weight is negligible.

Hydraulic jumps in sloping channels may occur in various forms as shown in Fig. 7.4. Let h_t be the tailwater depth (i.e. the depth produced by the downstream control), h_1 be the supercritical depth of flow on the slope which is assumed to be constant, h_2 is the sequent depth corresponding to h_1 when the jump occurs on horizontal channel given by Eq.(7.10) and h_2^* is the sequent depth when the jump occurs on a sloping channel. Then,

i) if $h_t \le h_2$, the jump forms on the horizontal bed and type A jump occurs,

ii) if $h_2^* > h_t > h_2$, the toe of the jump is on the slope and the end is on the horizontal bed and a type B jump forms,

iii) if $h_2^* = h_t > h_2$, the end of the jump coincides with the intersection of the sloping and the horizontal beds and a type C jump occurs, and

iv) if $h_t > h_2^* > h_2$, a type D jump occurs completely on the sloping channel.

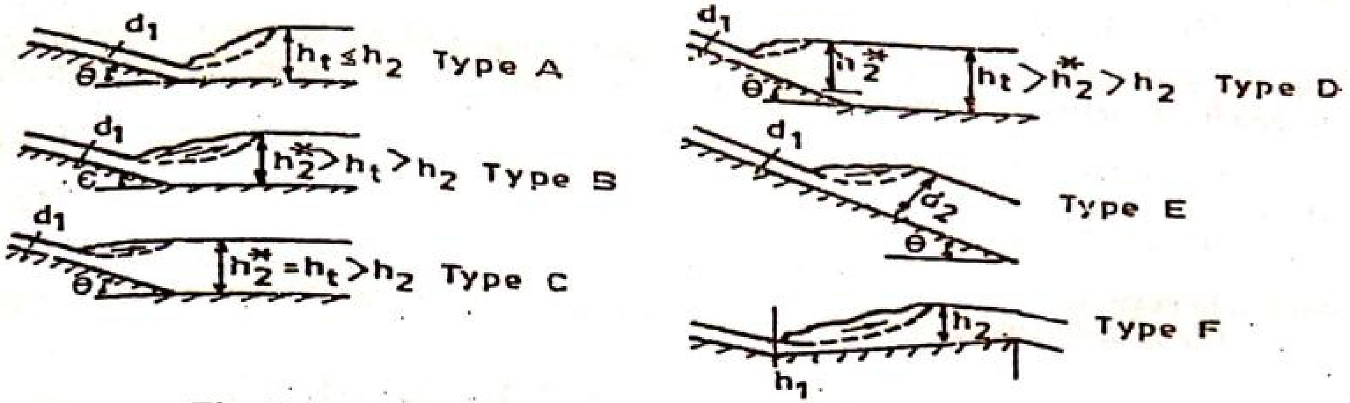


Fig. 7.4 Types of jumps that occur in sloping channels

The type E jumps occur on sloping beds which have no break in slope. The rare type F jump forms in adverse slope channels and is normally found in stilling basins below drop structures.

The forces which act on a type E jump in a rectangular channel are shown in Fig. 7.5.

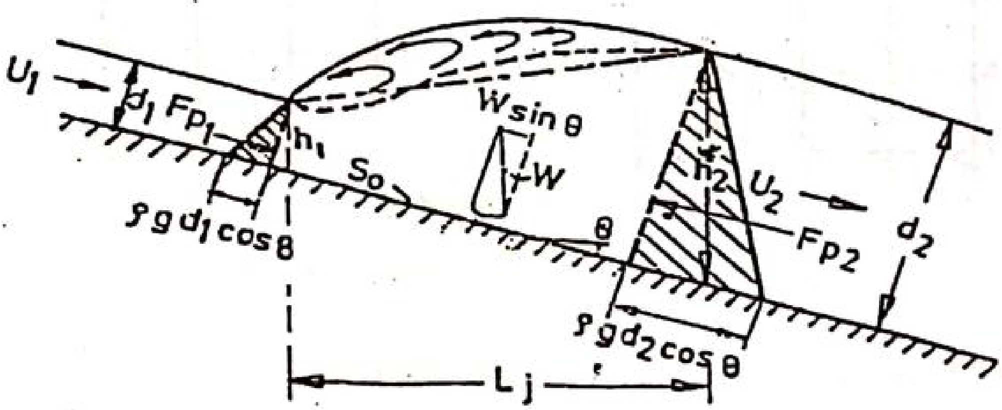


Fig. 7.5 Forces acting on a type E jump

Considering all forces parallel to the channel bottom, the momentum equation may be written as

$$\rho Q(\beta_2 U_2 - \beta_1 U_1) = F_{p1} - F_{p2} + W\sin\theta - F_f \tag{7.21}$$

Now, $Q = bd_1U_1$, $U_2 = U_1d_1/d_2$, $F_{p1} = 0.5\rho gbd_1^2\cos\theta$, $F_{p2} = 0.5\rho gbd_2^2\cos\theta$, $d_1 = h_1\cos\theta$, $d_2 = h_2^*\cos\theta$, the friction force F_f is negligible and β_1 and β_2 may be taken as unity. If the jump profile is a straight line, W can be computed easily. The difference between the straight line and the actual profile and the effect of slope may be corrected by a factor Γ. Thus,

$$W = 0.5\Gamma\rho gbL_j(d_1+d_2) \tag{7.22}$$

Substituting Eq.(7.22) in Eq.(7.21) and simplifying, it can be shown that

$$\frac{h_2^*}{h_1} = \frac{d_2}{d_1} = \frac{1}{2}(\sqrt{1+8G^2} - 1) \tag{7.23}$$

where

$$G = \frac{Fr_1}{\sqrt{\cos\theta - \dfrac{\Gamma L_j \sin\theta}{d_2 - d_1}}} \tag{7.24}$$

(7.25)

and $Fr_1 = U_1/\sqrt{gd_1}$

One may expect Γ and L_j to be functions of Fr_1 and θ. Hence, G, h_2^*/h_1 and d_2/d_1 are functions of Fr_1 and θ. The term G can be computed using the empirical relationship (Rajaratnam, 1967)

(7.26)

$$G^2 = k_1^2\, Fr_1^2 \qquad (7.27)$$

where $k_1 = 10^{0.027\theta}$

and θ is in degrees.

The relative length of jump L_j/h_2^* as a function of Fr_1 and S_o are presented in Fig 7.6.

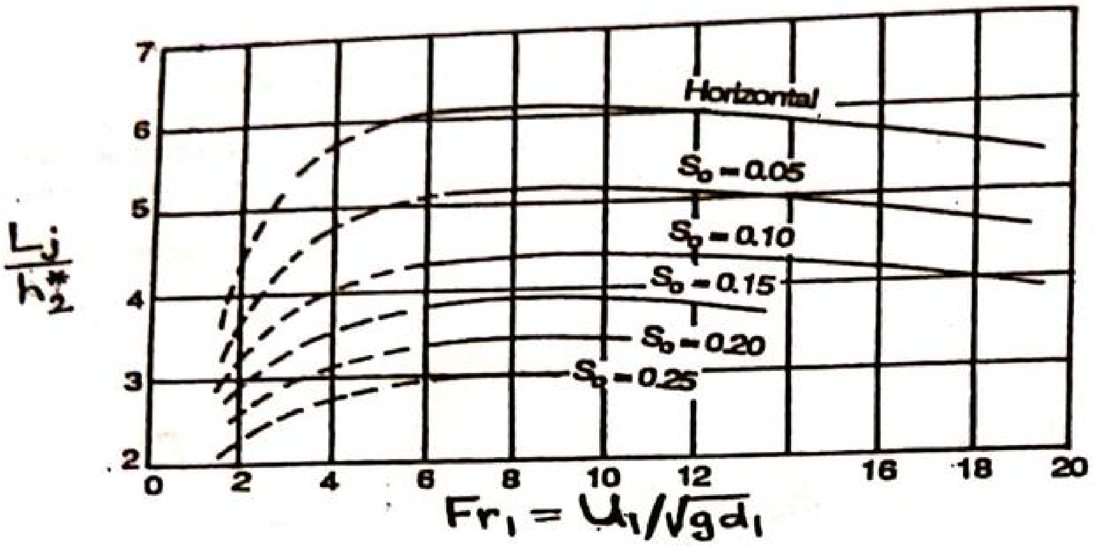


Fig. 7.6 Length of jumps in sloping channels

The energy loss involved in the jump (Fig. 7.5) is given by

$$h_L = H_1 - H_2 = z_{b1} + d_1\cos\theta + \alpha_1\frac{U_1^2}{2g} - z_{b2} - d_2\cos\theta - \alpha_2\frac{U_2^2}{2g}$$

$$= (z_{b1} - z_{b2}) + d_1\cos\theta - d_2\cos\theta + \alpha_1\frac{U_1^2}{2g} - \alpha_2\frac{U_2^2}{2g}$$

$$= L_j\tan\theta + (d_1 - d_2)\cos\theta + \left(\alpha_1\frac{U_1^2}{2g} - \alpha_2\frac{U_2^2}{2g}\right) \qquad (7.28)$$

Example 7.4

A rectangular channel is 1 m wide and inclined at an angle of 3.5^0 with the horizontal. Determine the type of jump when the discharge is 0.15 m^3/s, the initial depth of flow section (d_1) is 0.02 m and the tailwater depth is 0.70 m. Also, compute the energy loss in the jump if the length of the jump is 2 m.

Solution Rectangular channel, b = 1 m, $\theta = 3.5^0$, Q = 0.15 m^3/s, d_1 = 0.02 m, h_t = 0.70 m, L_j = 2 m

$h_1 = d_1/\cos\theta = 0.02/\cos 3.5^0 = 0.02$ m

$A_1 = bd_1 = 1 \times 0.02 = 0.02\ m^2$

$U_1 = Q/A_1 = 0.15/0.02 = 7.5$ m/s

$Fr_1 = U_1/\sqrt{gd_1} = 7.5/\sqrt{9.81 \times 0.02} = 16.93$

$$h_2 = \frac{h_1}{2}\left(\sqrt{1+8Fr_1^2} - 1\right) = \frac{0.02}{2}\left(\sqrt{1+8\times 16.93^2} - 1\right) = 0.467\ m$$

Since $h_t > h_2$, the jump occurs on the sloping channel and we have to compute h_2^*. Now,

$$k_1 = 10^{0.027\times3.5} = 1.243 \qquad G^2 = k_1^2 Fr_1^2 = 442.9$$

$$\therefore h_2^* = \frac{h_1}{2}(\sqrt{1+8G^2}-1) = \frac{0.02}{2}(\sqrt{1+8\times442.9}-1) = 0.585\ m$$

Since $h_t > h_2^* > h_2$, the jump occurs entirely on the sloping channel and a D type jump occurs. Then,

$$d_2 = h_2^*\cos\theta = 0.585\times\cos3.5^0 = 0.584\ \text{m}$$

$$A_2 = bd_2 = 1\times0.584 = 0.584\ \text{m}^2$$

$$U_2 = Q/A_2 = 0.15/0.584 = 0.257\ \text{m/s}$$

The energy loss in the jump is obtained using Eq. (7.28) as

$$h_L = L_j\tan\theta + (d_1-d_2)\cos\theta + \left(\alpha_1\frac{U_1^2}{2g} - \alpha_2\frac{U_2^2}{2g}\right)$$

$$= 2\times\tan3.5^0 + (0.02-0.584)\cos3.5^0 + \frac{1}{2\times9.81}(7.5^2-0.257^2)$$

$$= 0.122 - 0.563 + 2.864 = 2.423\ \text{m of water}$$

7.5 STILLING BASINS

A stilling basin is a paved channel placed at the end of a hydraulic structure to which a hydraulic jump used for energy dissipation is confined. To promote the formation of the jump, to make it stable in one position and to make it as short as possible, the basins are usually provided with special appurtenances which include chute blocks, sills and baffle piers.

Chute Blocks

The chute blocks are used to form a serrated device at the entrance to the basin. Their function is to furrow the incoming jet and lift a portion of it from the floor.

Sill, Dentated or Solid

The sill, dentated or solid, is usually provided at the end of the basin. In large stilling basins, the sill is usually dentated (also known as the Rehbock sill) to aid in the diffusion of the high velocity jet that may reach the end of the basin.

Baffle Piers or Floor Blocks

Baffle piers are placed at intermediate locations in the basin. Their primary function is to dissipate energy mostly by impact. When the approach velocities are low, baffle piers may be very effective. However, when incoming velocities are high, this type of appurtenance is unsuitable because of the possibility of cavitations. This is why the baffle piers are normally used when the incoming velocity is less than about 16 m/s (50 ft/sec).

Because of the widespread use of stilling basins, various types of generalized designs of stilling basins have been developed by the U.S. Bureau of Reclamation, based on model studies, experience and observation of existing stilling basins (Figs. 7.7 and 7.8). Of the many generalized designs of stilling basins, four typical designs are considered below.

USBR Basin II

The USBR Basin II (Fig. 7.7) is recommended for use on large structures, e.g. large high-dam and earth-dam spillways, large canal structures,etc. when $Fr_1 > 4.5$ and the incoming velocity U_1 exceeds about 18 m/s (60 ft/sec). The length of the basin is reduced by about 33 percent with the provision of chute blocks at the upstream end and a dentated sill at the downstream. This design may be safe and conservative for spillways with fall up to 60 m (200 ft) and for flows up to 46 m^3/s per m of basin width.

The elevation of the basin floor is set to utilize full tailwater depth plus an added factor of safety. The dotted lines in Fig. 7.7(b) are guides based on various ratios of the actual tailwater depth to sequent depth. There is a lower limit of this ratio which is determined by the curve labeled "minimum TW depth". The basin should never be designed for less than the sequent depth and a minimum safety factor of 5% should be added to the sequent depth.

The length of the basin is obtained from Fig.7.7(c). The height, width and spacing of the chute blocks is equal to h_1. A space equal to $0.5h_1$ is provided along each basin wall. The height of the dentated sill is equal to $0.2h_2$ and the width and the spacing of the dentates is approximately $0.15h_2$.

USBR Basin III

USBR Basin III (Fig. 7.8a) is similar to USBR Basin II , but it is used when the incoming velocity U_1 is less than 18 m/s (60 ft/sec). The major difference between the designs of Basin II and Basin III is that in the latter lower velocities allow the installation of baffle piers downstream of the chute blocks. The added resistance offered by the piers allows the use of a shorter basin. With the help of the appurtenances the basin length can be reduced about 60%.

USBR Basin IV

USBR Basin IV (Fig. 7.8b) is designed in conjunction with canal and diversion structures for the special purpose of suppressing the waves at their source generated in oscillating jumps when $2.5 < Fr_1 < 4.5$. This is achieved by intensifying the rollers which appear in the upper portion of the jump. The appurtenances used include a few chute blocks at the entrance and a solid sill at the end.

Saint Anthony Falls (SAF) Stilling Basin

The SAF Basin (Fig. 7.8c), developed at the Saint Anthony Falls Hydraulic Laboratory, University of Minnesota, for the U. S. Soil Conservation Service, is usually used in conjunction with small spillways, outlet works and canal structures. It is intended for much the same use as the USBR Basin III, but it is designed for a greater range of upstream Froude numbers, viz. $1.7 < Fr_1 < 17$. The reduction in basin length achieved through the use of appurtenances (chute blocks, baffle piers and a solid end sill) ranges from 70% to 90%. Thus, the SAF basin is shorter and more economical but, in consequence, has a lower factor of safety than the USBR Basin III.

Example 7.5

Proportion a USBR stilling basin II for the overflow spillway of Kaptai Hydro Station with the following data:

Design discharge = 15,870 m^3/s TW level = 17.26 m
Basin width = 227.1 m Elevation of ground = 0.00 m
Velocity at the foot of the spillway = 24.70 m/s

Solution $Q = 15{,}870\ m^3/s$, $h_t = 17.26$ m, $b = 227.1$ m, $U_1 = 24.70$ m/s

$$h_1 = \frac{Q}{bU_1} = \frac{15870}{227.1\times 24.70} = 2.83\ m$$

$$Fr_1 = \frac{U_1}{\sqrt{gh_1}} = \frac{24.70}{\sqrt{9.81\times 2.83}} = 4.69 > 4.50$$

Hence, the jump formed is a steady jump.

$$\frac{h_2}{h_1} = \frac{1}{2}(\sqrt{1+8Fr_1^{\,2}}-1) = \frac{1}{2}(\sqrt{1+8\times 4.69^2}-1) = 6.15$$

$$\therefore h_2 = 6.15\times 2.83 = 17.40\ m \quad and\ h_t = 17.26\,m$$

$$\therefore h_2 > h_t$$

So, when the basin floor is set at 0.00 m, the jump moves downstream and more basin length need to be provided. Hence, the floor must lowered. With 5% safety margin

$$h_t = 1.05h_2 = 1.05\times 17.40 = 18.27\,m$$

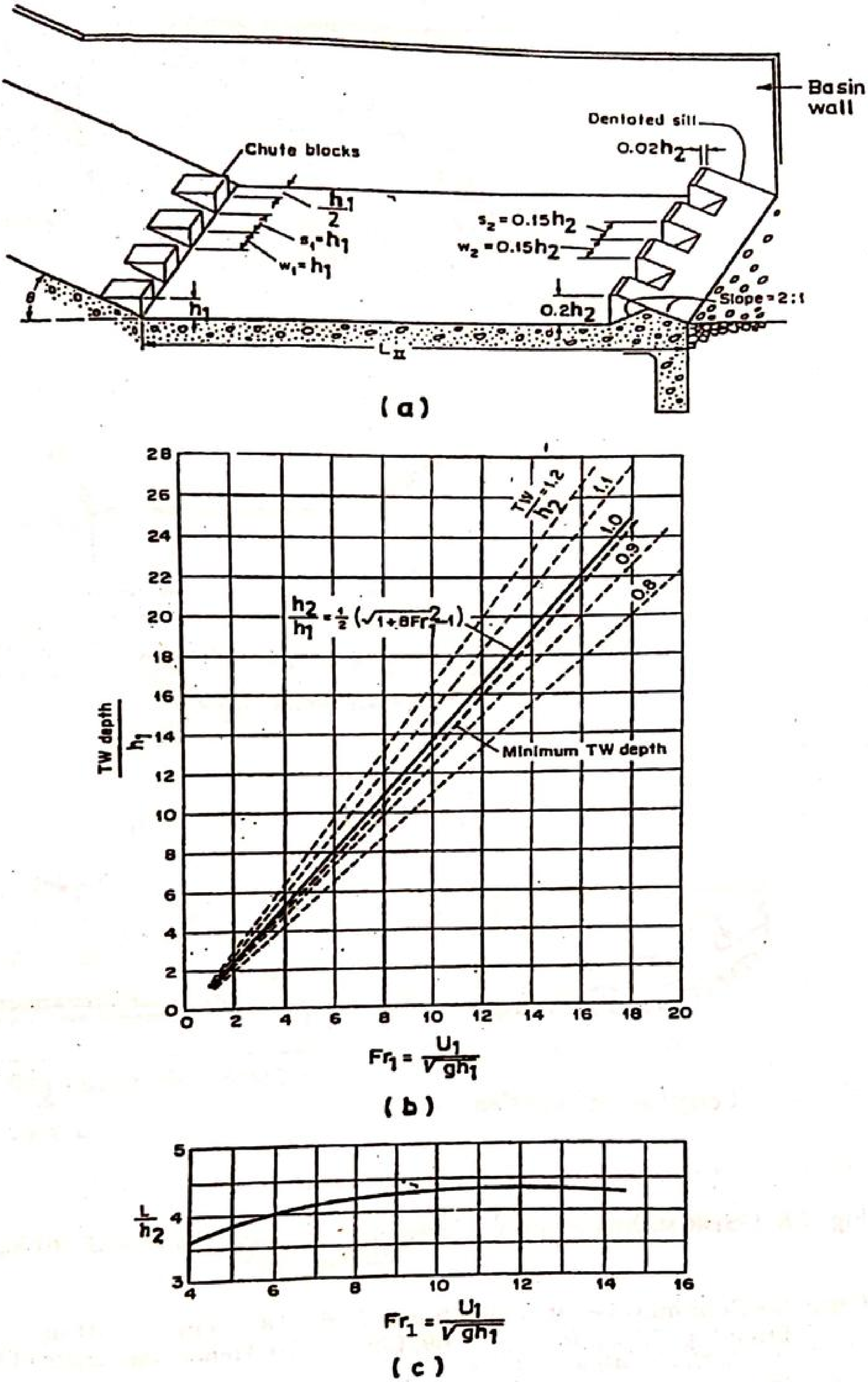
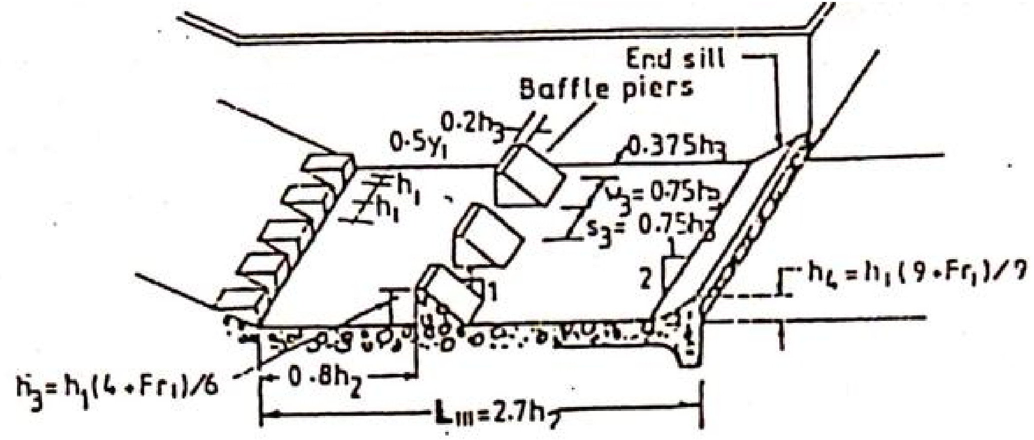
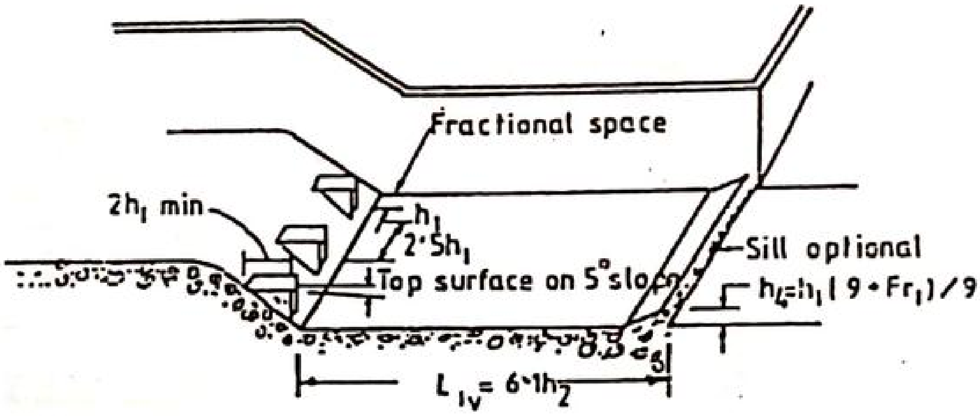


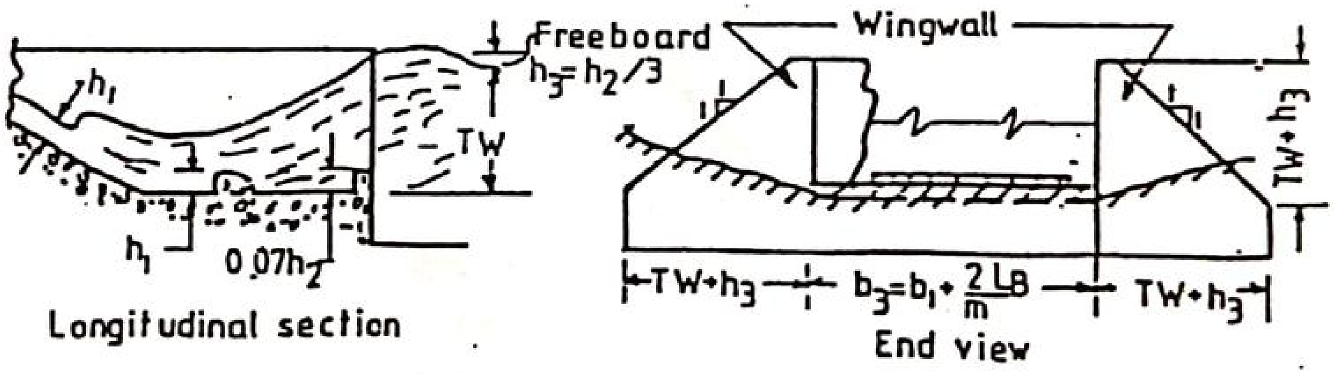
Fig. 7.7 USBR stilling basin II. (a) Recommended proportions, (b) Minimum tailwater depths, and (c) Length of hydraulic jump (Bradley and Paterka, (1957)



(a) USBR basin III ($Fr_1 \geq 4.5$)



(b) USBR basin IV ($2.5 < Fr_1 < 4.5$)



(c) SAF basin ($1.7 < Fr_1 < 17$)

Fig. 7.8 USBR stilling basin III, USBR stilling basin IV and SAF stilling basin(Bradley and Paterka, 1957)

Hence, the floor must be set at elevation (17.26 – 18.27) m = - 1.01 m

From Fig. 7.7(c), for $Fr_1 = 4.69$, $L/h_2 = 3.70$. Hence, the length of the basin $L = 3.70 \times 17.40 = 64.40\ m$

The height, width and spacing of the chute blocks = $h_1 = 2.83$ m

The height of the dentated sill = $0.2h_2 = 0.2 \times 17.40 = 3.48$ m and the width and spacing of the dentates = $0.15h_2 = 0.15 \times 17.40 = 2.61$ m

PROBLEMS AND EXERCISES

7.1 Define (i) hydraulic jump, (ii) initial depth, (iii) sequent depth, (iv) tailwater depth, (v) length of a jump, (vi) efficiency of a jump, and (vii) height of a jump.

7.2 Write five practical applications of hydraulic jumps.

7.3 Classify the hydraulic jumps in horizontal rectangular channels according to USBR.

7.4 Why is it advisable to avoid an oscillating jump in design?

7.5 State why the momentum equation, and not the energy equation, is initially used for the analysis of a hydraulic jump.

7.6 State the qualitative relationships among h_2^*, h_2 and h_t for different forms of jumps in sloping channels.

7.7 What is a stilling basin? Describe the special appurtenances usually provided with a stilling basin. Write the names of the appurtenances provided with USBR Basin II.

7.8 Verify Eqs. (7.10), (7.11), (7.15), (7.18), (7.20), and (7.23).

7.9 Water flows at a velocity of 6.1 m/s and a depth of 1 m in a horizontal rectangular channel 6.1 m wide. Find (i) the type of jump, (ii) the downstream depth necessary to form a jump, (iii) the height of the jump, (iv) the length of the jump, (v) the horsepower dissipation in the jump, and (vi) the efficiency.

7.10 Water flows at a depth of 1 m in a horizontal trapezoidal channel having a base width of 5 m, side slope of 1:1 and Q = 30 m^3/s. If a hydraulic jump occurs in this channel, compute the sequent depth and the energy loss involved in the jump.

7.11 A horizontal triangular channel having s = 2 carries a discharge of 20 m^3/s at a depth of 1 m. Compute the downstream depth that will form a hydraulic jump.

7.12 A horizontal parabolic channel contains a discharge of 10 m^3/s at a depth of 0.50 m. The profile of the channel is given by the equation $y^2 = 4z$. If a hydraulic jump occurs in this channel, compute the sequent depth.

7.13 A rectangular channel 6 m wide and inclined at an angle of 5^0 with the horizontal carries a discharge of 20 m^3/s. Determine the jump type if the upstream depth (normal to the direction of flow) is (i) 0.20 m, (ii) 0.30 m, and (iii) 0.40 m, when the tailwater depth is 3.20 m.

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Chapter 8

CHANNEL CONTROLS AND TRANSITIONS

8.1 INTRODUCTION

A control has been defined as a feature, natural or artificial, which establishes a definite relationship between the discharge and the depth of flow. Sluice gates, free overfalls, weirs and spillways are some familiar examples of controls. Many of these structures are used for flow measurement. In addition, these control structures govern the depth of flow upstream in case of subcritical flow and downstream in supercritical flow.

A transition has been defined as a change either in the direction or slope or cross-section of the channel that produces a change, either temporary or permanent, in the state of flow in the channel. Channel bends, humps, depressions, expansions and contractions are typical examples of channel transitions.

A transition producing a permanent change in the state of flow in effect becomes a control. Therefore, all controls are transitions, but all transitions are not necessarily controls under all conditions.

8.2 SLUICE GATE

The vertical sluice gate is one of the most commonly used types of underflow gates used for flow measurement. In its simplest form, it consists of a vertical gate that can be lifted up and down. It is sometimes used to raise the water level and maintain a constant water level in an irrigation canal. It acts as a control provided the height of the gate opening h_g is less than the critical depth of flow h_c in the channel. When this condition is fulfilled, the flows upstream and downstream of the gate are subcritical and supercritical, respectively, and decreasing h_g increases the upstream depth h_1 and decreases the downstream depth h_2 and vice versa (Fig. 8.1). As the water issues out of the sharp edge of the gate, the streamlines converge rapidly till the flowing water attains a minimum depth at the vena contracta approximately at a distance equal to h_g from the plane of the gate. The gate may operate under free or submerged condition depending on the tailwater depth.

Free Flow

Consider a vertical sluice gate in a horizontal rectangular channel as shown in Fig. 8.1. Flow under a sluice gate is an example of converging flow in which the energy losses between sections 1 and 2 are small and negligible. So, considering free flow and taking the channel bed as datum, we can write

$$h_1 + \frac{U_1^2}{2g} = h_2 + \frac{U_2^2}{2g} \qquad or, \; h_1 + \frac{U_1^2}{2g} = C_c h_g + \frac{U_2^2}{2g} \tag{8.1}$$

where C_c (= h_2/h_g) is the coefficient of contraction. From the continuity equation, we have

$$Q = C_c b h_g U_2 = b h_1 U_1 \tag{8.2}$$

Combining Eqs. (8.1) and (8.2) and simplifying

$$Q = C_d b h_g \sqrt{2gh_1} \tag{8.3}$$

in which the discharge coefficient C_d is given by

$$C_d = \frac{C_c}{\sqrt{1 + C_c h_g / h_1}} \tag{8.4}$$

Thus, the sluice gate is in effect a control because for a given gate opening there is a definite relationship between Q and the upstream depth h_1 as given by Eq.(8.3).

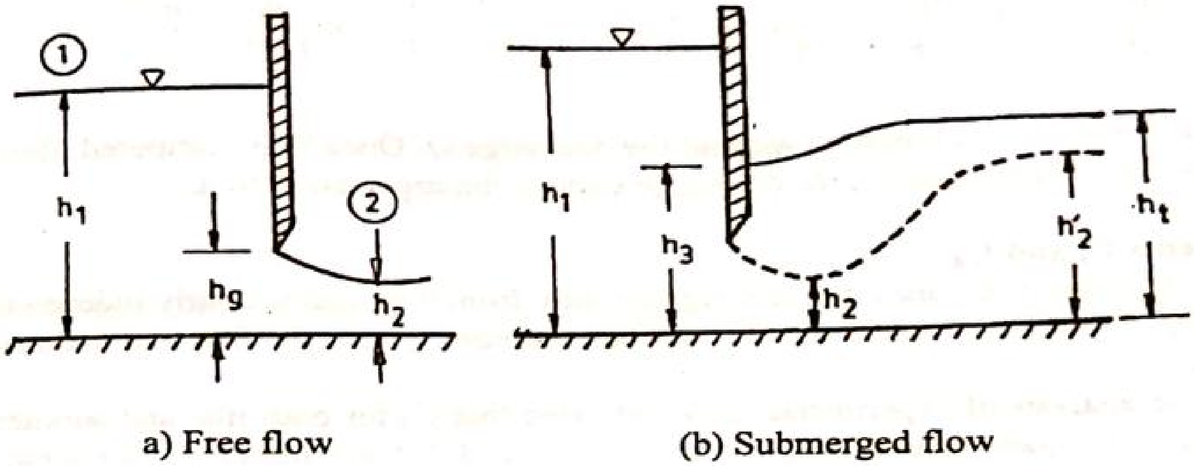


Fig. 8.1 Flow under a sluice gate

Submerged Flow

By operating a control located downstream, the tailwater depth h_t can be increased and a hydraulic jump can be formed and advanced upstream. Till the toe of the jump reaches the vena contracta, the discharge beneath the sluice gate is not affected and the free flow condition exists. Any further increase of h_t causes the jump to be submerged. The effect of submergence is to reduce the discharge in the channel and/or to increase the depth h_1.

Equation (8.3) may also be used for submerged flow. However, for submerged flow

$$C_d = f(h_1/h_g, h_t/h_g) \tag{8.5}$$

Rajaratnam and Subramanya (1967) proposed an alternative method of computing the discharge as

$$Q = C_d b h_g \sqrt{2g\Delta h} \tag{8.6}$$

where

$$\Delta h = h_1 - h_2 = h_1 - C_c h_g \tag{8.7}$$

for free flow, and

$$\Delta h = h_1 - h_3 \tag{8.8}$$

for submerged flow.

In order to compute the discharge under submerged flow condition using Eqs. (8.6) and (8.8), the prediction of h_3 is necessary. Applying the momentum principle and the equation of continuity, it can be shown that in horizontal rectangular channels

$$\frac{h_3}{h_t} = \left[1 + 2Fr_t^2\left(1 - \frac{h_t}{h_g}\right)\right]^{1/2} \tag{8.9}$$

where h_g is the height of the sluice gate opening, h_t is the tailwater depth, h_2 is the subcritical sequent depth of the free jump corresponding to h_g and Fr_t is the Froude number corresponding to h_t. Equation (8.9) can be used to compute h_3. However, computation of h_3 using this equation requires that Q must be known initially. Using the momentum principle, Rajaratnam and Subramanya(1967) gave the equation

$$\frac{h_3}{h_g C_d} = 2\left(1 - \frac{h_g C_d}{h_t}\right) + \sqrt{4\left(1 - \frac{h_g C_d}{h_t}\right)^2 + \left(\frac{h_t}{h_g C_d}\right)^2 - 4\left(\frac{h_1}{h_g C_d} - \frac{h_1}{h_t}\right)} \tag{8.10}$$

which can be used to compute h_3 without the discharge Q. Once h_3 is computed, Eqs.(8.6) and (8.8) may be used to compute the discharge under submerged condition.

Coefficients C_c and C_d

The value of C_c does not vary significantly from 0.61 and is nearly independent of the ratio h_1/h_g. Therefore, a value of 0.61 can be used for C_c both for free and submerged flow conditions.

The analysis of experimental data indicated that C_d for both free and submerged flow conditions is uniquely related to h_g/h_1 as shown in Fig. 8.2. The values of C_d for some values of h_g/h_1 are given in Table 8.1. From Fig. 8.2 and Table 8.1, it may be noted that the variation C_d in the range of h_g/h_1 from 0 to 0.30 is small and we can use a constant value of 0.60 for C_d when $h_g/h_1 \leq 0.40$.

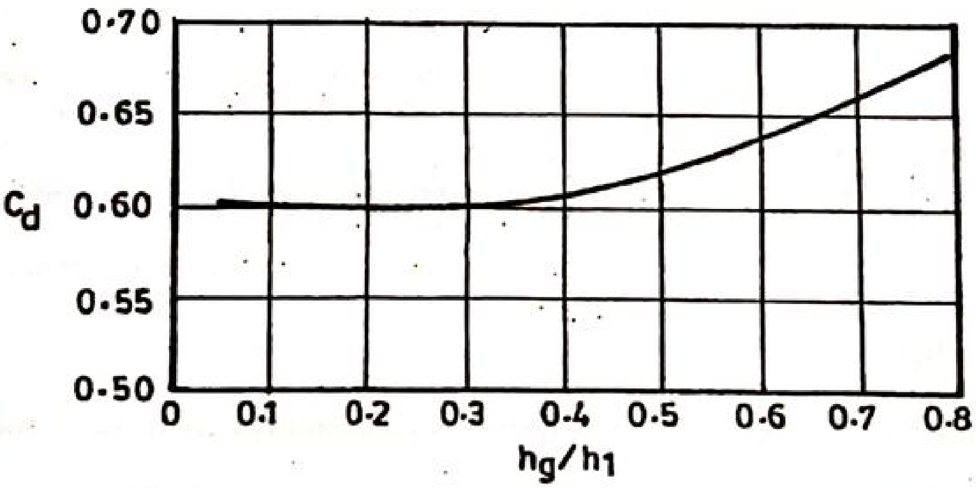


Fig 8.2 Coefficient of discharge for vertical sluice gates

Table 8.1 Values of C_d for some values of h_g/h_1 (Rajaratnam and Subramanya, 1967)

h_g/h_1	0.00	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70
C_d	0.61	0.60	0.60	0.605	0.605	0.607	0.620	0.640	0.660

Example 8.1

Compute the discharge through a vertical sluice gate in a horizontal rectangular channel 6 m wide and having a depth of 1 m at the vena contracta when the upstream head is 4 m, for (i) free flow condition and compare it with the discharge obtained in Problem 2.6, and (ii) submerged condition when the tailwater depth is 3.5 m.

Solution Horizontal rectangular channel, b = 6 m, h_1 = 4 m, h_2 = 1 m, h_t = 3.5 m
Take C_c = 0.61 so that $h_g = h_2/C_c$ = 1/0.61 = 1.64 m (< h_c = 1.86 m)

(i) For free flow condition, using Eq. (8.4), we get

$$C_d = \frac{C_c}{\sqrt{1+C_c h_g/h_1}} = \frac{0.61}{\sqrt{1+0.61\times 1.64/4}} = 0.546$$

Then, using Eq.(8.3)

$$Q = C_d b h_g \sqrt{2gh_1} = 0.546\times 6\times 1.64\times\sqrt{2\times 9.81\times 4} = 47.56\ m^3/s$$

which is exactly the discharge obtained in Problem 2.6 (Q = 47.54 m^3/s).

Again, h_g/h_1 = 1.64/4 = 0.41 and from Fig.8.2, C_d = 0.61. Hence, using Eq.(8.6), we get

$$Q = C_d b h_g\sqrt{2g\Delta h} = C_d b h_g\sqrt{2g(h_1-h_2)}$$
$$= 0.61\times 6\times 1.64\times\sqrt{2\times 9.81\times(4-1)} = 46.05\ m^3/s$$

which is about 3% lower than, but still considered to be very near to, the discharge obtained in Problem 2.6.

(ii) For submerged flow condition, we have $h_g C_d/h_t$ = 1.64×0.61/3.5 = 0.286. Then, using Eq.(8.10), we get

$$\frac{h_3}{h_g C_d} = 2\left(1-\frac{h_g C_d}{h_t}\right)+\sqrt{4\left(1-\frac{h_g C_d}{h_t}\right)^2+\left(\frac{h_t}{h_g C_d}\right)^2-4\left(\frac{h_1}{h_g C_d}-\frac{h_1}{h_t}\right)}$$

$$= 2(1-0.286)+\sqrt{4(1-0.286)^2+(\frac{1}{0.286})^2-4(\frac{4}{1.64\times 0.61}-\frac{4}{3.5})} = 3.341$$

$\therefore$ h_3 = 3.341 × 1.64 × 0.61 = 3.342 m

$$Q = C_d b h_g\sqrt{2g\Delta h} = C_d b h_g\sqrt{2g(h_1-h_3)}$$
$$= 0.61\times 6\times 1.64\times\sqrt{2\times 9.81\times(4-3.342)} = 21.57\ m^3/s$$

8.3 FREE OVERFALL

A free overfall occurs when the bottom of a channel is discontinued causing the flow to separate from the channel bed and forming a nappe as shown in Fig. 8.3. The pressure above and below the nappe is atmospheric and the water surface profile of the nappe is a parabola.

The flow downstream of brink or edge of channel is supercritical. Now, if the flow in the channel is subcritical, then there is a section at which the flow is critical. Theoretically, the critical section or the section of minimum specific energy should occur at the brink. However, the determination of critical depth using Eq.(3.9) is based on the assumption of parallel flow and applicable only approximately to gradually varied flow. The flow in the brink is actually curvilinear having appreciable curvature of the streamlines which causes the pressure distribution at the brink section to depart from hydrostatic. The actual situation is that the brink section is the true section of minimum specific energy, but it is not the critical section as computed by the assumption of parallel flow. The critical flow section occurs a short distance (approximately $3h_c$ to $4h_c$) upstream of the brink and the brink depth is less than the critical depth.

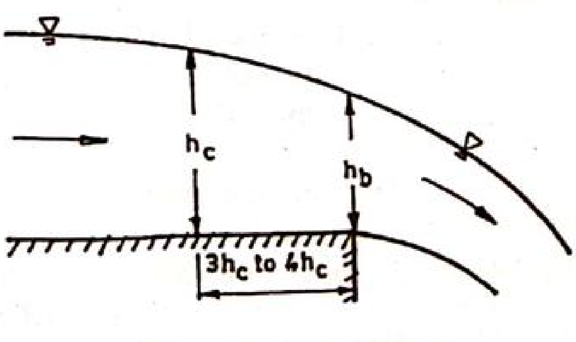


Fig. 8.3 Free overfall at the end of a channel with subcritical flow

Rouse (1936) was probably the first to determine experimentally that the value of h_b/h_c for subcritical flow in a horizontal rectangular channel is 0.715, i.e. $h_c = 1.4h_b$. Since then a large number of experimental and semi-theoretical studies have confirmed that the value of h_b/h_c obtained by Rouse (1936) for horizontal rectangular channels is not significantly different from 0.715. For other slopes, the relative brink depth can be expressed as

$$h_b/h_c = f(S_0/S_c, \text{channel shape})$$

Thus, for a given channel shape, the variation of h_b/h_c can be expressed as a unique function of S_0/S_c. The variation of h_b/h_c with channel slope, experimentally determined by Delleur al.(1956) using data on adverse and mild slopes, is shown in Fig. 8.4.

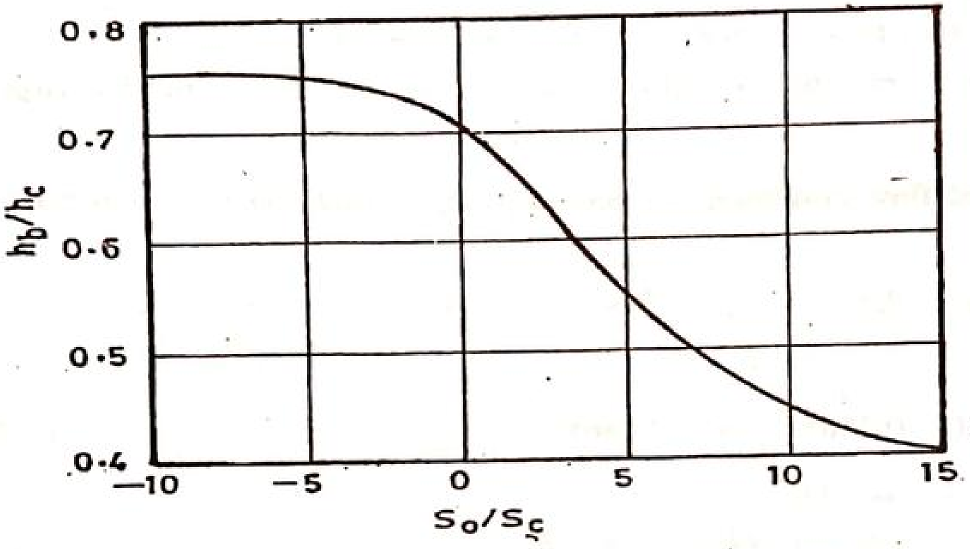


Fig. 8.4 Brink depth in rectangular channels for different slopes

Flow Measurement Using Brink Depth

The unique relationship between h_c and h_b for a given channel shape gives rise to a unique relationship between the brink depth and the discharge for that channel shape. This is of considerable practical interest, because it permits the use of a free overfall for flow measurement. For the purpose of flow measurement, the channel must be level in the lateral direction and preceded by a channel of length not less than $15h_c$. The depth should be measured at the brink section on the centerline of the channel accurately. A channel with horizontal or small slope must be used.

The following discharge formulas are to be used when the channel is horizontal or of small slope:

1. Rectangular channel

$$h_b/h_c = 0.715 \qquad Q = 5.18bh_b^{1.5} \tag{8.11}$$

2. Triangular channel:

$$h_b/h_c = 0.795 \qquad Q = 3.93sh_b^{2.5} \tag{8.12}$$

3. Parabolic channel

$$h_b/h_c = 0.772 \qquad Q = 5.72\,c^{-1/2}\,h_b^{2} \tag{8.13}$$

The values of h_b/h_c for other slopes can be obtained from Fig. 8.4 and the corresponding discharge can be estimated. However, in this case a trial procedure is needed.

For circular and trapezoidal channels, explicit relationship between the brink depth and the discharge can not be determined. However, in this case by determining h_b, h_c can be estimated and the discharge Q can be estimated. The values of h_b/h_c for free overfalls in circular conduits for different bottom slopes are given in Table 8.2. The value of h_b/h_c for a horizontal circular channel is 0.725.

Table 8.2 Brink depth in circular channels for different slopes

S_0/S_c	-4.00	0.00	2.00	4.00	6.00	8.00
h_b/h_c	0.75	0.725	0.61	0.53	0.491	0.487

For a trapezoidal channel, the general functional relationship is

$$h_b/h_c = f(S_0/S_c, sh_c/b) \tag{8.14}$$

Rajaratnam and Muralidhar (1970) performed experiments on trapezoidal outfalls and analysed the data with those of Diskin (1961) in accordance with Eq.(8.14). The relationship developed by them is shown in Fig. 8.5. For a horizontal trapezoidal channel the ratio h_b/h_c varies from 0.715 to 0.795. The determination of discharge corresponding to a measured brink depth involves a trial procedure.

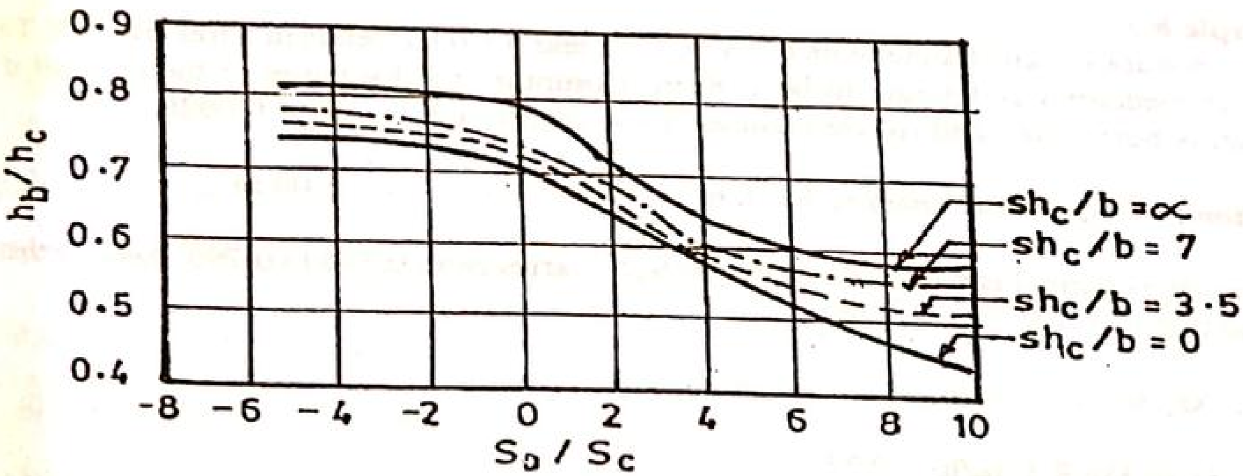


Fig. 8.5 Brink depth in smooth trapezoidal channels for different slopes

Example 8.2

A rectangular channel 6 m wide is made of concrete (n = 0.013) and ends in a free overfall. Compute the discharge in the channel if the brink depth is 0.75 m, when (i) the channel is horizontal, and (ii) the channel is laid on a slope of 0.0020.

Solution

Rectangular channel, b = 6 m, n = 0.013, h_b = 0.75 m

(i) When the channel is horizontal, using Eq.(8.11), we get

$$Q = 5.18bh_b^{1.5} = 5.18 \times 6 \times 0.75^{1.5} = 20.19 \text{ m}^3/\text{s}$$

or, alternatively, since $h_b/h_c = 0.715$, $h_c = h_b/0.715 = 0.75/0.715 = 1.049$ m, we get

$$Q = \sqrt{g}bh_c^{1.5} = \sqrt{9.81} \times 6 \times 1.049^{1.5} = 20.19 \ m^3/s$$

(ii) When the channel slope is 0.0020, let us take Q = 20.19 m^3/s as a first approximation to determine the critical slope. Then, applying the Manning formula

$$Q = \frac{1}{n} AR^{2/3} S_c^{1/2}$$

we have

$$20.19 = \frac{1}{0.013} \times (6 \times 1.049) \times \left(\frac{6 \times 1.049}{6 + 2 \times 1.049} \right)^{2/3} \times S_c^{1/2}$$

which gives the critical slope as $S_c = 0.0024$ so that

$$S_0/S_c = 0.0020/0.0024 = 0.82$$

Then, from Fig 8.5, we get $h_b/h_c = 0.70$ and $h_c = 0.75/0.70 = 1.08$ m. For this critical depth, Q = 21.08 m^3/s and $S_c = 0.0024$ which is the same as we obtained previously.

$$\therefore Q = 21.08 \text{ m}^3/\text{s}$$

Example 8.3

A trapezoidal channel with b = 6 m, s = 1 and n = 0.015 ends in a free overfall. The brink depth is measured and found to be 1.00 m. Compute the discharge in the channel if (i) the channel is horizontal, and (ii) the channel has a longitudinal slope of 0.0020.

Solution Trapezoidal channel, b = 6 m, s = 1, n = 0.015, $h_b = 1.00$ m

(i) When the channel is horizontal, the ratio h_b/h_c varies from 0.715 to 0.795. Assume that $h_b/h_c = 0.75$ so that $h_c = 1/0.75 = 1.33$ m.

$$\therefore sh_c/b = 1 \times 1.33/6 = 0.22$$

Then, from Fig.8.5, $h_b/h_c = 0.73$

$$\therefore h_c = 1/0.73 = 1.37 \text{ m and } sh_c/b = 1 \times 1.37/6 = 0.23$$

Therefore, the value of h_c may be taken as 1.37 m.

$$\therefore A_c = (6 + 1 \times 1.37) \times 1.37 = 10.10 \text{ m}^2$$

$$B_c = 6 + 2 \times 1 \times 1.37 = 8.74 \text{ m}$$

$$D_c = A_c/B_c = 10.10/8.74 = 1.16 \text{ m}$$

$$U_c = \sqrt{gD_c} = \sqrt{9.81 \times 1.16} = 3.37\ m/s$$

$$\therefore Q_c = A_cU_c = 33.99\ m^3/s$$

(ii) When the channel slope is 0.0020, assume that $h_b/h_c = 0.70$ so that $h_c = 1/0.70 = 1.428$ m.

$$\therefore A_c = (b + sh_c) \times h_c = (6 + 1 \times 1.428) \times 1.423 = 10.612\ m$$

$$B_c = b + 2sh_c = 6 + 2 \times 1 \times 1.428 = 8.857\ m$$

$$P_c = b + 2h_c\sqrt{1+s^2} = 6 + 2 \times 1.428 \times \sqrt{1+1^2} = 10.041\ m$$

$$R_c = A_c/P_c = 10.612/10.041 = 1.057\ m$$

$$D_c = A_c/B_c = 10.612/8.857 = 1.979\ m$$

$$U_c = \sqrt{gD_c} = \sqrt{9.81 \times 1.979} = 3.428\ m/s$$

$$\therefore Q_c = A_cU_c = 36.37\ m^3/s$$

Now, from the Manning formula

$$Q = \frac{1}{n} AR^{2/3} S_c^{1/2}$$

we have

$$36.37 = \frac{1}{0.015} \times 10.612 \times 1.057^{2/3} \times S_c^{1/2}$$

which gives the critical slope as $S_c = 0.002456$.

$$\therefore S_0/S_c = 0.0020/0.002456 = 0.814 \quad \text{and} \quad sh_c/b = 1 \times 1.428/6 = 0.237$$

From Fig 8.5, $h_b/h_c = 0.70$. Hence, the assumed value of h_b/h_c is correct and

$$Q = 36.37\ m^3/s$$

8.4 FLOW OVER A WEIR

A weir is an overflow structure built across a channel whose primary function is to estimate the discharge in the channel. Weirs may be of different cross-sectional shapes like rectangular, trapezoidal, triangular, circular, parabolic and so on.

Depending on the value of h_1/L, where h_1 is the upstream depth measured above the weir crest and L is the length of the weir crest along the direction of flow (Fig.8.6), the flow over a weir may be classified as follows.

1. $h_1/L < 0.10$: Flow over the weir crest is subcritical and resistance effects are appreciable. Critical flow section occurs at the downstream end of the weir. This type of weir is termed *long-crested* and cannot be used for flow measurement. However, it can be used for

flow measurement based on the brink depth as discussed in Art. 8.3 provided the crest is long enough to develop the full drawdown profile.

2. $0.10 \le h_1/L \le 0.35$: A region of parallel flow and the critical flow (control) section occur in the vicinity of the midpoint of the crest. The weir can precisely be defined as *broad–crested* and is widely used for flow measurement. In general, the coefficient of discharge has a constant value in this rage of h_1/L.

3. $0.35 \le h_1/L \le 1.5$: The water surface is generally curvilinear all over the crest and the control section occurs near the upstream end of the weir. The weir is termed *short-crested* or *narrow-crested*.

4. $h_1/L > 1.5$: The flow may separate completely at the upstream edge of the weir. The flow above the crest has curved streamlines. The weir is similar to a *sharp-crested* weir ($h_1/L > 15$) and can be used for flow measurement.

Broad-Crested Weir

It was shown in Chapter 3 that if the height of a hump provided on the channel bed Δz is equal to or greater than a limiting value Δz_c, the flow over the hump will be critical. However, the length of the crest in the direction of flow must be adequate so that parallel flow with hydrostatic pressure distribution develops over the hump. As started earlier, such a situation exists when $0.10 \le h_1/L \le 0.35$ and the hump, in effect, because a broad-crested weir.

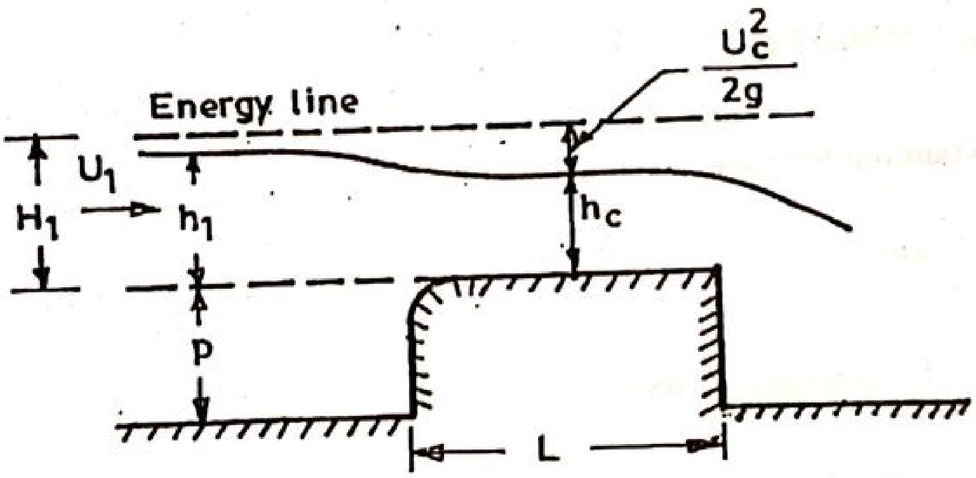


Fig. 8.6 Free flow over a rectangular broad-crested weir

A broad-crested weir may operate under free-flow or submerged condition. Under free-flow condition the discharge does not depend on the depth downstream of the weir. Under submerged condition the discharge depends on the downstream water level. Under free-flow condition the flow over the weir is critical and the discharge is maximum. Such a weir provides an excellent means of measuring the discharge in open channels.

Rectangular broad-crested weir

Free flow: Consider a rectangular broad-crested weir with vertical faces and rounded upstream corner under free-flow condition spanning over the entire width of the channel (Fig. 8.6). Neglecting the frictional losses and applying the energy equation between the upstream section and the critical flow section, one obtains

$$H_1 = h_c + \frac{U_c^{\,2}}{2g} = h_c + \frac{h_c}{2} = \frac{3}{2}h_c \qquad (8.15)$$

or

$$h_c = \frac{2}{3} H_1 \tag{8.16}$$

For critical flow condition

$$Fr = U_c / \sqrt{gh_c} = 1$$

and hence

$$U_c = \sqrt{gh_c} \tag{8.17}$$

The discharge over the broad-crested weir is $Q = A_c U_c = bh_c U_c$, or

$$Q = b\sqrt{g}\, h_c^{1.5} = 3.132 b h_c^{1.5} \tag{8.18}$$

where b is the width of the channel.

Equation (8.18) can be used for computing the discharge in an open channel when frictional losses over the weir crest are negligible, the flow over the weir is parallel, i.e. the water pressure over the weir crest is hydrostatic and the upstream velocity of approach is negligible. However, the usual difficulty in using Eq.(8.18) for computing discharge in an open channel lies in locating the critical flow section and measuring the critical depth accurately. These difficulties are, however, avoided by measuring the depth of flow upstream of the weir where the flow is not affected by the presence of the weir.

Combination of Eqs.(8.16) and (8.18) yields

$$Q = \left(\frac{2}{3}\right)^{1.5} b\sqrt{g} H_1^{1.5} = 1.705 b H_1^{1.5} \tag{8.19}$$

Introducing a discharge coefficient C_d, the equation for actual discharge over a rectangular broad-crested weir becomes

$$Q = \left(\frac{2}{3}\right)^{1.5} C_d b\sqrt{g} H_1^{1.5} = 1.705 C_d b H_1^{1.5} \tag{8.20}$$

The discharge coefficient C_d takes into account the effects of fluid viscosity, surface tension, non-uniform velocity distribution, curvature and friction and usually varies from 0.80 to 1.00. It is a function of the parameters h_1/L and $h_1/(h_1 + p)$. However, for $0.10 < h_1/L < 0.35$ and $h_1/(h_1 + p) < 0.35$ (i.e. $h_1/p < 0.54$), the value of C_d is found to remain constant at 0.848, a value which is termed as the *basic discharge coefficient*. When the above conditions are not met, the basic discharge coefficient must be multiplied by a correction factor f which can be obtained using Fig.8.7 (French, 1986).

Equation (8.20) is rather inconvenient for use as it contains the energy head H_1. If the velocity of approach is small so that $U_1^2/2g \ll h_1$, one can use the equation

$$Q = \left(\frac{2}{3}\right)^{1.5} C_d b\sqrt{g} h_1^{1.5} = 1.705 C_d b h_1^{1.5} \tag{8.21}$$

for estimating the discharge. However, if the velocity of approach U_1 is appreciable, Eq.(8.20) has to be used, initially neglecting the velocity of approach and then, improving the solution by including the velocity of approach.

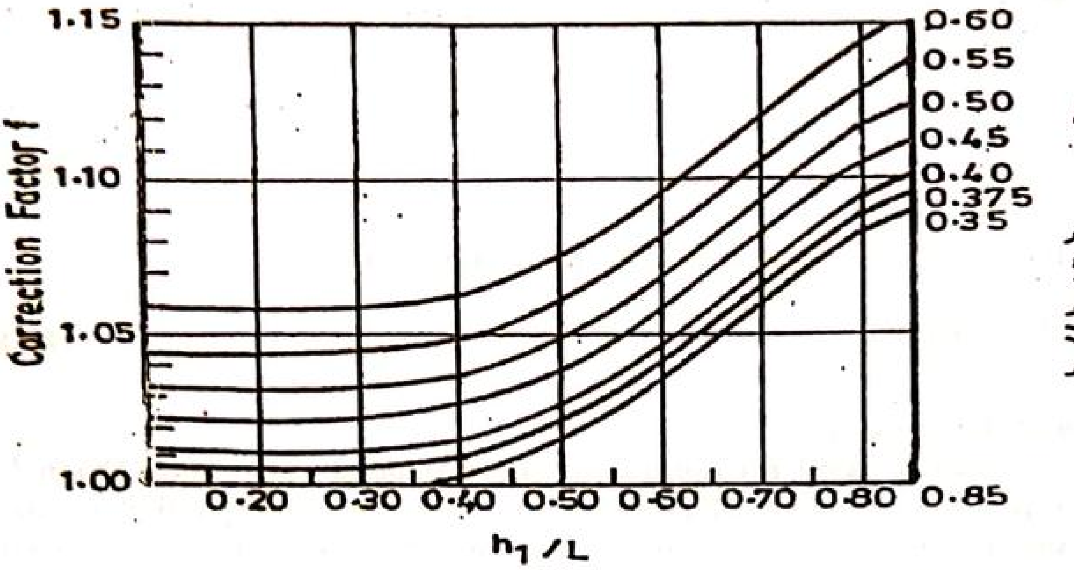


Fig. 8.7 Correction factor f as a function of h_1/L and $h_1/(h_1+p)$ for rectangular broad-crested weir (French, 1986)

Submerged flow: The effect of submergence is to drown the critical depth on the weir crest and make the flow over the weir crest subcritical. Under submerged conditions the discharge over the weir depends on the downstream depth h_2 (Fig.8.8). However, the flow is affected by downstream depth only when h_2/h_1 is greater than about 0.80, and when $h_2/h_1 < 0.80$ the weir discharges freely with no submergence effects.

The discharge formula for a submerged weir may be written as

$$Q = \left(\frac{2}{3}\right)^{1.5} C_d C_s b\sqrt{g} h_1^{1.5} = 1.705 C_d C_s b h_1^{1.5} \tag{8.22}$$

where the coefficient C_s takes into account the effect of submergence and depends on h_2/h_1 and the geometry of the weir. For a rectangular broad-crested weir with vertical upstream and downstream faces, the numerical value of C_s varies from 1.00 at $h_2/h_1 = 0.80$ (no submergence effect) to 0 at $h_2/h_1 = 1$ as given in Table 8.3.

Table 8.3 Variation of C_s with h_2/h_1 for weirs with vertical faces (Ranga Raju, 1993)

h_2/h_1	≤ 0.80	0.85	0.90	0.95	0.98	0.99	1.00
C_s	1.00	0.95	0.82	0.63	0.35	0.20	0

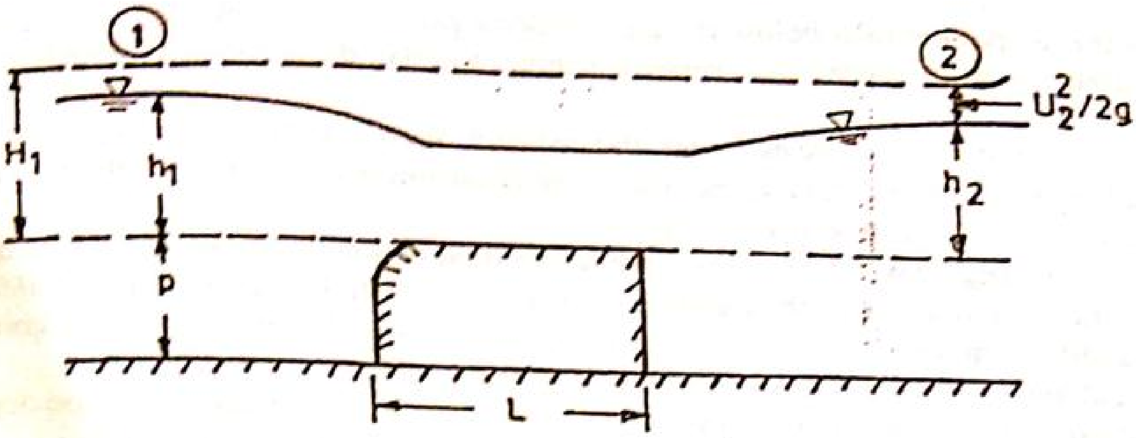


Fig. 8.8 Submerged flow over a rectangular broad-crested weir

Example 8.4

A rectangular broad-crested weir with vertical faces is 1 m high, has crest length of 2 m and span over the entire width of the channel. If the head over the weir is 0.50 m, (i) compute the discharge per unit width. (ii) What would be the discharge if the depth downstream of the weir is 0.45 m?

Solution

(i) In this case, the weir operates under free flow condition. We have, p = 1 m, L = 2 m and h_1 = 0.50 m.

$$\therefore \frac{h_1}{L} = \frac{0.50}{2} = 0.25 < 0.35$$

$$\frac{h_1}{h_1 + p} = \frac{0.50}{0.50 + 1} = 0.33 < 0.35$$

Hence, the weir is practically a broad-crested weir with C_d = 0.848. The computation of discharge is carried out with Eq.(8.20) as follows.

h_1	U_1	$U_1^2/2g$	H_1	q
0.50	-	-	0.50	0.511
0.50	0.3407	0.0059	0.5059	0.520
0.50	0.3468	0.0061	0.5061	0.521
0.50	0.3471	0.0061	0.5061	0.521

Hence, the discharge per unit width, q = 0.521 m^2/s

ii) Since h_2/h_1 = 0.45/0.50 = 0.90 > 0.80, the weir operates under submerged condition. From Table 8.3, we obtain C_s = 82. Therefore, q = 0.521 × 0.82 = 0.427 m^2/s.

Sharp-Crested Weir

If the length of the weir in the direction of flow is such that h_1/L > 15 (Fig.8.9), then the weir is termed sharp-crested. In practice, the crest length provided in a sharp-crested weir is only 1 mm to 2 mm and the downstream end is bevelled at an angle of 45° to 60°. In this case, the flow springs clear of the weir body downstream of the weir and an air pocket is formed beneath the nappe from which air is continuously removed by the overflowing jet. As the flow continues, the pressure in the air pocket falls below the atmospheric pressure and the nappe is depressed. For flow measurement, the atmospheric pressure is maintained in the air pocket through the provision of air vents.

In the case of a sharp-crested weir, the concept of critical flow is not applicable. For this type of discharge measuring device, the discharge equation is derived by assuming that the weir behaves as an orifice with free water surface.

A weir having its width B (transverse to the flow) equal to the width of the channel b, so that only vertical contraction of the nappe takes places, is called a *suppressed or full-width weir*. When the width of the weir is less than the width of the channel so that the nappe contracts both in the vertical and lateral directions, the weir is termed a *contracted weir*.

A sharp-crested weir may operate under free-flow or submerged condition depending on whether the water level downstream of the weir lies below or above the crest.

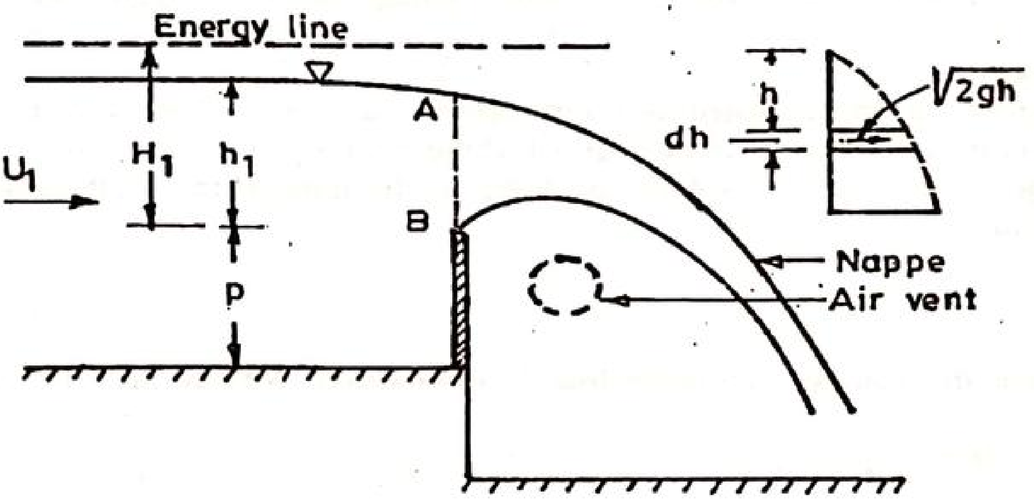


Fig. 8.9 Free flow over a rectangular sharp-crested suppressed weir

Rectangular sharp-crested suppressed weir

Free flow: Consider a rectangular sharp-crested weir spanning the full width b of a rectangular channel as shown in Fig.8.9. It is assumed that the flow does not contract as it passes over the weir and the pressure is atmospheric across the whole section AB. The velocity at any depth h below the energy line is equal to $\sqrt{2gh}$ and the discharge through an elementary strip of thickness dh is given by

$$dQ = b\sqrt{2gh}\,dh \tag{8.23}$$

The total discharge Q is then

$$Q = b\sqrt{2g}\int_{U_1^2/2g}^{h_1+U_1^2/2g}\sqrt{h}\,dh$$

$$= \frac{2}{3}\sqrt{2g}\,b\left[\left(h_1+\frac{U_1^2}{2g}\right)^{1.5}-\left(\frac{U_1^2}{2g}\right)^{1.5}\right] \tag{8.24}$$

$$= \frac{2}{3}\sqrt{2g}\, bh_1^{1.5}\left[\left(1+\frac{U_1^2}{2gh_1}\right)^{1.5} - \left(\frac{U_1^2}{2gh_1}\right)^{1.5}\right] \quad (8.25)$$

where

$$U_1 = \frac{Q}{b(h_1 + p)} \quad (8.26)$$

is the approach velocity.

The effect of flow contraction is taken into account by a coefficient of contraction C_c.

Then

$$Q = \frac{2}{3} C_c \sqrt{2g}\, bh_1^{1.5}\left[\left(1+\frac{U_1^2}{2gh_1}\right)^{1.5} - \left(\frac{U_1^2}{2gh_1}\right)^{1.5}\right] \quad (8.27)$$

Introducing a discharge coefficient C_d, Eq.(8.27) can be written in a more compact form as

$$Q = \frac{2}{3} C_d \sqrt{2g}\, bh_1^{1.5} \quad (8.28)$$

where

$$C_d = C_c\left[\left(1+\frac{U_1^2}{2gh_1}\right)^{1.5} - \left(\frac{U_1^2}{2gh_1}\right)^{1.5}\right] \quad (8.29)$$

If the Reynolds number of the flow is sufficiently high and the upstream depth h_1 is at least 0.11 m so that the surface tension and viscosity effects are negligible, then C_d becomes independent of the Reynolds and Weber numbers and depends only on the ratio h_1/p. The variation of C_d for rectangular sharp-crested weirs is given with satisfactory accuracy using the well-known *Rehbock formula*

$$C_d = 0.611 + 0.08\, h_1/p \quad (8.30)$$

which is valid for $h_1/p \le 5$.

When p becomes very large, C_d becomes equal to 0.611. Since in this case $U_1^2/2gh_1$ becomes negligibly small, Eq.(8.29) shows that C_c also becomes equal to 0.611.

When p = 0 so that h_1/p is infinite, the situation becomes a free overfall, considered in Art. 8.3. For a very low weir, in the range $h_1/p > 20$, critical flow occurs just upstream of the weir. Such a weir is termed as a *sill*. In this case

$$h_1 + p = h_c = \left(\frac{Q^2}{gb^2}\right)^{1/3}$$

or

$$Q = \sqrt{g}\, bh_c^{1.5} = \sqrt{g}\, b(h_1 + p)^{1.5} = \frac{2}{3} C_d b\sqrt{2g}\, h_1^{1.5} \quad (8.31)$$

so that

$$C_d = 1.06\left(1 + \frac{p}{h_1}\right)^{1.5} \quad (8.32)$$

In the range $20 > h_1/p > 5$, the intermediate region between a weir and a sill, the coefficient of discharge C_d is expected to have a smooth transition from Eq. (8.30) to Eq. (8.32).

Submerged flow: The discharge over a broad-crested weir is affected by the tailwater level downstream of the weir if it is above the weir crest. Such a flow is called a submerged flow. Under submerged conditions, the discharge over the weir depends on the submergence ratio h_2/h_1, where h_2 is the water surface elevation measured above the weir crest (Fig. 8.10), and is given by the *Villemonte equation*

$$Q_s = Q\left[1-\left(\frac{h_2}{h_1}\right)^n\right]^{0.385} \quad (8.33)$$

where Q is the free-flow discharge under head h_1 (Eq.(8.28)) and n is the exponent of head in the head-discharge relationship $Q = kh^n$. For rectangular weirs, n = 1.5 and for triangular weirs, n = 2.5.

To ensure free flow over a broad-crested weir, the water level downstream of the weir must be kept a few centimeters below the weir crest.

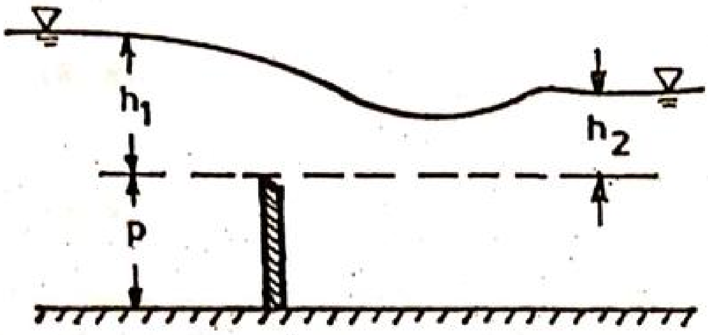


Fig. 8.10 Submerged sharp-crested weir

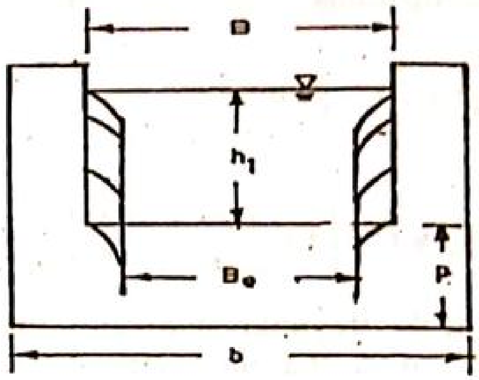


Fig. 8.11 Weir with end contractions

Rectangular sharp-crested contracted weir

In a contracted weir (Fig.8.11), the effective width of the weir (transverse to the direction of flow) is reduced and is given by the well-known *Francis formula*

$$B_e = B - 0.1nh_1 \quad (8.34)$$

where B is the width of the weir and n is the number of end contractions. The discharge equation for a contracted weir may be developed in the same way as for a suppressed weir. However, Kindsvater and Carter (1957), based on their extensive experimental investigation, modified the theoretical equation so that it would apply to all rectangular sharp-crested weirs regardless of whether they are suppressed or contracted. The *Kindsvater and Carter formula* for discharge over a sharp-crested weir is

$$Q = \frac{2}{3}C_{de}\sqrt{2g}\,B_e h_{1e}^{\ 1.5} \quad (8.35)$$

where C_{de} is the effective coefficient of discharge and B_e and h_{1e} are the effective width and head of the weir, and

$$C_{de} = K_1 + K_2 h_1/p \quad (8.36a)$$

$$B_e = B + K_b \quad (8.36b)$$

$$h_{1e} = h_1 + K_h \quad (8.36c)$$

where the parameters K_b and K_h represent the combined effects of viscosity and surface tension on the flow. Usually K_h is taken to be equal to 0.001 m. The parameters K_1, K_2 and K_b, which depend on B/b, are given in Table 8.4.

Table 8.4 Values of K_1, K_2 and K_b for broad-crested weirs (Kindsvater and Carter, 1957)

B/b	K_1	K_2	K_b (m)
1.0	0.602	0.0750	-0.0009
0.9	0.599	0.0640	0.0037
0.8	0.597	0.0450	0.0043
0.7	0.595	0.0300	0.0041
0.6	0.593	0.0180	0.0037
0.5	0.592	0.0110	0.0030
0.4	0.591	0.0058	0.0027
0.3	0.590	0.0020	0.0025
0.2	0.589	-0.0018	0.0024
0.1	0.588	-0.0021	0.0024

Example 8.5

A rectangular sharp-crested weir spanning the full width of a rectangular channel 2 m wide is 1 m high. Compute the discharge over the weir under an upstream head of 0.75 m.

Solution Rectangular channel, b = 2 m, p = 1 m, h_1 = 0.75 m

$$C_d = 0.611+0.08h_1/p = 0.611+0.08\times0.75/1 = 0.671$$

$$\therefore Q=\frac{2}{3}C_d\sqrt{2g}bh_1^{1.5}=\frac{2}{3}\times0.671\times\sqrt{2\times9.81}\times2\times0.75^{1.5}=2.574\,m^3/s$$

Example 8.6

Compute the discharge over a sharp-crested contracted weir 1 m wide and 1 m high set in a rectangular channel 2 m wide if the head over the weir is 0.75 m.

Solution b = 2 m, B = 1 m, p = 1 m, h_1 = 0.75 m

$$\frac{B}{b}=\frac{1}{2}=0.50$$

∴ From Table 8.4, K_b = 0.0030, K_1 = 0.592 and K_2 = 0.0110.

$$\therefore\ C_{de} = K_1 + K_2\times\frac{h_1}{p} = 0.592 + 0.0110\times\frac{0.75}{1} = 0.60025$$

$$B_e = B + K_b = 1 + 0.0030 = 1.0030$$

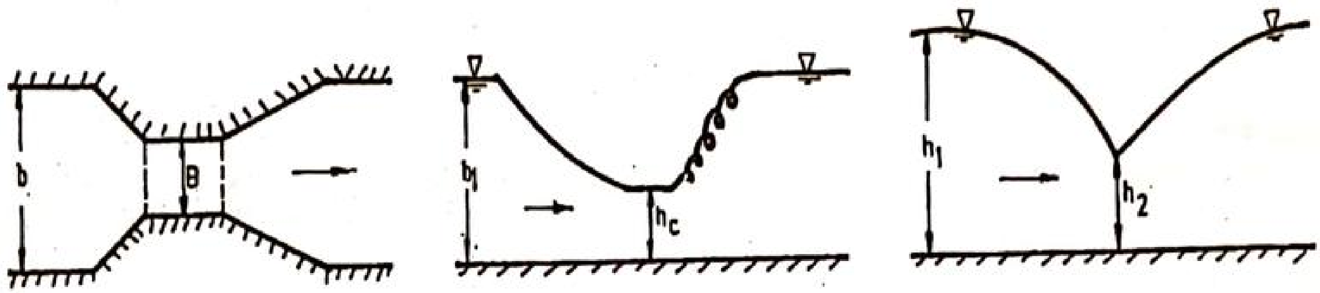
$$h_{1e} = h_1 + 0.001 = 0.75 + 0.001 = 0.751$$

$$\therefore Q=\frac{2}{3}C_{de}\sqrt{2g}B_eh_{1e}^{1.5}=\frac{2}{3}\times0.60025\times\sqrt{2\times9.81}\times1.0030\times0.751^{1.5}=1.161\,m^3/s$$

8.5 CRITICAL FLOW FLUMES

It has been stated in Chapter 3 that a critical flow section is a control section and can be used for flow measurement. Based on critical flow, various devices have been used for flow measurement. In these devices, critical flow is produced either by raising the channel bottom, as in a broad-crested weir, or by reducing the channel width, as in a critical flow flume. The use of a broad-crested weir for flow measurement has been considered in Art. 8.4. It has, however, the disadvantage of having a dead water region upstream in which silt and debris can accumulate. This difficulty can be overcome by the use of a critical flow flume, in which the occurrence of critical flow is forced by a contraction in channel width, followed by a short length of supercritical flow and a hydraulic jump. Obviously, the width at the contracted section or throat must be equal to or less than the width required for producing critical flow, i. e. $b \le b_c$, as stated in Chapter 3.

The critical flow flume, also known as the *Venturi flume*, has been designed in various forms. It is usually operated with a free-flow condition having the critical depth at the contracted section or throat and a hydraulic jump in the exit section. Under certain conditions of flow, however, the jump may be submerged.



(a) Plan (b) Free flow (c) Submerged flow

Fig.8. 12 Flow in a rectangular critical flow flume

Let us consider a rectangular flume as shown in Fig. 8.12. For free-flow (Fig. 8.12b) condition, the discharge through the flume is given by the equation

$$Q = B\sqrt{g}\, h_c^{1.5} = 3.132 B h_c^{1.5} \tag{8.37}$$

and when it is difficult to locate the critical flow section and measure the critical depth accurately, by the equation

$$Q = \left(\frac{2}{3}\right)^{1.5} B\sqrt{g} H_1^{1.5} = 1.705 B H_1^{1.5} \tag{8.38}$$

where B is the throat width and h_c is the critical depth at the throat. Equations (8.37) and (8.38) are the same as Eqs. (8.18) and (8.19), respectively, used for computing the discharge over a broad-crested weir under free flow condition.

For drowned or submerged condition (Fig. 8.12c), the discharge is given by

$$Q = A_2\sqrt{2g(h_1 - h_2)} / \sqrt{1 - r^2} \tag{8.39}$$

where A_2 is the throat area (= Bh_2), $r = A_2/A_1$ and A_1 is the upstream area (= bh_1).

The most extensively used critical flow flume is the Parshall flume, developed by R. L. Parshall in 1920 and named after him. Detailed designs for this flume have been developed for a wide range of discharges, and the usual difficulties of locating the critical flow section and measuring the critical depth accurately are readily disposed of by suitable choice of a standard section (not necessarily the critical one) at which the depth is measured. Figure 8.13 shows the

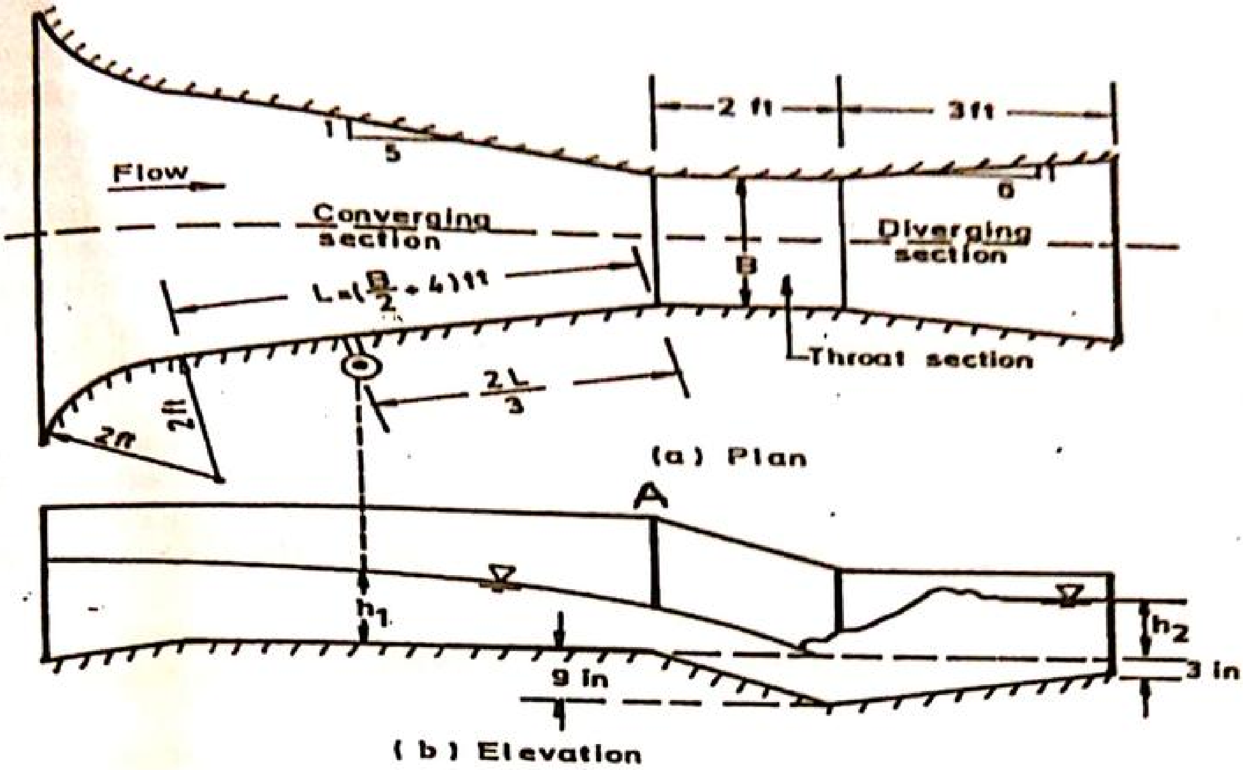


Fig. 8.13 Dimensions of Parshall flume for widths B of 1 ft to 8 ft (Henderson, 1966)

standard design of Parshall flume for throat widths b of 1 ft (0.30 m) to 8 ft (2.67 m). The size of the flume is determined by the throat width in inch or ft. The sidewall convergence and sudden dip in the bed are provided to promote the formation of critical flow near section A followed by a short length of supercritical flow and a hydraulic jump at the exit section. For the above range of throat widths, reliable empirical formula for discharge has been established by calibration and is given by

$$Q = 4Bh_1^{1.522B^{0.026}} \tag{8.40}$$

where B is the throat width and h_1 is the depth measured at the upstream location as shown in Fig. 8.13 and all quantities are in English units.

The above formula is true for values of the submergence ratio, h_b/h_a, up to 0.7. When the submergence ratio exceeds 0.7, the flow becomes submerged. The effect of submergence is to reduce the discharge, i. e. the discharge for submerged flow is less than the free flow discharge. In this case the discharge given by the above formula must be corrected by a negative quantity.

Other designs and the discharge formulae for Parshall flumes covering a range of widths from 3 in to 50 ft are also given by Parshall (see Chow, 1959; French, 1986).

Example 8.7

Compute the theoretical discharge through a Venturi flume having a throat width of 0.25 m, (i) when the critical depth measured at the throat is 0.35 m under free-flow condition, and (ii) when the upstream and downstream depths measured are 0.50 m and 0.45 m, respectively, and the channel width is 0.60 m, under submerged condition.

Solution $B = 0.25$ m

(i) Under free-flow condition, the theoretical discharge through the Venturi flume is given by

$$Q = B\sqrt{g}\, h_c^{1.5} = 0.25 \times \sqrt{9.81} \times 0.35^{1.5} = 0.162\ m^3/s$$

(ii) $b = 0.60$ m, $h_1 = 0.50$ m, $h_2 = 0.45$ m, $A_1 = bh_1 = 0.60 \times 0.50 = 0.30\ m^2$, $A_2 = Bh_2 = 0.25 \times 0.45 = 0.1125\ m^2$ and $r = A_2/A_1 = 0.1125/0.30 = 0.375$. Hence, the theoretical discharge under submerged condition is

$$Q = A_2\sqrt{2g(h_1 - h_2)}/\sqrt{1-r^2} = 0.1125 \times \sqrt{2 \times 9.81 \times (0.50 - 0.45)}/\sqrt{1 - 0.375^2} = 0.103\ m^3/s$$

Example 8.8

Determine the discharge through a 4-ft Parshall flume if the gage reading h_1 is 1.25 m under free-flow condition.

Solution $B = 4$ ft $\quad h_1 = 1.25\text{ m} = 1.25 \times 3.28 = 4.10$ ft

$$\therefore Q = 4Bh_1^{1.522B^{0.026}} = 4 \times 4 \times 4.10^{1.522 \times 4^{0.026}} = 148.254\ ft^3/\text{sec} = 4.202\ m^3/s$$

PROBLEMS AND EXERCISES

8.1 State the difference between a control and a transition.

8.2 (a) What is the use of a sluice gate? When does a sluice gate act as a control?

(b) Show that the discharge under a vertical sluice gate in a horizontal rectangular channel for free-flow condition is maximum when $h_2/h_1 = 2h_1/3$.

8.3(a) State why a free overfall can be used for flow measurement.

(b) Derive Eqs.(8.11) to (8.13).

8.4(a) What is the use of a weir? What is the difference between (i) sharp-crested and broad-crested weirs, (ii) free-flow and submerged weirs, and (iii) suppressed and contracted weirs?

(b) Derive the formula for discharge over a rectangular sharp-crested suppressed weir for free-flow condition.

(c) What do we get from (i) the Rehbock formula, (ii) the Villemonte formula, (iii) the Francis formula, and (iv) the Kindsvater and Carter formula?

8.5 Find the discharge through a vertical sluice gate in a horizontal rectangular channel 6 m wide and having a gate opening of 1 m under an upstream head of 4 m (i) for free flow condition, and (ii) for submerged condition when the tailwater depth is 3.25 m. (iii) Also, compute the depth of submergence by Eq.(8.9).

8.6(a) A horizontal rectangular channel carries a discharge of 1.30 m^2/s. There is a vertical sluice gate in the channel. What would be the height of the gate opening to pass the stated flow when the upstream head is 4 m and the gate operates under free flow condition?

(b) Assuming that the flow though the sluice gate in Prob. 8.6(a) occurs under submerged condition and the tailwater depth is 3.2 m, what would be the upstream depth h_1 and the submergence depth h_3 if the discharge in the channel remains the same?

(c) Compute the force on the sluice gate in Problems 8.6(a) and 8.6(b).

8.7 A horizontal channel ends in a free overfall. The brink depth is measured and found to be 0.50 m. Compute the discharge if the channel is

i) rectangular with b = 6 m
ii) triangular with s = 2
iii) parabolic with perimeter equation $y^2 = 4z$
iv) circular with $d_0 = 2$ m
v) trapezoidal with b = 6 m and s = 2.

8.8. An open channel having a slope of 0.0025 ends in a free overfall. The brink depth is measured and found to be 0.50 m. Compute the discharge if the channel is (i) rectangular with b = 6 m, (ii) triangular with s = 2, and (iii) trapezoidal with b = 6 m and s = 2.

8.9 A broad-crested weir with vertical faces is 1 m high, has a crest length of 2 m and spans the entire width of the channel. If the head over the weir is 0.80 m, compute the discharge per unit width. What would be the discharge per unit width if the depth downstream of the weir is (i) 0.60 m, and (ii) 0.75 m?

8.10 A broad-crested weir with sharp square corner and vertical faces at the upstream and spanning the full width of a rectangular channel has a crest length of 2.50 m. The height of the weir is 1.50 m and the channel is 6 m wide. Calculate the depth of flow upstream of the weir when the discharge is (i) 6 m^3/s, and (ii) 10 m^3/s, assuming free flow.

8.11 A rectangular channel 10 m wide is to carry a discharge of 15 m^3/s. It is desired to place a broad-crested weir across the channel. Compute the height of the weir and its crest length.

8.12 Compute the discharge over a suppressed sharp-crested weir of height 0.50 m and constructed in 2 m wide rectangular channel. The head over the weir is 0.50 m.

8.13 Compute the discharge over a contracted sharp-crested weir 2 m wide and 0.50 m high constructed in a 2.5 m wide rectangular channel. The head over the weir is 0.50 m.

8.14 The depth and discharge in a rectangular channel 2 m wide are 0.75 m and 0.50 m^3/s, respectively. Find the height of a suppressed sharp-crested weir that will pass the channel discharge.

8.15 In Problem 8.13, if a contracted sharp-crested weir of width 1.6 m is used, what would be the height of the weir?

8.16 Estimate the discharge over a suppressed sharp-crested weir spanning the full width of a rectangular channel 2 m wide when the depths upstream and downstream of the weir above the weir crest are 1.6 m and 1.3 m, respectively, and the height of the weir is 1m.

8.17 Show from Eqs.(8.29) and (8.32) that $C_c = 0.715$ for the completely free overfall (p = 0).

8.18 Compute the theoretical discharge through a Venturi flume having a throat width of 0.65 m, (i) when the critical depth measured at the throat is 0.45 m under free-flow condition, and (ii) when the upstream and downstream depths measured are 0.60 m and 0.50 m, respectively, and the channel width is 0.75 m, under submerged condition.

8.19 Determine the discharge through a 5-ft Parshall flume if the gage reading h_1 is 1.32 m under free-flow condition.

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WRE 301 OPEN CHANNEL FLOW (4 hours/week)

Open channel flow and its classification. Velocity and pressure distributions. Energy equation, specific energy and transition problems. Critical flow and control. Principles of flow measurement and devices. Concept of uniform flow, Chezy and Manning equations, estimation of resistance coefficients and computation of uniform flow. Momentum equation and specific momentum. Hydraulic jump. Theory and analysis of gradually varied flow. Computation of flow profiles. Design of channels.

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4.15 $h_n = 1.8285$ m, $U_n = 1.4953$ m/s
4.16 (i) 3.30 1 m^3/s (ii) 4.994 m^3/s (iii) 0.557 m^3/s (iv) 1.2 38 m^3/s (v) 0.48 m^3/s
4.17 (i) $h_n = 1.36$ m, $U_n = 2.022$ m/s (ii) $h_n = 1.55$ m, $U_n = 2.127$ m/s
(iii) $h_n = 1.84$ m, $U_n = 2.016$ m/s (iv) $h_n = 1.30$ m, $U_n = 1.387$ m/s
4.18 (i) 9.21 m (ii) 10. 37 m
4.19 $S_n = 0.0062$ (ii) $S_c = 0.0072$ (iii) $S_c = 0.0063$
4.20 17.675 m^3/s (i) 22.49 m^3/s (ii) 22.895 m^3/s (iii) 33.99 m^3/s
4.21 0.21 m^3/s
4.22 Q = 1052.35 m^3/s, U = 1.571 m/s, n = 0.026, $\alpha = 1.621$, $\beta = 1.221$
4.23 30,690 m^3/s

Chapter 5

5.7(a)(i) P = 17.889 m, R = 2.236 m (ii) P = 17.889 m, R = 2.236 m
(iii) P = 16.647 m, R = 2.403 m (iv) P = 15.853 m, R = 2.523 m
(v) P = 17.369 m, R = 2.303 m Circular section has the minimum wetted perimeter
5.8 h = 2.408 m, b = 1.994 m 5.9 (i) h = 3.098 m (ii) h = 1.824 m, b = 18.124 m
5.10 (i) b = 9.67 m, h = 1.61 m (ii) b = 14.76 m, h = 1.30 m 5.11 b = 21.150 m, h = 1.163 m

Chapter 6

6.10 (i) M1, M3 (ii) C1, C3 (iii) S1, S3 (iv) H2, H3 (v) A2, A3
6.11(a) (i) M2 (ii) C2 (iii) None (iv) H2 (v) A2
(b) M2 when $h_d < h_n$ and M1 when $h_d > h_n$
(c) Jump and S1 when $h_d > h_c$ and none when $h_d < h_c$
6.12 (a) M1, H2 (b) H2 (c) S1 or H3, H2 (d) C1, H2
(e) H2, A2 (f) M2,C2 (g) A2 (h) C2, S2
(i) M2 (j) H2, C2 (k) C3 (l) A2,S2
6.13 (a) M 1, M2, S2 (b) C2, S2, S1 or M3 (c) S1 or M3, M1 (d) H2, M2, C2
(e) C1, A2, H2 (f) H2, A2, S2 (g) M1, A2, H2 (h) M2, C2, C1,C3,C2, S2
6.14 (a) (i) M1 (ii) S3 (b)(i) M2 (ii) S2
6.15 (i) M2, S2 (ii) M2 (in upstream channel) (iii) S1 or H3
(iv) S3 (in downstream channel) (v) H2
6.16 M2, S2, S3, S1 or M3 6.17 - 6,484.22 m 6.1 8 43.85 m
6.19(a) 60.09 m upstream (b) 2.22 m (c) (i) 2.3396 m (ii) 2. 2115 m

Chapter 7

7.9 (i) 2.30 m (ii) weak jump (iii) 1.30 m (iv) $L_j = 9.66$ m using Fig. 7.3 and $L_j = 9.28$ m by Silvester formula (v) 116. 50 (vi) 91.78%
7.10 1.88 m, 0.12 m 7.11 3.05 m 7.12 2.45 m
7.13(i) Jump type A (ii) Jump type B (iii) Jump type D

Chapter 8

8.5(i) 30.20 m^3/s using Eq.(8.3) and 29.60 m^3/s using Eq.(8.6) (ii) 16.85 m^3/s (iii) 3.07 m
8.6(a) 0.25 m (b) 6.70 m, 2.83 m (c) 67,706.46 N , 180,557.30 N
8.7(i) 10.99 m^3/s (ii) 1.39 m^3/s (iii) 2.86 m^3/s (iv) 2.137 m^3/s (vi) 12.20 m^3/s
8.8(i) 11.85 m^3/s (ii) 1.51 m^3/s (iii) 13.93 m^3/s
8.9 1.093 m^2/s (i) 1.093 m^2/s (ii) 0.732 m^2/s 8. 10(i) 0.77 m (ii) 1.07 m
8.11 2.50 m, 3.50 m 8. 12 1.443 m^3/s 8.1 3 1.347 m^3/s 8.1 4 0.49 m 8.1 5 0.45 m
8.16 5.318 m^3/s 8.1 8(i) 0.614 m^3/s (ii) 0.218 m^3/s 8.1 9 5.80 m^3/s

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