PROJECT PLANNING AND CONTROL

WITH

PERT

AND

CPM

Dr. B.C.PUNMIA
K.K.KHANDELWAL

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Preface to the First Edition

Complex research and development projects can be managed effectively if the project managers have the means to plan and control the schedules and costs of the work required to achieve their technical performance objectives. When the planning of a project is undertaken, a host of questions arise : How should the work be accomplished ? What resources will be needed ? How long will it take ? How much will it cost ? The answers to all these questions can be found by adopting the modern techniques of project management.

It would be difficult to find in the history of management methods any techniques which has received such widespread attention as that accorded to network methods for planning, scheduling and controlling. The network techniques are called by various names such as PERT, CPM, UNETICS, TOPS and SCANS. However, these and other systems have emerged from two major network systems, namely PERT and CPM.

The aim of this book is to present the basic principles of PERT and CPM in such a way that they can be effectively applied to the solution of management problems. Attempt has been made to present the subject-matter which lays emphasis on fundamentals. General statements of important principles, methods and procedures are almost invariably given by practical illustrations. Unsolved problems with answers have also been incorporated at the end of each chapter to enable the student/reader to test his reading at different stages of his studies.

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B.C. PUNMIA
K.K. KHANDELWAL

Preface to the Fourth Edition

In the Fourth Edition, the book has been revised and updated.

1-7-2002

B.C. PUNMIA

K.K. KHANDELWAL

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1

Project Management

1.1. INTRODUCTION

A project is composed of jobs, activities, functions or tasks that are related one to the other in some manner, and all of these should be completed in order to complete the project. Every project has one specific purpose : it starts at some specific moment and it is finished when its objectives have been fulfilled. For completion of a project, two basic things are required :

(*i*) material resources,

(*ii*) manpower resources.

Many countries, rich in material resources are exceedingly poor in terms of level of production or plan achievement, while there are other countries which have very limited natural resources but have achieved higher level of productivity mainly because of talents, skills, experience and know-how of their people. Availability, quality and use of human resources is a single determinant factor in accomplishing project objectives.

Rapid accumulation of scientific technique in the recent past has not been matched by a corresponding improvement in the sphere of human group relations. In other words, sociology has not kept pace with technology. We are not in a position to utilize fully our technology advancement unless we are also able to advance in social sphere. Here comes the role of *management*. While *technology* deals with *material things, management* deals with both *material things* as well as *human-beings*.

Management increases the productivity through technological innovation taking into account human factors involved in these advances.

Each project, whether big or small has three *objectives* :

(*i*) The project should be completed with a minimum of elapsed time.

(*ii*) It should use available manpower and other resources as sparingly as possible, without delay.

(*iii*) It should be completed with a minimum of capital investment, without delay.

Project management is a highly specialised job, to achieve the above objectives. Project management involves, the following three *phases* :

1. Project *planning*
2. Project *scheduling*
3. Project *controlling*.

Out of the above three phases of project management, the first two phases are accomplished before the actual project starts. The third phase is operative during the execution of the project, and its aim is to recognize the difficulties during the execution and to apply measures to deal with these difficulties.

1.2. PROJECT PLANNING

Planning is the most important phase of project management. Planning involves defining objectives of the project, listing of tasks or jobs that must be performed, determining gross requirements for material, equipment and manpower and preparing estimates of costs and durations for the various jobs or activities to bring about the satisfactory completion of the project.

Planning is important because :

(*i*) It provides direction

(*ii*) It provides unifying frame-work

(*iii*) It helps to reveal future opportunities and threats

(*iv*) It provides performance standards.

In the planning phase, *plan* is made and *strategies* are set, taking into consideration the company's *policies, procedures* and *rules*.

Plan

It is a statement of intent, *i.e.*, what is to be done. It is interpreted in terms of what has to be done to resources to achieve the intent. The resources to be used may be : office staff, tradesmen, labour, materials, plant and machinery, space and funds. *Plans* are detailed methods, formulated before hand for doing or making something. Plans simply list the goals (target) and define the means of achieving them. These listed goals are called *events* and means of achieving these goals are known as operations or *activities* in attaining final target set aside by the plan. The size of the *activities* depends on the nature and scale of project ; however, each activity should be sufficiently well defined, for work on them to proceed without interruption from other tasks. *Activities are those operations of the plan which take time to carry out and on which resources are expended.*

Strategies

Strategy is one important type of plan. It specifies the central concept or purpose of the enterprise as well as the *means* by which it intend to carry that purpose.

Policies, procedures and rules

Policies, procedures and rules differ from each other in degree of specificity. *Policies* usually set broad guide-lines for the enterprise. For example, it might be the policy of a departmental store that if a customer is dissatisfied with any of its sale article, his/her money will be refunded.

Procedure specify how to proceed in some situation. For example, 'before refunding the money of the customer, the salesman should carefully inspect the article to be returned and then obtain approval from the manager for the refund'.

A *rule* is even more specific guide for action. For example, the departmental store may have a rule that 'under no conditions will the money be refunded to the customer if he/she brings the defective article after 15 days of the purchase'.

Thus, plans should be finalised and strategies should be set only after taking into considerations the company's policies, procedures and rules.

Steps in project planning

Following eight steps are generally recognised in the planning process of a project :

1. DEFINE : the objectives of the project in definite words.
2. ESTABLISH : goals and stages intermediate to attain the final target.
3. DEVELOP : forecast and means of achieving goals, *i.e., activities*.

4. EVALUATE : organization's resources—financial, managerial and operational—to carry out activities and to determine what is feasible and what is not.
5. DETERMINE : alternatives—individual courses of action that will allow to accomplish goals.
6. TEST : for consistency with company's policy.
7. CHOOSE : an alternative which is not only consistent with its goals and concept but also one that can be accomplished with the evaluated resources.
8. DECIDE : on a plan.

During the planning phase, the information needed is about all those operations or activities, which have to be carried out before the project is completed, their sequence and their logical inter relationship.

Resources

In running a project, there is a basic need of *resources*. These resources can be classified as under :

(*i*) Material resources (*what*)
(including financial resources)

(*ii*) Equipment resources (*how*)

(*iii*) Space resources (*where*)

(*iv*) Effort or manpower resources (*who*)

(*v*) Time resources (*when*).

Resources are the starting point of many problems that have to be solved by the manager in the planning phase, before proceeding for scheduling phase of the project.

1.3. SCHEDULING

Scheduling is the allocation of resources. These resources, in conceptual sense, are time and energy, but in practical sense are *time, space, equipment* and *effort* applied to material. More specifically, *scheduling* is the mechanical process of formalising the planned functions, assigning the starting and completion dates to each part (or activity) of the work in such a manner that the whole work (or project) proceeds in a logical sequence and in an orderly and systematic manner. In other words, scheduling is the *laying out* of the actual activities of the project in time order in which they are to be performed, and *calculating* the manpower and material requirements (or resources requirements, in general) needed at each stage of production, along with the expected completion time of each of the activity.

Steps in Project Scheduling Phase

Scheduling is done in the following steps :

1. CALCULATE : detailed control information.
2. ASSIGN : timings to events and activities.
3. GIVE : consideration to the resources. The manager is generally concerned with those resources whose availability is limited and which thereby impose a constraint on the project. The important ones are usually skilled, technical and supervisory manpower and capital investment.
4. ALLOCATE : the resources.

In traditional techniques, the term scheduling a project is some-what misleading because actually some attempt at planning and scheduling are performed as one step.

1.4. CONTROLLING

As stated earlier, the planning and scheduling phases of a project are undertaken before the actual project starts while the controlling phase is undertaken during the actual project operations. *Controlling* consists of reviewing the difference between the schedule and actual performance once the project has begun. *Project control is the formal mechanism established to determine deviations from the basic plan, to determine the precise effect of these deviations on the plan, and to replan and reschedule to compensate for the deviations.*

Steps in Control Process

Controlling is accomplished in the following well recognised steps :

1. ESTABLISH	:	standards or targets. These targets are generally expressed in terms of time.
2. MEASURE	:	performance against the standards set down in the first step.
3. IDENTIFY	:	the deviations from the standards.
4. SUGGEST AND SELECT	:	correcting measures. This will involve all the problems-identifying, decision-making and organising and leadership skill of the decision-maker.

1.5. ROLE OF DECISION IN PROJECT MANAGEMENT

While planning, organising, staffing, leading, scheduling and controlling are the basic functions of *management,* each of these clearly involve *decisions*—decision as to which plan to implement, what goals to achieve, what ways to use, and so forth. The success or failure of *management* is judged from the decisions it takes at various stages. A poor or erroneous decision may lead to the failure of a project. *If a poor decision is made and a wrong road is choosen none but luckiest survive.*

Steps in Decision-making

Following are steps for a better decision-making :

1. IDENTIFY	: the central problem.
2. DEVELOP	: alternatives.
3. ANALYSE	: the alternatives.
4. MAKE	: final decision.

1.6. TECHNIQUES FOR ANALYSING ALTERNATIVES : OPERATION RESEARCH

Generally, we have several *alternatives*, and it is essential to evaluate them before we can choose the best out of these. This can be successfully done through *operation research.* The term 'operation research' usually refers to a set of mathematical techniques through which a variety of organisational problems can be analysed and solved.

Steps in Operational Research Techniques :

1. FORMULATE	:	the problem.
2. CONSTRUCT	:	a mathematical model to represent the system under study.
3. DERIVE	:	a solution for the model.
4. TEST	:	the model and solution derived from it.
5. ESTABLISH	:	controls over the solution.
6. PUT	:	the solution to work and implementation.

1.7. METHODS OF PLANNING AND PROGRAMMING

As stated earlier, complex research and development projects can be managed effectively if project managers have the means to plan and control the schedules and costs of the work required to achieve their technical performance objectives.

When the planning of a project is undertaken a host of questions arise : How should the work be accomplished ? What resources will be needed ? How long will it take ? How much will it cost ? The answers to all these questions can be found by following the modern techniques of project management.

Managers at all levels need improved techniques at all stages in a project to :

(*i*) define the work to be performed.

(*ii*) develop more realistic schedule and cost estimates based or resources planned to perform the work.

(*iii*) determine where resources should be applied to best achieve the time, cost and technical performance objectives.

(*iv*) identify those areas developing potential delays or cost overruns, in time to permit corrective action.

Following are some of the tools or techniques of project management :

(1) Bar charts and Milestone charts.

(2) Net work diagrams.

1. Bar Charts and Milestone Charts

Bar charts were introduced by Henery Gantt around 1900 AD. Bar charts represent pictorial representation in two dimensions of a project by breaking it down into a number of manageable units or activities for planning and control shown on one dimension or axis and the durations assigned to these activities on the other dimensions or axes. Bar charts were later modified to yield the milestone charts. While the bar chart represents the *activities*, a milestone chart represents the *events* which mark either the beginning or the end of an activity. The bars of the bar chart are broken into a number of pieces, each one of which represent an identifiable major event. It should be noted that each event is a point in time which the management has identified as important reference point during the completion of the project.

2. Network Methods

Network diagram is an outcome of the improvements in the milestone charts (see chapter 2). The network technique is a major advance in management science. This technique is based on the basic *characteristics* of all projects, that all work must be done in well-defined steps. For example, for completing a foundation, the various steps are : (*i*) layout, (*ii*) digging, (*iii*) placing side boards and (*iv*) concreting. The network technique exploits this characteristics by representing the steps of the project objective graphically in the form of a network or arrow diagram. It would be difficult to find in the history of management methods any technique which has received such widespread attention as that accorded to network methods for planning, scheduling and controlling.

The network techniques are called by various names such as PERT, CPM, UNETICS, LESS, TOPS and SCANS. However, these and other systems have emerged from the following two major *network systems* :

(*i*) PERT

(*ii*) CPM

The other systems, by and large, differ from their parents only in non-essentials.

(*i*) PERT

PERT stands for 'Program Evaluation and Review Technique'. The method was basically developed by the Navy

Special Projects Office in co-operation with Booz, Allen and Hamilton, a management consulting firm and Lockheed Missile System Division for evaluating the feasibility of existing schedules on Polaris missile program and for reporting progress.

The PERT system uses a network diagram consisting of *events* which must be established to reach project objectives. An *event* is that particular instant of time at which some specific part of a plan is to be achieved. It indicates a point in time and does not require any resources. PERT uses *event oriented* network diagrams in which successive events are joined by arrows. For example, in a foundation construction project, the various events may be 'foundation layout started', 'foundation excavated', 'side boards fixed', 'concreting completed' etc. etc. The approach of event-orientation in network diagram grew out of the desire to report on the project progress via discernible management milestones.

PERT system is preferred for those projects or operations which are of non-repetitive nature or for those projects in which precise time determination for various activities cannot be made. In such projects, managements cannot be guided by the past experience. They are referred to as *once-through* operations or projects. For example, the project of *launching a space craft* involves the work never done before. For such a project the range of possible technical problem is immense in such research and development projects, the times estimates made for use may be little more than guesses. PERT system is best suited for such projects.

(ii) **CPM**

CPM stands for 'Critical Path Method'. In CPM networks, the whole project consists of a number of clearly recognisable jobs or operations, called *activities*. Activities are usually operations which take time to carry out, and on which resources are expended. Junctions between activities are termed as *events*. The CPM networks are often referred to as *activity oriented* diagrams in which each activity is represented by an arrow, and the sequence in which the activities are performed is shown by the sequence of the arrows. For example, in a foundation construction project, the various activities may be ; 'lay out the foundation trench', 'excavate the foundation', 'put side boards', 'concrete the foundation base', etc. etc.

CPM network are generally used for repetitive type projects, or for those projects for which fairly accurate estimate of time for completion of each activity can be made ; and for which cost estimations can be made with fair degree of accuracy. For example, CPM is very useful for construction projects. However, it is not suitable for research and development projects.

Comparison of PERT and CPM

As stated earlier, PERT uses *event-oriented* network diagram while CPM uses *activity oriented* network diagram. This does not mean that PERT system does not have activities and CPM system does not have events. In PERT, emphasis is given on events while in CPM, emphasis is given on activities. Both PERT and CPM have many things in common. However, they differ on the following :

(*i*) In CPM, time estimates for completion of activities are with fair degree of accuracy, while in PERT system, time estimates are not so accurate and definite.

(*ii*) In CPM, cost optimisation is given prime importance. The time duration for completion depends on this cost optimisation. The cost is not directly proportional to time. On the other hand, in PERT, it is assumed that cost varies directly with time. Attention is, therefore, paid to minimise the time so that minimum cost results. Thus, in CPM, cost is the direct controlling factor while in PERT, time is controlling factor.

To conclude, CPM is preferred in those projects where time can be estimated fairly well and when costs can be calculated in advance. On the other hand, PERT is used in those projects where there is an extreme degree of uncertainty, and where control over time overweighs control over cost. PERT is, therefore, more frequently used for research and development type of projects which has uncertainties about the times required for development research, engineering design and ultimate construction. CPM is used on those projects which employ long-developed and well-seasoned components and which are based on more or less stable technology. For such works, changes occur mainly in design—sizes, shapes and arrangement rather than in design concepts.

PROBLEMS

1. Discuss in brief the role of management in project execution.
2. Describe various phases of project management.
3. Explain why plannings is necessary. Describe various steps for planning a project.
4. Explain in brief the difference between PERT and CPM networks. Explain the circumstances under which one is preferred over the other.

2

Bar Charts and Milestone Charts

2.1. INTRODUCTION

A project generally consists of a number of well-defined manageable units or activities which should be performed or completed in a definite sequence, for the successful completion of the project. These *activities* or *jobs* are those operations of the project plan which take time to carry out and on which resources are expanded. Out of the various tools or techniques of project management, *bar charts technique* was probably one of the earliest one.

Bar charts were introduced by Henery Gantt around 1900 AD. In his work on production control, Gantt developed the famous Gantt chart still used on many projects of moderate

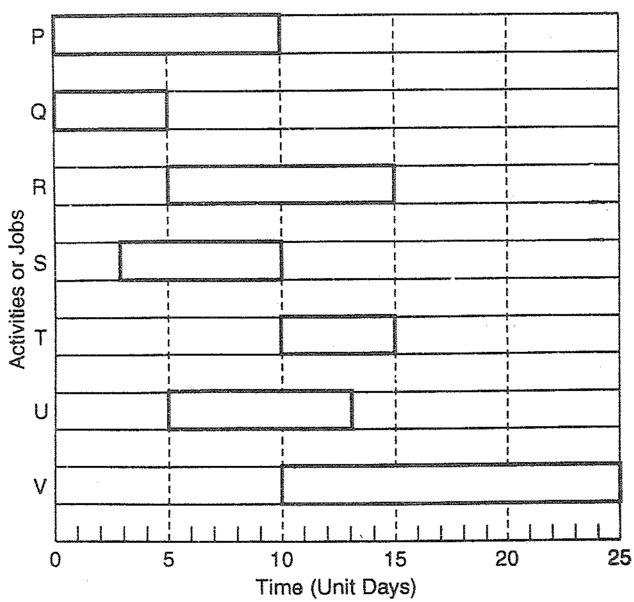


FIG. 2.1. BAR CHART.

magnitude. A *bar chart* consists of two co-ordinate axes, one (usually horizontal axis) representing the time elapsed and the other (the vertical axis) represent the jobs or activities to be performed. Each bar represent one specific job or activity of the project. The beginning and end of each bar represent the time of start and time of finish of that activity ; the length of bar, therefore, represents the time required for the completion of that job or activity.

Fig. 2.1 shows the bar chart for a project which has seven distinct jobs or activities (P, Q, R, S, T, U, V) to be performed for its completion. The time durations required for the completion of these activities are 10, 5, 10, 7, 5, 8 and 15 unit days respectively. From the chart, we conclude the following :

(*i*) Activities P and Q can start simultaneously, at zero time. Both the activities are independent. However, activity Q is completed much earlier than activity P.

(*ii*) Activity R starts only when activity Q is complete.

(*iii*) However, activity S is independent of activity R. It starts earlier than R and is completed earlier.

(*iv*) Activity T starts only when activity S is complete.

(*v*) Activities U and R can start simultaneously, when activity Q is complete.

(*vi*) Activity V can start when activity P and S are complete. End of activity V marks the completion of the project.

Fig. 2.2 shows another bar chart for the project related to purchase and installation of a lathe. The complete project consists

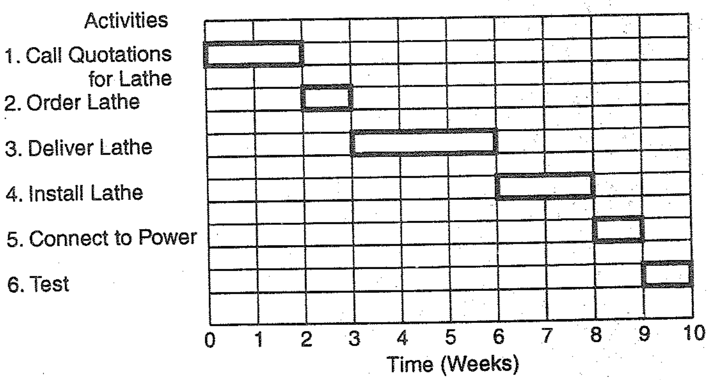


FIG. 2.2

of five distinct activities. Each activity cannot be started unless the previous activity is finished.

From the above two examples, we find that there are some operations or activities which can take place concurrently while there are some activities that succeed a preceding activity and cannot be started unless the preceding activity is complete. The concurrent activities or jobs are represented by bars running parallel or overlapping each other *time-wise*. The other types of activities have bars that run serially one after the other (Fig. 2.2).

In either case, each bar can be represented either by a set of two lines running parallel to each other (*i.e.* by a rectangle) or by a thick, solid line. The first form is preferred.

2.2. DEVELOPMENT OF BAR CHART

The following are important stages in developing a bar chart :

1. BREAK DOWN : the project into its various activities or jobs or operations, each representing manageable unit for planning and control.
2. DECIDE : the method to be employed in execution of the project, as well as for each activity or operation or task ; also decide above the sequence in which the activities are to be completed.
3. ASSIGN : duration of time for the completion of each activity. Once the activities are separated and choice of method is made, it is possible to estimate the time required for the completion of each activity.
4. REPRESENT : the above information in the bar chart, indicating the relative positions of the each activity.

2.3. ILLUSTRATIVE EXAMPLES

Example 2.1. *Draw the bar chart for 'finalisation of designs and work order' for a building project.*

Activity	*Description*	*Time for completion*
A	*Site selection and survey*	*4 weeks*
B	*Design*	*6 weeks*
C	*Preparation of drawings*	*3 weeks*
D	*Preparation of specifications and tender document*	*2 weeks*
E	*Tendering (NIT)*	*4 weeks*
F	*Selection of contractor*	*1 week*
G	*Award of work order*	*1 week*

Solution. In the above project, activities *A* and *B* can start concurrently, since some parts of the architectural and structural designs can be done even if complete survey data is not available. Similarly, drawing work can also be started, as soon as survey work is over, though all the designs are still not complete. Specifications can be finalised when once the designs are complete. Activity *E* can be started only when activity *D* is complete. Activities *E*, *F* and *G* are to be completed in sequential order.

The bar chart representing the above sequence of activities is shown in Fig. 2.3 (on next page). From the figure, we conclude that the total time required for this phase in 14 weeks.

Example 2.2. *The activity breakdown for a certain project is as under.*

Activity No.	*Duration (weeks)*
1	*1*
2	*2*
3	*4*
4	*3*
5	*1*
6	*2*
7	*4*

Activity 2 and activity 3 can be done concurrently, and both must follow activity 1. Activity 2 must precede activity 4. Activity 5 cannot begin until both activities 2 and 3 are completed. Activity 6 can be started only after activities 4 and 5 are complete.

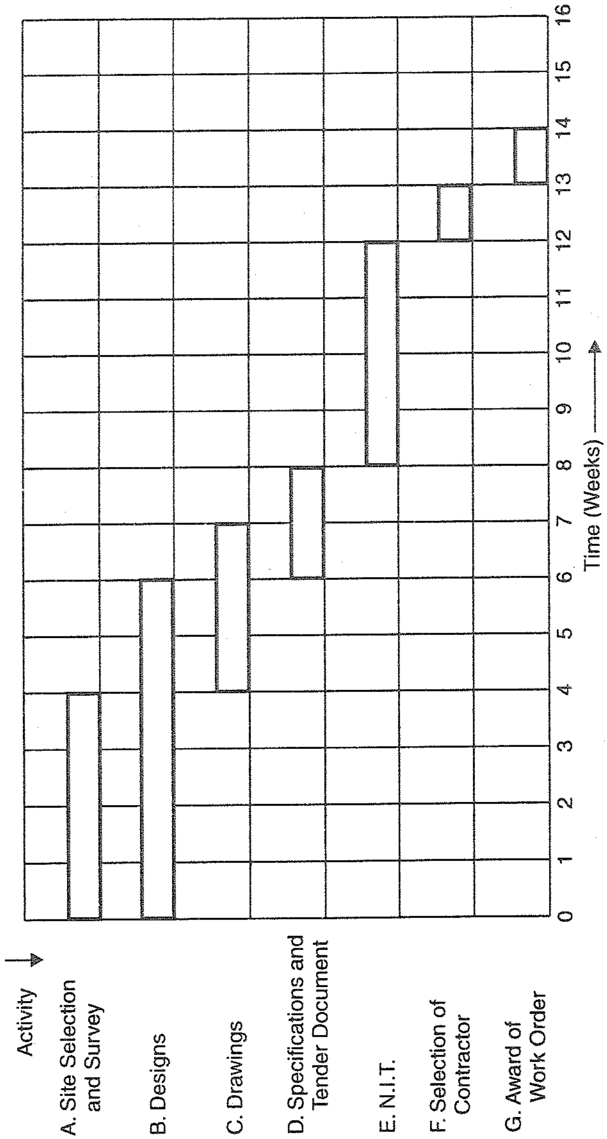


FIG. 2.3. BAR CHART FOR 'FINALISATION OF DESIGNS AND WORK ORDER' FOR A BUILDING PROJECT.

Activity 7 is the last activity which can be started only after completion of activity 5. Prepare the bar chart for the project.

Solution. The resultant bar chart is shown in Fig. 2.4. We find that the project can be completed in 10 weeks duration.

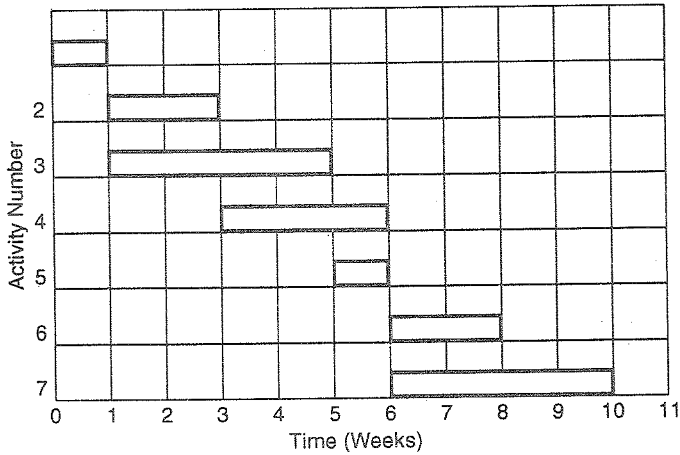


FIG. 2.4

Example 2.3. *A typical small house construction project consists of the following operations along with the time set for its completion.*

S. No.	*Operation*	*Time (in days)*
1.	*Survey, design and layout*	*3*
2.	*Construction of foundations*	*5*
3.	*Construction of superstructure*	*11*
4.	*Roofing*	*5*
5.	*Fixing doors and window frames*	*2*
6.	*Plumbing and house drainage*	*3*
7.	*Electric fitting*	*3*
8.	*Plastering*	*4*
9.	*Flooring*	*4*
10.	*Carpentry work*	*4*
11.	*Construction of boundary wall and other minor items*	*3*
12.	*Land shaping and clearing*	*2*
13.	*White washing of walls and painting of doors*	*3*
14.	*Inauguration*	*1*

The project commences on Wednesday, 14th October. Assuming five working days in a week, prepare bar chart of the project.

State the assumptions made. Also determine (a) total time, and date of completion of the project (b) expected progress by 10th November.

Solution. The bar chart is shown in Fig. 2.5, prepared with

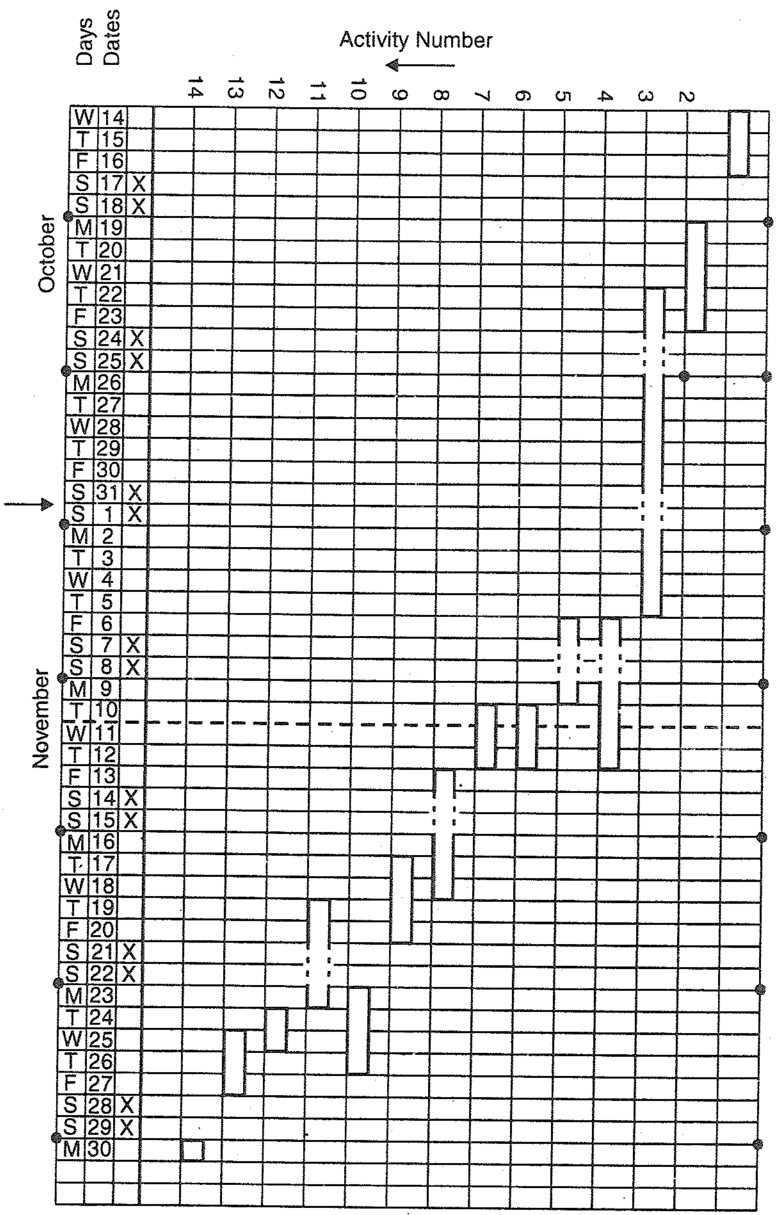


FIG. 2.5. BAR CHART FOR CONSTRUCTION OF A HOUSE.

the following assumptions regarding the sequence of various activities :

(*i*) Activity 2 can start only after activity is 1 over.

(*ii*) Activity 3 can start even when half the work of activity 2 is over.

(*iii*) Activity 4 and 5 can start concurrently, but only after activity 3 is over.

(*iv*) Activity 6 and 7 can start concurrently, but only after activity 5 is complete.

(*v*) Activity 8 can start only after activities 6 and 7 are complete.

(*vi*) Activity 9 can start even when half of activity 8 is over.

(*vii*) Activity 10 can start only when activity 9 is over.

(*viii*) Activity 11 can start only when activity 8 is over.

(*ix*) Activity 12 can start only when activity 11 is over.

(*x*) Activity 13 can start even when activity 10 is half over.

(*xi*) Activity 14 is the last activity which marks the completion of the project.

Cross (×) denotes the day on which there will be no construction work.

From the bar chart shown in Fig. 2.5, we find that project will be complete on 30th November—48 days after its start. Also, the progress upto 10th November will be as follows :

(*a*) Activities 1, 2, 3 and 5 will be completely over.

(*b*) Activities 4, 6 and 7 will have 2 days work left.

2.4. SHORTCOMINGS OF BAR CHARTS AND REMEDIAL MEASURES

Bar charts have following shortcomings. These shortcomings can be partly overcome by the following suggested remedial measures.

1. Lack of Degree of Details

On bar chart, only major activities are shown. If too many activities or tasks are separately shown, it becomes clumsy. Due to this, bar charts are not very useful for big projects. A particular activity, whether big or small, is shown by one bar, without any details of sub-activities contained in it. These sub-activities cannot be separated out. Due to this, effective control over the activities cannot be achieved.

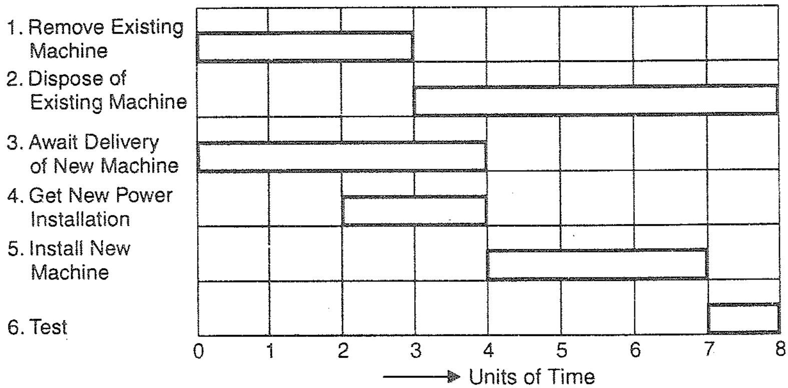


FIG. 2.6. ORIGINAL BAR CHART.

For example, consider the activity '*dispose of existing machine*'. In the bar chart (Fig. 2.6) prepared for the task of obtaining a new machine, this item will be represented in the chart by one bar, though the following functions control the completion time for the activity :

A. Dumping machine in store

B. Notice inviting bidding

C. Finalisation of highest bid

D. Final disposal of the machine.

For effective completion of the main activity, these sub-activities should be scheduled properly. The above information can be

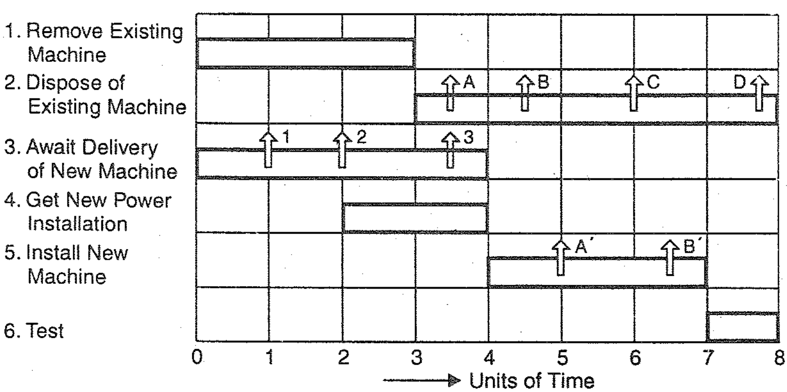


FIG. 2.7. IMPROVED BAR CHART.

shown effectively by marking stages (called '*mile stones*') on the activity bar, shown in Fig. 2.7. Similarly, sub-activities of other activities of each bar can be marked with stages or mile stones 1, 2, 3 etc. A', B', C' etc.

2. Review of Project Progress

A bar chart does not show the progress of work and hence it cannot be used as a control device. For proper control of the project, information of the progress made at a particular instant of time should be available. '*Controlling*' is essential for *re-scheduling* the remaining activities. However, an existing bar chart can be *modified* to depict the progress made. This can be done by showing the progress of each activity, by hatched lines along the corresponding bar of the activity. Generally, hatching is done in half the width of the bar.

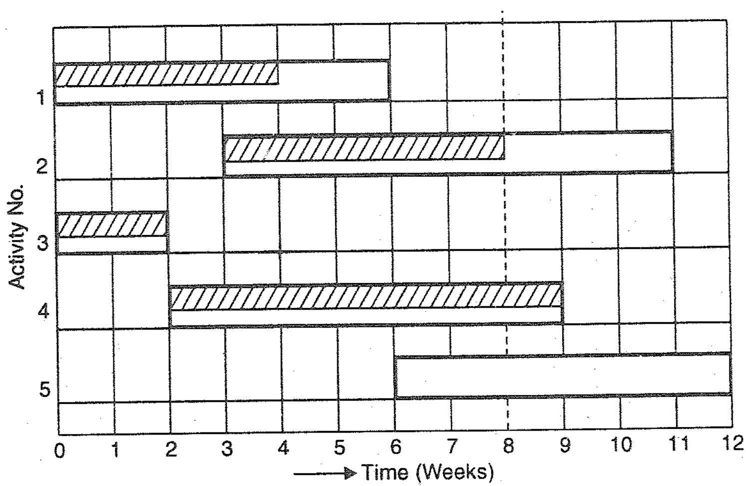


FIG. 2.8. PROGRESS OF ACTIVITIES ON BAR CHART.

For example, let us mark the progress made on the bar chart of Fig. 2.8, after 8 weeks of the start of the progress. Activity 1 had a total time allocation of 6 weeks. At the end of 8th week, only 4 week's work has been done ; that means that activity 1 is 4 weeks behind schedule. Activity 5 was wholly dependent on the completion of activity 1 ; the beginning of activity 5 will now be delayed by 4 weeks. Hence rescheduling of activity 5 is essential. Activities 2 and 3 are perfectly as per schedule. However, activity 4 is 1 week *ahead* of the schedule.

Sometimes, different colours are filled in the bars to show various 'control informations', as indicated below :

Control information	*Colour*
Anticipated progress	black
Actual progress	green
Progress behind schedule	red

3. Activity Inter-relationships

As indicated earlier, there are some activities of a project which are taken up concurrently, while there are others which can be taken up only after the completion of some other activity. The concurrent activities are represented by bars which run parallel to each other, or which overlap. The activities whose start and end depend on other activities are shown serially. In a project, there may be large number of activities which can start with a certain degree of concurrency. By merely depicting them by parallel lines, the inter-relationships between them cannot be *clearly* depicted. One cannot draw the conclusion that if two activities are scheduled for simultaneous or overlapping times, they are inter-dependent or completely independent. For example, take the project of laying a pipe-line, consisting of following activities :

A. Excavating the trench	12 weeks
B. Laying and jointing the pipe	10 weeks
C. Refilling and compacting	6 weeks

Activity *C* is dependent on *B* and *A*, while activity *B* is dependent on activity *A*. If all the activities are scheduled *serially*, it will take a very long time—28 weeks for completion. However, the activities can be staggered as shown in Fig. 2.9.

From bar chart of Fig. 2.9, we find that if activity *B* is started 4 weeks after activity *A*, activity *B* has 2 weeks work left after

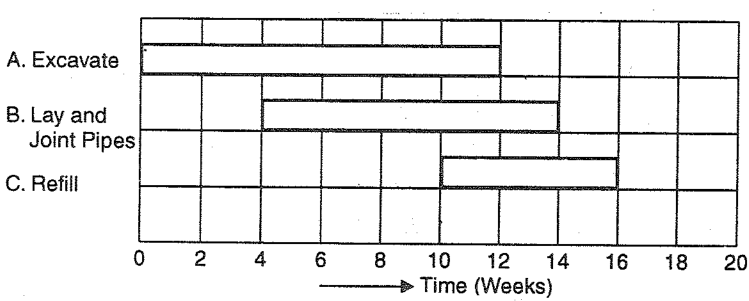


FIG. 2.9

completion of activity A. Similarly, activity C has 2 week's works left after completion of activity B. Now, if due to some circumstances, time of completion of activity A is delayed by 1 or 2 weeks, how will the activities B and C be affected ? This is not clearly portrayed by the bar chart, since inter-dependencies of the activities are not clearly indicated by bar charts.

This difficulty can be partly overcome by breaking each activity into a number of sections, so that the corresponding sections of various activities are *precisely* depicted inter-dependently. For example, let the jobs of Fig. 2.10 be divided into 4 *sections*. Since activity B is faster than A, and activity C is still faster, the shifting or staggering of these activities can be for more than 1 section. The modified bar chart is shown in Fig. 2.10 by depicting the completion of each section by 'milestones' 1, 2, 3 and 4.

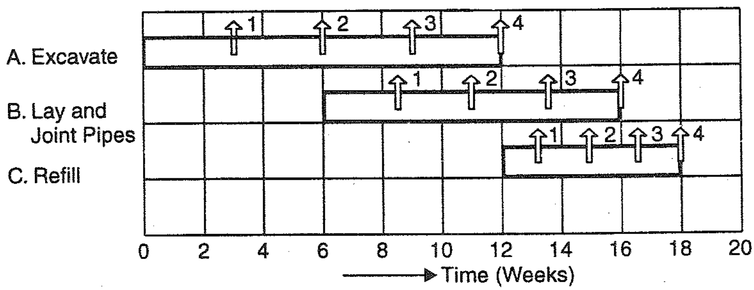


FIG. 2.10. MODIFIED BAR CHART.

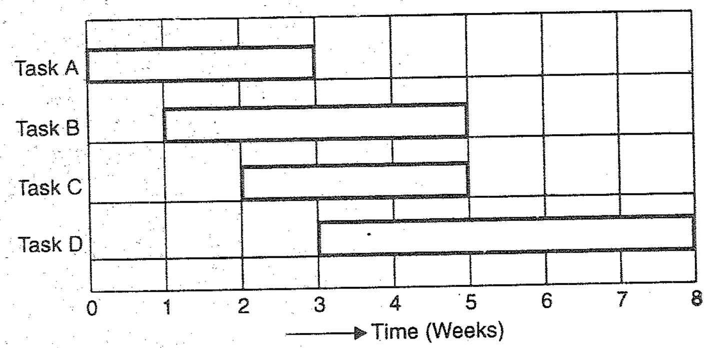
For activity A (excavate), each section will require 3 weeks time. For activity B, each section will require $2\frac{1}{2}$ weeks time while for activity C, each section will require $1\frac{1}{2}$ weeks time of completion. If activity B is started 6 weeks after the start of activity A, it will mean that activity B will start after section 2 of activity A is already complete, and that activity B will require 4 weeks time after the completion of section 4 of activity A. Now the effects of delay in the work of any *section* of activity A on activity B can be easily found, and necessary control measures can be taken.

4. Time Uncertainties

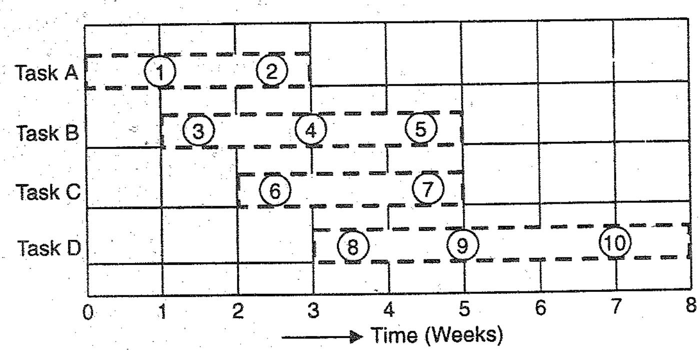
Bar charts are not at all useful in those projects where there are uncertainties in determination or estimation of time required for the completion of various activities. Such uncertainties are always there in all research and development projects and for space vehicle launch projects. Because of uncertainties in time determinations in these projects, some of the activities may require rescheduling. Such rescheduling flexibility cannot be reflected in the bar chart diagrams. Hence bar chart diagrams are useful for only small size conventional projects, specially construction and manufacturing projects, in which time estimates can be made with fair degree of certainty.

2.5. MILESTONE CHARTS

Milestone chart is a modification over the original Gantt chart. *Milestones* are key events of a main activity represented by a bar : these are specific points in time which mark the completion of certain portions of the main activity. These points are those which can be easily identified over the main bar. We have already seen that when a particular activity, represented by a bar on a bar-chart



(a) Gantt Bar Chart



(b) Gantt Milestone Chart

FIG. 2.11. MODIFICATION OF BAR CHART INTO MILESTONE CHART.

is very long, the details lack. If, however, the activity is broken or sub-divided into a number of sub-activities, each one of which can be easily recognised during the progress of the project, controlling can be easily done and inter relationships between other similar activities can be easily established. The beginning and end of these sub-divided activities or tasks are termed as *milestones*.

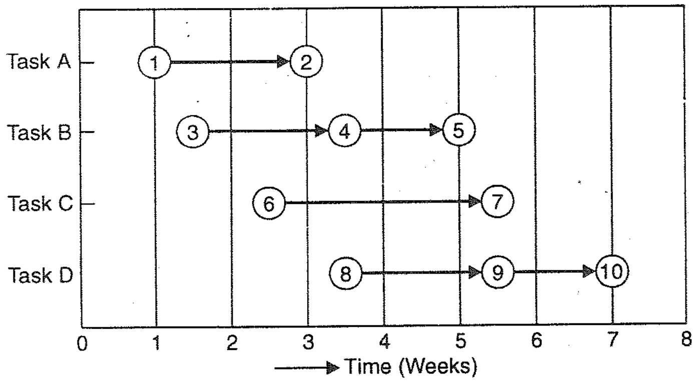
For example, consider a bar chart diagram shown in Fig. 2.11 (*a*). It consists of four jobs or tasks or activities—Task *A*, task *B*, task *C* and task *D*.

Fig. 2.12 (*b*) shows some 'milestones' on each bar. Each main task contains some specific points in time which can be recognised, and through which *controlling* can be achieved. Each milestone can be considered to be specific *event* along the main activity or job or task. This chart is, therefore, called the *milestone chart*. Each milestone is represented either by a circle or by a square, and is serially marked.

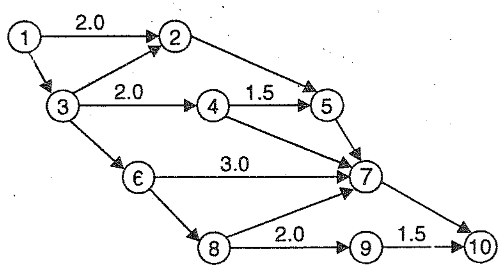
Though controlling can be better achieved with the help of milestone chart, it still possess the same deficiency contained by the bar chart—it does not show the inter-dependencies between the events. Within a task, the relationship between two specific milestones is revealed, but the relationship between and among milestones contained in different task is not indicated on the chart. For example, Gantt milestone chart [Fig. 2.11 (*b*)] does not indicate whether milestone 6 can be started before milestone 2 has been completed, or whether milestones of task *C* are at all dependent on milestones of task *A*.

2.6. DEVELOPMENT OF PERT NETWORK

The above deficiency of milestone chart (*i.e.* its inability to depict the relationships between milestones of different tasks) has been removed effectively by developing the milestone chart into a network diagram. Since the milestone chart has milestones which can be considered as 'events' along a main task, since a PERT network is always event oriented (chapter 1), the resulting network diagram is called the PERT network.



(a) Milestone Chart with Arrows



(b) PERT Network

FIG. 2.12. DEVELOPMENT OF PERT NETWORK.

Fig. 2.12 (*a*) shows the milestone chart, in which the milestones of *each bar* (or task) have been joined serially by arrows, since these are to be accomplished one after the other, serially. The next transition step is achieved by joining those milestones of different bars (tasks) which are inter-related, by arrows, so that it may clearly show the necessary flow of work. This modification naturally replies the main question about what milestone should precede what other milestones, and which milestones can be completed independently without being co-ordinated with others.

In Fig. 2.12, milestone 4 is dependent on milestone 1, because it cannot be started unless 3 is complete, and 3 cannot start unless 1 is complete. Similarly, milestone 7 is dependent on milestone 4 and 5. Thus, the inter-relationships between and among all the milestones can be established in the network diagram, regardless of original tasks to which they belonged. Due to this reason, the main task designations do not have any importance, and they are omitted in the network diagram. Similarly, the horizontal time scale is also omitted, and the times between the milestones (now called the *events*) are indicated on the arrows between them.

The network diagram so developed has the following *advantages* :

(*i*) The network diagram clearly shows the inter-relationships between the milestones (events).

(*ii*) The designation of tasks becomes redundant and the project is viewed as an integrated whole, consisting of a number of milestones (events) and not on number of tasks. This helps very much in controlling the project.

(*iii*) Network can be used even for highly complicated projects consisting of a large number of activities.

(*iv*) It directly indicates the time required in between two milestones or activities. This is helpful in rescheduling if required.

(*v*) Wherever there are uncertainties, network diagrams allows the use of probability theory for time estimation.

PROBLEMS

1. What is a Gantt bar chart ? Explain, with the help of a suitable example, the method of preparing a bar chart.
2. What is a milestone chart ? How does it differ from a bar chart ? How can milestone chart be developed into a network ?
3. What are the shortcomings of bar charts ? How are these removed ?
4. A project consists of 8 activities A, B, C, D, E, F, G and H with their times of completion as follows :

Activities	*Duration (weeks)*
A	2
B	4
C	2
D	4
E	6
F	4
G	5
H	4

The precedence relationships are as follows :

A and *B* can be performed in parallel.

C and *D* cannot start until *A* is complete.

E cannot start until half the work of activity *C* is complete.

F can start only after activity *D* is complete.

G succeeds *C*.

H is the last activity, which should succeed *E*.

(*a*) Draw the bar chart.

(*b*) What is the total time of completion of the project ?

(*c*) If there is increase of 2 weeks in time of completion of activity *A*, what will be the corresponding increase in the total time of the completion of the project ?

3

Elements of Network

3.1. INTRODUCTION

We have already seen that network technique is one of the most modern tools of project management. It is always possible to break up the entire project into a number of distinct, well defined jobs or tasks (called *activities*). The beginning or end of each such activity constitutes an *event* of the project. A network is a flow diagram consisting of *activities* and *events*, connected logically and sequentially. In the network diagram, an activity is represented by arrows while events are represented, usually, by circles, as shown in Fig. 3.1.

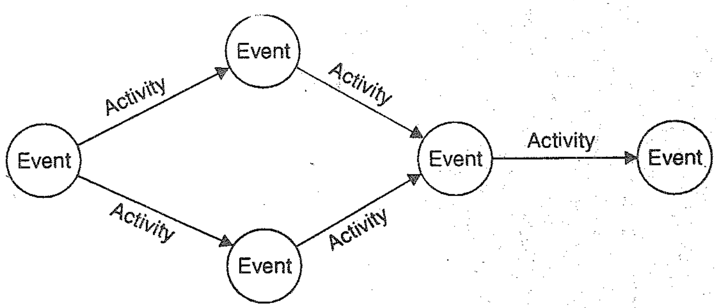


FIG. 3.1. NETWORK DIAGRAM.

Networks are of two types : PERT network and CPM network. PERT network is *event-oriented*, while CPM network is *activity-oriented*. Fundamentally, both CPM and PERT networks are techniques of project management involving graphical and diagrammatic representation, which management can use as an aid in planning, scheduling and controlling of operations in a project.

Characteristics of CPM/PERT Projects

A project to be analysed by CPM or PERT technique should have the following characteristics :

1. The project to be planned by network technique should consist of clearly recognizable jobs or operations, usually called *activities*.

2. These jobs, operations or activities must have definite commencement and completion. The start or end of a job or operation or activity is called an *event*.

3. The *events* must occur in a definite pattern and must be performed in a technological sequence.

Thus, the basic elements of a project network are :

(*i*) Event

(*ii*) Activity.

As an example, consider the project of laying a foundation. The project consists of the following well defined operations :

(*a*) Excavation of foundation

(*b*) Laying side boards

(*c*) Concreting foundation.

All the three operations are to be performed in a sequential order. The simple network will be as shown in Fig. 3.2.

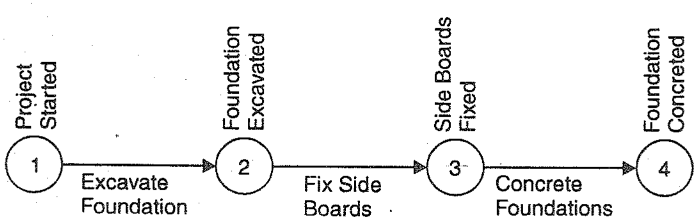


FIG. 3.2

In Fig. 3.2, the *activities* (*i.e.* excavate foundation, fix side boards, concrete foundations) have been shown by arrows. The beginning and end of activities are events and they are shown by circles provided at the nodes. The events of the above project are : (1) project started or excavation started, (2) foundation excavated, (3) side boards fixed and (4) foundation concreted.

As another example, consider the project of purchasing a new heavy duty lathe and disposing of the old lathe. The project consists of the following activities :

(*i*) Await delivery of lathe

(*ii*) Remove existing lathe

(*iii*) Install power supply

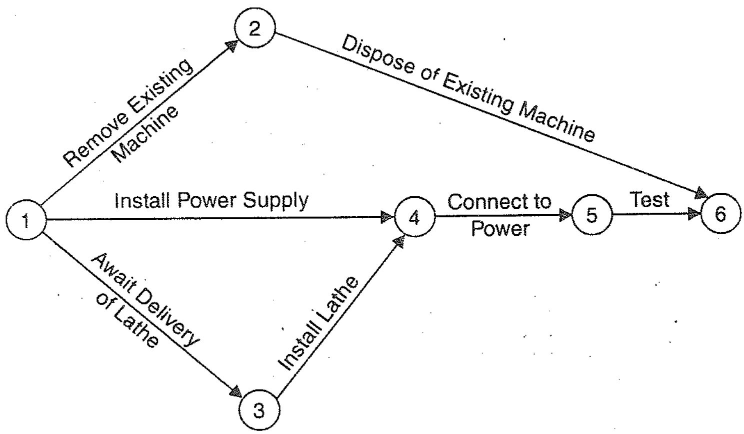
(*iv*) Install lathe

(*v*) Connect to power

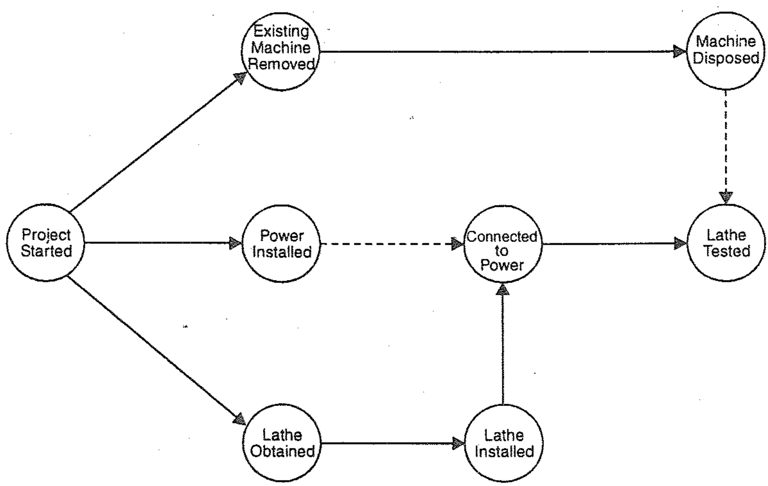
(*vi*) Test

(*vi*) Dispose of existing lathe.

The above project can be represented by a network shown in Fig. 3.3 (*a*) which is activity oriented diagram. Fig: 3.3 (*b*) shows an alternative network which is event oriented.



(a) Activity Oriented Network



(b) Event Oriented Network

FIG. 3.3

3.2. EVENT

3.2.1. Definition

The commencement or completion of an activity is called an event. An event is that particular instant of time at which some specific part of a plan has been or is to be achieved. More specifically an event is a specific definable accomplishment in a project plan, recognizable at a particular instant of time.

Examples :

DESIGN COMPLETED	: is an event
EXCAVATION COMPLETED	: is an event
LATHE INSTALLED	: is an event
PARTS ASSEMBLED	: is an event
EXCAVATE FOUNDATION	: is *not* an event
PIPE LINE LAID	: is an event

An *event* has three basic *properties* :

(*i*) An event is either the start or completion of an activity.

(*ii*) An event represents a noteworthy, significant and recognizable point in the project. Events act as control points in a project.

(*iii*) An event is an accomplishment occurring at an instantaneous point in time, but requiring no time or resources itself.

An event must satisfy the following *requirements* :

(*a*) A significant event must be positive, specific, tangible and meaningful to the project.

(*b*) It should be definitely distinguishable as a specific point in time.

(*c*) It should be readily understood by all concerned with the project.

3.2.2. Representation of Events

In a network diagram, events are represented by nodes. The shape of the nodes may be (*i*) circular, (*ii*) square, (*iii*) rectangular,

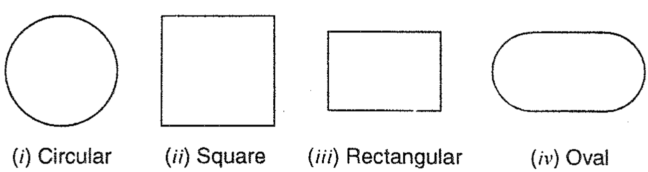


FIG. 3.4. WAYS OF REPRESENTING EVENTS.

(*iv*) oval, or (*v*) any other regular geometrical figure, as shown in Fig. 3.4.

In this book, events have been represented by a circular node.

Events are numbered for their identification. The number of an event is written inside the node or circle. Events may also be given verbal description whenever meaningful.

3.2.3. Specifying the Events

A particular event out of various events on the network diagram may be specified as :

1. Tail event
2. Head event
3. Dual role event.

1. Tail event

A *tail event* is the one which marks the beginning of an activity. If a particular tail event represents the commencement of the project, it is known as the *initial event*.

Fig. 3.5 (*a*) shows a tail event (No. 10), representing the beginning of a certain activity, while Fig. 3.5 (*b*) shows an *initial event* (No. 1) representing the commencement of the project. Fig. 3.5 (c) shows a tail events marking the *beginning* of three

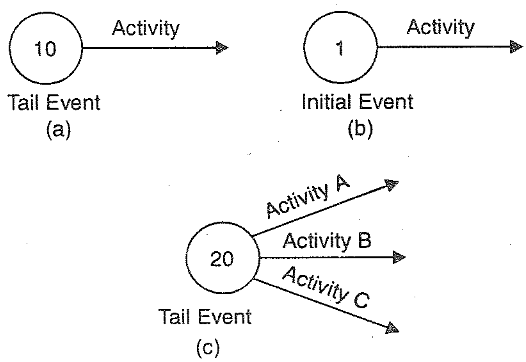


FIG. 3.5. TAIL EVENTS.

activities *A*, *B* and *C*. Suppose activity *A* commences at 6 units of time, *B* commences at 4 units of time and *C* commences at 7 units of time. Hence the *earliest occurrence time is 4 units of time. A tail event, representing the beginning of more than one activity, is said to occur when the first activity starts from it.*

2. Head event

All activities have an ending *i.e.* again a specific point of time and is marked by an event. Such an event is known as head event, because in a network diagram, it is connected to the head or barbed end of an arrow. If a particular head event marks the completion of the project it is known as the *final event* or *end event*.

Fig. 3.6 (*a*) shows a head event while Fig. 3.6 (*b*) shows a final or end event ; in each case they mark the completion of the activity.

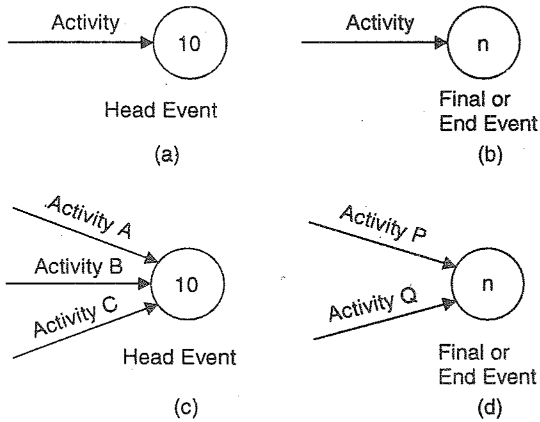


FIG. 3.6. HEAD EVENTS.

Fig. 3.6 (*c*) shows a head event, marking the completion of three activities. Similarly, Fig. 3.6 (*d*) shows a final or end event, having two activities ending in it.

When a head event occurs at the end of more than one activity, *the event is said to have occurred when all activities leading to it are completed*. For example, if activity *A* [Fig. 3.6 (*c*)] is completed at 20 units of time, activity *B* is completed at 16 units of time and activity *C* is completed at 22 units of time, the *earliest occurrence time* for event 10 is 22 units of time.

3. Dual role events

Actually, most of the events serve dual function *i.e.*, they are head event to some activity and tail event to other activity. *All events except initial and final events are dual role events.*

Thus, in Fig. 3.7 (*a*), event 11 is *head event* for activity *A* and *tail event* for activity *B*. Similarly, in Fig. 3.7 (*b*), event 26 is *head event* for activities *A*, *B* and *C*, while it is tail event for activities *P* and *Q*.

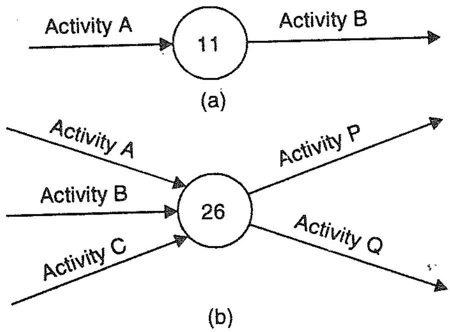


FIG. 3.7. DUAL ROLE EVENTS.

Another example is given in Fig. 3.8, in which :

Event 1 is	:	(*i*) initial event
		(*ii*) tail event for activities *A* and *B*.
Event 2 is	:	(*i*) head event for activity *A*
		(*ii*) tail event for activity *D*.
Event 3 is	:	(*i*) head event for activity *B*
		(*ii*) tail event for activities *C* and *E*.
Event 4 is	:	(*i*) head event for activity *C*
		(*ii*) tail event for activity *F*.
Event 5 is	:	(*i*) head event for activities *D, E, F*
		(*ii*) final or end event.

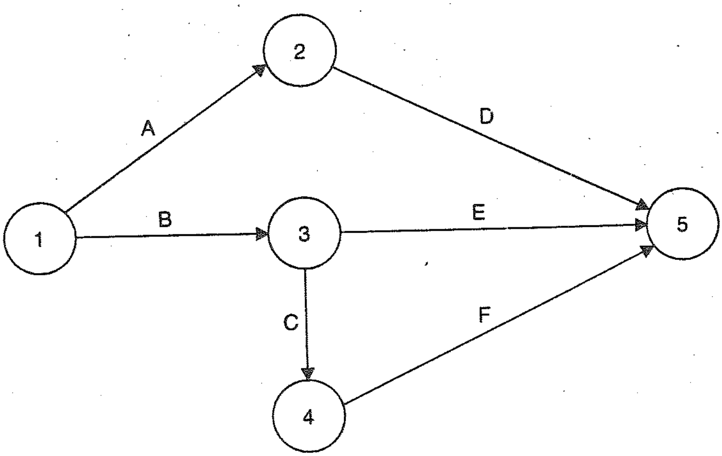


FIG. 3.8

3.2.4. Inter-relationship between Events

The completion of a project, which has been split into a number of activities, passes through a number of events. These events must occur at definite time and in a particular sequence or order. In preparing event oriented network diagram, one must think of the following questions regarding the sequence of events :

(*i*) What event or events must be completed before a particular event under consideration can be started ?

(*ii*) What event or events must follow the particular event under consideration ?

(*iii*) What activities can be accomplished simultaneously ?

The order or sequence relates various events as

(*a*) Successor events

(*b*) Predecessor events.

Successor events

The event or events that follow another event are called *successor events* to that event. Also, the event or events that immediately follow another event without any intervening ones are called *immediate successor events* to that event. In most of the cases, the successor events that are of greater concern are the immediate successor events ; some authors prefer to call the '*immediate successor event*' simply as '*successor event*'.

Predecessor events

The event or events that occur before another event are called predecessor events to that event. Also, the event or events that immediately come before another event without any intervening ones are called *immediate predecessor events* ; some authors prefer to call these simply as predecessor events.

As an illustration, consider network of Fig. 3.9.

(*i*) Events 2, 3, 4 and 5 are *successor events* to event 1.

(*ii*) Events 2, 3 and 4 are *immediate successor events* to event 1.

(*iii*) Event 5 is the *immediate successor event* to events 2, 3 and 4 each.

(*iv*) Events 1, 2, 3 and 4 are *predecessor events* to event 5.

(*v*) Events 2, 3 and 4 are *immediate predecessor events* to event 5.

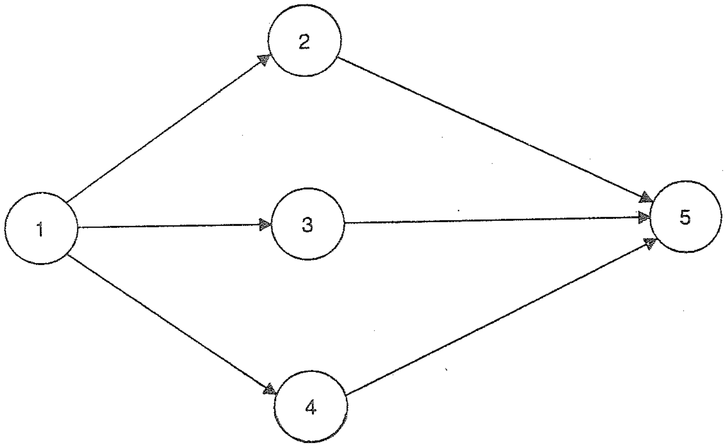


FIG. 3.9

(*vi*) Event 1 is *immediate predecessor event* to events 2, 3 and 4, each.

3.3. ACTIVITY

3.3.1. Definition

An *activity* is the actual performance of a task. It is the work required to complete a specific event. An activity is a recognizable part of a work project that requires *time* and *resources* (manpower, material, space, facilities etc.) for its completion.

Example :

EXCAVATE TRENCH	: is an activity
MIX CONCRETE	: is an activity
PREPARE SPECIFICATIONS	: is an activity
ASSEMBLE PARTS	: is an activity
LATHE INSTALLED	: is *not* an activity
DESIGN COMPLETED	: is *not* an activity
PREPARE BUDGET	: is an activity

A significant activity must be :

(*a*) a positive, specific, tangible and meaningful effort.

(*b*) such that the primary responsibility of effort can be determined, (*c*) having a description understandable by all concerned with the project, and

(*d*) having a time span.

3.3.2. Representation and Identification

In a network diagram, activities are represented by simple arrows, usually drawn from left to right. The length of arrow does neither represent the magnitude of work involved nor the time required for its completion. It is thus not a vector quantity. The length of the arrow is chosen to suit the drafting convenience.

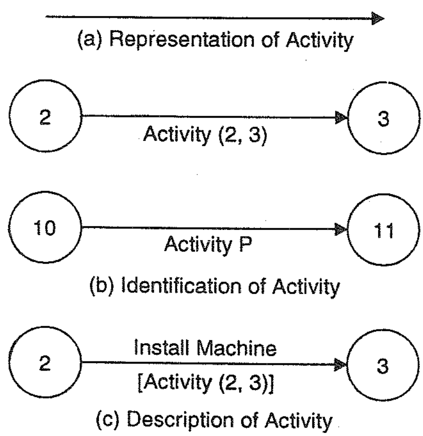


FIG. 3.10. ACTIVITIES.

The identification or description of an activity can be done in either of the following ways :

(*a*) The activities can be identified in terms of the events they connect, by the use of event numbers. Thus, in Fig. 3.10 (*b*), the activity connecting events 2 and 3, is designated as activity (2, 3).

(*b*) Activity can be identified by use of English alphabets, such as activity *P* in Fig. 3.10 (*b*). Such an identification must clearly define or describe the activity. For example :

ACTIVITY A	: excavate trench
ACTIVITY B	: fix side boards
ACTIVITY C	: concrete foundations
ACTIVITY P	: construct roof.

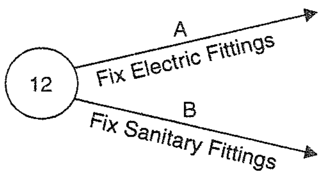
(*c*) Activity can also be described by writing actual performance over the arrow. For example, in Fig. 3.10 (*c*), the activity joining events 2 and 3 may be described as '*install machine*' by writing on the arrow.

3.3.3. Inter-relationships

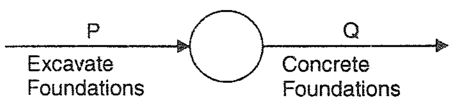
A project may consist of a number of activities or jobs. Depending upon the interdependency, we can categorise activities as (*i*) parallel activities and (*ii*) serial activities.

Parallel activities

Those activities which can be performed simultaneously and independently to each other are known as parallel activities. For example, in Fig. 3.11 (*a*), activities *A* and *B* are parallel activities since they can be taken up concurrently and executed simultaneously.



(a) Parallel Activities



(b) Serial Activities

FIG. 3.11. PARALLEL AND SERIAL ACTIVITIES.

Serial activities

Serial activities are those which are to be performed one after the other, in succession. These activities cannot be performed independently to each other. For example, activities *P* and *Q* in Fig. 3.11 are serial activities. Activity *Q* cannot be started, unless activity *P* is complete.

Activity *P* is known as *preceding activity*, while activity *Q* is known as *succeeding activity*, in relation to each other.

Predecessor activity

Activity or activities that are required to be performed before another job or activity can begin are called *predecessor activities* to that activity. The activity or activities that are required to be performed immediately before another activity, without an intervening activity are known as *immediate predecessor activities* to that activity.

Successor activity

Activity or activities that can be performed after the performance of other activity are known as *successor activities* to that activity. The activity or activities that immediately follows another activity, without any intervening activity are known as *immediate successor activities* to that activity.

Redundancy exists when among the number of predecessor activities of any given activity, one of the activity is a predecessor to some other activity in the same set. For example, the predecessor activities to activity P contain activities A, B and M, while activity M has A and B as predecessors. The activities A and B are *redundant* in the predecessors list of activity P, and these can safely be eliminated. In other words, jobs A and B are not immediate predecessor to P but are distant ones. *Hence predecessor list should contain only immediate predecessor only*; some writers prefer to call these immediate predecessors by simply 'predecessors'.

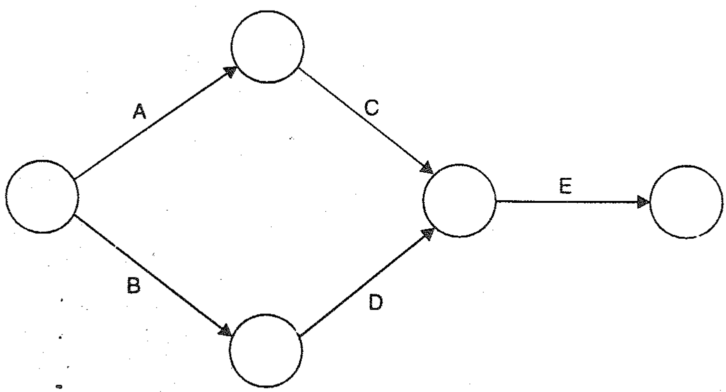


FIG. 3.12

As an example, consider network shown in Fig. 3.12. Table 3.1 gives the list of predecessor and successor activities to each activity.

Table 3.1

Activity	*Predecessor*	*Successor*
A	—	C^* ; E
B	—	D^* ; E
C	A^*	E^*
D	B^*	E^*
E	A ; C^* ; B ; D^*	—

Note. * Indicates *immediate* predecessor or successor.

3.4. DUMMY

A *dummy* is a type of operation in the network which neither requires any time nor any resources, but is merely a device to identify a *dependence* among operations. A dummy is thus a connecting link for control purposes or for maintaining uniqueness of activity.

A dummy is also represented by arrow ; but since it is not an activity, it is represented by dashed arrow. A dummy is identified by the numbers of the terminal node.

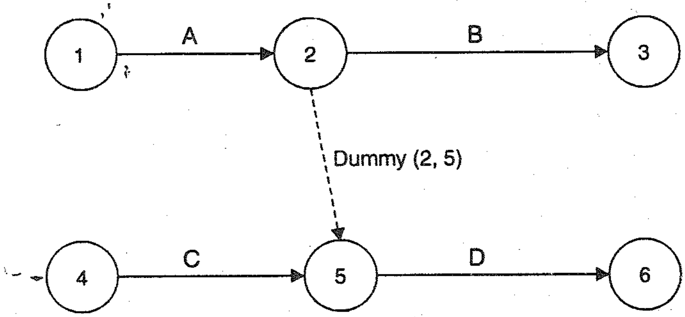


FIG. 3.13. DUMMY OPERATION.

For example, consider the two sets of activities shown in Fig. 3.13.

Set 1. *A*. A wait delivery of new machine.

B. Install new machine.

Set 2. *C*. Remove existing machine.

D. Dispose of existing machine.

Activities A and B are to be performed serially. Similarly, activities C and D are to be performed serially. Both the sets are performed simultaneously. However, from practical considerations, we find that activity D of set 2 cannot be performed unless activity A of set 1 is completed. Hence a *dummy link* is used, joining node 2 to node 5, indicating that activity D cannot be started unless event 2 is over.

Uses of dummies

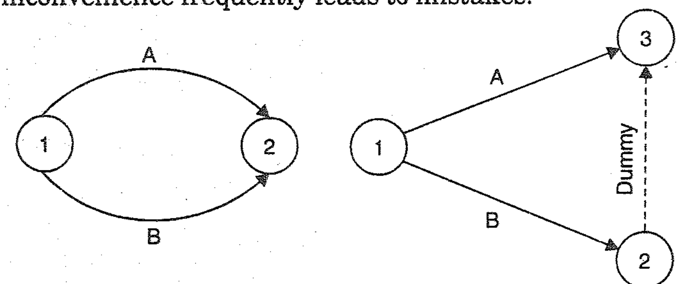
Dummies serve two purposes in a network :

(*a*) Grammatical purpose

(*b*) Logical purpose.

(*a*) **Grammatical purpose**

A dummy is used to prevent two arrows having common beginning and end points. For example, consider the arrows of activities A and B [Fig. 3.14 (*a*)] ; both start from node 1 and end at node 2. Due to this an inconvenience results when the network is used for computations, *i.e.*, uniqueness in the identification is lost. This inconvenience frequently leads to mistakes.



(a) Ambiguous Representation

(b) Grammatically Clear Representation

FIG. 3.14

This trouble can be avoided by using a dummy link as illustrated in Fig. 3.14 (*b*), giving thereby a grammatically correct and clear representation.

(*b*) **Logical purpose**

Dummies are also used to give logical clear representation in a network having an activity common to two sets of operations running parallel to each other. For example, consider two activities Q and R having common end node. Activity Q has O and P as successor activities, while activity R has P and N as successor activities. Fig. 3.15 (*a*) shows the illogical representation of the activities, because activity P cannot have *dual identity*. It should have unique identity.

This uniqueness can be maintained by introducing two dummies P_1 and P_2 as shown in Fig. 3.15 (*b*).

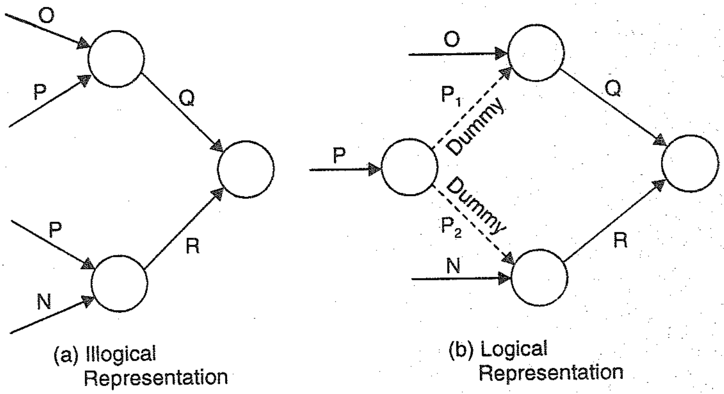


FIG. 3.15. USE OF DUMMIES.

Rules for Provision of Dummies

While planning a network, a natural question that arises is where to provide dummies. Provision of redundant dummies in the network may create confusion. For that, the simple rule is that during the *initial stage* of developing a network, *liberal use* of dummies should be made to fulfil the requirements of inter-relationships between various activities and between various sets of activities. This may result in the introduction of some unnecessary dummies which can be *removed* by the use of following rules :

1. If a dummy job is the only one emanating from its initial

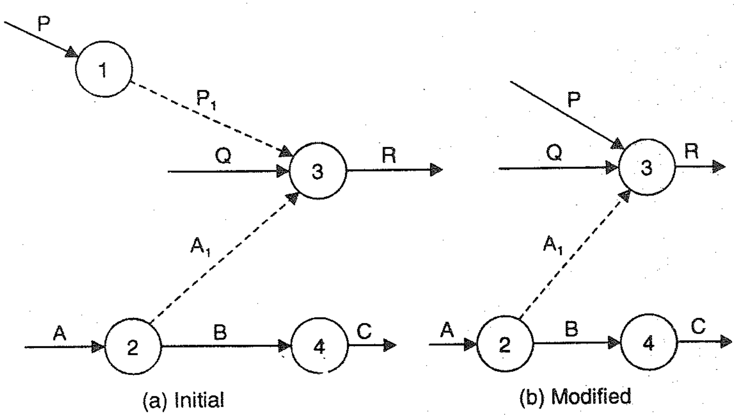


FIG. 3.16

node, it can be removed and the activity terminating at that node can be directly connected to that node to which the dummy was terminating.

For example, consider the initial drawing of the partial network shown in Fig 3.16 (*a*), the dummy P_1 is the only job emanating from its initial node ; it can therefore be removed and activity P can be directly connected to the forward node (3) as shown in Fig. 3.16 (*b*). It should be noted that the same treatment cannot be given to dummy A_1, since other jobs or activities (such as B) are also emanating from the same node.

2. If a dummy job is the only one terminating into a node, the dummy can be removed and the two node at the two ends of the dummy can be merged into one.

For example, activity A was initially joined to activities C, D and B by three dummies A_1, A_2 and A_3 respectively. Since dummy A_3 is the only one terminating into node 2, it can be removed, and nodes 1 and 2 situated at the two ends of dummy A_3 can be combined, as shown in Fig. 3.17 (*b*).

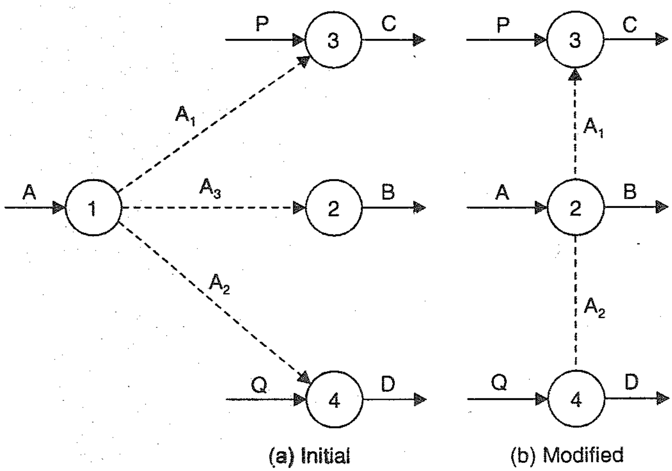


FIG. 3.17

3. If two or more activities, emanating from different nodes, have identical set of predecessors some of which also appear in different predecessor sets of other activities, the two activities

should emanate from a single node. This node can then be connected to their predecessor activities by dummies.

For example, consider a partial network situation shown in Fig. 3.18 (*a*), in which two activities *B* and *D* emanating from two different nodes 5 and 6 have identical sets of predecessors, some of which also appear in different predecessor sets of other activities. In such case, *B* and *D* can be made to emanate from a single node (5) to which the predecessor activities can be joined through dummies. This is shown in Fig. 3.18 (*b*) in which two dummies A_3 and C_3 have been completely eliminated.

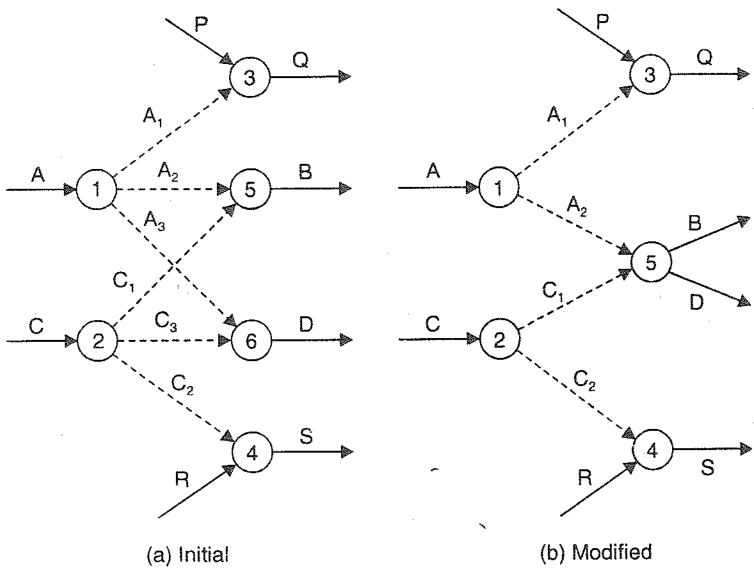


FIG. 3.18

4. If two or more activities, terminating into different nodes, have identical set of successors, the latter having other predecessors as well, the two activities should terminate into one single node. This node can then be connected to their successors through appropriate dummies.

For example, consider two activities *A* and *B*, terminating into two different nodes 1 and 2, as shown in a partial network situation of Fig. 3.19 (*a*). Each one of these have identical set of successors *Q* and *S* having their other predecessors *P* and *R* respectively. In such a case, both *A* and *B* can be made to terminate into one common node (2) and connected to their successors through dummies A_1 and B_1, as shown in Fig. 3.19 (*b*). Thus two dummies A_2 and B_2 have been eliminated.

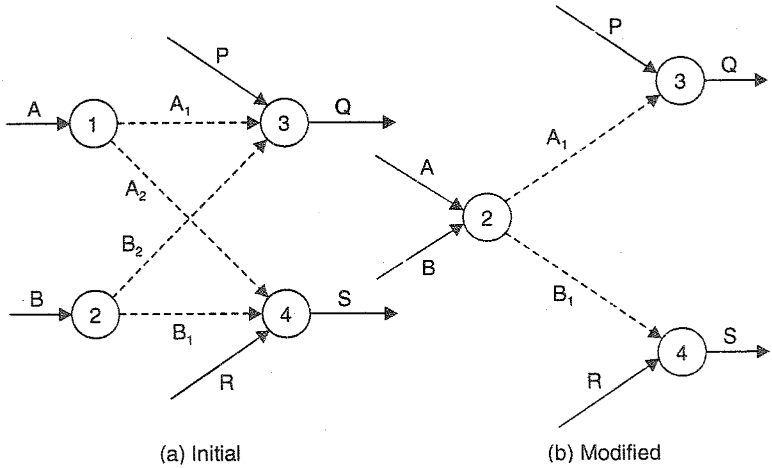


FIG. 3.19

5. Such dummies which are used to show predecessor relations already implied by other activities are known as *redundant dummies*, and can be removed.

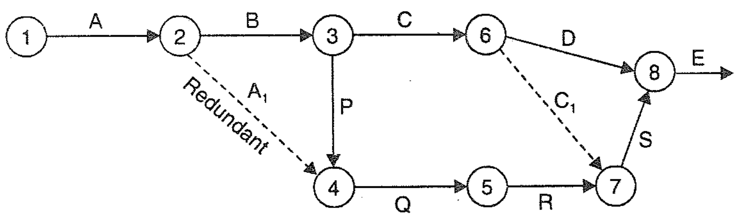


FIG. 3.20

For example, dummy A_1 used to show that activity A is predecessor to activity Q, is redundant since the predecessor relation is already implied by activity P. Hence A_1 can be removed. However, dummy C_1 is not redundant since it has been used to show that activity C is predecessor to activity S ; this predecessor relationship was not implied otherwise.

Example 3.1. *Separate out activities and events from the following list :*

1. Prepare budget.

2. Design completed.

3. *Lay railway track.*
4. *Cure concrete cubes.*
5. *Commence testing cubes.*
6. *Material received at site.*
7. *Distribute invitation cards.*
8. *Service reservoir filled.*
9. *Test pipe line.*
10. *Payment made.*
11. *Assemble parts of the machine.*
12. *Prepare estimate.*
13. *Survey the site.*
14. *Collect data and prepare check-list.*
15. *Show room inaugurated.*
16. *Install pump.*
17. *Drive piles for right pier of bridge.*
18. *Specifications prepared.*

Solution.

Activities : 1, 3, 4, 7, 9, 11, 12, 13, 14, 16, 17.

Events : 2, 5, 6, 8, 10, 15, 18.

Example 3.2. *Prepare a table showing immediate predecessor activities and events, predecessor activities and events, immediate successor activities and events and successor activities and events for various activities and events of the network shown in Fig. 3.21.*

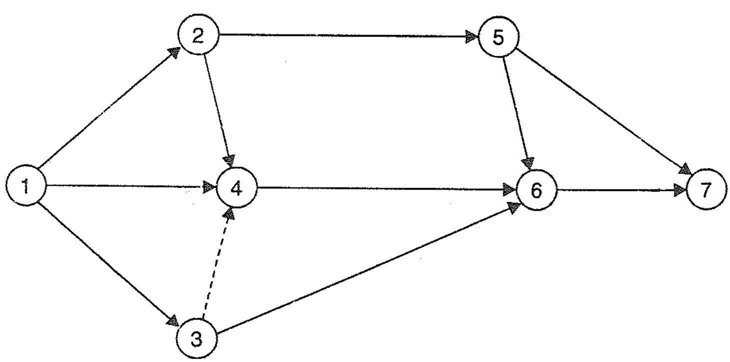


FIG. 3.21

Solution. Table 3.2.

Table 3.2

	Immediate predecessor	*Predecessor*	*Immediate Successor*	*Successor*
(*a*) **Activities**				
(1–2)	—	—	(2–5)	(2–5), (5–6), (6–7), (5–7)
(1–4)	—	—	(4–6)	(4–6), (6–7)
(1–3)	—	—	(3–6), (3–4)	(3–6), (6–7), (3–4), (4–6)
(2–4)	(1–2)	(1–2)	(4–6)	(4–6), (6–7)
(2–5)	(1–2)	(1–2)	(5–6), (5–7)	(5–6), (5–7), (6–7)
(3–4) Dummy	(1–3)	(1–3)	(4–6)	(4–6), (6–7)
(3–6)	(1–3)	(1–3)	(6–7)	(6–7)
(4–6)	(1–4), (2–4), (3–4)	(1–4), (2–4), (3–4), (1–2), (1–3)	(6–7)	(6–7)
(5–6)	(2–5)	(2–5), (1–2)	(6–7)	(6–7)
(5–7)	(2–5)	(2–5), (1–2)	—	—
(6–7)	(3–6), (4–6), (5–6)	(3–6), (4–6), (5–6), (1–3), (1–4), (1–2), (2–5), (2–4), (3–4)	—	—
(*b*) **Events**				
1	—	—	2, 3, 4	2, 3, 4, 5, 6, 7
2	1	1	4, 5	4, 5, 6, 7
3	1	1	4, 6	4, 6, 7
4	1	1	6	6, 7
5	2	2, 1	6, 7	6, 7
6	5, 4, 3	5, 4, 3, 2, 1	7	7
7	5, 6	5, 6, 2, 4, 3, 1	—	—

Example 3.3. *Introduce dummies in the network shown in Fig. 3.22, to identify each activity uniquely.*

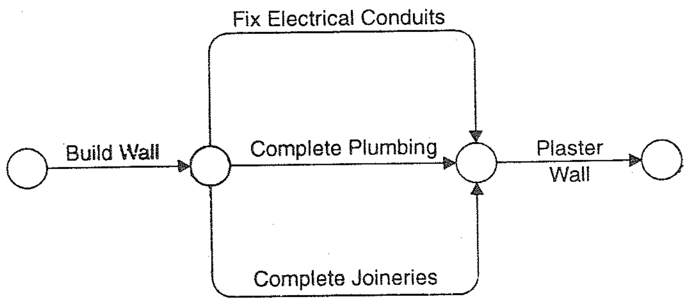


FIG. 3.22

Solution. In the above network, we find that the three activities (fix electrical conduits, complete plumbing and complete joineries) start from one common node and end at some other common node. This gives ambiguous representation, and is grammatically incorrect. Hence, two dummies A_1 and A_2 may be introduced as shown in Fig. 3.23.

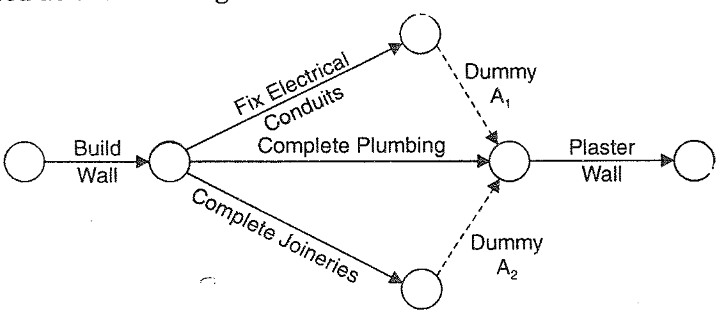


FIG. 3.23

Example 3.4. *In a retaining wall construction, A, B, C represent the shuttering operations for three bays of the wall and P, Q, R represent corresponding concreting operations. Assume that in each case shuttering has to precede concreting. Only one crew for form work operation and another crew for concreting operation is available. Using dummies, show the network, representing the restraints and the consequent interdependencies.*

Solution. The network is shown in Fig. 3.24. Four dummies D_1, D_2, D_3 and D_4 have been used to show the interdependencies of the operations.

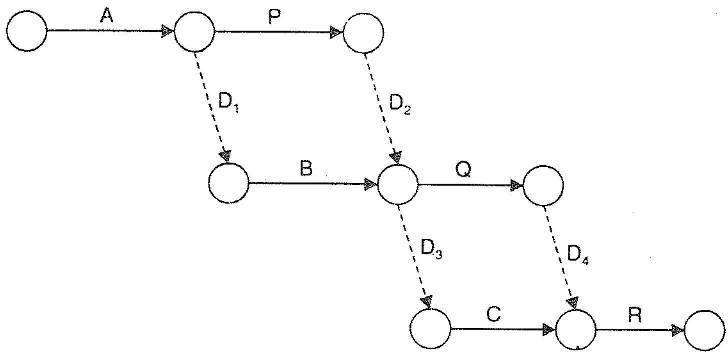


FIG. 3.24

3.5. NETWORK RULES

The following network rules are noteworthy :

1. Initial node has only outgoing arrows. There must be only single initial node in a network.

2. An event cannot occur until all the activities leading to it are completed.

3. An event cannot occur twice, *i.e.* there cannot be any network path looping back to previously occurred event. No event depends, for its occurrence upon the occurrence of a succeeding event. Thus, the network shown in Fig. 3.25 is wrong.

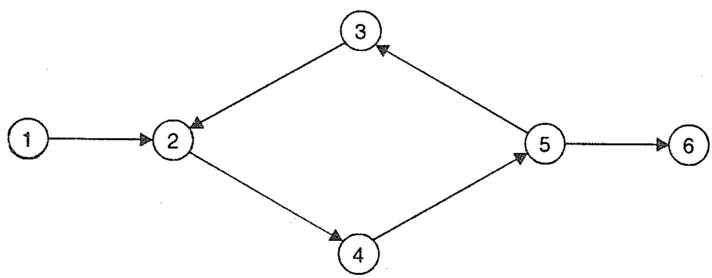


FIG. 3.25. INCORRECT NETWORK.

4. There must not be any dead end left except the final node. Final node has only incoming arrows. There must be only single final node. Thus, the network shown in Fig. 3.26 is wrong because there are two final nodes.

5. No activity can start until its tail end event (preceding event) has occurred.

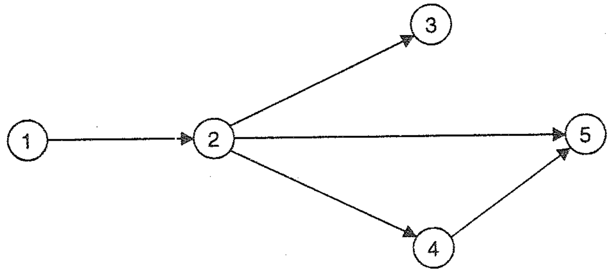


FIG. 3.26. INCORRECT NETWORK.

6. Any arrow should represent singular situation, *i.e.* individuality and separate entity of an activity must be maintained in a network diagram. Particular arrow can emanate from a single event only. Number of arrows should be equal to number of activities in the project. Thus, the network shown in Fig. 3.27 is wrong since activity *P* has two arrows.

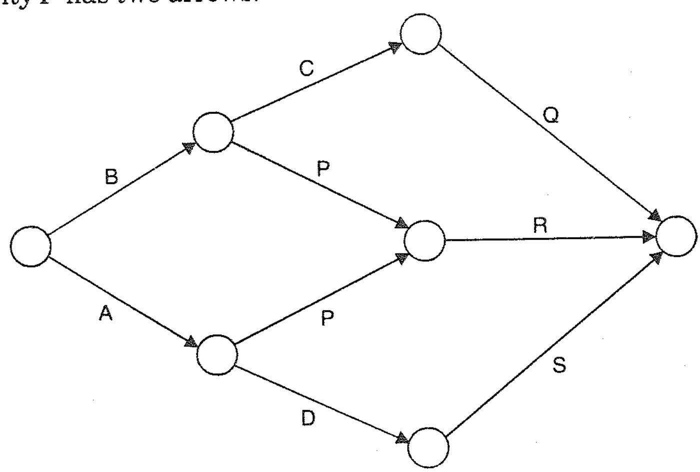


FIG. 3.27. INCORRECT NETWORK.

7. Representation of the network should be such that every activity is completed to reach the end objective.

8. All constraints and interdependencies should be shown properly on the network by use of appropriate dummies.

9. Logic of network should always be maintained, *i.e.* arrow heads point correct way to indicate the true control situation.

10. It is usual practice to show the time flow from left to right.

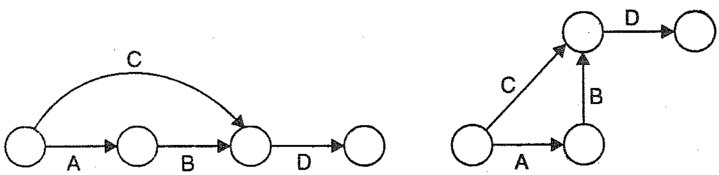
3.6. GRAPHICAL GUIDELINES FOR NETWORK

The following guidelines should be followed for better drawing of network diagrams :

1. Arrows are not vectors. They are never used to indicate duration (or time) through its length. All arrows should be of nearly equal size wherever possible, *i.e.* wide variation is length of arrows should be avoided. Length should be chosen to suit drafting requirements.

2. Orientation or angle between arrows should be chosen to suit drafting convenience. The angle between arrows leaving or joining nodes, should be as large as possible so that more space is available for the addition of other relevant information in the diagram.

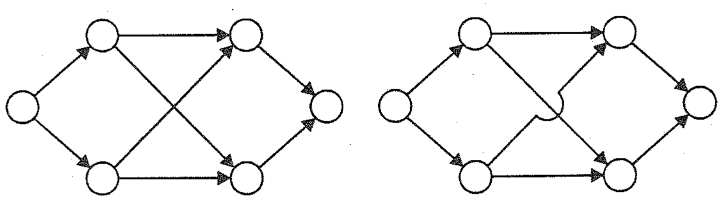
3. As far as possible, straight arrows should be used. Curved arrows are not preferred.



(a) Curved Arrows (b) Straight Arrows

FIG. 3.28

4. Arrows should normally not cross each other. If this crossing is not avoidable, length of the arrow should be broken to bridge over the other, as shown in Fig. 3.29.

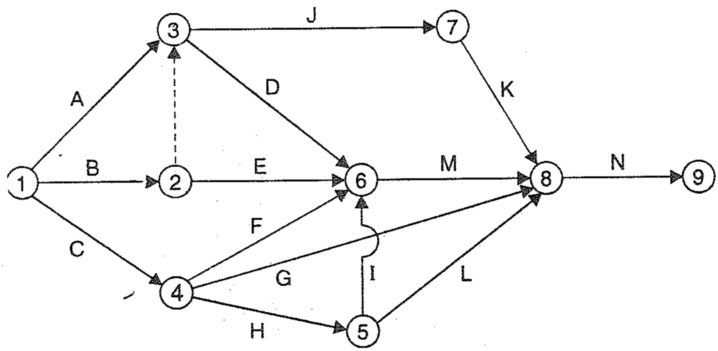


(a) Not Desirable (b) Bridging

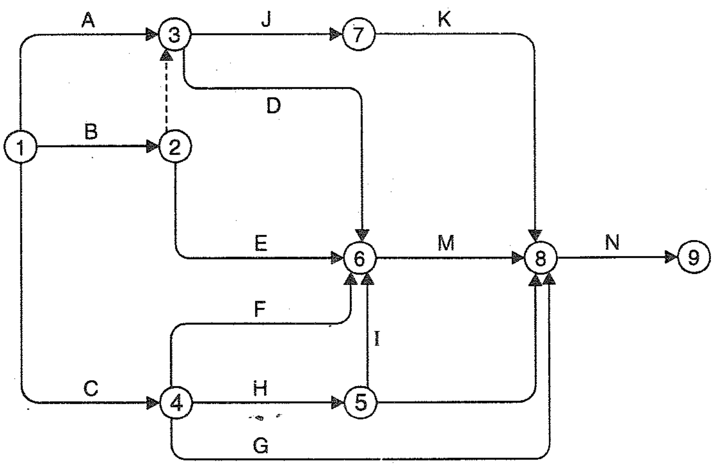
FIG. 3.29

Rectangular Network Diagram

The graphical guidelines given above are applicable for an *angular* network diagram in which the arrows are at an angle with horizontal or vertical direction. Some people prefer a cigar-shaped or *rectangular network diagram* in which the arrows are either horizontal or vertical. In such a situation, an arrow may, sometimes, be required to give a right angled turn to obtain requisite 'flow' in the network diagram. The network diagram so obtained is always very compact. However, the angular network diagram, using straight arrows gives a better understanding of the various activities of the project. Fig. 3.30 shows the two types of network diagrams for the same project.



(a) Angular Network Diagram



(b) Rectangular Network Diagram

FIG. 3.30

3.7. COMMON PARTIAL SITUATIONS IN NETWORK

Fig. 3.31 gives some common partial situations in a network.

Partial Situation

1. B is controlled by A. Operation B cannot begin until operation A is completed.

2. C is controlled by A and B. Operation C cannot begin until operations A and B are completed.

3. Activities B and C are controlled by activity A. Neither of activities B and C can start unless A is completed.

4. Activities C and D are controlled by activities A and B. Neither of activities C and D can start until A and B are completed. However, C and D can be started independent of each other.

5. Activity B is controlled by A and C. However, activity D is controlled by activity C only.

6. Activity D is controlled by A and B, while activity E is controlled by activity B and C.

7. Activity D is controlled by A, B and C. However, activity E is controlled by B and C.

Representation

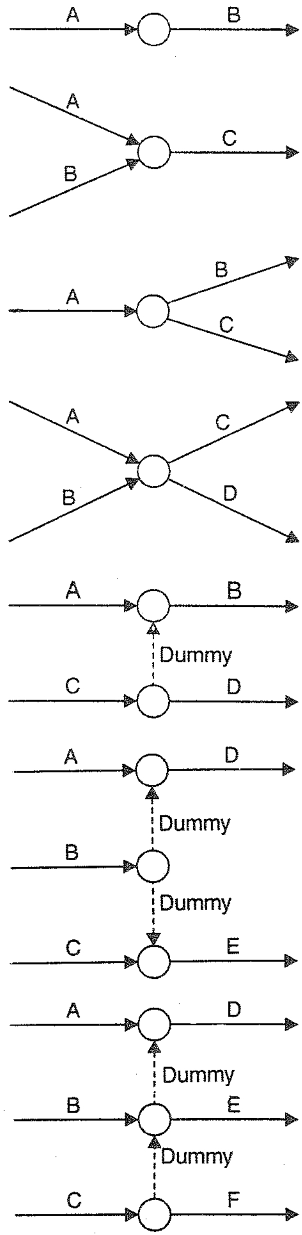
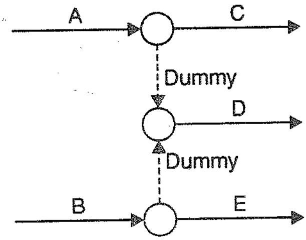


FIG. 3.31 (*a*)

Partial Situation **Representation**

8. Activity A controls C and D, while activity B controls D and E. Thus, D is controlled by both A and B.



9. Activity X is controlled by D and A ; activity Y is controlled by A, B and C, while activity Z is controlled by D only.

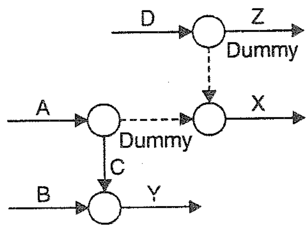


FIG. 3.31 (*b*)

3.8. NUMBERING THE EVENTS

It is essential to number the events or node points. The activities joining the nodes can better be identified on the network by the event numbers or node numbers at the tail and head of the activity. The event numbering should be scientifically done so that they reflect their logical sequence. In a big network, the problem of numbering can be simplified if the rules devised by D.R. Fulkerson are followed. The sequential numbering to the events may be assigned in the following steps :

1. There is a single initial event in a network diagram. This initial event will have arrows coming out of it and none entering it. Number this initial event as 1.

2. Neglect all the arrows emerging out of the initial event numbered 1. Doing so will apparently provide one or more *new* initial events.

3. Number these apparently produced new initial events as 2, 3, 4 etc.

4. Again neglect all emerging arrows from these numbered events ; this will create few more initial events.

5. Follow step 3.

6. Continue this operation until the last event, which has no emerging arrows, is numbered.

Skip Numbering

As a rule, a tail event must have a lower number than the head event. In bigger networks, where extensive modifications are frequently required to be made, *renumbering* can be avoided by numbering the events in the multiple of 10, *i.e.*, numbering the events as 10 (initial), 20, 30, 40 etc. If an event is added later, it can be assigned a number (such as 31, say) which lies between the number of immediate predecessor event and immediate successor event. This process of numbering is known as *skip numbering*. Alternatively, skip numbering can also be achieved by leaving out successive number such as (8, 9), (18, 19), (28, 29) etc. from the numbering of initial network. These left out numbers can later be assigned to newly added events arising out of the modification of the initial network.

Example 3.5. *Using Fulkerson rule, number the events of the network shown in Fig. 3.32.*

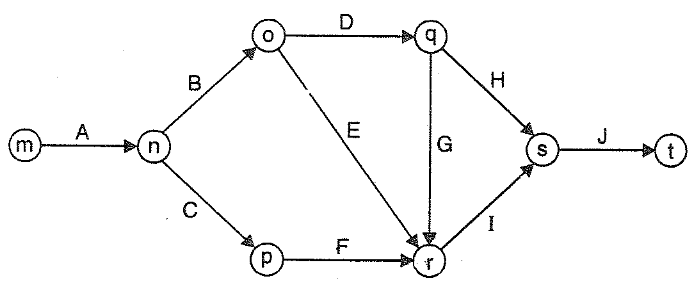


FIG. 3.32

Solution.

1. Event m is the initial event ; hence number it as 1.

2. Neglect the arrow (A) emerging out of the initial event. Due to this event n will be the new initial event. Number it as 2.

3. There are two arrows B and C emerging out of event No. 2. By neglecting these, two more new initial events are obtained at nodes o and p ; number these as 3 and 4 respectively.

4. Consider event 3. Neglect two arrows D and E emerging out of it. This will result in a new event at nodes q ; assign number 5 to it. Note that event r will still have arrows entering to it.

5. Consider event 5 and neglect arrows G and H coming out of it. This will result in two new initial events at nodes r and s. Since node r is predecessor event to node s, number it as 6. Number node s as 7.

6. Finally, event at node t has no arrow emerging out of it. Hence number this final event as 8.

The numbered network diagram is shown in Fig. 3.33.

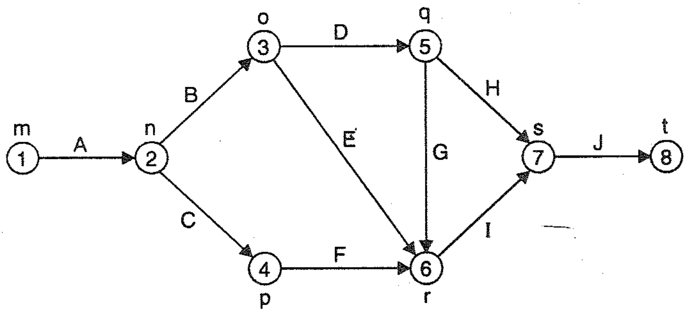


FIG. 3.33

Example 3.6. *Draw the network for a project having four activities labelled A, B, C and D, and related as below :*

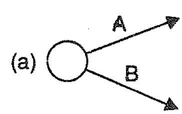
(i) Activity A and activity B can be done concurrently.

(ii) Activity A is the immediate predecessor of activity C, and so is the relation between B and D.

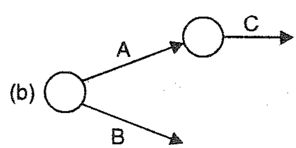
(iii) Accomplishment of C and D marks the completion of the project.

Solution. Arrow network diagram can be drawn step by step as illustrated below (Fig. 3.34) :

(*i*) Activity A and B being done concurrently. Fig. 3.34 (*a*).

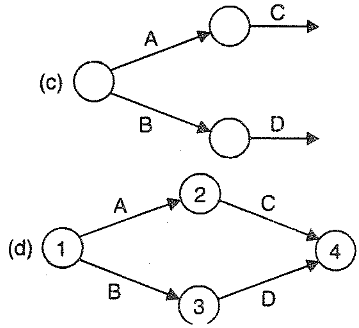


(*ii*) Activity C is the immediate successor activity A. Fig. 3.34 (*b*).



(*iii*) Activity D is the immediate successor to activity B. Fig. 3.34 (*c*).

(*iv*) Accomplishment of C and D marks the completion of the project. Fig. 3.34 (*d*).



FINAL DIAGRAM
FIG. 3.34

Thus, final network is shown in Fig. 3.34 (*d*), after numbering the events.

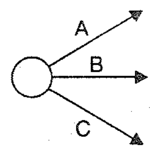
Example 3.7. *Draw the network of a project having seven activities. Activities A, B and C run concurrently. Activities predecessor relationships are as follows :*

Activity	*Immediate Predecessor*
D	A
E	B
F	C

Activity G is the last operation of the project, and is also immediate successor to D, E and F.

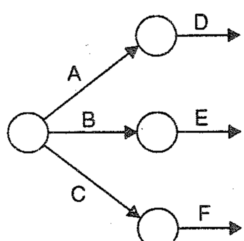
Solution. The network diagram can be developed step by step as follows (Fig. 3.35) :

(*i*) Activities A, B and C run in parallel [Fig. 3.35 (*a*)].



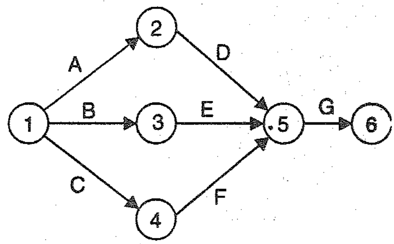
(a)

(*ii*) Activities D, E and F are immediate successors to activities A, B and C respectively [Fig. 3.35 (*b*)].



(b)

(*iii*) The last activity *G* is immediate successor to *D*, *E* and *F* [Fig. 3.35 (*c*)].



(c) Final Net Work

FIG. 3.35

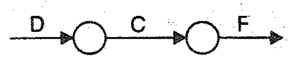
Thus, Fig. 3.35 (*c*) shows the final network, after properly numbering the events.

Example 3.8. *Draw a network diagram for the project having 9 activities, with the following inter-relationships :*

(*i*) *C follows D but precedes F.*
(*ii*) *C follows B but precedes H.*
(*iii*) *G follows F but precedes I.*
(*iv*) *E follows A but precedes I.*
(*v*) *D follows A.*
(*vi*) *H and I terminate at the same time.*
(*vii*) *A and B start at the same time.*

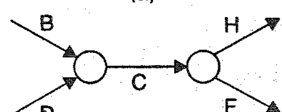
Solution. The network diagram can be developed step by step as under (Fig. 3.36).

(*i*) *C* follows *D* but precedes *F* [Fig 3.36 (*a*)].



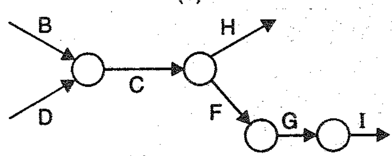
(a)

(*ii*) *C* follows *B* but precedes *H* [Fig. 3.36 (*b*)].



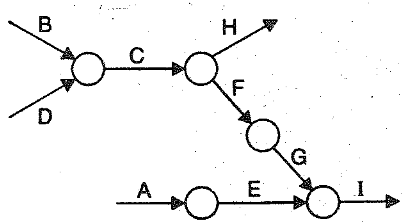
(b)

(*iii*) *G* follows *F* but precedes *I* [Fig. 3.36 (*c*)].



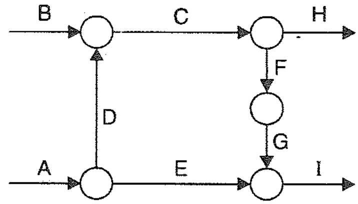
(c)

(*iv*) *E* follows *A* but precedes *I* [Fig. 3.36 (*d*)].



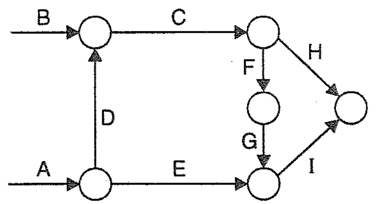
(d)

(*v*) *D* follows *A* [Fig. 3.36 (*e*)].



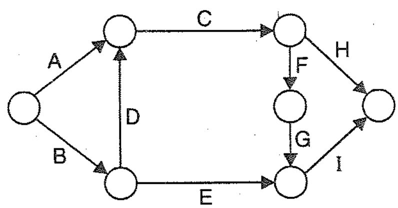
(e)

(*vi*) *H* and *I* terminate at the same time [Fig. 3.36 (*f*)].



(f)

(*vii*) *A* and *B* start at the same time [Fig. 3.36 (*g*)].



(g)

FIG. 3.36

The final network diagram obtained in Fig. 3.36 (*g*) is shown in Fig. 3.37 after numbering the events.

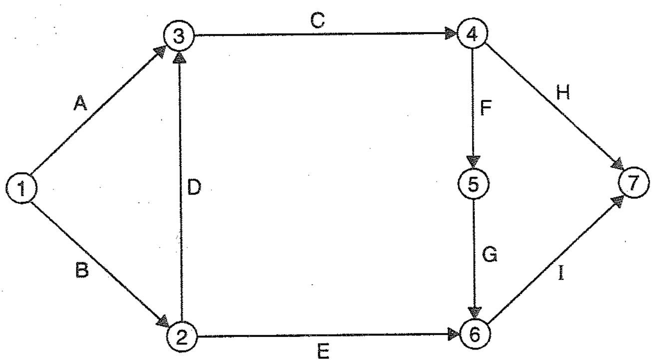


FIG. 3.37

Example 3.9. *A project consists of six activities (jobs) designated from A to F, with the following relationships :*

(*i*) *A is the first job to be performed.*

(*ii*) *B and C can be done concurrently, and must follow A.*

(*iii*) *B must precede D.*

(*iv*) *E must succeed C, but it cannot start until B is complete.*

(*v*) *The last operation F is dependent on the completion of both. Draw the network diagram.*

Solution. The step by step development of the diagram is shown in Fig. 3.38. The final diagram has its events numbered.

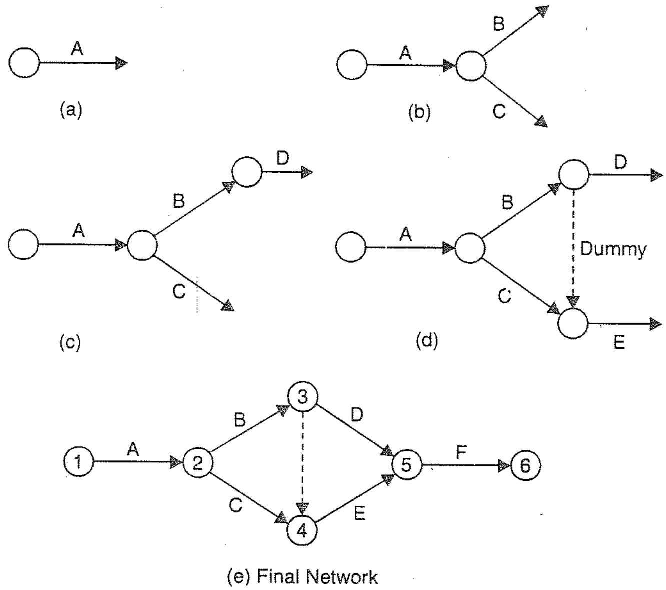


FIG. 3.38

Example 3.10. *The maintenance project of a building consists of ten jobs. The predecessor relationships are identified by their node numbers, as indicated below :*

Job	*Identification*	*Job*	*Identification*
A	(1, 2)	*F*	(4, 5)
B	(2, 3)	*G*	(4, 7)
C	(2, 4)	*H*	(5, 8)
D	(3, 6)	*I*	(6, 8)
E	(3, 5)	*J*	(7, 8)

Draw the network diagram for the project.

Solution. The step by step development of the network is given in Fig. 3.39.

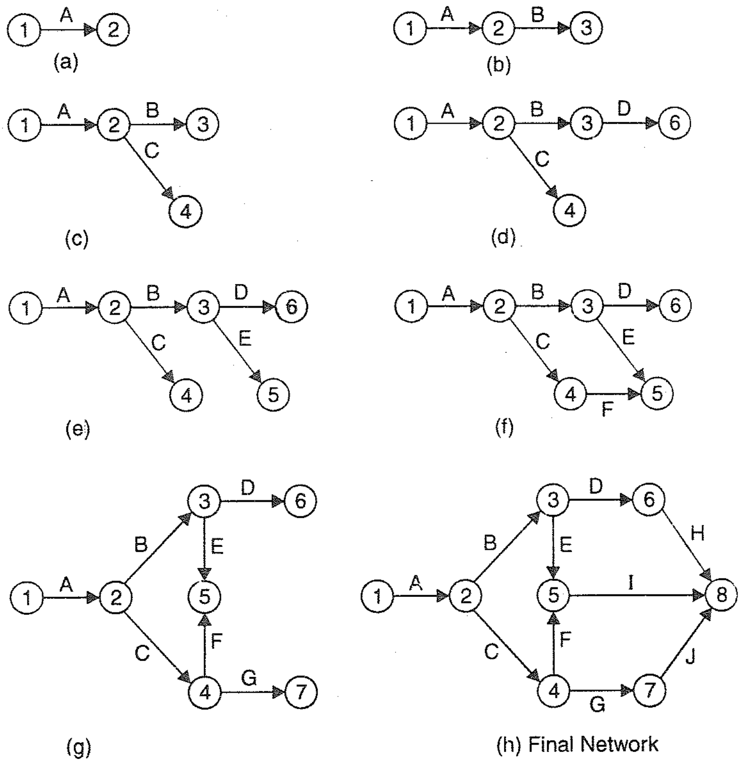


FIG. 3.39

Example 3.11. *A project plan consisting of ten events have predecessor relationships as under :*

Event	*Immediate Predecessor*	*Event*	*Immediate Predecessor*
1	—	*6*	*3, 5*
2	*1*	*7*	*3, 4*
3	*2*	*8*	*3, 7*
4	*2*	*9*	*7*
5	*2*	*10*	*3, 6, 8, 9*

Draw the network diagram for the project plan.

Solution. The step by step development of the network is shown in Fig. 3.40.

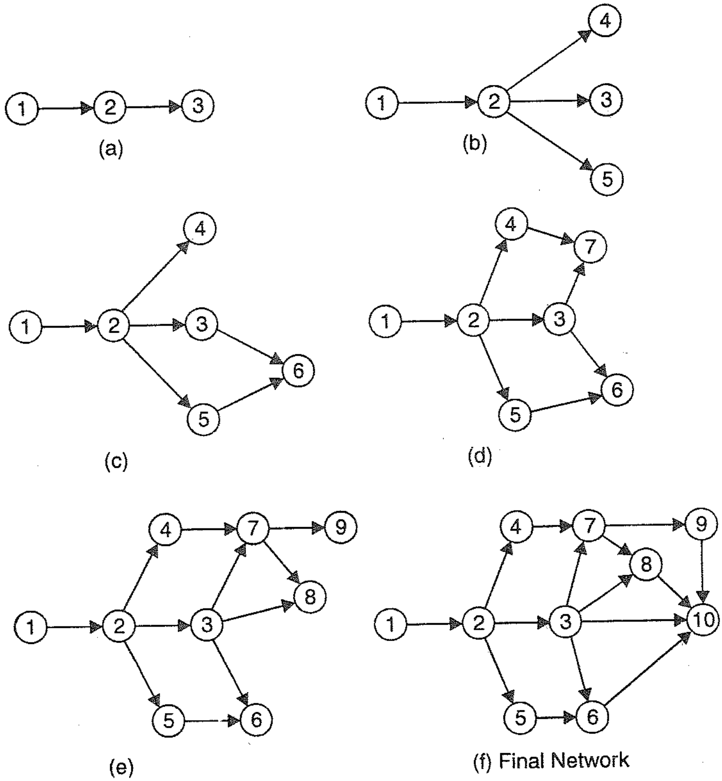


FIG. 3.40

Example 3.12. *A project has fourteen activities A through M. The relationships which obtain amongst these activities are given below.*

(i) A is the first operation.

(ii) B and C can be performed in parallel and are immediate successor to A.

(iii) D, E and F follow B.

(iv) G follows E.

(v) H follows D, but it cannot start until E is complete.

(vi) I and J succeed G.

(vii) F and J precede K.

(viii) H and I precede L.

(ix) M succeeds L and K.

(x) The last operation N succeeds M and C.

Construct the network diagram.

Solution. The step by step development of the network is shown in Fig. 3.41. Note that a dummy operation has been introduced in Fig. 3.41 (*c*) to fulfil the requirements mentioned in (*v*) above. In the final diagram [Fig. 3.41 (*e*)], the events have been numbered according to Fulkerson's rule.

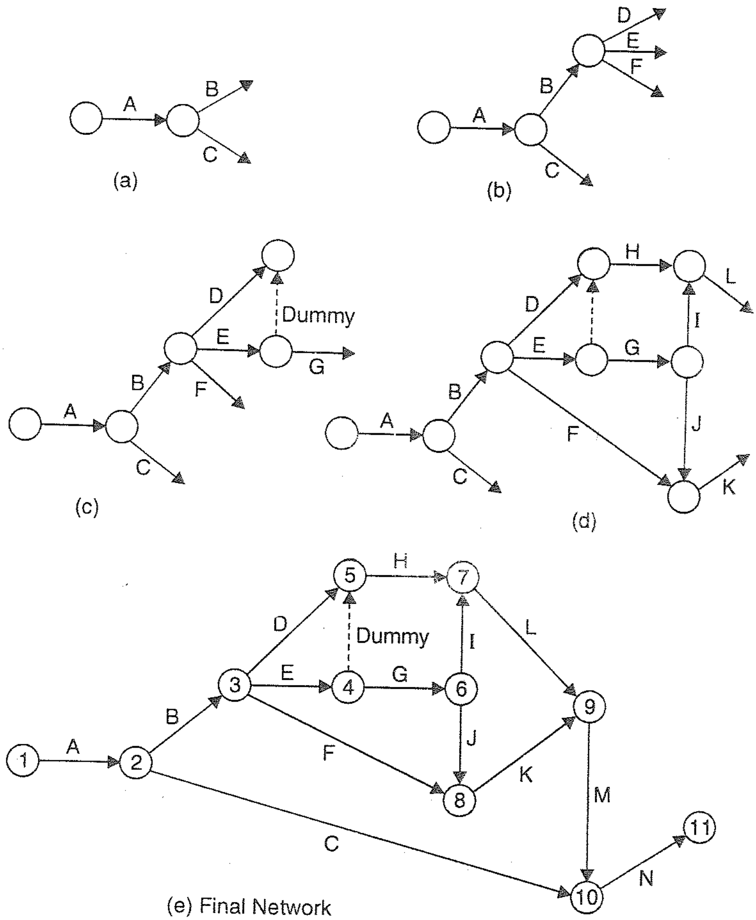


FIG. 3.41

3.9. CYCLES

A *loop network* or *cycle* is any path of activities that leads back into itself. In such a circumstance, an activity shows up a distant predecessor of itself. Cycles cause logical errors in the network. Such a situation *may* occur in a complicated network ; however, cycles should be removed before network calculations are made.

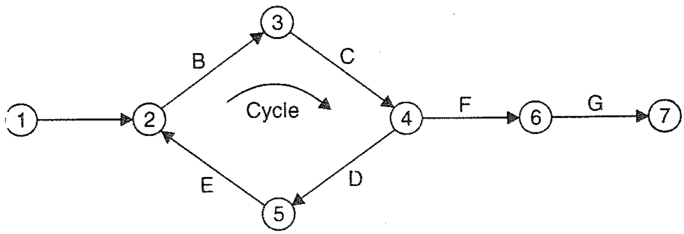


FIG. 3.42. CYCLE.

In Fig. 3.42, we find a cycle between nodes 2, 3, 4 and 5. Event 2 cannot take place unless activity E is complete. Activity E depends upon the completion of event 5. However, event 5 depends upon the completion of event 2. Thus, a loop or cycle is obtained. Such a cycle can be very easily located by numbering the events in logical sequence. Cycles are more serious than simple redundancies. Once a cycle is discovered, the person who compiled the project data must recheck for accuracy of the stated predecessor relationships of jobs in the cycle.

PROBLEMS

1. Define an 'event' and an 'activity'. Differentiate clearly between the two.
2. Differentiate between PERT network and CPM network. Illustrate your answer by drawing the two types of networks for a project.
3. What do you understand by a 'dummy' ? What are its uses ?
4. Choose 'Events' and 'Activities' from the following list :

   (*a*) Survey the site.

   (*b*) Maps prepared.

   (*c*) Invitations mailed.

   (*d*) Print the minutes.

   (*e*) Audit the accounts.

   (*f*) Fabricate screws.

   (*g*) Office inaugurated.

   (*h*) Start interior decoration.

   (*i*) Electrical design completed.

   (*j*) Assemble parts.
5. Define 'head event', 'tail event', 'dual role event', 'successor event' and 'predecessor event'.
6. Write notes on (*a*) redundancies and (*b*) cycles, with reference to a network.

7. Discuss various rules for providing dummies in a network. What are redundant dummies ?
8. Explain Fulkerson's rules for numbering the events of a network.
9. Discuss various network rules.
10. For the network shown in Fig. 3.43, prepare a table showing a list of predecessors, immediate predecessors, successors and immediate successors to each of the events.

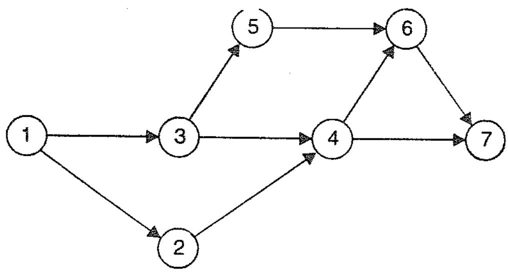


FIG. 3.43

11. A project consists of three operations E, L and F. The network (Fig. 3.44) shows the sequence.

FIG. 3.44

Each of the three operations can be completed in three sections. Draw a network, assuming that the completion of one section in an operation allow beginning of the next operation.

12. Using Fulkerson's rule, number the events of the network shown in Fig. 3.45.

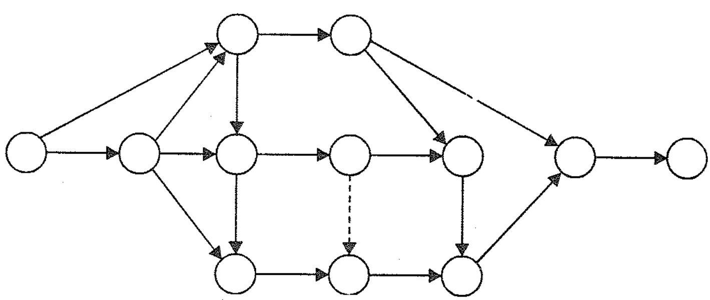


FIG. 3.45

13. Using Fulkerson's rule, number the events of the network shown in Fig. 3.46.

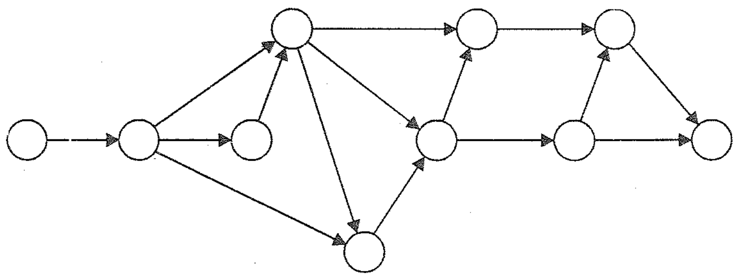


FIG. 3.46

14. A project consists of 16 activities having their predecessor relationship as follows :

(i) A is the first activity of the project.

(ii) B, C and D follow A and can be done concurrently.

(iii) E and G cannot begin until C is completed, and can be performed simultaneously.

(iv) F is the immediate successor to activities B and E.

(v) H and K run in parallel, and both succeed G.

(vi) L succeeds F and H.

(vii) I and J are immediate successor activities to activity D.

(viii) M and N are immediate successor to I and K. However, both M and N can be performed concurrently.

(ix) Activities O and P are the last activities. Activity O is the immediate successor to N and L. Activity P is the immediate successor to M and J. Draw the network and number the events.

15. A construction project consists of 12 activities. The predecessor relationships are identified by their node numbers as indicated below :

Activity	*Identification*	*Activity*	*Identification*
A	(1, 2)	G	(4, 6)
B	(2, 4)	H	(5, 6)
C	(2, 3)	I	(5, 7)
D	(2, 7)	J	(7, 8)
E	(3, 4)	K	(6, 8)
F	(3, 5)	L	(8, 9)

Draw the network diagram.

16. A project consists of eight activities M, N, O, P, Q, R, S and T. Draw the network and number the events if :

(a) Activities M, N and Q can start concurrently.

(*b*) Activities O and P are concurrent, and depend on the completion of both M and N.

(*c*) Activities R and S are concurrent and depend on the completion of O.

(*d*) Activity T depends upon the completion of P, Q and R.

(*e*) The project is complete when S and T are done.

17. A project consists of eight events having predecessor relationships as under :

Event	*Immediate Predecessor*	*Event*	*Immediate Predecessor*
1	—	5	3, 4
2	1	6	3, 5
3	1	7	6
4	2, 3	8	4, 7

Draw the network.

4

Development of Network

4.1. INTRODUCTION

In the previous chapters, we have studied the basic principles of network, and have developed the terminology necessary for the understanding of CPM/PERT networks. As stated earlier, a project to be planned by network technique should consist of well specified and clearly definable jobs, operations or activities. In this chapter, we will develop a concept of breaking down a project into activities in order to draw the network. This decomposition is necessary for performing the basic calculations required for scheduling and analysis purpose. The concept associated with the construction and understanding the network diagram, portraying each of the activities and predecessor-successor relations amongst them, will be our major concern.

4.2. PLANNING FOR NETWORK CONSTRUCTION

Depending upon the sense of thinking with respect to the end configuration of the plan, networks can be constructed either by *forward planning* or by *backward planning* or by combination of both forward and backward planning.

1. Forward planning

In this method, the planner starts from the initial event and builds up the events and activities logically and sequentially until the end event is reached. In this method, while considering an activity, a planner asks himself the following questions :

What event comes next ?

What are dependent events ?

What events can take place concurrently ?

The answer to these questions is not that simple, specially in a complex situation.

2. Backward planning

In this method, the planner starts with the end event, and arranges the events and activities until the initial event is reached. Keeping the goal in view, the planner asks himself 'if we want to achieve this, what events or activities should have taken place ?'

3. Combined planning

In practice, a combination of both forward planning and backward planning is followed. At any stage, the planner may need traverse the network back and forth several times until it is found to be satisfactory. In this method, the planner must ask himself the following questions, at any stage of network planning :

(*a*) What event or events must be completed before the particular event can start ?

(*b*) What event or events follows this ?

(*c*) What activities can be accomplished simultaneously ?

4.3. MODES OF NETWORK CONSTRUCTION

There are basically two modes of network diagrams :

(*i*) Event oriented diagrams

(*ii*) Activity oriented diagrams.

Fig. 3.3 (*a*) and (*b*) show respectively the activity oriented diagram and event oriented diagram for a project of purchasing a new heavy duty lathe and disposing the old lathe.

Event Oriented Diagrams

PERT users prepare event oriented network diagram, in which emphasis is placed on the events of the project. One first selects the events that are to be included in the plan. The interest is focussed upon the start or completion of events rather than the activities. The activities that take place between events are not specified. This approach grew out of the desire to report on the project progress via discernible management milestones, as events represent them.

The events in such a network fall in a logical sequence. Where ambiguities are not caused, the 'start' circle is omitted and only the completed event is recorded. It is understood that the start event must have taken place before the completed event can occur. The enumerated events are then connected by arrows to show how they are related to current plans for accomplishing the project.

Activity Oriented Diagrams

CPM users prepare activity oriented diagrams in which emphasis is placed on activities of the project. The activities are arranged in sequential and logical order. A description of the activity is written on the arrow representing it. As diagramming proceeds, additional activities are selected as suggested by the nature of the work and the diagram grows into a co-ordinated whole.

Generally, the importance of events is minimised in such diagrams. However, after a diagram is drawn, one can easily select certain key events one wishes to index and name for the progress reporting purposes.

In a particular network, whether event oriented or activity oriented, one can include both events and activities. Both of these are important for planning. While events are used to show the milestones or stages in a project, activities are the actual performance of a task to achieve the event.

4.4. STEPS IN DEVELOPMENT OF NETWORK

The following steps are found to be useful in the development of a network for a proposed project :

1. OBJECTIVE	:	set down in words.
2. PLAN BREAK-DOWN	:	depending upon the management level of use, activities and events identified and listed in general list.
3. SEQUENCING	:	the activities and events thus prepared, *i.e.* marshalling the data.
4. DEVELOPMENT	:	of predecessor and successor relationship in events through *location* of nodes in rough layout, giving events usual relative time effect through position.
5. DRAWING	:	activities by connecting pair of events with arrows.
6. CHECK	:	network diagram (*a*) in respect of content, sequence and sense, and (*b*) for degree of detail.

7. REDRAW	:	network diagram to eliminate errors and attain style.
and		
INTRODUCE	:	uniqueness dummies for grammar of network.
8. NUMBER	:	events for identification.

Each of the steps in the construction of network require some discussion and will involve some perception to establish the conditions under which task will be performed.

STEP 1 : OBJECTIVE

During the planning of a project, the first and foremost step is (*i*) to define the project, and (*ii*) to decide the way in which it is to be carried out. The task to be undertaken requires to be set down as *specific, definite, complete and well-defined verbal statement*. Specific verbal statement means the specific description of particular dimensions, type of materials, plants etc. necessary for the project.

Objective specifies the task to be undertaken and policy of its execution. This *specification* defines the project and determines the way in which it is to be carried out.

Example. ***Specification for a lathe installation project.***

"A new lathe is to be installed at a location by removing the existing machine to clear the floor. Existing machine is to be disposed off and the complete installation is to be tested."

STEP 2 : PLAN BREAKDOWN

After establishing objective of the task, the planner has to adopt either forward planning or backward planning (or mixed planning) to achieve the goal. This backward or forward thinking will give a list of activities or jobs to be performed to achieve the task and also stages in the project execution. To obtain a list of activities and/or events, the specification has to be examined under three headings :

(*a*) What are the difficulties that will have to be overcome.

(*b*) What facilities will assist in the removal or circumvention of the obstacles.

(*c*) What safeguards will have to be taken in using the facilities.

The result of the examination will create a list of activities and/or events.

Example. ***For the lathe installation project, plan break-down is done by examining specification under the above mentioned questions :***

(*a*) *What is the way in reaching the objective ?*

New lathe is to be purchased. Existing machine is to be removed and disposed off.

(*b*) *What is the help available ?*

Power supply for testing.

Finance for purchasing the lathe.

Implements for removal of existing machine.

Disposal facilities.

(*c*) *What safeguards are necessary ?*

Removal of existing machine is necessary to clear off the site for installation. Also, installation of power supply is necessary.

From the above, the following list is obtained.

General List

Events	*Activities*
Order for new machine placed.	Await delivery of lathe
Existing machine removed.	Install lathe
Power supply installed.	Remove existing machine
Existing machine disposed.	Dispose off existing machine
Lathe installed.	Install power supply ; connect the lathe to power.
Installation tested.	Test complete installation.

Note that activities in the above list are not in any particular order. It is convenient, however, to enter the earlier activities near the head of the list.

In large projects, several methods of carrying out projects may be possible, but only one sequence must be followed and used in a particular case, best suited to the specific set of conditions. This often occurs in construction industry, where identical buildings can be erected in very different activity orders.

STEP 3 : SEQUENCING—MARSHALLING THE DATA

In the second step we have obtained a general list of various activities and events necessary for the completion of the project.

This general list is to be reviewed so that in each of the main group, those with definite similarities can be put in suitable subgroups. The marshalled list for the lathe project is given below :

Marshalled List

Events	***Activities***
(*i*) *Subgroup 1E*	(*i*) *Subgroup 1A*
Order for new lathe placed	Await delivery of lathe
Lathe received	Install lathe
Lathe installed	Connect to power
Lathe tested	Test the installation.
(*ii*) *Subgroup 2E*	(*ii*) *Subgroup 2A*
Existing machine removal	Remove existing machine
commenced	Dispose existing machine
Existing machine removed	
Existing machine disposed	
(*iii*) *Subgroup 3E*	(*iii*) *Subgroup 3A*
Power supply applied	Obtain power supply
Power supply obtained	Install power supply
Power supply installed	

STEP 4 : LOCATION OF NODES

Now the events listed above are required to be located on paper so that a visual effect of movement along a time scale is obtained. Events should be located in such a way that they represent initial picture of the relation amongst them. This relationship results from the proposed use of manpower, money, material and other resources during a particular period of time. For obtaining the relationships amongst events, the planner must ask himself the following three questions regarding the sequence :

1. What event or events must be completed before the particular events can occur ?

2. What event or events should follow this ?

3. What events can be accomplished simultaneously ?

This way, he will be able to get the logical sequence of events in the network.

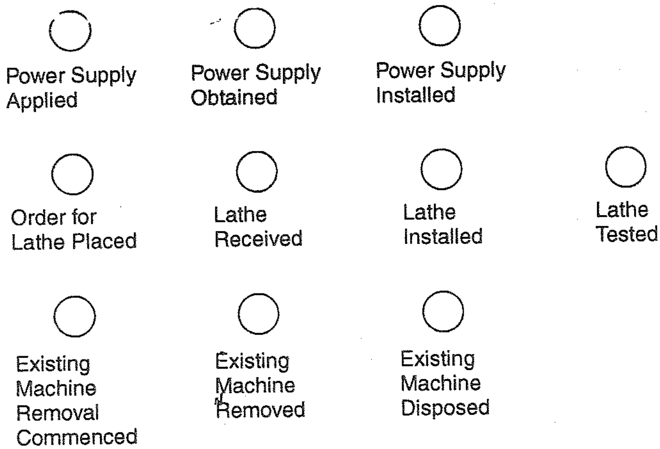


FIG. 4.1. LOCATION OF NODES.

The following points must be kept in view while locating the nodes :

(*i*) First event must be located by the left most node.

(*ii*) Events occurring earlier must be represented to the left side with respect to the events occurring later.

(*iii*) Serial events must be represented by nodes along horizontal plane while simultaneous events are represented up or down the vertical plane.

(*iv*) No event should be located in position to the left of (*i.e.* behind) the vertical axis of any preceding event.

For the example of lathe project, the location of nodes is shown in Fig. 4.1.

STEP 5 : DRAWING ARROWS

Events having close and direct relationship are joined to each other by arrows representing activity to be performed for passing from one stage of the project to the other. These activities should fall in a logical sequence. The planner should ask himself the following three questions regarding the sequence of activities :

1. What activity or activities should be completed before a particular activity can start ?

2. What activity or activities follow this ?

3. What activities can be accomplished simultaneously ?

For the lathe project, the network diagram, after drawing of arrows, is shown in Fig. 4.2. Note that the activity inter-relationships between the three groups are also obtained at this stage by

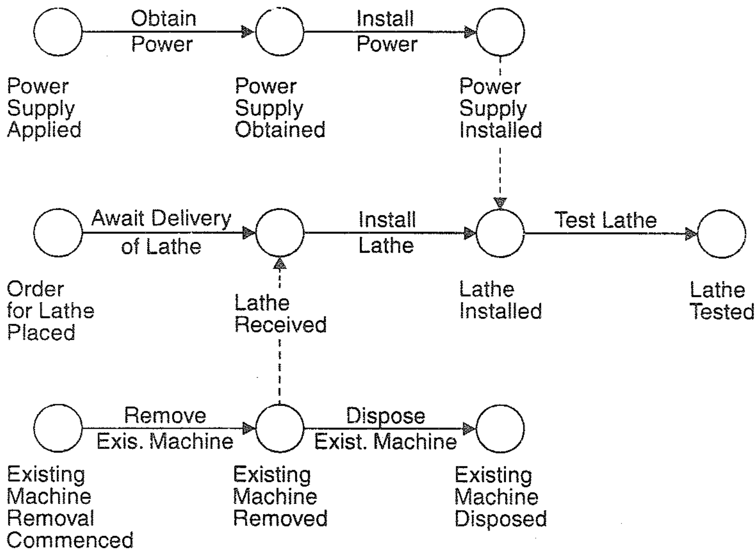


FIG. 4.2. DRAWINGS ARROWS.

suitable use of dummies (represented by dotted arrows). The activities may be listed on the corresponding arrows.

STEP 6 : CHECKING

At this stage, the diagram is checked with respect to

(*a*) Content, sequence and sense.

(*b*) Degree of detail.

CONTENT : In the above diagram, the operation 'test lathe' cannot commence before connecting the lathe to power. Hence one operation 'connect to power' is missing.

SEQUENCE : It is essential to check the diagram for events and activities in respect of (*i*) logic and (*ii*) accuracy. Particular attention should be paid to multiple events, *i.e.* those events at which more than one arrows enters and/or more than one arrows leave, since it is at this point that errors are most likely to occur. The checking ensures that the network correctly represents the sequence.

SENSE : It should be ensured that the network does not contain 'loops' or 'cycles'. If located, these should be removed.

Also, it should be checked whether there is any event (other than the first) which has only outgoing arrows, or whether there is any event (other than the last one) which has only incoming arrows. Such situation, if found, should be rectified. There should be no dead ends left. With this check, it is revealed that :

Start Project event indicates commencing of operations : (*i*) obtain power, (*ii*) 'await delivery of lathe' and (*iii*) 'remove existing machine'. Thus, these three operations should start from the same node.

Project Completed event indicates completion of operation : (*i*) 'test lathe' and (*ii*) dispose existing machine. Thus, these two operations should end at the same node.

DEGREE OF DETAILS : An arrow should always represent singular situation but an event may represent commencement of more than one operations. In respect of sufficient detail, a ratio, known as *E*/*A* ratio defined as under :

$$\frac{E}{A} \text{ ratio} = \frac{\text{Total no. of events}}{\text{Total no. of activities}}.$$

In a good network, its value should lie between 1 to 1.6.

STEP 7 : REDRAW

The errors found in the previous step are removed, and the diagram is redrawn, by introducing uniqueness dummies, if necessary. The redrawn diagram is shown in Fig. 4.3.

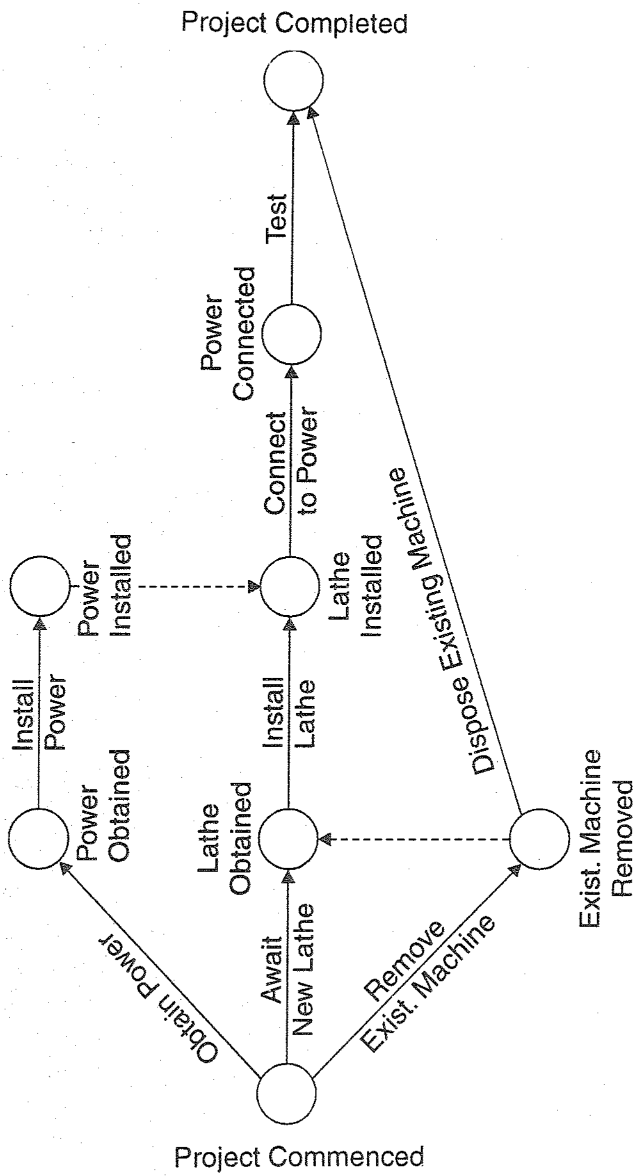


FIG. 4.3

STEP 8 : NUMBER

After having drawn the final network, the events are numbered, using Fulkerson's rule.

4.5. WORK BREAKDOWN STRUCTURE

In Art. 4.4 we have studied various steps in the development of network, out of which the second step is the *plan breakdown* to identify the activities and events necessary for the completion of the project. This is a very important step, which should be studied in greater details. In the first step, the *objective(s)* should be specified in terms of 'end items'. The subsequent division of each end item creates a work break-down structure (or schedule) which serves as the *framework* for developing the network. Work breakdown structure or schedule is a pictorial representation of the entire program. It is a preliminary diagram illustrating the way in which all the supporting objectives go together and mesh to ensure the attainment of the major objective. Such a breakdown structure is more essential in complex projects consisting of hundreds of events and activities.

In work breakdown structure, the *top-down* approach to planning is adopted. Such an approach ensures that the total project is fully planned, and that all derivative plan contribute directly to the desired end objectives. The work breakdown schedule aids in the identification of objectives and allows the planner to see the total picture of the project.

The development of the work breakdown structure begins at the highest level of the program with the identification of project end items. The major end items are then divided into their sub-component parts (*i.e.* systems, sub-systems, components) and the component parts are further divided into their more detailed units. The subdivision of the work breakdown structure continues to successively lower levels, reducing the value (cost) and complexity of the units at each level, until it reaches the level where the end item subdivisions finally become manageable units for planning and controlling purposes. The end items subdivisions appearing at this last level in work breakdown structure are then divided into major work packages (*i.e.* engineering, manufacturing, testing etc.). A typical work breakdown structure of a project is shown in Fig. 4.4.

Work packages at the lowest level are generally represented by a number of activities that are used in the preparation of network. A work package may be represented either by one activity, with a beginning and event, or by a number of activities separated by

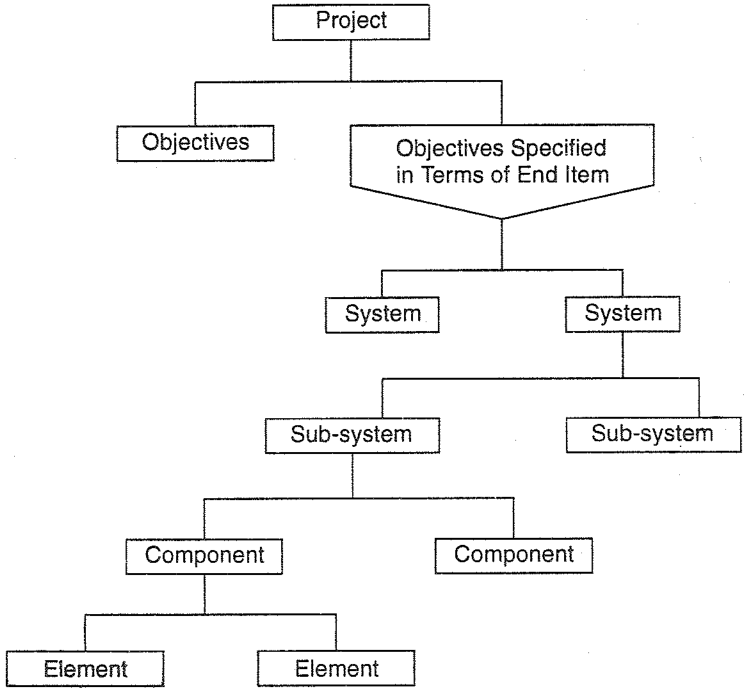


FIG. 4.4. WORK BREAKDOWN STRUCTURE.

events which serve as beginning or end points for other activities in the project.

Let us now take a typical example of house construction project. The major or total objective in this case is '*house constructed*' which goes on the top of the work breakdown structure (level 1). The supporting objectives or stages are : (*i*) Survey and land levelling, (*ii*) Masonry work, (*iii*) Carpentry work, (*iv*) Electric fitting, (*v*) Water and sanitary fittings, (*vi*) Finishing and interior decoration. All these objectives are at level 2.

Now if we take any one of the second level objective, such as masonry work, there may be following phases : (*i*) foundations, (*ii*) pillars, (*iii*) walls, (*iv*) partitions, (*v*) roof etc. All these phases are thus at level 3.

Now take any one of the 3rd level phase, say foundation, which can be further subdivided into following sub-phases : (*i*) excavation, (*ii*) laying forms, (*iii*) mixing concrete and placing concrete, (*iv*) curing concrete, (*v*) removal of forms etc. These sub-phases are at level 4.

All these stages are shown below.

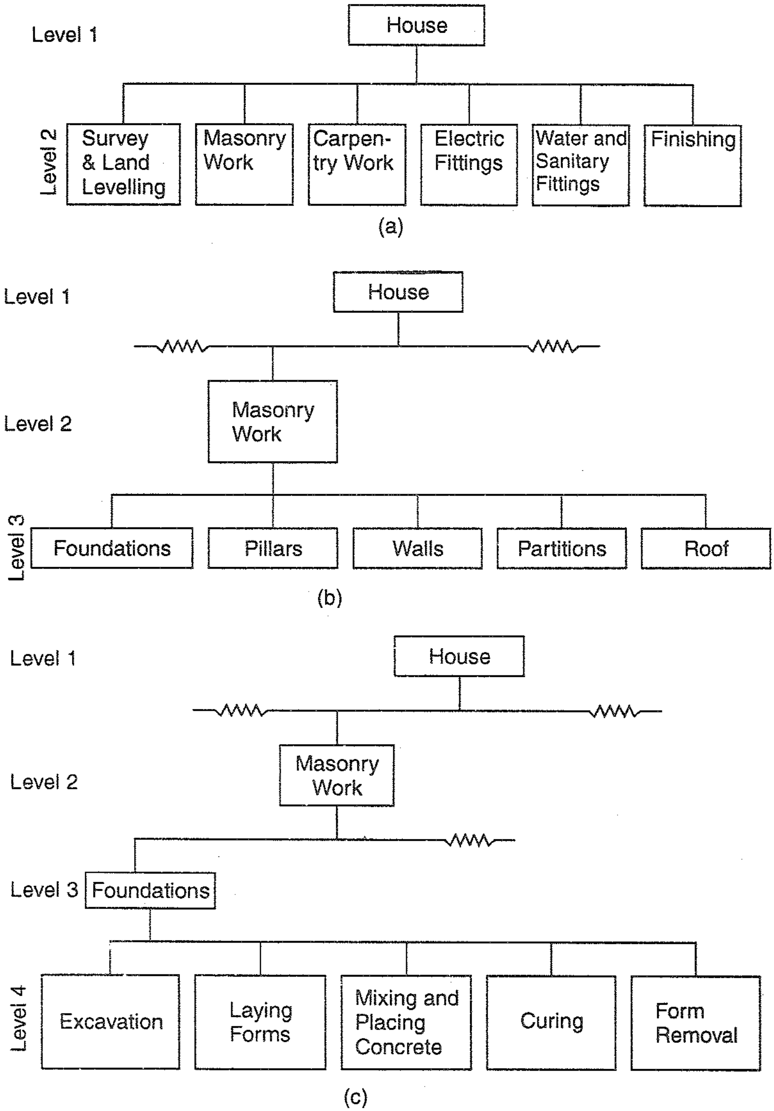


FIG. 4.5. WORK BREAKDOWN STRUCTURE FOR HOUSE CONSTRUCTION PROJECT.

This way, the planner can go from one level to the other. The number of levels into which the project has to be splitted depends upon the type and complexity of the project itself. The basic requirement is that the work breakdown schedule should be detailed enough to allow the eventual construction of PERT/CPM network which will precisely reflect the inter-relationship among all the events and activities which make-up the entire project. In a large complex project, there may be ten or even more levels of sub-objectives. The work breakdown schedule so obtained presents the entire project in a systematic way so that inter-relationships among all phases of the projects are easily seen. As the work progresses, the project work breakdown schedule or structure also serves as the framework for summarising data from the bottom up, so that the amount of detail presented at any level in the project is commensurate with the decision-making requirements of management at that level.

4.6. HIERARCHIES

Large and complex networks contain more than two to five hundred work operations. If all of these operations are represented on one single network diagram, it will become clumsy. Since all of these work operations are to be represented, one can successfully use a *hierarchy or family of networks* of increasing detail. There may be several stages in the hierarchy, each to be used by different set of people, *i.e.* top management people, middle, and upper-management people etc. The number of stages in the hierarchy may reflect not only the complexity of the project but also the structure of the company management, and the systems of control and reporting that are in use.

The hierarchy may have generally two or three *stages* or *levels*. The number of *arrows* (work operations) at each stage of the hierarchy will depend on the following factors :

(*i*) Purpose of the diagram,

(*ii*) Degree of control desired,

(*iii*) Extent of available information, and

(*iv*) How the diagram is to be used.

The *first level diagrams* are primarily used for general information. The purpose is to describe in general terms to top management, clients or the public, the over all nature of the project and how it is to be accomplished. The diagram will contain very few arrows, each representing a piece of work of fairly broad scope. The diagram at this stage need not be strictly correct logically. Each arrow may represent a *project* itself for the second level diagram.

As an example, consider the network of Fig. 4.6 for the project of development and marketing of a new product of a company. The *First Stage* network is intended for the top management of the company. The activities *'develop product'* and *'prepare marketing plan'* run concurrently, and are operated by different groups or divisions of the company.

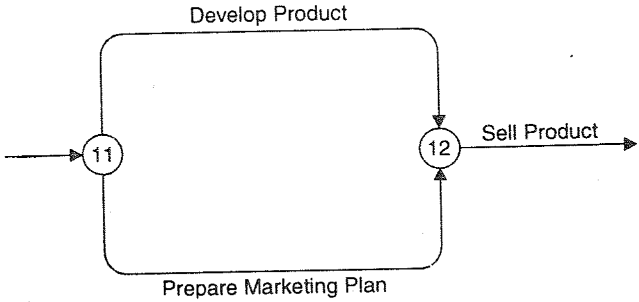


FIG. 4.6. FIRST STAGE NETWORK.

Each division concerned can now breakdown the corresponding activity into finer details, at the *Second Stage* of the network. Thus the activity *'develop product'* becomes a project *'develop product'*. Similarly, the activity *'prepare marketing plan'* becomes another project, *'prepare marketing plan'*. Each one of these second stage networks may contain several activities. The details of second stage network for each of the two activities is shown in Fig. 4.7.

It will be seen that both the second stage networks are *independent* and they run parallel. However, they have common starting and finishing events. The network diagrams at the second stage are used by middle management with the information for cost and time control in classical sense. The work packages at this stage are better defined than at first level, and the diagram should be logically correct.

Small and middle size projects may contain only two stages. However, large projects may have even the *third stage*. The third level of detail has the purpose of directing the efforts of small groups of individuals at the site (construction or manufacturing). It is only at this level that true project control can be achieved.

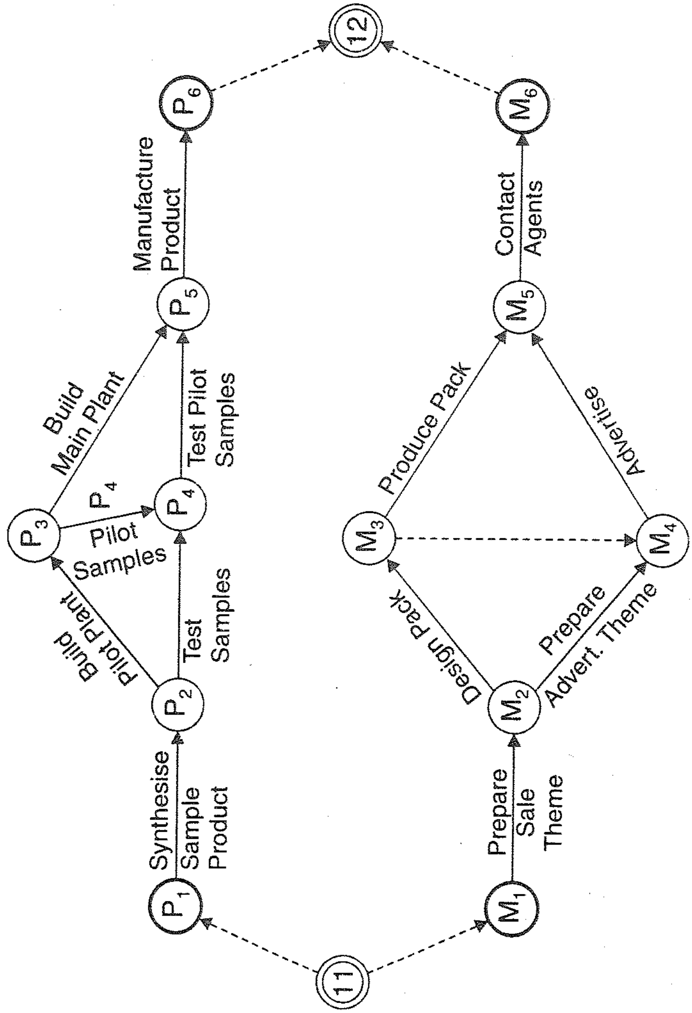
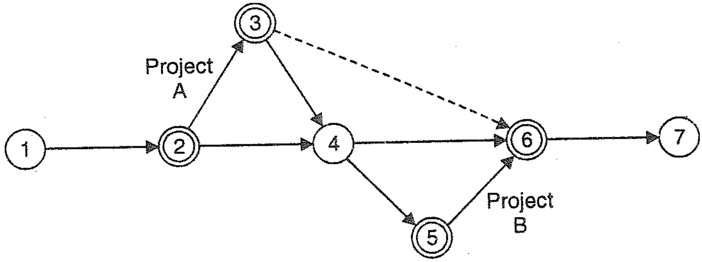


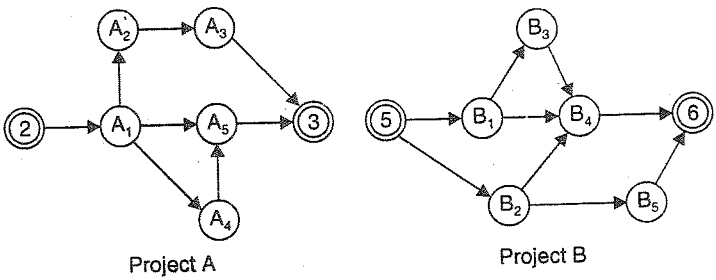
FIG. 4.7. SECOND STAGE NETWORK.

Fig. 4.8 shows the *three stages* of networks for a certain building construction company.

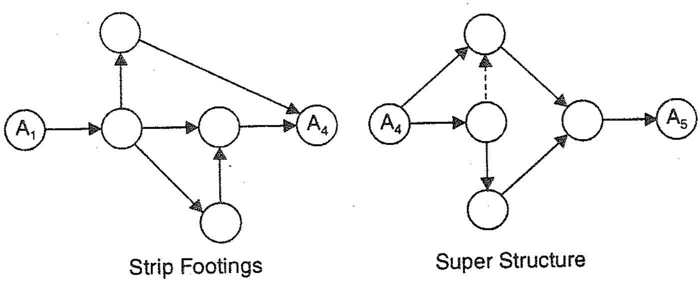
When hierarchy principle is used for large project to split it up, the first level or overall network should be prepared by the top administrator, in consultation with the representatives of various divisions so that inter-departmental constraints are introduced properly. The representatives of each division will then prepare more detailed sub-networks for middle level (project control) and lower level (site/production control) implementation.



(a) Top Level (Management of Several Projects)



(b) Middle Level (Project Control)



(c) Lower Level (Site Control)

FIG. 4.8. HIERARCHIES.

4.7. ILLUSTRATIVE EXAMPLES

Example 4.1. *Write specification, determine plan breakdown and prepare network for the project of 'casting a concrete beam over verandah opening'.*

Solution.

Specification. A concrete beam is to be designed and cast at the site. Mixing to be done by mechanical mixer which is available.

Plan breakdown. The project will consist of the following activities :

Design the beam

Order concrete materials (cement, sand, aggregate)

Order steel

Order timber

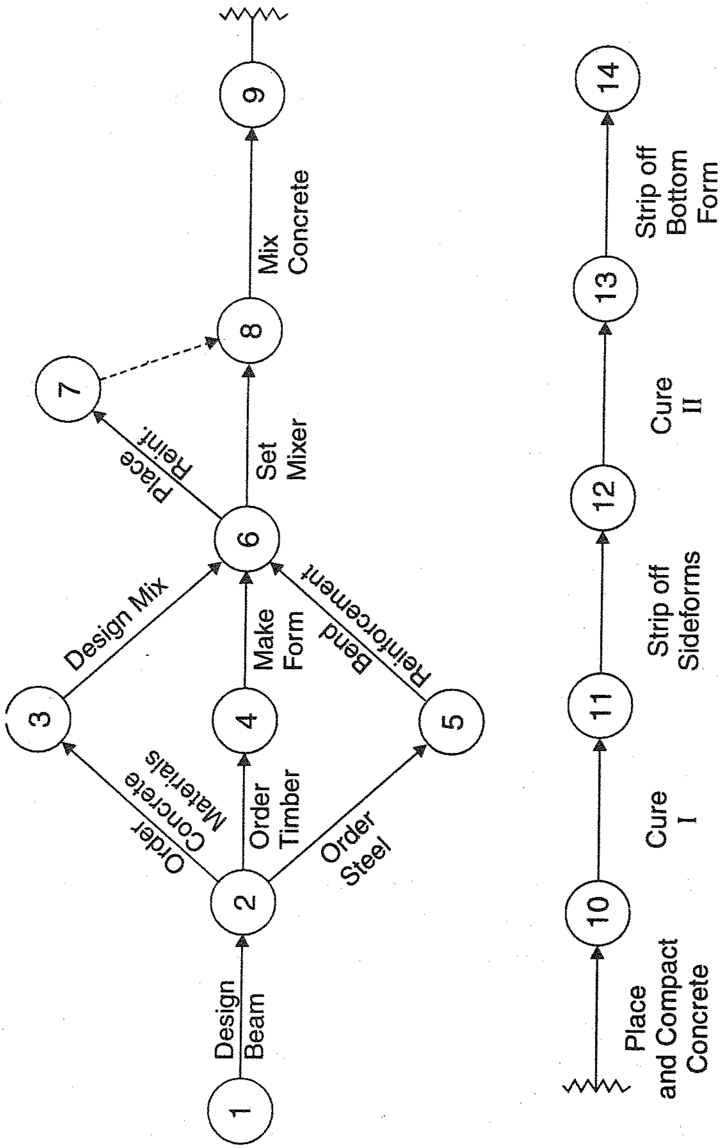


FIG. 4.9

Design mix proportion

Make and fix form

Bend reinforcement

Place reinforcement

Set mixer at site

Mix concrete

Place and compact concrete

Cure stage I

Strip off forms

Cure stage II

Net work Diagram. Shown in Fig. 4.9.

Example 4.2. *Construct the network for the manufacture of a storage cabinet, given the following specification :*

'A simple storage cabinet is to be manufactured by fabrication and assembly of frame and panels. The cabinet is to be painted. Panels and paint are available from the store.'

Solution. The following is the list of activities :

Order material for frame work

Await delivery of material

Obtain panels and paint from store

Set up tools

Fabricate frame

Fix panels

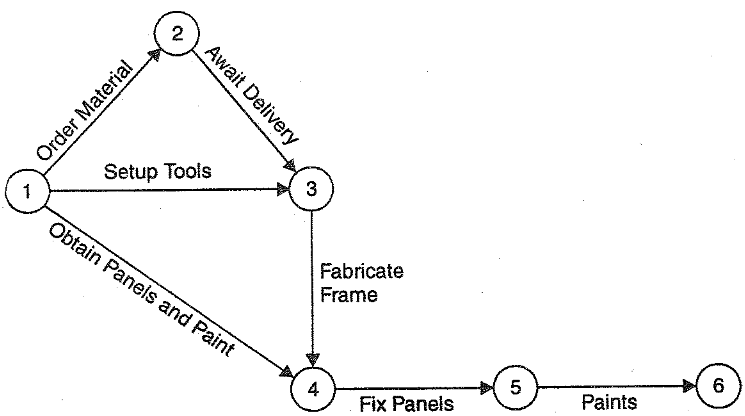


FIG. 4.10

Obtain paint

Paint cabinet.

The network is shown in Fig. 4.10.

Example 4.3. *Construct PERT network for the project using the listed events. The events are not in logical sequence.*

'Building of an aircraft'

Event a	:	*Air craft equipment received.*
Event b	:	*Final fuselage (body of an aircraft to which the engine, wings and tails are fitted) drawing completed.*
Event c	:	*Air craft tested and commissioned.*
Event d	:	*Tail assembly received.*
Event e	:	*Sub-contract for tail assembly awarded.*
Event f	:	*Procurement of engine initiated.*
Event g	:	*Programme go ahead.*
Event h	:	*Wings from sub-contractor received.*
Event i	:	*Plans and specifications completed.*
Event j	:	*Fuselage-engine assembly completed.*
Event k	:	*Air craft equipment submitted.*
Event l	:	*Manufacture of fuselage completed.*

Solution. After logical sequencing, the network shown in Fig. 4.11 is obtained.

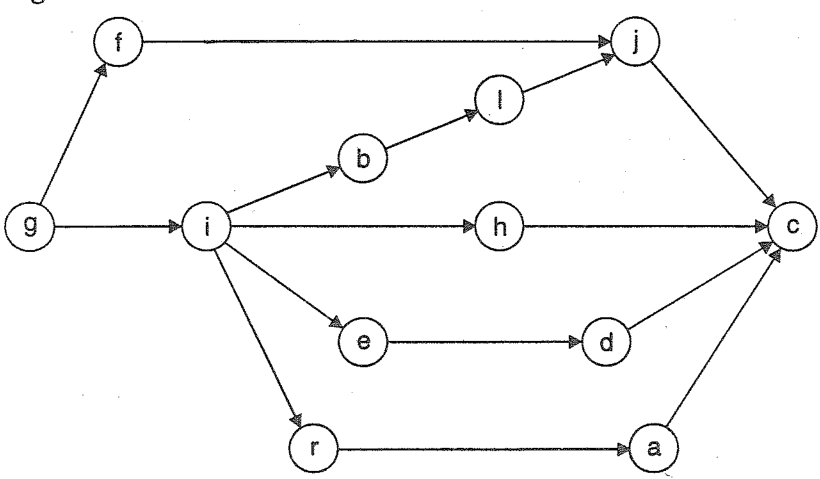


FIG. 4.11

Example 4.4. *Assume that a statue is to be erected in a village square on a stone platform which is to be built on a cement concrete foundation. The statue is to be prepared at another place, moved and erected. The various operations of the entire project are given below. These operations are not in logical sequence.*

A. Make statue

B. Shift statue

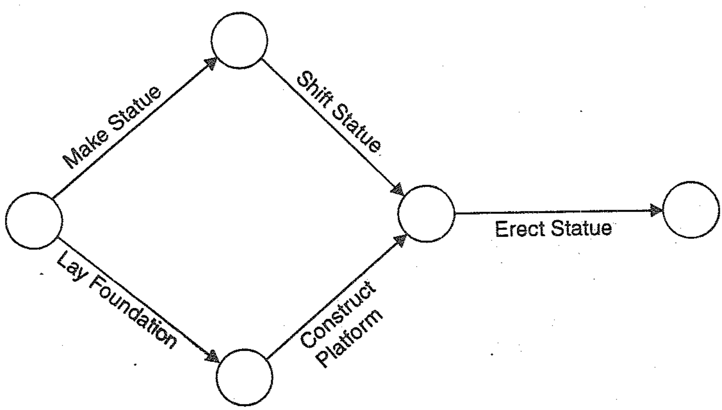
C. Erect statue

D. Lay Foundation

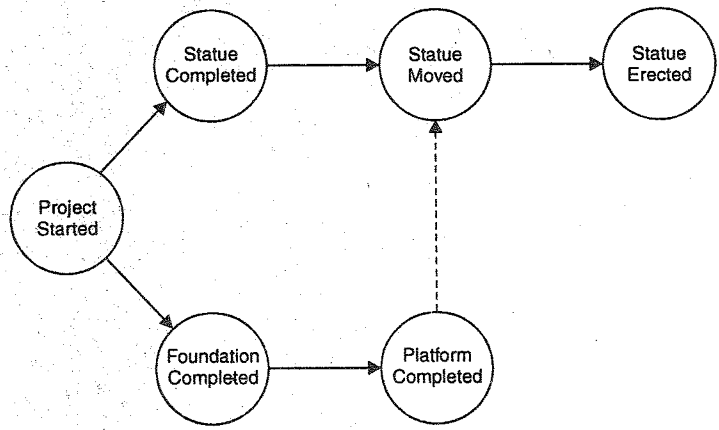
E. Construct Platform.

Represent the above project by (a) activity oriented network and (b) event oriented network.

Solution. The *activity-oriented diagram* is shown in Fig. 4.12 (*a*) while *event oriented diagram* is shown in Fig. 4.12 (*b*).



(a) Activity Oriented Diagram



(b) Event Oriented Diagram

FIG. 4.12

PROBLEMS

1. Differentiate between 'forward planning', 'backward planning' and 'combined planning'.
2. Differentiate between 'activity oriented diagram' and 'event oriented diagram'.
3. Write a note on 'development of networks'.
4. What do you understand by work breakdown structure ? What is its importance in network planning ?
5. Write a note on 'hierarchies'.
6. Write specification, determine plan breakdown and complete by 'numbering the network', for the work 'Answering an official letter'.
7. Write specifications, determine plan breakdown and complete by numbering the network for a 'Budgeting Project' of a large manufacturing firm.
8. Write specification, determine plan breakdown and complete the network for the project 'overhaul of diesel generator'.

5

PERT : Time Estimates

5.1. INTRODUCTION

PERT stands for Programme Evaluation and Review Technique, which can be applied to any field requiring planned, controlled and integrated work efforts to accomplish established goods. The method was basically developed by Navy special projects office in co-operation with Booz, Allen and Hamilton, a management consulating firm and Lockheed Missile System Division for evaluating the feasibility of existing schedule, on Polaris Missile Program and for reporting progress. A project is composed of many diversified activities which contribute to its completion according to a predetermined schedule. There may be many uncertainties associated with these activities. PERT is a technique that statistically presents knowledge about these activities and the activity uncertainties.

PERT may be elaborately *defined* as

1. A MANAGER'S TOOL : for defining a project and co-ordinating various operations involved in it.
2. A DIRECTION : what must be done to successfully accomplish the objective of the project.
3. A PROFILE : that aids the decision-maker, but does not make decision for him.
4. A WAY : for synchronizing various parts of the overall job.
5. A TECHNIQUE : that presents statistical information regarding the uncertainties about computation time of various activities associated with the project.

6. A METHOD	:	for focussing managerial attention in (*a*) latent problems that require quick decisions (*b*) procedure and adjustments regarding time, resources, or performance which may improve the capabilities of meeting target dates, *i.e.* a method of scheduling and budgeting resources as to accomplish a predetermined job on schedule, and (*c*) of minimising the production delays, interruptions and conflicts.
7. AN OUTSTANDING APPROACH	:	of expending the completion of project.
8. A COMMUNICATION FACILITY	:	in that it can report developments both favourable and unfavourable to manager, and in that it can keep the manager posted and informed.

The PERT system uses a network diagram consisting of *events* which must be established to reach project objectives. An *event* is that particular instant of time at which some specific part of a plan is to be achieved. It indicates a point in time and does not require any resources. The approach of event-orientation in network diagram grew out of the desire to report on the project progress *via* discernible management milestones.

5.2. UNCERTAINTIES : USE OF PERT

As soon as the work of network construction for a project is over, the item of concern is the determination of time required for the occurrence of each event. The planner is faced with questions concerning how long the project will take and when specific activities may be performed. Since the activities are to be performed in future, the time period required for the execution of each activity or job can only be estimated. This factor presents no problem if the project consists of activities with which the estimator or planner is thoroughly familiar, perhaps because of his wide personal experience. However, because of certain uncertainties, an exact estimation of time for completion of an activity may be difficult.

Two approaches may be used for the assessment of duration for activity completion. The first approach is the *deterministic approach* in which we may assume that we know enough about each job or operation, so that a single estimate of their durations is sufficiently accurate to give reasonable results. This approach is followed by CPM users. The second approach is the *non-deterministic approach* or the *probabilistic approach* in which one may only be able to state limits within which it is virtually certain that the activity duration will lie. Between these limits we must guess what is the probability of executing the activity. The second approach is followed by PERT planners.

PERT was developed and has been used most frequently in the research and development type projects, such as space industry, aerospace industry, defence products industry etc. PERT system is preferred for these projects or operations which are of non-repetitive nature or for those projects in which correct time determination for various activities cannot be made. PERT application is favourable in projects where much of their design and construction or production requires new developments in materials and technology. All this is to say that there is a large amount of uncertainties in the development of new systems. These uncertainties may be about the times required for developmental research, engineering designs, ultimate construction and may be for specific activity or sometimes about the configuration of end product itself. There is little past history on which to base network construction and time estimates. In such projects, management cannot be guided by past experience. They are referred to as once-through operations or projects. For example, the project of *launching a space craft* involves the work never done before. For such a project, the range of possible technical problems is immense. In such research and development projects, the time estimates made for use may be little more than guesses. PERT system is best suited for such projects.

5.3. TIME ESTIMATES

Time is the most essential and basic variable in PERT system of planning and control. We have seen that PERT is mostly used for research and development type projects which are referred to as once-through. In these projects, there is uncertainty about the times required for the completion of various activities. Exact estimation of times of completion for various activities is difficult. In the PERT network an estimate is made of not only the *most probable time* required to complete the activity, but some measure of uncertainty is also *incorporated* in this estimate to consider two more time estimates : the *pessimistic estimate* and the *optimistic estimate.*

Thus, to take the uncertainties into account, PERT planners make three kinds of time estimates :

(*i*) The optimistic time estimate,

(*ii*) The pessimistic time estimate, and

(*iii*) The most likely time estimate.

1. The Optimistic Time Estimate

This is the shortest possible time in which an activity can be completed, under ideal conditions. This particular time estimate represents the time in which we could complete the activity or job if everything went along perfectly, with no problems or adverse conditions. Better than normal conditions are assumed to prevail. This time estimate is denoted by t_O.

2. The Pessimistic Time Estimate

It is the best guess of the maximum time that would be required to complete the activity. This particular time estimate represents the time it might take us to complete a particular activity if every thing went wrong and abnormal situations prevailed. However, this estimate does not include possible effects of highly unusual catastrophies such as earthquakes, floods, fires etc. This time estimate is denoted by t_P.

3. The Most Likely Time Estimate

The most likely time or *most probable time* is the time that, in the mind of the estimator, represents the time the activity would most often require if normal conditions prevail. This time estimate lies between the optimistic and pessimistic time estimates. This time estimate reflects a situation where conditions are normal, things are as usual and there is nothing exciting. This time estimate is denoted by t_L.

These time estimates, though look simple, are not always easy to prepare. However, they give useful information about the expected uncertainties in an activity. These time estimates are usually expressed in days, weeks or months, and represent calender dates and not actual working days.

5.4. FREQUENCY DISTRIBUTION

As stated above, the three time estimates are very difficult to prepare, unless some guidance is available. The planner should base the estimations on available information and past experience. For example, consider a certain activity '*A*' under diverse conditions. The time required for the completion of this activity under each condition is known. Naturally, the time of completion will be short (optimistic time) if better than normal conditions exist. The number of cases when such normal conditions exist for completion of an activity *A* will be naturally small. Similarly, time of completion will be long (pessimistic time) if adverse conditions are there, and such cases will also be small in number.

If a curve is now plotted between the 'time' of completion and the number of jobs completed in that 'time', *a frequency distribution curve*, such as the one shown in Fig. 5.1 will be obtained. From the curve, it is clear that there are large number of cases of the activity that are completed in the *most likely time*. Point *P* corresponds to the optimistic time (t_O), point *R* corresponds to the pessimistic time (t_P) while point *Q* corresponds to the most likely time (t_L). Such a curve is also called *unimodal curve*, since it has single hump.

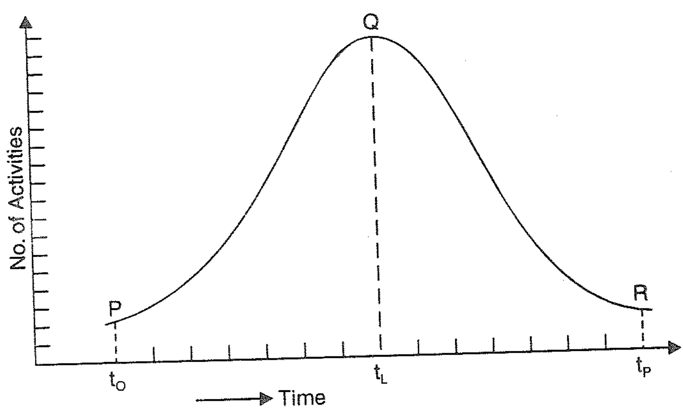


FIG. 5.1. FREQUENCY DISTRIBUTION CURVE.

The curve shown in Fig. 5.1 is symmetrical on either side of point *Q* ; such a curve is known as the *normal curve*. It is not necessary that a frequency distribution curve may be normal ; it may have *skew* due to which it is not symmetrical about the peak Q.

Fig. 5.2 (a) shows the frequency distribution curve for job A, having skew to the left ; the difference between t_L and t_O is only 1 day while the difference between t_P and t_L is $7 - 4 = 3$ days. Fig. 5.2 (c) shows the frequency distribution curve for activity B,

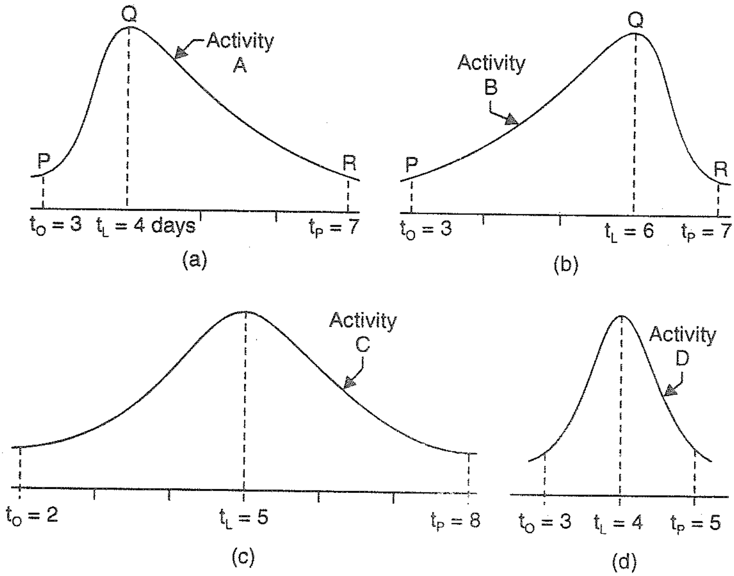


FIG. 5.2. FREQUENCY DISTRIBUTION CURVES.

having skew to the right ; the difference between t_L and t_O is equal to $6 - 3 = 3$ days while the difference between t_P and t_L is only 1 day. The frequency distribution curves for activities C and D, shown in Fig. 5.2 (c) and (d) respectively are symmetrical about the peak, and they are therefore *normal curves*. However, curve (c) has wider variation between t_P and t_O and has therefore *greater uncertainty* in time estimate. On the other hand, curve (D) has smaller variation between t_P and t_O and hence more reliable time estimates are expected. To conclude, *a wide range in time estimates represents, greater uncertainty and hence less confidence in our ability to correctly anticipate the actual time that the activity will require.*

To summarise, statistical data for varying durations of time that jobs of a particular type consumed in the past can be expressed in the form of a frequency distribution curve. The method of preparing a *frequency distribution curve* will now be explained with the help of an example.

Example 5.1. *In a certain project, the times required for digging 54 trenches of fixed dimensions are recorded below. The trenches were excavated by different parties, each consisting of the same number of persons. Plot the frequency distribution curve.*

Table 5.1

TIMES OF COMPLETION OF TRENCH (DAYS)

8	11	14	9	10	8
10	9	12	11	9	10
12	8	7	13	11	9
6	10	9	10	10	11
9	14	13	14	7	
11	16	10	9	13	
10	12	8	12	11	
13	16	11	15	8	
15	15	17	14	12	
12	10	13	9	11	

Solution. From Table 5.1, we find that the minimum time taken for completion of trench is 6 days which corresponds to the optimistic time (t_O), while the maximum time taken is 17 days which corresponds to the pessimistic time (t_P). The time varies between 6 days to 17 days. Table 5.2 gives the No. of trenches completed in 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16 and 17 days respectively.

Table 5.2

Days of completion	*No. of trenches completed during these days*	*Days of completion*	*No. of trenches completed during these days*
6	1	12	6
7	2	13	5
8	5	14	4
9	8	15	3
10	9	16	2
11	8	17	1

The data of Table 5.2 can now be plotted to get the frequency distribution curve between No. of days of completion and No. of trenches completed during this period, as shown in Fig. 5.3. From the curve, the most likely time (t_L), corresponding to the peak of the curve, comes out to be 10 days. The frequency curve so obtained is not symmetrical ; it has skew to the left.

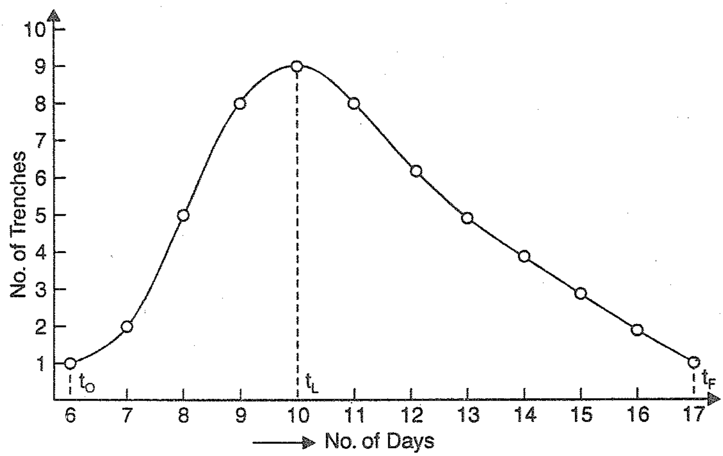


FIG. 5.3

5.5. MEAN, VARIANCE AND STANDARD DEVIATION

From the previous article we find that the frequency distribution curve can be drawn if data about varying durations of time taken for the completion of jobs of a particular type are available. If this curve is symmetrical, then it is called the *normal curve* ; otherwise it is said to have a *skew* which could be either to the left or to the right. Whatever may be the form of the curve, the following aspects of the characteristics of the distribution are important :

(*i*) Mean time or average time (called the mean of the distribution),

(*ii*) Deviation,

(*iii*) Variance and

(*iv*) Standard deviation.

Mean. Mean of the distribution may be defined by the algebraic sum of time durations taken by various jobs divided by the number of the jobs :

$$t_m = \frac{\Sigma t}{n} \qquad ...(5.1)$$

Deviation. Deviation is the difference between the time under consideration and the mean time. This difference may be either positive or negative.

Thus $$\delta = t - t_m \qquad ...(5.2)$$

where δ = deviation of any time t from the mean

t = time under consideration, for which deviation is being found.

Variance. Variance is the mean of the squared deviations. It is expressed by σ^2.

Thus, $$\sigma^2 = \frac{\Sigma\delta^2}{n} = \frac{\Sigma(t - t_m)^2}{n} \qquad ...(5.3)$$

Variance is calculated in the following steps :

(*i*) Obtain the *mean* of the distribution, by Eq. (5.1).

(*ii*) Determine the deviation of each time from the mean.

(*iii*) Find square of these individual deviations.

(*iv*) Find the mean of the squared deviations.

It is to be noted that though the deviations may be negative also, but their squares will always be positive. Hence variance will always be positive. It cannot have zero value unless each individual deviation is zero.

Variance is commonly used in statistics as measure of *variability* of the distribution.

Standard deviation. It is simply the square root of the variance. Standard deviation is denoted by symbol σ.

Thus, $$\sigma = \sqrt{\frac{\Sigma(t - t_m)^2}{n}} \qquad ...(5.4)$$

Let us now calculate mean, variance and standard deviation for the data of example 5.1. The computation are arranged in Table 5.3.

From Table 5.3, consisting of 54 time observations, we get

$$\Sigma t = 597$$

Table 5.3

Time taken (t)	*Deviation* $\delta = t - t_m$	δ^2	*Time taken* (t)	*Deviation* $\delta = t - t_m$	δ^2
8	– 3.06	9.364	17	+ 5.94	35.284
10	– 1.06	1.124	13	+ 1.94	3.764
12	+ 0.94	0.884	9	– 2.06	4.244
6	– 5.06	25.604	11	– 0.06	0.004
9	– 2.06	4.244	13	+ 1.94	3.764
11	– 0.06	0.004	10	– 1.06	1.124
10	– 1.06	1.124	14	+ 2.94	8.644
13	+ 1.94	3.764	9	– 2.06	4.244
15	+ 3.94	15.524	12	+ 0.94	0.884
12	+ 0.94	0.884	15	+ 3.94	15.524
11	– 0.06	0.004	14	+ 2.94	8.644
9	– 2.06	4.244	9	– 2.06	4.244
8	– 3.06	9.364	10	– 1.06	1.124
10	– 1.06	1.124	9	– 2.06	4.244
14	+ 2.94	8.644	11	– 0.06	0.004
16	+ 4.94	24.404	10	– 1.06	1.124
12	+ 0.94	0.884	7	– 4.06	16.484
16	+ 4.94	24.404	13	+ 1.94	3.764
15	+ 3.94	15.524	11	– 0.06	0.004
10	– 1.06	1.124	8	– 3.06	9.364
14	+ 2.94	8.644	12	+ 0.94	0.884
12	+ 0.94	0.884	11	– 0.06	0.004
7	– 4.06	16.484	8	– 3.06	9.364
9	– 2.06	4.244	10	– 1.06	1.124
13	+ 1.94	3.764	9	– 2.06	4.244
10	– 1.06	1.124	11	– 0.06	0.004
8	– 3.06	9.364	$\Sigma t = 597$		$\Sigma\delta^2 = 338.856$
11	– 0.06	0.004	$t_m = \frac{\Sigma t}{n} = \frac{597}{54} \approx 11.06$		$\sigma^2 = \frac{\Sigma\delta^2}{n} = \frac{338.856}{54} = 6.275$

$$\therefore \qquad t_m = \frac{\Sigma t}{n} = \frac{597}{54} \approx 11.06$$

Also, $\quad \Sigma\delta^2 = 338.856$

$$\therefore \text{ Variance} \quad \sigma^2 = \frac{\Sigma\delta^2}{n} = \frac{338.856}{54} = \mathbf{6.275}$$

Hence standard deviation

$$= \sqrt{6.275} \approx \mathbf{2.5}.$$

From Fig. 5.1, we observed that the most likely time (t_L) was 10 days, while the mean time t_m is 11.06 days. The tallest peak of the distribution curve is called the *mode*, corresponding to the most likely time (t_L). Both *mean* and *mode* do not coincide because the distribution curve is not symmetrical about its peak. In the case of a symmetrical curve (*i.e.* normal distribution curve) the mean (centre of gravity) coincides with the mode.

5.6. PROBABILITY DISTRIBUTION

Probability is connected with *chance* and *uncertainty*. The three time estimates that the estimator selects either from his experience or from the *frequency distribution* has inherent uncertainties. In probability analysis, and in consequent probability distribution, we try to associate numbers with uncertainties. In the frequency distribution one studies the group behaviour, while in the probability distribution, we have the distribution of probability values for all possible outcomes. The probability measures are always between 0 to 1. If an event has probability of 1, it is certain to occur, while if the probability is zero it will not occur. Closer the probability value is to 1, more certain is the occurrence of the event.

Let us take an example of manufacture of steel trusses by a factory. Let us assume that the factory manufactures 50 trusses in all, under varying circumstances, and the duration of time taken are as follows :

5 trusses in 12 days each
12 trusses in 14 days each
13 trusses in 15 days each
8 trusses in 16 days each
12 trusses in 18 days each.

Let us now find the probability of manufacturing a truss in 12 days. This is evidently equal to the ratio of number of trusses manufactured in 12 days each to the total number of trusses manufactured. Thus, probability

$$= \frac{5}{50} = 0.1 \text{ or } 10\%.$$

Similarly, the probability of manufacturing the truss in 15 days

$$= \frac{15 + 12 + 13}{50} = 0.6 \text{ or } 60\%.$$

Thus, probability number can always be assigned to the estimated time, if sufficient data is available. Generally, the available data (frequency distribution) is used to plot *probability distribution.*

Probability distribution is the curve, with its height so standardised that the area under the curve is equal to unity. The height or the ordinate of the curve at any point x, is denoted by function $f(x)$, usually called the *probability density function.*

Thus $$\int_{-\infty}^{+\infty} f(x)\, dx = 1 \qquad ...(5.5)$$

Fig. 5.4 shows the probability curve. It is to be noted that the ordinate $f(x)$ to the curve at any point x does not give the probability. The probability of completion of work in 8 days is equal to the ratio of the shaded area (area to the left of 8 days) to the total area of the curve. Since the total area of the curve is equal to unity, the

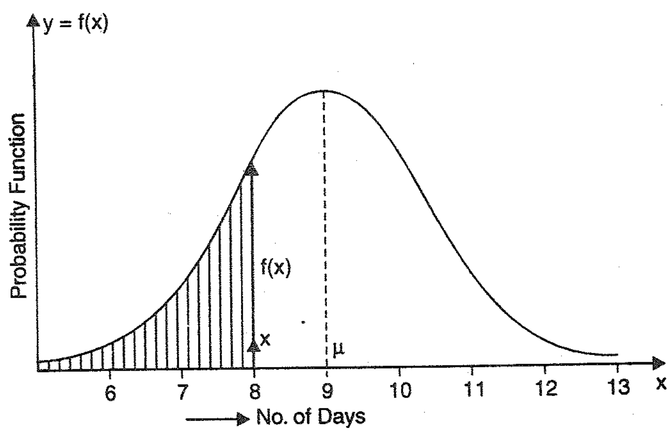


FIG. 5.4

probability of completion of the job in 8 days is equal to the shaded area itself.

Normal Probability Distribution

The probability curve is not necessarily symmetrical about its apex. If the curve is symmetrical, then it is known to have *normal* or Gaussian distribution, shown in Fig. 5.5.

The mean of the *normal probability distribution* is denoted by μ (*i.e.* $x = \mu$). It can be proved that :

(*a*) Approximately 68% of the values of the normal distribution lie within $\pm\ \sigma$ from the average, where σ is the standard deviation. This means that the shaded area of the curve (Fig. 5.5) between $x = \mu - \sigma$ to $x = \mu + \sigma$ is 68% of the total area.

(*b*) Approximately 95% of all the values lie within $\pm\ 2\sigma$ from the average. This means that the area of the curve between $x = \mu - 2\sigma$ to $x = \mu + 2\sigma$ is 95% of the total area.

(*c*) Approximately 99.7% of all the values lie within $\pm\ 3\sigma$ from the average. This means that the area of the curve between $x = \mu - 3\sigma$ to $x = \mu + 3\sigma$ is 99.7%.

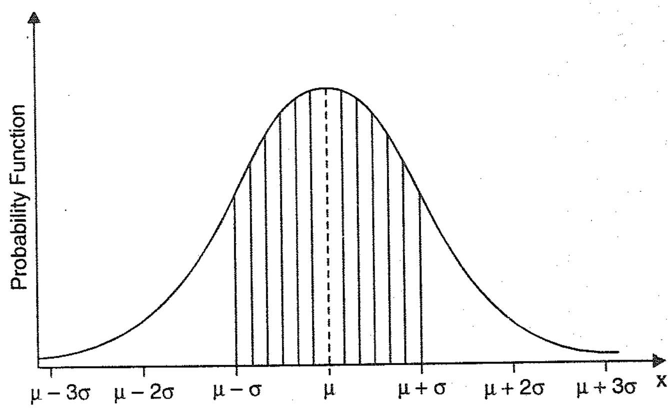


FIG. 5.5. NORMAL PROBABILITY DISTRIBUTION.

The last property (*c*) can be used to calculate the *standard deviation* directly if the minimum time (t_O) and maximum time (t_P) are known. Let us say that the minimum time is 6 days and maximum time is 18 days for the completion of a job. If 99.7% of all the values (*i.e.* possible completion times) are assumed to lie between 6 and 18 days then the distance between the extreme left value (6 days) and extreme right value (18 days) should be equal to $\pm\ 3\sigma$ or 6σ in total. The standard deviation

$$= \frac{18-6}{6} = 2 \text{ days.}$$

Hence we conclude, in general, that standard deviation is given by

$$\sigma = \frac{t_P - t_O}{6} \qquad ...(5.6)$$

or variance

$$\sigma^2 = \left(\frac{t_P - t_O}{6}\right)^2 \qquad ...[5.6\ (a)]$$

It is seen that the standard deviation is affected by the relative distance from the most optimistic estimate to the most pessimistic estimate. It is not influenced by the most likely estimate (t_L).

The above method of calculating standard deviation is *approximate.* A more exact method is by frequency distribution, explained in example 5.1. However, in PERT problems, the emphasis is one-time, non-repetitive projects for which there is no history of the activity. Hence we must base computations for σ on the given time estimates of the estimator. If the estimator feels them his range of t_O and t_P includes about all the possible values under the curve, then the standard deviation can be computed from Eq. 5.6 with reasonable accuracy.

5.7. THE BETA DISTRIBUTION

The beta distribution is a typical type of probability distribution, which fits well for PERT analysis. A beta distribution is the one which is not symmetrical about its apex. Fig. 5.6 shows two beta distributions, one having skew to the left (*beta distribution for optimistic estimator*) and the other having skew to the right (*beta distribution for the pessimistic estimator*).

The originators of PERT were interested in finding that type of probability distribution which satisfies the following conditions :

1. The distribution should have a small probability of reaching the most optimistic time (shortest time).

2. The distribution should have a small probability of reaching the most pessimistic time (longest time).

3. The distribution should have one and only one most likely time (*i.e.* unimodal) which would be free to move between the two extremes mentioned in 1 and 2 above.

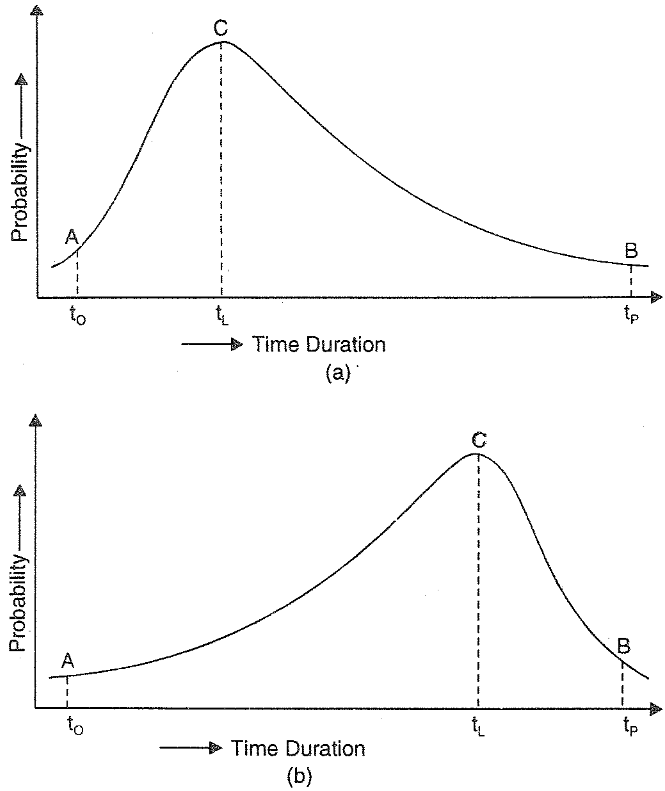


FIG. 5.6. BETA DISTRIBUTIONS.

4. The distribution should be such that the amount of uncertainty in the estimating can be measured easily.

The above mentioned four requirements are met with beta distribution. Hence this distribution is used in PERT analysis.

It can be shown that for Beta distribution, the standard deviation is given by

$$\sigma = \frac{t_P - t_O}{6}$$

The variance $\sigma^2 = \left(\frac{t_P - t_O}{6}\right)^2.$

We have already seen that *variance* is the measure of *uncertainty*. Greater the variance, greater will be the uncertainty.

5.8. EXPECTED TIME

The three time estimates t_O (optimistic time), t_P (pessimistic time) and t_L (most likely time) are identified on the Beta-distribution. The variance and standard deviation can be computed using t_O and t_P. However, one must combine the three time estimates into one single time—the average time taken for the completion of the activity or job. This average time or single workable time is commonly called the *expected time* and is denoted by t_E. If the exact shape of the probability distribution curve is known, the average time or expected time could be accurately calculated. However, since the precise curves are never available (specially for non-repetitive jobs) we must use *approximation.* This is done algebraically, using a weighted average derived by statisticians. In computing the expected time, a weightage of 1 is given to the optimistic time t_O, weightage of 4 to the most likely time (t_L) and weightage of 1 to the most pessimistic time (t_P).

Thus, $$t_E = \frac{t_O + 4t_L + t_P}{6} \qquad \text{...(5.7)}$$

The above expression for t_E, based on weighted average method, is reasonable since the chance of completion of the job in t_O or t_P is much less than the most likely time (t_L).

Let us take examples of estimated times of completion of two jobs A and B, as under.

	t_O	t_L	t_P (days)
Job A	4	6	11
Job B	5	10	12

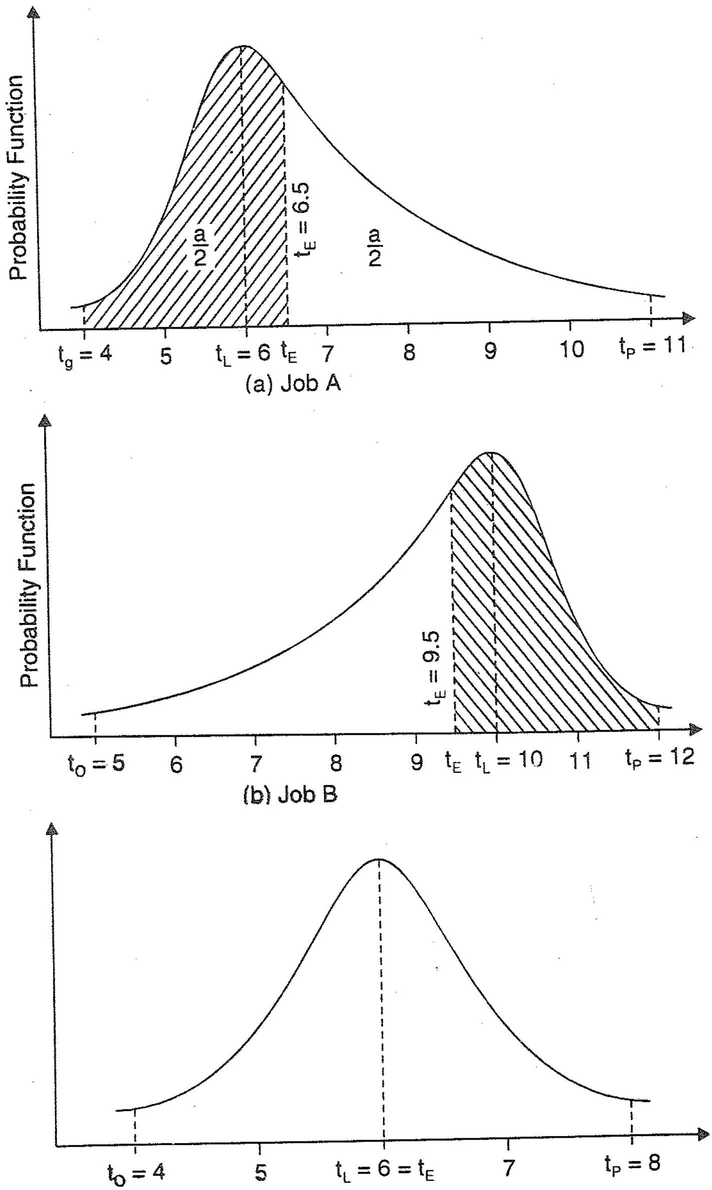
The expected time for these jobs are

$$(t_E)_A = \frac{t_O + 4t_L + t_P}{6} = \frac{4 + (4 \times 6) + 11}{6} = 6.5 \text{ days}$$

$$(t_E)_B = \frac{t_O + 4t_L + t_P}{6} = \frac{5 + (4 \times 10) + 12}{6} = 9.5 \text{ days}$$

Thus, for job A, the expected time falls to the right of the most likely time, though the curve has skew to the left [Fig. 5.7 (a)]. For job B, the expected time t_E falls to the left of the most likely time, though the curve has skew to the right.

One important point should be noted about the expected time t_E. The expected time t_E represents the *average value* while the most likely time t_L represents the *mode* of the β-distribution. The expected time represents a particular value on the distribution curve, that has both a 50-50 chance of being exceeded and a 50-50 chance of



(c)

FIG. 5.7

being met. There is the same chance of the actual time taken will be *greater* than t_E as there is it will be *less* than t_E. Naturally, the vertical ordinate through t_E will divide the probability curve into two equal areas, as shown in Fig. 5.7.

If the estimated time (t_O, t_L and t_P) are such that the expected time t_E computed with these comes out to be equal to t_L, the distribution curve will be symmetrical about the mode (t_L). For example, if $t_O = 4$, $t_L = 6$ and $t_P = 8$, for a job C, we have

$$t_E = \frac{t_O + 4t_L + t_P}{6}$$

$$= \frac{4 + 4 \times 6 + 8}{6} = 6$$

$$\therefore \qquad t_E = t_L$$

Such a situation is shown in Fig. 5.7 (c), in which the curve is symmetrical.

For the three jobs A, B and C mentioned before, the standard deviations and variance are as under :

For activity A,

Standard deviation $\sigma_A = \dfrac{t_P - t_O}{6} = \dfrac{11 - 4}{6} = 1.167$

Variance $\quad \sigma_A^2 = (1.167)^2 = 1.36$

For activity B,

$$\sigma_B = \frac{t_P - t_O}{6} = \frac{12 - 5}{6} = 1.167$$

Variance $\quad = (\sigma_B)^2 = (1.167)^2 = 1.36$

For activity C,

$$\sigma_C = \frac{t_P - t_O}{t} = \frac{8 - 4}{6} = 0.667$$

Variance $(\sigma_C)^2 = (0.667)^2 = 0.444.$

EXPECTED TIME FOR ACTIVITIES IN SERIES

When a number of activities are in series, the expected time for the path, along the activities, can be found by first finding the t_E for each activity, and then taking their sum. Alternatively, the optimistic time (t_O), the most likely time (t_L) and the pessimistic times (t_P) of the path can be calculated first by taking the sum of all t_O, t_L and t_P respectively and then t_E can be computed.

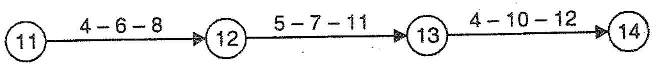


FIG. 5.8

For example, consider the three activities. 11—12, 12—13 and 13—14 shown in Fig. 5.8 with their individual time estimates (t_O, t_L and t_P) marked.

t_E for the path = Σt_E

Computations are arranged below (Table 5.4)

Table 5.4

Activity	t_O	t_L	t_P	t_E
11—12	4	6	8	6
12—13	5	7	11	7.333
13—14	4	10	12	9.333
				Σt_E = 22.666

Alternatively,

$$\Sigma t_O = 4 + 5 + 4 = 13$$

$$\Sigma t_L = 6 + 7 + 10 = 23$$

$$\Sigma t_P = 8 + 11 + 12 = 31$$

$$\therefore \qquad \Sigma t_E = \frac{\Sigma t_O + 4\Sigma t_L + \Sigma t_P}{6}$$

$$= \frac{13 + 4 \times 23 + 31}{6} = 22.67$$

Thus, t_E for the series of activities computed by both the methods is the same.

The *standard deviation* for the last event (network ending event) in a series of activity, is given by

$$\sigma_{t_E} = \sqrt{\sigma_1^2 + \sigma_2^2 + \ldots. + \sigma_n^2} \qquad \ldots(5.8)$$

where $\sigma_1, \sigma_2, \ldots.., \sigma_n$ are the standard deviations of each of the activities.

σ_{t_E} = standard deviation of network ending event.

For the above example,

$$\sigma \text{ for } 11\text{—}12 = \frac{8-4}{6} = 0.667$$

$$\sigma \text{ for } 12\text{—}13 = \frac{11-5}{6} = 1$$

$$\sigma \text{ for } 13\text{—}14 = \frac{12-4}{6} = 1.337$$

Hence the standard deviation of event 14, symbolised by σ_{t_E} is

$$= \sqrt{(0.667)^2 + (1)^2 + (1.333)^2}$$
$$= \sqrt{3.222} = 1.795.$$

A similar approach can be made for a network consisting of several paths, each path with a number of activities in series. When t_E for path in a network is known, the *critical path* can be chosen easily. A *critical path* is the one which consumes maximum of time resources. This is illustrated in example 5.3.

Example 5.2. *For a particular activity of a project, time estimates received from two engineers X and Y are as follows :*

	Optimistic time	*Most likely time*	*Pessimistic time*
Engineer X	*4*	*6*	*8*
Engineer Y	*3*	*5*	*8*

State who is more certain about the time of completion of the job.

Solution.

The degree of uncertainty (or otherwise) is indicated by the variance of the time estimates.

The variance of time estimates given by engineer *X* is

$$\sigma_X^2 = \left(\frac{t_P - t_O}{6}\right)^2 = \left(\frac{8-4}{6}\right)^2 = 0.4356$$

The variance of time estimate of engineer *Y* is

$$\sigma_Y^2 = \left(\frac{t_P - t_O}{6}\right)^2 = \left(\frac{8-3}{6}\right)^2 = 0.69.$$

Thus, the variance of time estimates given by *Y* is more. Since greater the variance, greater will be uncertainty, engineer *X*'s time estimates have more certainty.

Example 5.3. *The network for a certain project is shown in Fig. 5.9. Determine the expected time for each of the path. Which path is critical ?*

Solution.

In the network, event 1 is the starting event while event 8 is the end event. There are following four paths from the starting event to the end event :

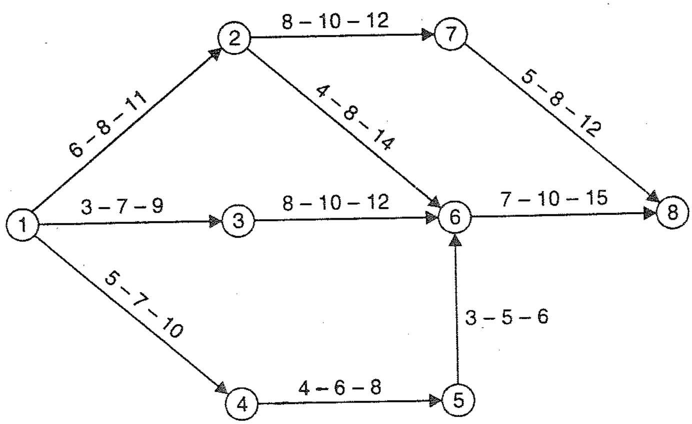


FIG. 5.9

Path A : 1—2—7—8

Path B : 1—2—6—8

Path C : 1—3—6—8

Path D : 1—4—5—6—8

The optimistic time for each of the path is equal to the sum of all t_O's of activity of that path. Similarly, the pessimistic time and most likely time of each of the paths can be found. These times are shown in Table 5.5.

Table 5.5

	Optimistic time	*Most likely time*	*Pessimistic time*
Path A 1—2—7—8	19	26	35
Path B 1—2—6—8	17	26	40
Path C 1—3—6—8	18	27	36
Path D 1—4—5—6—8	19	28	39

From the point of view of pessimistic time, path *B* is *critical* since it takes longest duration. From the point of view of optimistic time, paths A and D are equally critical. However, from the most likely time point of view, path D is the most critical.

In the PERT analysis, the *expected time* t_E is taken as the basis for finding the *critical path*. As indicated earlier, the expected time t_E is computed from the equation

$$t_E = \frac{t_O + 4t_L + t_P}{6}$$

Thus, expected time for each activity can be found. The expected time (t_E) for any path is equal to Σt_E of all activities. The computations are shown in Table 5.6.

Table 5.6

Path	*Activity*	t_O	t_L	t_P	t_E	Σt_E
	1—2	6	8	11	8.17	
A	2—7	8	10	12	10.00	26.34
	7—8	5	8	12	8.17	
	1—2	6	8	11	8.17	
B	2—6	4	8	14	8.33	26.83
	6—8	7	10	15	10.33	
	1—3	3	7	9	6.67	
C	3—6	8	10	12	10.00	27.00
	6—8	7	10	15	10.33	
	1—4	5	7	10	7.17	
D	4—5	4	6	8	6.00	28.33
	5—6	3	5	6	4.83	
	6—8	7	10	15	10.33	

From Table 5.6, we find that path D is *critical* since Σt_E for this path is the maximum.

PROBLEMS

1. Define 'optimistic time estimate', 'pessimistic time estimate' and 'most likely time estimate'.
2. Differentiate clearly between most likely time estimate (t_L), mean time (t_m) and expected time (t_E).

3. What do you understand by frequency distribution ? How do you determine (*i*) most likely time, (*ii*) variance and (*iii*) standard deviation from the frequency distribution ?

4. What is meant by probability distribution curve ? Differentiate clearly between normal probability distribution curve and beta distribution.

5. How do you use the normal probability curve for determining standard deviation ?

6. Explain how beta distribution is suitable for PERT analysis. Explain how do you determine the expected time and standard deviation.

7. The time estimates for three activities *A*, *B* and *C* are as follows :

	Optimistic time	*Most likely time*	*Pessimistic time*
A	10	12	14
B	6	8	12
C	5	10	12

Determine expected time and variance for each activity. Which activity has more reliable time estimates ?

8. A path of a certain network is shown in Fig. 5.10 with the time estimates for its activities as mentioned along each activity. Determine the expected time for the path. What is the standard deviation for the path ?

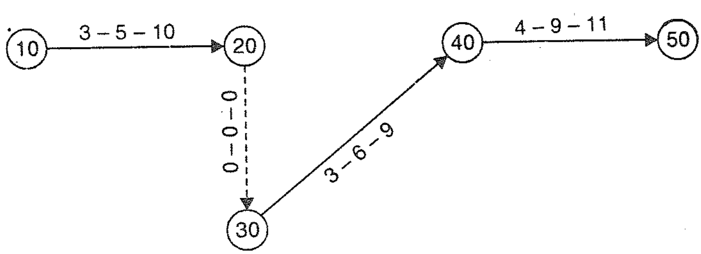


FIG. 5.10

9. The network for a certain project is shown in Fig. 5.11. Determine the expected time for each path. Which path is critical ?

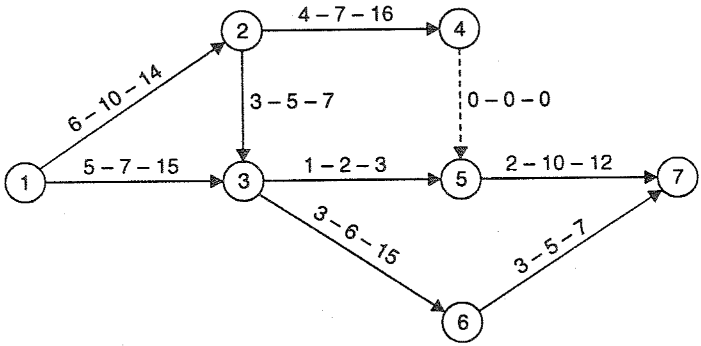


FIG. 5.11

6

PERT : Time Computations

6.1. INTRODUCTION

In the previous chapter, we have considered *time estimates* from which the *expected time* t_E was determined. All these times (*i.e.* optimistic time, pessimistic time, most likely time and expected time) refer to the completion of an activity. Let us now concentrate on the time of occurrence of an *event*. Though, for simple networks, the *expected time* or *average time* of completion of activities enables us to find the critical path, but in complex network, it is necessary to follow a systematic method for determining the critical path. This is achieved by first computing for each event and the following two time estimates are :

(*a*) Earliest expected time (T_E).

(*b*) Latest allowable occurrence time (T_L).

The three times estimates t_O, t_L and t_P, as well as the expected or average time t_E, which refer to an activity or job are designated by small t while the above two times (*i.e.* earliest expected time T_E and latest allowable occurrence time T_L) which refer to an event are symbolised by capital T.

6.2. EARLIEST EXPECTED TIME

The *earliest expected time* is the time when an event can be expected to occur. It is represented by symbol T_E and appear above or below the node (event circle) in a network.

The earliest expected time (T_E) is computed by adding the *expected times* (t_E) of all the *activities* along an *activities path* leading to that event. If more than one activity paths lead to that event, then the *maximum* of the *sum* of t_E's along the various paths will give the earliest expected time.

Let us first consider a simple network shown in Fig. 6.1 in which there is only one *activity path*. The three time estimates (t_O,

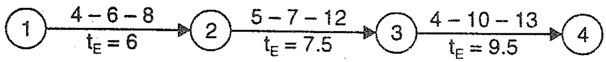


FIG. 6.1

t_L and t_P) of each activity are normally expressed above activity arrow while the activity expected time (t_E) is written below it.

Let us assume that event 1 is the initial event, which occurs at zero time. The activity 1—2, which connects events 1 and 2, has $t_E = 6$. Hence the earliest expected time for event 2 will be

$$= 0 + 6 = 6.$$

Event 3 is connected to event 2 by activity 2—3 which has $t_E = 7.5$. Hence T_E for 3 = T_E for 2 + t_E for activity (2—3)

$$= 6 + 7.5 = 13.5.$$

Similarly, T_E for the last event 4 = $(T_E)_3 + (t_E)_{3\text{-}4}$

$$= 13.5 + 9.5 = 23.$$

Hence we conclude that the earliest expected time for any event is the sum of the activities expected times (t_E) of the activity path leading to the event under consideration. The expected time T_E for each event is entered near the event circle (normally above the circle) as shown in Fig. 6.2.

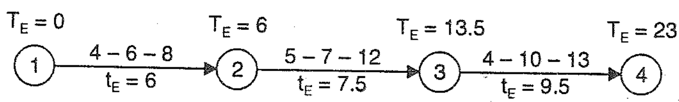


FIG. 6.2

Let us now consider a network shown in Fig. 6.3. The expected time (t_E) for each activity is shown on the activity arrow.

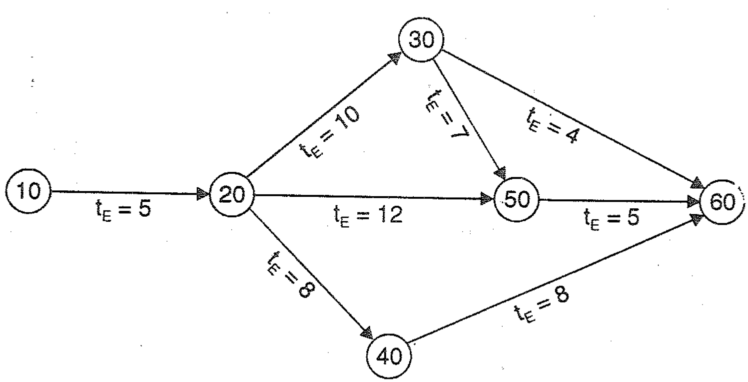


FIG. 6.3

Consider event 50. It has two activity paths : (*i*) path 10—20—30—50 and (*ii*) 10—20—50. In both these paths, events 10 and 20 are common. Since event 10 is the initial event, $T_E = 0$ for this. For event 20, $T_E = 0 + 5 = 5$ units. The time computations along each path leading to event 50 are as under :

Events		T_E
10		0
20		5
30		15
50	(*i*) Path 10—20—30—50	22
	(*ii*) Path 10—20—50	(17)

From the above, we note that T_E for event 50 are different along the two activity paths. However, *no event can be considered to have reached until all activities leading to the event are completed.* Hence event 50 cannot be considered to have occurred until all the activities along both the paths are complete. Thus, T_E will be the greater of the two values obtained from the two paths. Thus, T_E for 50 is 22 days.

Now let us come to the last event 60. It has four paths leading to it :

(*i*) Path 10—20—30—50—60.

(*ii*) Path 10—20—50—60.

(*iii*) Path 10—20—30—60.

(*iv*) Path 10—20—40—60.

Out of these, the first two paths pass through event 50. Since T_E for event 50 is 22, T_E for event 60 along both the paths will be

$$= 22 + 5 = 27.$$

For the third path 10—20—30—60,

$$T_E = 0 + 5 + 10 + 4 = 19.$$

Similarly, for the fourth path 10—20—40—60,

$$T_E = 0 + 5 + 8 + 8 = 21.$$

Since the earliest expected time for event 60 has to be the largest of the above, $T_E = 27$.

The following table gives T_E for each event of the network shown in Fig. 6.3. Fig. 6.4 shows the T_E's marked on the event circles.

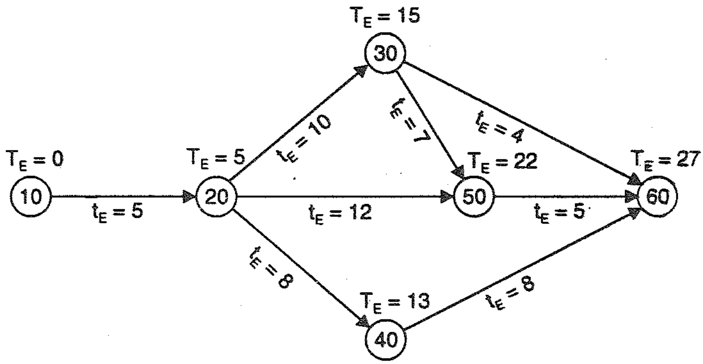


FIG. 6.4

Event	T_E	*Remarks*
10	0	
20	5	
30	15	
40	13	
50	22	Path 10—20—30—50
60	27	Path 10—20—30—50—60

6.3. FORMULATION FOR T_E

The method described above may be all right for small networks, but for large or complicated networks in which an event under consideration may have many predecessor events, it is better

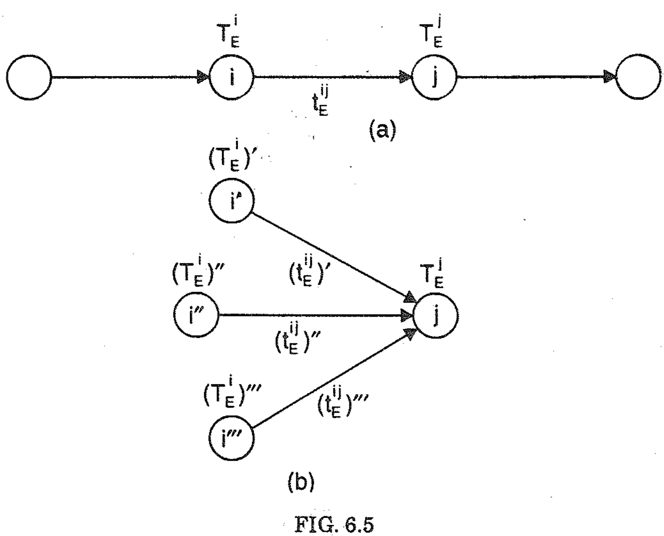


FIG. 6.5

to formulate a rule for computation of T_E so that frequent references to the network may not be necessary.

Let us represent an activity symbolically by ij where i is the predecessor event and j is the successor event, and i—j is the activity connecting the two events. Since T_E for a successor event is equal to T_E for the predecessor event plus the expected activity time (t_E), we have

T_E (successor event) = T_E (predecessor event) + t_E (activity).

Expressed symbolically,

$$T_E^j = T_E^i + t_E^{ij} \qquad \text{...(6.1)}$$

The above formulation is true if there is only one predecessor event. If, however, there are more than one predecessor events to the successor event (*i.e.* event under consideration), the above rule needs modification since the event j cannot occur unless all activities leading to it are completed. Hence T_E for the event will be equal to *maximum* of $(T_E^i + t_E^{ij})$ along various activity paths. Hence

$$T_E^j = (T_E^i + t_{ij})_{max}. \qquad \text{...(6.2)}$$

Thus, in Fig. 6.5, event j has three predecessor events i', i'' and i'''. With the three activities $(ij)'$, $(ij)''$ and $(ij)'''$ leading to it. The earliest expected time T_E^j for the event will be the maximum of $(T_E^i + t_E^{ij})'$, $(T_E^i + t_E^{ij})''$ and $(T_E^i + t_E^{ij})'''$.

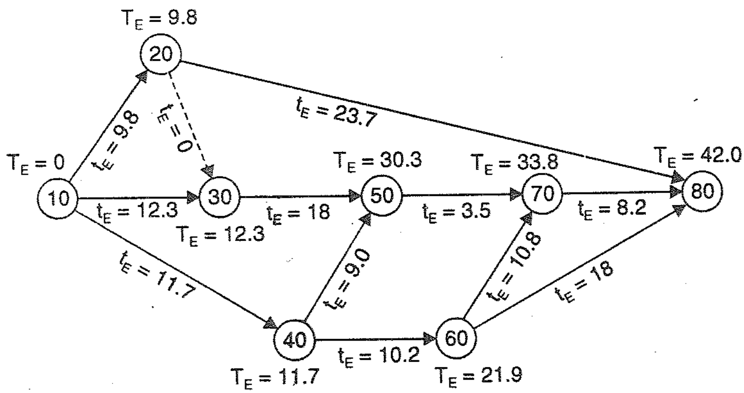


FIG. 6.6

Let us now apply the above formulation to the network shown in Fig. 6.6.

Event 10 has no predecessor event. Hence

$$T_E^{10} = 0$$

For event 20,

$$T_E^{20} = T_E^{10} + T_E^{10-20}$$

or $$T_E^{20} = 0 + 9.8 = 9.8$$

Event 30 has two predecessor events, 10 and 20. For each of these,

$$T_E^{30} = T_E^{10} + t_E^{10-30}$$
$$= 0 + 12.3 = 12.3$$

and $$T_E^{30} = T_E^{20} + t_E^{20-30}$$
$$= 9.8 + 0 = 9.8.$$

Out of these, the maximum value is 12.3. Hence $T_E^{30} = 12.3$. The maximum value is often *underscored* by dash (—). Similarly, the computations for other events are as under :

Event 40 : $T_E^{40} = T_E^{10} + t_E^{10-40} = 0 + 11.7 = 11.7$

Event 50 : (*i*) $T_E^{50} = T_E^{30} + t_E^{30-50} = 12.3 + 18 = \underline{30.3}$

(*ii*) $T_E^{50} = T_E^{40} + t_E^{40-50} = 11.7 + 9 = 20.7$

$\therefore$ $T^{50} = \underline{30.3}$

Event 60 : $T_E^{60} = T_E^{40} + t_E^{40-60} = 11.7 + 10.2 = 21.9$

Event 70 : (*i*) $T_E^{70} = T_E^{50} + t_E^{50-70} = 30.3 + 3.5 = 33.8$

(*ii*) $T_E^{70} = T_E^{60} + t_E^{60-70} = 21.9 + 10.8 = 32.7$

$\therefore$ $T_E^{70} = \underline{33.8}$

Event 80 : (*i*) $T_E^{80} = T_E^{20} + t_E^{20-80} = 9.8 + 23.7 = 33.5$

(*ii*) $T_E^{80} = T_E^{70} + t_E^{70-80} = 33.8 + 8.2 = \underline{42.0}$

(*iii*) $T_E^{80} = T_E^{60} + t_E^{60-80} = 21.9 + 18 = 39.9$

$\therefore$ $T_E^{80} = \underline{42.0}$

Computations are usually done in a tabular form. This is illustrated in Table 6.1, in which events are tabulated, starting with the end event, for convenience in computations. Thus, event 80 is entered first in the successor event column. Since it has three predecessor events (70, 60 and 20), these are entered next in their numerical order, with the high numbered event first. For all these three entries, the successor event is the same (*i.e.* 80). The *next lower event* 70 is then entered in the successor event column. It has two predecessor events 60 and 50 which are entered in the predecessor event column in the decreasing order of their numerical value. This procedure is followed till all the events are entered. The last successor event that is entered is 20 which has a predecessor event 10. The initial event 10 is not entered in the table since it does not have any predecessor event.

After having entered the successor and predecessor events, the expected time t_E^{ij} for each activity is entered. The column for *earliest expected time* t_E^j is then filled, starting from the bottom.

Table 6.1
Computation of Earliest Expected Time

Successor event j	*Predecessor event* i	*Activity* i—j	t_E^{ij}	t_E^j
80	70	70—80	8.2	<u>42.0</u>
	60	60—80	18.0	39.9
	20	20—80	23.7	33.5
70	60	60—70	10.8	32.7
	50	50—70	3.5	<u>33.8</u>
60	40	40—60	10.2	<u>21.9</u>
50	40	40—50	9.0	20.7
	30	30—50	18.0	<u>30.3</u>
40	10	10—40	11.7	<u>11.7</u>
30	20	20—30	0	9.8
	10	10—30	12.3	<u>12.3</u>
20	10	10—20	9.8	<u>9.8</u>

For example, ***for event 20***,

$$T_E^{20} = T_E^{10} + t_E^{10\text{-}20} = 0 + 9.8 = 9.8.$$

Since there is only one path to event 20, only one value of T_E^{20} has been obtained. This value is entered in the table and is underscored.

Next, ***for the activity 30***, we have

$$T_E^{30} = T_E^{10} + t_E^{10\text{-}30} = 0 + 12.3 = \underline{12.3}$$

also $$T_E^{30} = T_E^{20} + t_E^{20\text{-}30} = 9.8 + 0 = 9.8.$$

Here we obtain two values for T_E^{30} out of which the greater value (*i.e.* 12.3) is underscored in the table, to *identify* it, so that this value is used for further computations.

For event 40, $T_E^{40} = T_E^{10} + t_E^{10-40} = 0 + 11.7 = \underline{11.7}$

Since this is the only value, it is underscored.

For event 50, $T_E^{50} = T_E^{30} + t_E^{30-50} = 12.3 + 18.0 = 30.3$

It is to be noted that for T_E^{30}, greater of the two values (shown underscored) has been used.

Also $\quad T_E^{50} = T_E^{40} + t_E^{40-50} = 11.7 + 9.0 = 20.7.$

Out of the two values (*i.e.* 30.3 and 20.7) of T_E^{50}, the greater value (*i.e.* 30.3) is underscored.

For event 60, $\quad T_E^{60} = T_E^{40} + t_E^{40-60} = 11.7 + 10.2 = 21.9.$

For event 70, $\quad T_E^{70} = T_E^{50} + t_E^{50-70} = 30.3 + 3.5 = 33.8.$

Here also the greater of the two values of T_E^{60} (*i.e.* 30.3) has been used.

Also, $\quad T_E^{70} = T_E^{60} + t_E^{60-70} = 21.9 + 10.8 = 32.7$

Out of the two values of T_E^{70} so obtained, the greater value (*i.e.* 33.8) is underscored.

For event 80,

$$T_E^{80} = T_E^{20} + t_E^{20-80} = 9.8 + 23.7 = 33.5$$

$$T_E^{80} = T_E^{60} + t_E^{60-80} = 21.9 + 18.0 = 39.9$$

$$T_E^{80} = T_E^{70} + t_E^{70-80} = 33.8 + 8.2 = \underline{42.0}.$$

Here also, for T_E^{70}, the greater of the two values (*i.e.* 33.8) has been used. Thus, we obtain three values of T_E^{80}. Out of these, the maximum values (*i.e.* 42.2) is underscored.

In the computations of T_E^j for the above table, we proceeded from the *initial event* and ended with the end event. This is known as the *forward pass*, in which the network is traversed from initial event node to the final event node.

6.4. LATEST ALLOWABLE OCCURRENCE TIME

A planner is equally concerned with the completion of the project within the scheduled time. For each event, therefore, some time limit is allotted by which that event must occur. The latest time by which an event must occur, to keep the project on schedule is called the *latest allowable occurrence time*. It is denoted by symbol T_L. This is, therefore, another *event time*.

Whenever a project is taken in hand, decision is made regarding the completion time of the project and the accepted figure is called the *scheduled completion time* (or *the contractual obligation time*) and is denoted by symbol T_S. Naturally, T_S refers to the latest time of the last event (*i.e.* $T_S = T_L$).

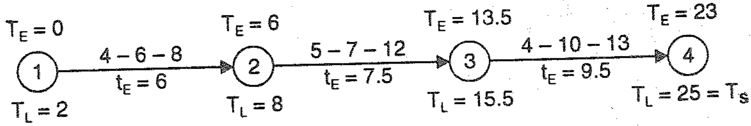


FIG. 6.7

To compute the latest allowable occurrence time for various events, let us again consider the simple network of Fig. 6.2. Let us assume that *scheduled completion time* T_S for the project is 25, meaning thereby that the end event 4 must occur, latest by 25 units of time after the project is initiated.

Thus, for the last event, $T_L^4 = T_S = 25$.

The activity 3—4 takes 9.5 units of time for its completion. Hence event 3 cannot occur later than $25 - 9.5 = 15.5$. Thus, $T_L^3 = 15.5$.

Similarly, for event 2, $T_L^2 = T_L^3 - t_E^{2\text{-}3} = 15.5 - 7.5 = 8.0$

and for the initial event.

$$T_E^1 = T_L^2 - t_E^{1\text{-}2} = 8.0 - 6 = 2.0.$$

These values of latest occurrence time for each event are indicated below the corresponding event circles.

Let us now consider the network of Fig. 6.4, reproduced in Fig. 6.8, where some events may have more than one successor events.

Let us presume that the *scheduled completion time* T_S for the project is 27 units of time. The latest occurrence time T_L for the last event is therefore 27.

For event 50, latest occurrence time is given by

$$T_L^{50} = T_L \text{ for } 60 - t_E \text{ for } (50\text{—}60)$$
$$= 27 - 5 = 22.$$

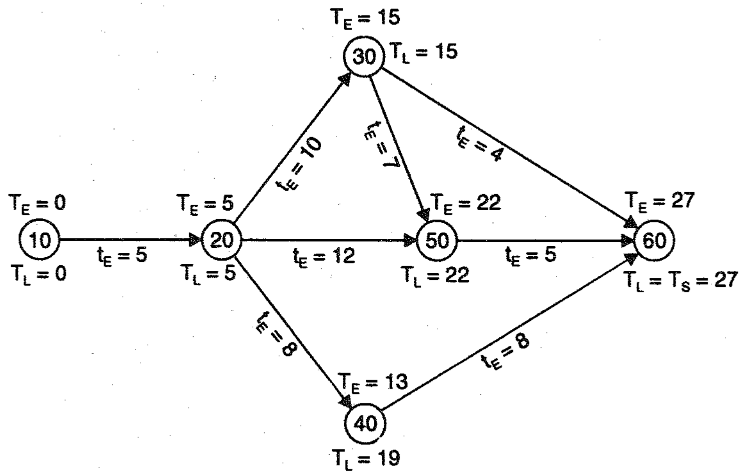


FIG. 6.8

For event 40,

$$T_L^{40} = T_L \text{ for } 60 - t_E \text{ for } (40\text{—}60)$$
$$= 27 - 8 = 19.$$

Event 30 has two successor events : event 40 and event 50. Hence two values of T_L are obtained as under :

$$T_L^{30} = T_L \text{ for } 60 - t_E \text{ for } (30\text{—}60)$$
$$= 27 - 4 = 23$$

and
$$T_L^{30} = T_L \text{ for } 50 - t_E \text{ for } (30\text{—}50)$$
$$= 22 - 7 = 15.$$

Out of this, the *minimum* value (*i.e.* 15) will be the appropriate value of T_L^{30}. This is because if event 50 cannot occur later than 22 units of time after the beginning of the project, event 30 cannot occur later than 15 units of time after the initiation of project since activity 30—50 takes 7 units of time for its completion. If a higher value of T_L^{30} (= 23) is permitted, T_L^{50} will be = 23 + 7 = 30 and the event 50 will be late by 8 units of time. Hence a minimum value, out of the various values, is to be selected.

Similarly, event 20 has three successor events : 50, 40 and 30 having T_L's as 22, 19 and 15 respectively. Therefore, we get three values of T_L^{20} as under :

(*i*) $T_L^{20} = T_L$ for $50 - t_E$ for (20—50)

$= 22 - 12 = 10$

(*ii*) $T_L^{20} = T_L$ for $40 - t_E$ for (20—40)

$= 19 - 8 = 11$

(*iii*) $T_L^{20} = T_L$ for $30 - t_E$ for (20—30)

$= 15 - 10 = 5.$

Out of above, the minimum value (*i.e.* 5) is the appropriate value of T_L^{20}.

For the initial event,

$T_L^{10} = T_L$ for $20 - t_E$ for (10—20)

$= 5 - 5 = 0.$

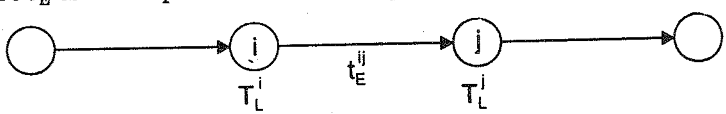
6.5. FORMULATION FOR T_L

Consider an activity $i\ j$ [Fig. 6.9 (*a*)], in which i is the predecessor event and j is the successor event.

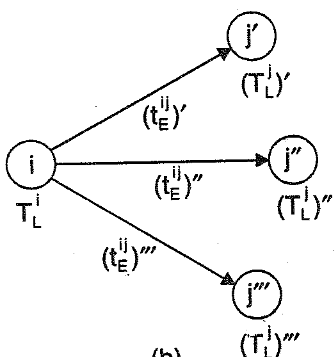
Let the latest occurrence time T_L^j be known. The latest occurrence time T_L^i for predecessor event is given by

$$T_L^i = T_L^j - t_E^{ij} \quad \text{...(6.3)}$$

where t_E^{ij} is the expected time of completion of activity $i\ j$.



(a)



(b)

FIG. 6.9

The above formulation is useful when the event i under consideration has only one successor event (j). If however, there are more than one successor events ($j', j'' j'''$ etc.), the *minimum* of ($T_L^j - t_E^{ij}$) will be the appropriate latest occurrence time T_L^i for event i. This is so because if the higher of the various values is taken, the latest occurrence time for the successor events will be also increased, suggesting a delay in the project completion.

Thus, $$T_L^i = (T_L^j - t_E^{ij})_{min} \qquad ...(6.4)$$

Thus, in Fig. 6.9 (b), there are three successor events (j', j'' and j''') to event i. The latest occurrence time T_L^i for the event i will be the *minimum* of $(T_L^j - t_E^{ij})'$, $(T_L^j - t_E^{ij})''$ and $(T_L^j - t_E)'''$.

Let us now apply this formulation to the network of Fig. 6.6, reproduced in Fig. 6.10.

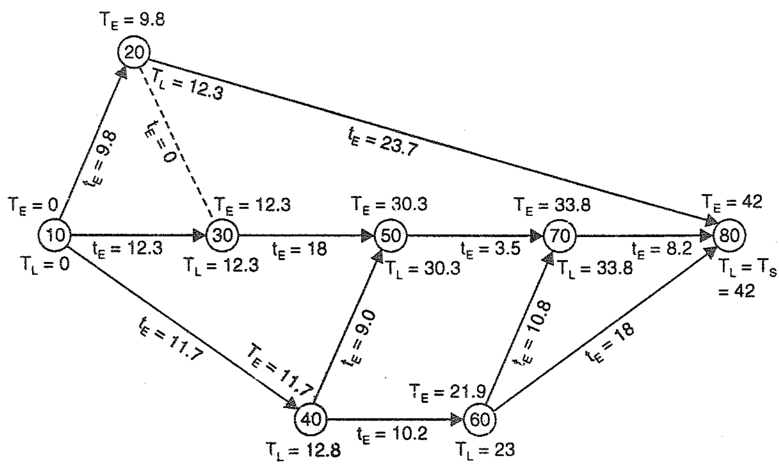


FIG. 6.10

Here, the *scheduled completion time* (T_S) is not given. In such a circumstance, T_S can be taken equal to T_E of end event.

Thus, $$T_S = T_E^{80} = 42$$

Hence $$T_L^{80} = T_S = 42$$

For computation of T_L for other events, we start backwards from the end event 80 and apply the rule

$$T_L^i = (T_L^j - t_E^{ij})_{min}.$$

This is done till the initial event is reached. The procedure is known as the *backward pass*, in contrast to the procedure of *forward pass* used for the computation of earliest expected time T_E.

(*i*) ***Event 70***

It has only one successor event (80).

$$\therefore \quad T_L^{70} = T_L^{80} - t_E^{70-80}$$
$$= 42 - 8.2 = \underline{33.8}.$$

This value is underscored for further use.

(*ii*) ***Event 60***

This has two successor events : 80 and 70

$$T_L^{60} = T_L^{80} - t_E^{60-80}$$
$$= 42 - 18 = 24$$

and
$$T_L^{60} = T_L^{70} - t_E^{60-70}$$
$$= 33.8 - 10.8 = \underline{23.}$$

Out of these, minimum value (*i.e.* 23) is the appropriate value of T_L^{60} and is underscored for further use.

(*iii*) ***Event 50***

It has only one successor event (70)

$$\therefore \quad T_L^{50} = T_L^{70} - t_E^{50-70}$$
$$= 33.8 - 3.5 = \underline{30.3.}$$

(*iv*) ***Event 40***

It has two successor event : 60 and 50

$$\therefore \quad T_L^{40} = T_L^{60} - t_E^{40-60} = 23 - 10.2 = \underline{12.8}$$

and
$$T_L^{40} = T_L^{50} - t_E^{40-50}$$
$$= 30.3 - 9 = 21.3$$

Out of these two, the lower value of 12.8 is adopted.

(*v*) ***Event 30***

It has also one successor event (50)

$$\therefore \quad T_L^{30} = T_L^{50} - t_E^{30-50}$$
$$= 30.3 - 18 = \underline{12.3}$$

(*vi*) ***Event 20***

It has two successor events : 80 and 30.

$$\therefore \quad T_L^{20} = T_L^{80} - t_E^{30-80}$$
$$= 42 - 23.7 = 18.3$$

and
$$T_L^{20} = T_L^{30} - t_E^{20-30}$$
$$= 12.3 - 0 = \underline{12.3}$$

Out of these, the minimum value (12.3) is adopted.

(*vii*) ***Event 10***

It has three successor events 40, 30 and 20.

$$\therefore \quad T_L^{10} = T_L^{40} - t_E^{10-40}$$
$$= 12.8 - 11.7$$
$$= 1.1$$
$$T_L^{10} = T_L^{30} - t_E^{10-30}$$
$$= 12.3 - 12.3$$
$$= \underline{0}$$
and
$$T_L^{10} = T_L^{20} - t_E^{10-20}$$
$$= 12.3 - 9.8$$
$$= 2.5.$$

Out of these, the minimum value is 0, and is adopted as the appropriate value of T_L for the initial event.

The computation of T_L is usually done in Tabular form. This is illustrated in Table 6.2, for the network shown in Fig. 6.10.

The first two columns are for predecessor events (i) and successor events (j) respectively. Here also, the tabulation of events is done with the highest numbered events, decreasing downwards in numerical order. Column 3 is for the activity $i—j$ while column 4 is for the corresponding activity expected times t_E^{ij}.

The computation of T_L^i is started *in the backward* direction, starting with the highest numbered predecessor event. Thus, for event 70,

$$T_L^{70} = T_L^{80} - t_E^{70-80}$$
$$= 42 - 8.2$$
$$= \underline{33.8}.$$

This is the only value ; it is underscored for further use. For activity 60, there are two successor events, 80 and 70.

$$\therefore \quad T_E^{60} = T_E^{80} - t_E^{60-80}$$
$$= 42 - 18.0$$
$$= 24.0$$
and
$$T_E^{60} = T_E^{70} - t_E^{60-70}$$
$$= 33.8 - 10.8$$
$$= \underline{23.0}.$$

Table 6.2
Computation of Latest Allowable Occurrence Time

Predecessor event (i)	*Successor event (j)*	*Activity i–j*	t_E^{ij}	T_L^i
70	80	70—80	8.2	33.8
60	80	60—80	18.0	24.0
	70	60—70	10.8	23.0
50	70	50—70	3.5	30.3
40	60	40—60	10.2	12.8
	50	40—50	9.0	21.3
30	50	30—50	18.0	12.3
20	80	20—80	23.7	18.3
	30	20—30	0.0	12.3
10	40	10—40	11.7	1.1
	30	10—30	12.3	0.0
	20	10—20	9.8	2.5

The minimum value (*i.e.* 23.0) is underscored.

For activity 50, the successor event is 70 which has a T_L of 33.8.

$$\therefore \quad T_L^{50} = T_L^{70} - t_E^{50-70}$$
$$= 33.8 - 3.5 = 30.3.$$

The computations are thus continued till the first event is reached.

6.6. COMBINED TABULAR COMPUTATIONS FOR T_E and T_L

The computation of *earliest expected time* (T_E) is done by *forward pass*, starting from the *initial event*. For such computations, predecessor events (*i* events) form the base. Table 6.1 is used for such computations. On the other hand, the computation of *latest allowable occurrence time* (T_L) is done by *backward pass*, starting from the *end event*. For such computations, successor events (*j* events) form the base. Table 6.2 is used for such computations. However, for most of the networks, computations of both T_E and T_L

for each event is required. None of the two tables suggested earlier are convenient. A combined tabular form (Table 6.3) is therefore suggested, with the help of which both T_E and T_L can be computed.

For illustration, let us take the same example of network shown in Fig. 6.6.

Table 6.3
Computations for T_E and T_L

Event No.	*Earliest expected time* (↓)				*Latest occurrence time* (↑)			
	Predecessor event (i)	t_E^{ij}	T_E^j	T_E	*Successor event (j)*	t_E^{ij}	T_L^i	T_L
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
10	—	—	0	0	20 30 40	9.8 12.3 11.7	2.5 <u>0</u> 1.1	0
20	10	9.8	<u>9.8</u>	9.8	30 80	0 23.7	<u>12.3</u> 18.3	12.3
30	10 20	12.3 0	<u>12.3</u> 9.8	12.3	50	18.0	<u>12.3</u>	12.3
40	10	11.7	11.7	11.7	50 60	9.0 10.2	21.3 <u>12.8</u>	12.8
50	30	18.0	<u>30.3</u>	30.3	70	3.5	<u>30.3</u>	30.3
60	40	10.2	<u>21.9</u>	21.9	70 80	10.8 18.0	<u>23.0</u> 24.0	23.0
70	50 60	3.5 10.8	<u>33.8</u> 32.7	33.8	80	8.2	<u>33.8</u>	33.8
80	20 60 70	23.7 18.0 8.2	33.5 39.9 <u>42.0</u>	42.0	—	—	<u>42.0</u>	42.0

Column 1 of Table 6.3 gives the event number, starting with the initial event, and proceeding in the direction of increasing numbers of the events.

Column 2 gives the predecessor events while column 6 gives the successor events to the events of column 1. These columns are therefore completed first, using the network diagram. An event under consideration (column 1) may have one or more than one predecessor events (column 2), and one or more than one successor events (column 6). A horizontal line is drawn after entering all the predecessor events and successor events to every event of column 1.

Then, computations are done for the *earliest expected times of the event,* in columns (3), (4) and (5). Column 3 is for the activity times t_E^{ij} where j is the event under consideration (column 1) and i is the predecessor event (column 2). T_E^j is computed by using Eq. 6.1

$$T_E^j = T_E^i + t_E^{ij}$$

Where there are more than one predecessor events, several values of T_E^j are obtained, which are entered in column (4). The maximum value of $\underline{T_E^j}$ is underscored. This underscored value is the appropriate value of the *earliest expected time* for the event under consideration (column 1), and is entered as T_E in column 5. For the computations of T_E, thus, we use the *forward pass*, starting with the initial event, and proceeding in the downward direction (↓) in the Table. We observe that T_E for last event comes out to be 42.

Then, we compute the *latest occurrence time* of the event under consideration (column 1), in columns 7, 8 and 9. Column (7) is the activity time t_E^{ij}, where i is the event under consideration (column 1) and j is the successor event (column 6). T_L^i is computed by using Eq. 6.3

$$T_L^i = T_L^i - T_E^{ij}$$

Computations are done by *backward pass*, starting with the end event and proceeding upwards (↑) in the table. The scheduled completion time T_S is taken equal to T_E. Hence $T_L = T_S = T_E = 42$ for the last event.

When there are more than one successor events, several values of T_L^i are obtained, which are entered in column (8). The minimum value of T_L^i is underscored. This underscored value is the appropriate value of the *latest allowable occurrence time* for the event under consideration (column 1) and is entered as T_L in column 9.

Thus, for each of the activities of column 1, T_E is given in column 5 and T_L is given in column 9.

PROBLEMS

1. Explain the term 'earliest expected time'. Formulate an expression for determining the same.
2. What do you understand by the 'latest allowable occurrence time'? How do you determine it ?
3. Explain with the help of a tabular form, how do you determine the 'earliest expected time' and the 'latest allowable occurrence time' for a network. Differentiate clearly between the 'forward pass' and 'backward pass'.
4. A network is shown in Fig. 6.11, with the expected time of completion of each activity. Determine the earliest expected time and latest occurrence time for each event.

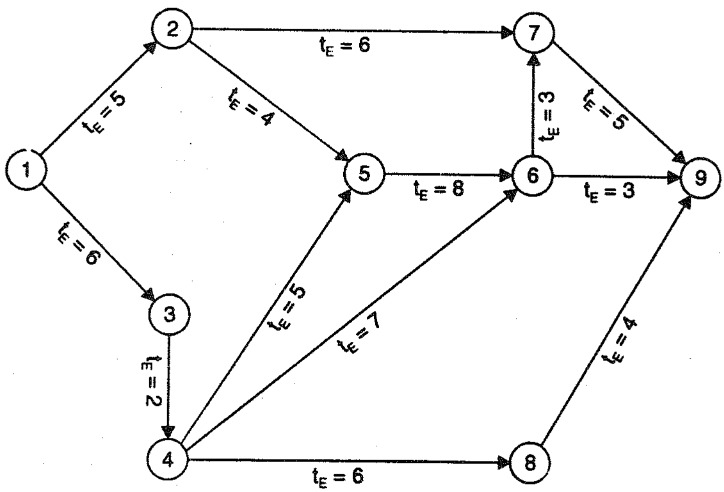


FIG. 6.11

5. The network for a construction project is shown in Fig. 6.12. The three time estimates for each activity are given along each activity arrow. Compute (a) expected time of completion of each activity, (b) earliest expected time for each event, (c) latest allowable occurrence time for each event.

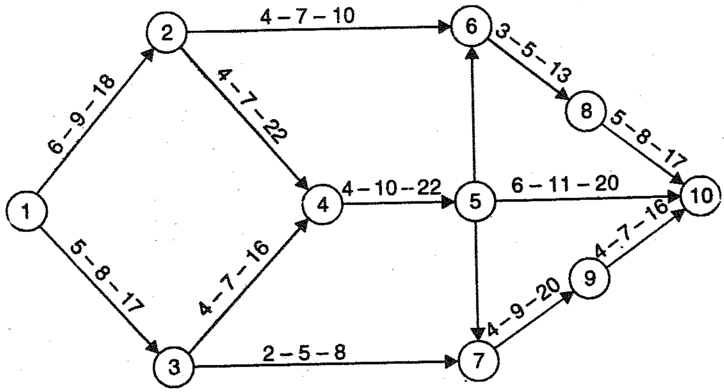

1
2
3
4
5
6
7
8
9
10
6 – 9 – 18
5 – 8 – 17
4 – 7 – 10
4 – 7 – 22
4 – 7 – 16
2 – 5 – 8
4 – 10 – 22
3 – 5 – 13
5 – 8 – 17
6 – 11 – 20
4 – 7 – 16
4 – 9 – 20

FIG. 6.12

7

PERT : Network Analysis

7.1. SLACK

In the previous chapter, we have computed two important times for the events : (*i*) the earliest expected time (T_E) and (*ii*) the latest allowable occurrence time T_L. The difference between the two times of an activity indicates the range between which the occurrence time of an event can vary. *Slack may be simply defined as the difference between the latest allowable time and the earliest expected time of an event.*

$$\therefore \qquad S = T_L - T_E \qquad ...(7.1)$$

where S is the slack for any event.

Let us take the case of a simple network of Fig. 6.3 (chapter 6), reproduced in Fig. 7.1, with T_E and T_L for each event marked.

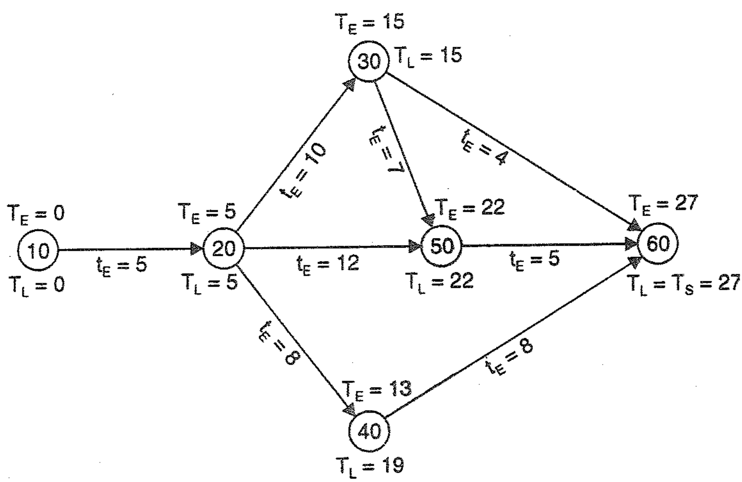


FIG. 7.1

The values of T_E, T_L and S for each event is tabulated in Table 7.1.

Table 7.1
Computation of Slack

Event No.	T_E	T_L	$S = T_L - T_E$
10	0	0	0
20	5	5	0
30	15	15	0
40	13	19	6
50	22	22	0
60	27	27	0

Here we observe that the slack for all events, except for event 40, is zero. The slack for event 40 is 6 units of time. This means that even if event 40 occurs 6 units of time *late*, the scheduled completion date of the project will not be affected. After the completion of event 20, event 40 can occur at 8 + (0 to 6) later. The other events 10, 20, 30, 50 and 60 do not have any slack time and hence their *occurrence is critical*.

Thus, we find that *slack* gives the idea of 'time to spare'. Slack means more time to work, less to worry about. It reveals about those areas which have an excess of resources from which trade-offs can be rearranged. It also spots those areas which are potential trouble areas, *i.e.* those areas of zero or minimum slack.

Slack can be *positive*, *zero* or *negative*, depending upon the relationship between T_L and T_E.

Positive slack. Positive slack is obtained when T_L is more than T_E for an event. It is an indication of an *ahead of schedule condition* (excess resources).

Zero slack. Zero slack is obtained when T_L is equal to T_E for an event. It is an indication of a *on schedule condition* (adequate resources).

Negative slack. Negative slack is obtained when the scheduled time of completion, T_S (and hence T_L) is less than the T_E. It is an indication of a *behind of schedule condition* (lack of resources).

Example 7.1. *Analyse with respect to resources the network shown in Fig. 7.2. Values are in days.*

Solution. The computations of T_E, T_L and S are arranged in Table 7.2. First T_E for all the events are computed by the *forward pass*, taking $T_E^1 = 0$. From the Table, we find $T_E^5 = 20$ for the last event. Since the scheduled time of completion (T_S) is not given, it is taken equal to $T_E^5 = 20$ days. Thus, $T_L^5 = T_S = 20$ days. Having found T_L for the last event, T_L for other events are computed by the *backward pass*.

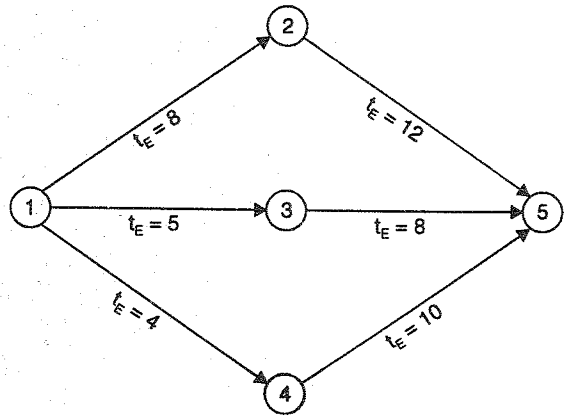


FIG. 7.2

Table 7.2

Event No.	*Earliest expected time* ↓				*Latest occurrence time* ↑				Slack $S = T_L - T_E$
	Predecessor event (i)	t_E^{ij}	T_E^i	T_E	*Successor event (j)*	t_E^{ij}	T_L^i	T_L	
1	—	—	0	0	2 3 4	8 5 4	0 7 6	0	0
2	1	8	8	8	5	12	8	8	0
3	1	5	5	5	5	8	12	12	7
4	1	4	4	4	5	10	10	10	6
5	2 3 4	12 8 10	20 13 14	20	—	—	20	20	0

The values of T_E and T_L, so obtained, for each event are marked on the network diagram, reproduced in Fig. 7.3.

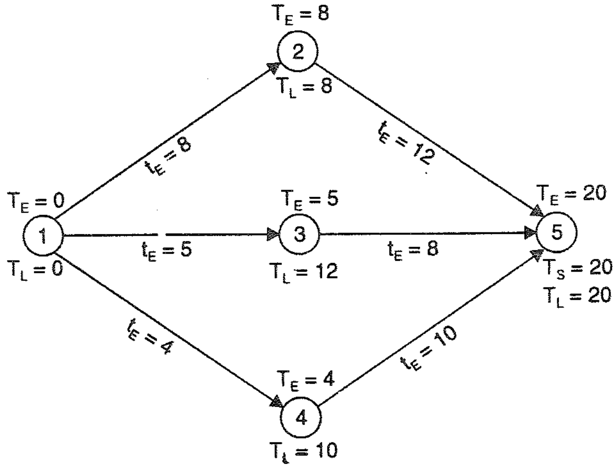


FIG. 7.3

The following conclusions are drawn :

1. Events 1, 2 and 5 have zero slack. Any delay in the activities connecting them would cause corresponding delay in the completion of the project. The occurrence of these events is *critical.*

2. Events 3 and 4 have slack of 7 and 6 days respectively. The activities connecting either of these events can be *delayed* by the slack value without affecting the scheduled completion time of the project.

3. Event 1 must start exactly on schedule. However, the above analysis suggests that the resources of activities 1—3 and 1—4 can be partly shifted to aid activities 1—2 and 2—5.

7.2. CRITICAL PATH

It is important to note that the value of slack, associated with an event, determine how critical that event is. The less the slack (more negative), the more critical an event is. A *critical path* is the one which connects the events having zero or minimum slack times. All the events along the *critical path* are considered to be *critical* in the sense that any delay in their occurrence will result in the delay in the scheduled completion of the project. *Eventually, a critical path is the longest path (time wise) connecting the initial and end event.*

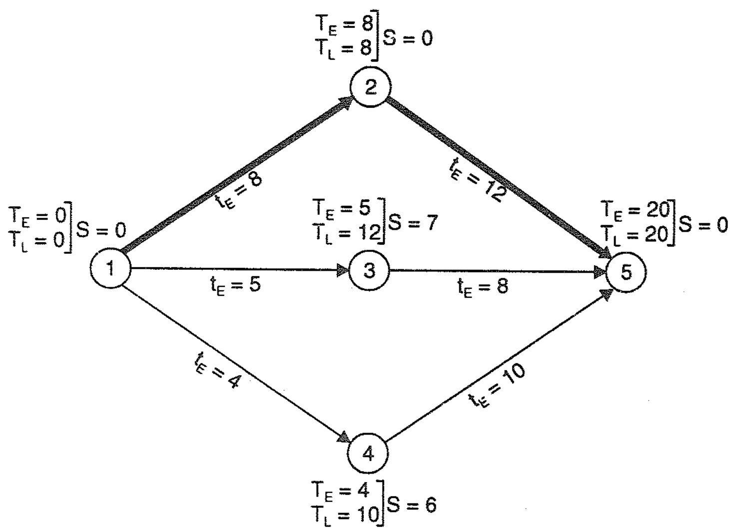


FIG. 7.4. THE CRITICAL PATH.

A critical path is distinctly marked in the network, usually by thick line.

For example, consider the network of Fig. 7.3 (Example 7.1). The path 1—2—5 is along the events having zero slack times. This path is therefore the critical path, shown by thick lines in Fig. 7.4. Note that Σt_E of all the activities along the critical path is equal to T_E of that last event. Along any other path, Σt_E is less than T_E of last event. *Thus, critical path is the longest path.*

7.3. ILLUSTRATIVE EXAMPLES

Example 7.2. *Determine the critical path for the network of Fig. 6.10.*

Solution. The earliest occurrence time (T_E) and latest allowable occurrence time (T_L) for each event are marked in Fig. 6.10. Slack for each event can be computed by taking the difference between T_L and T_E for each event, as indicated in Table 7.3.

Table 7.3

Event	T_E	T_L	*Slack* $S = T_L - T_E$
10	0	0	0
20	9.8	12.3	2.5
30	12.3	12.3	0
40	11.7	12.8	1.1
50	30.3	30.3	0
60	21.9	23.0	1.1
70	33.8	33.8	0
80	42.0	42.0	0

From the above table, we find that events 10, 30, 50, 60 and 80 have zero slack. Hence the path joining the events is the *critical path*. This is shown by thick lines in Fig. 7.5. Note that the critical path is the longest path, timewise. Also, note that Σt_E of all activities along the critical path is equal to T_E of last event.

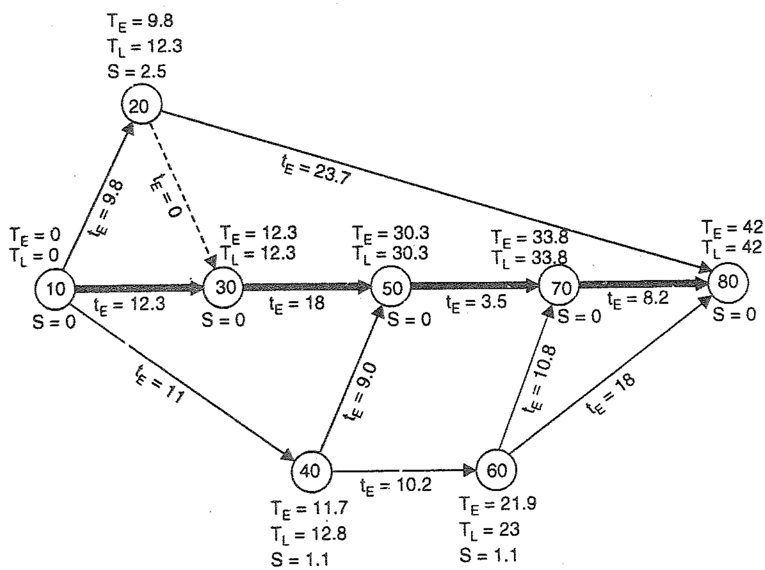


FIG. 7.5

Example 7.3. *Determine the critical path for the network shown in Fig. 7.6. Numbers indicate time in weeks.*

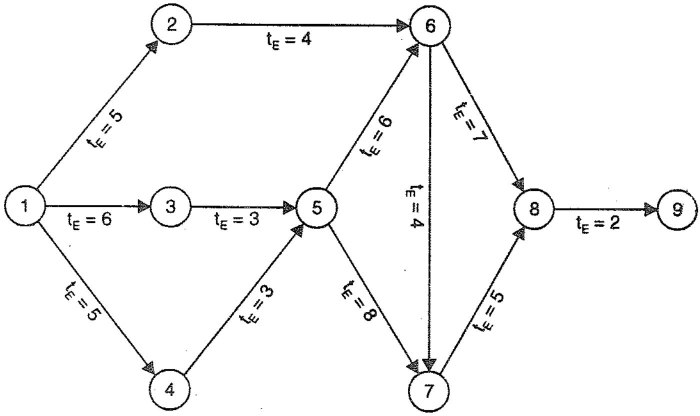


FIG. 7.6

Solution. For the determination of the critical path, it is essential to first determine T_E, T_L and S for each event. This is conveniently done in a tabular form. In Table 7.4, the earliest expected time for each event is first computed using forward pass and column 5 is obtained. This way T_E for the last event (9) is obtained. Since the scheduled time of completion of the project is not given, it is taken equal to $T_E^9 = 26$ weeks. Thus

$$T_L^9 = T_S = T_E^9 = 26 \text{ weeks.}$$

Knowing T_L^9, we calculate T_L for other events, using, backward pass, till the initial event is reached. Column 9 gives the latest allowable completion time for each event. Knowing T_E and T_L for each event, slack time S is found by taking the difference between T_L and T_S. This is tabulated in column 10.

From Table 7.4, we find that slack $S = 0$ at events 1, 3, 5, 6, 7, 8 and 9. Hence *critical path* is along 1—3—5—6—7—8—9, as shown by thick lines in Fig. 7.7. *Note that Σt_E of all activities along the critical path is equal to T_E of the last event.* Along any other path Σt_E is *less* than T_E of last event. Thus, the critical path is the longest path.

Table 7.4

Event No.	*Earliest expected time* ↓				*Latest occurrence time* ↑				*Slack* $S = T_L - T_E$
	Predecessor event (i)	t_E^{ij}	T_L^i	T_E	*Successor event (j)*	t_E^{ij}	T_L^i	T_L	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	—	—	0	0	2 3 4	5 6 5	6 $\underline{0}$ 1	0	0
2	1	5	$\underline{5}$	5	6	4	$\underline{11}$	11	6
3	1	6	$\underline{6}$	6	5	3	$\underline{6}$	6	0
4	1	5	$\underline{5}$	5	5	3	$\underline{6}$	6	1
5	3 4	3 3	$\underline{9}$ 8	9	6 7	6 8	$\underline{9}$ 11	9	0
6	2 5	4 6	9 $\underline{15}$	15	7 8	4 7	$\underline{15}$ 17	15	0
7	5 6	8 4	17 $\underline{19}$	19	8	5	$\underline{19}$	19	0
8	6 7	7 5	22 $\underline{24}$	24	9	2	$\underline{24}$	24	0
9	8	2	$\underline{26}$	26	—	—	26	26	0

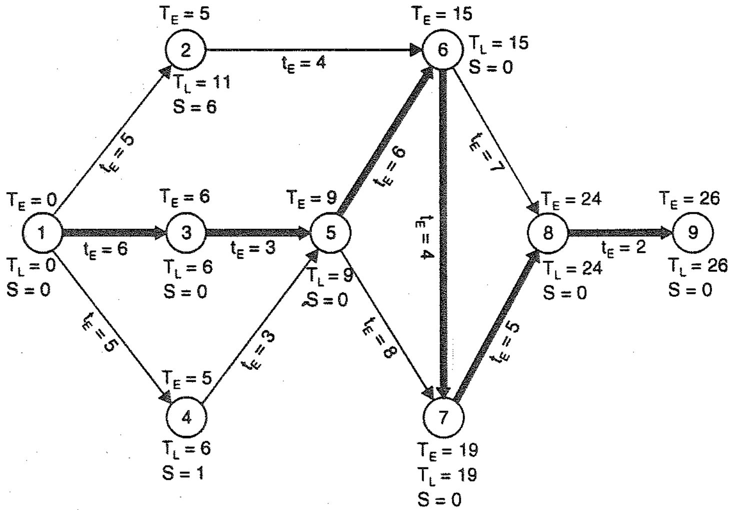


FIG. 7.7

Example 7.4. *The expected time of completion (in days) for each activity of a network is shown in Fig. 7.8. Determine the critical path. It is given that the scheduled completion time is 21 days.*

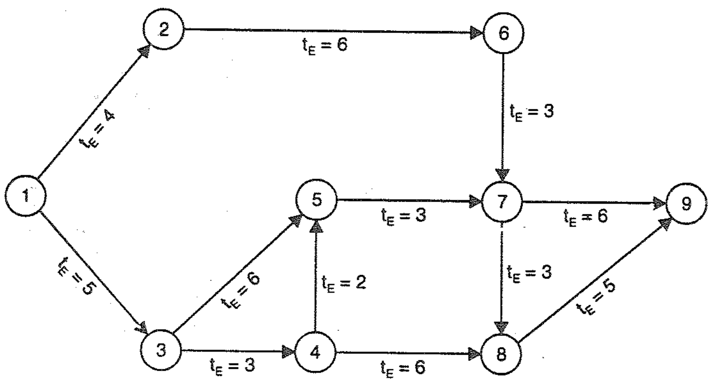


FIG. 7.8

Solution. The computations are arranged in the tabular form (Table 7.5). First, T_E for each activity is computed, as shown in column 5, from which we find that T_E for the last event comes out to be 22 days. The scheduled completion time (T_S) is 21 days only. This means that the project will have to be completed 1 day earlier than what it would normally take for its completion. This, of course, will have to be done by allocation of extra resources. The latest allowable occurrence time for the last event (T_L^9) will, therefore, be equal to T_L = 21 days. With this value, T_L is computed for all other events, by using *backward pass*. Column 9 gives the T_L values for all the events.

The slack for each event is determined by finding algebraic difference between T_L and T_E. This is tabulated in column 10 of Table 7.5. From this column, we find that the minimum value of slack is –1 day, and this occurs at events 1, 3, 5, 7, 8 and 9. Hence *critical path* is along 1—3—5—7—8—9, as shown by thick lines in Fig. 7.9. Note that Σt_E of the activities along the critical path is equal to T_E of the last event. This is not so along any other path.

Table 7.5

Event No.	*Earliest expected time* ↓				*Latest occurrence time* ↑				$S = T_L - T_E$
	Predecessor event (i)	t_E^{ij}	T_E^j	T_E	*Successor event (j)*	t_E^{ij}	T_L^i	T_L	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	—	—	0	0	2 3	4 5	0 – 1	– 1	– 1
2	1	4	4	4	6	6	4	4	0
3	1	5	5	5	4 5	3 6	5 4	4	– 1
4	3	3	8	8	5 8	2 6	8 10	8	0
5	3 4	6 2	11 10	11	7	3	10	10	– 1
6	2	6	10	10	7	3	10	10	0
7	5 6	3 3	14 13	14	8 9	3 6	13 15	13	– 1
8	4 7	6 3	14 17	17	9	5	16	16	– 1
9	7 8	6 5	20	22	—	—	21	21	– 1

From column (10), we observe that next minimum value of slack is 0 which occurs at events 2, 4 and 6. Therefore, the path connecting events 2 and 6, *i.e.* path 1—2—6—7 is the *sub-critical* or *semi-critical path*. Similarly, the path connecting event 4, *i.e.* path 3—4—5 is also sub-critical or semi-critical path. These two sub-critical paths are shown by dotted lines drawn adjacent to the corresponding activity arrows, in Fig. 7.9.

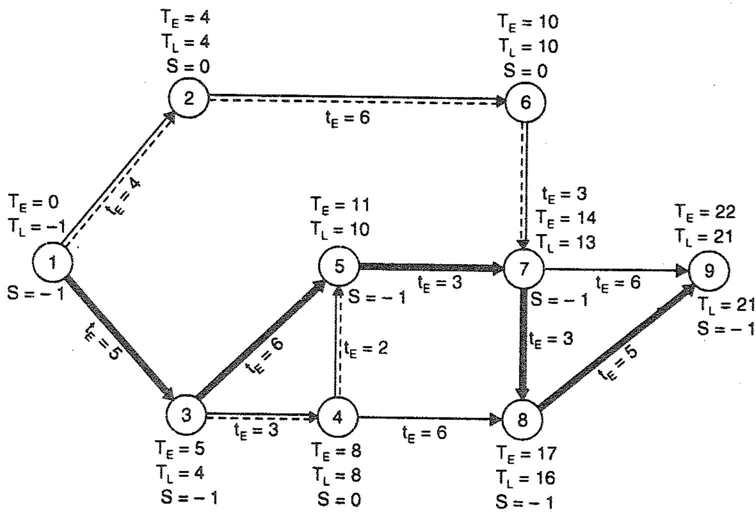


FIG. 7.9

7.4. PROBABILITY OF MEETING SCHEDULED DATE

After having calculated, the latest allowable occurrence time with the help of the assumed or given *scheduled completion time* of the project, and after having determined the critical path, the next question that remains to be answered is 'what is the probability of meeting the scheduled time ?' The answer to this question is sought by applying the probability theory to the network analysis.

We know that *critical path* is timewise the *longest path*. The critical path is along a number to activities of the path. In chapter 5, we considered three time estimates for any activity : (*i*) the optimistic time estimate (t_O), (*ii*) the most likely time estimate (t_L) and (*iii*) the pessimistic time estimate (t_P). We have also seen that these times have beta distribution (Fig. 5.6). Assuming the beta distribution, the *expected time* (t_E) for the completion of any activity is computed from Eq. 5.7.

$$t_E = \frac{t_O + 4t_L + t_P}{6} \qquad ...(5.7)$$

The expected time (t_E) is such that there is fifty-fifty chance of completion of the activity in this time. Fig. 7.10 shows the beta distribution, with t_O, t_L, t_E and t_P marked. If we draw a vertical line through t_E, the shaded area to its left is *half* of the total area. In other words, this vertical line through t_E divides the beta distribution curve in two-halves. Hence the activity *ij*, for which the above probability distribution curve refers, has fifty per cent probability of its completion within time t_E.

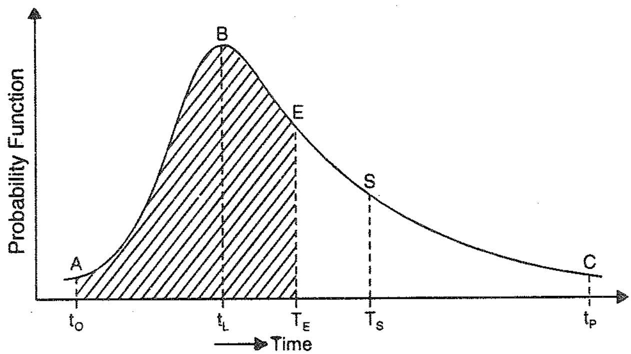


FIG. 7.10. BETA DISTRIBUTION.

Again the probability of completion of the activity within some other time t_S will be equal to the area under the curve upto vertical line through t_S divided by the total area of the curve :

$$\text{Probability} = \frac{\text{area under } ABS}{\text{area under } ABC}$$

We have seen in the previous article that Σt_E of all activities along the critical path is equal to T_E of the last event. Though t_E of individual activities has random probability distribution (assumed as beta distribution), the variation of T_E for the project as a whole has fortunately a *normal distribution* for all practical purposes. This assumption of normal distribution is based on the *central limit theorem* as given below.

Central Limit Theorem

This theorem states that if there are n activities, each having its own β-distribution with means $\mu_1, \mu_2, \mu_3, ..., \mu_n$ and standard deviations $\sigma_1, \sigma_2, \sigma_3, ..., \sigma_n$ respectively, then the distribution of time for the *project as a whole* will approximately be a *normal distribution curve*. The normal distribution curve will have a mean μ and variance σ^2 given by

$$\mu = \mu_1 + \mu_2 + \mu_3 + ... + \mu_n \qquad ...(7.2)$$

and

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + ... + \sigma_n^2 \qquad ...(7.3)$$

Normal Distribution Curve

Fig. 7.11 shows the normal distribution curve, which is symmetrical about its apex. Since $T_E = \Sigma t_E$, and since each t_E has

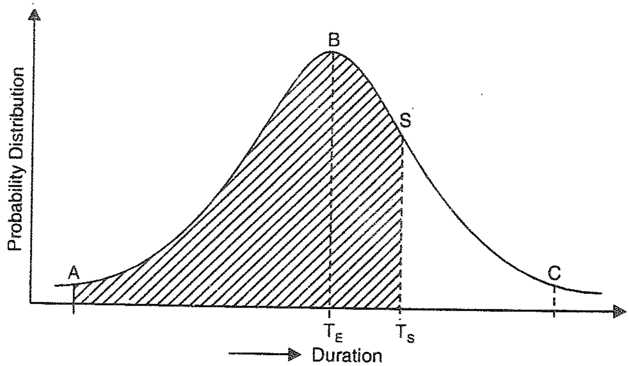


FIG. 7.11. NORMAL DISTRIBUTION.

fifty-fifty probability of completion of the activity, T_E should also have fifty-fifty probability. Hence we have to make the T_E value for the end event coincide with the modal value of the normal distribution curve. When once this is achieved, the random curve derived from a particular network will be reduced to the *normal form*.

Now, if T_S is the scheduled completion time of the project, the probability of completion of the project within the time T_S is given by

$$\text{Probability} = \frac{\text{Area under } ABS}{\text{Area under } ABC} \qquad ...(7.4)$$

In order to find the shaded area, we take the help of the important property of the normal distribution curve discussed in § 5.6. Refer Fig. 5.5 and 7.12.

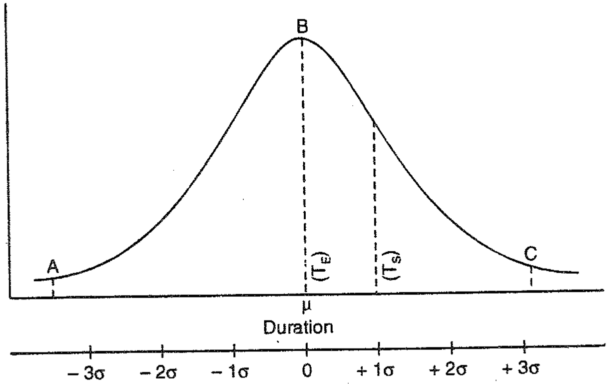


FIG. 7.12

If mean of the normal distribution is denoted by μ, it can be proved that

(*a*) Approximately 68 per cent of the value of normal distribution lie within $\pm\ \sigma$ from the average, where σ is the standard deviation. This means that the area of the curve between $x = \mu - \sigma$ to $x = \mu + \sigma$ is 68% of the total area (Fig. 5.5).

(*b*) Approximately 95 per cent of all the values lie within $\pm\ 2\sigma$ from the average. This means that area of the curve between $x = \mu - 2\sigma$ to $x = \mu + 2\sigma$ is 95% of the total area.

(*c*) Approximately 99.7 per cent of all the values lie within $\pm\ 3\sigma$ from the average. This means that the area of the curve between $x = \mu - 3\sigma$ to $x = \mu + 3\sigma$ is 99.7%.

Table 7.6 gives the values of probability corresponding the various values of the *normal deviate Z* (*i.e.* distance from the mean expressed in terms of σ).

Procedure for finding the probability of meeting the scheduled time of completion

The above properties of the normal distribution curve, and the % probability given in Table 7.6 can be used only if the random curve obtained from a particular network is reduced to the *normalized form*. This is obtained by making T_E value for the end event coincide with the modal value of the normal distribution curve. The following procedure is adopted for determining the probability of meeting the scheduled time of completion :

Table 7.6
Standard Normal Distribution Function

$Z(+)$	*Probability* (P_r) (%)	$Z(-)$	*Probability* (P_r) (%)
0	50.0	0	50.0
+ 0.1	53.98	− 0.1	46.02
+ 0.2	57.93	− 0.2	42.07
+ 0.3	61.79	− 0.3	38.21
+ 0.4	65.54	− 0.4	34.46
+ 0.5	69.15	− 0.5	30.85
+ 0.6	72.57	− 0.6	27.43
+ 0.7	75.80	− 0.7	24.20
+ 0.8	78.81	− 0.8	21.19
+ 0.9	81.59	− 0.9	18.41
+ 1.0	84.13	− 1.0	15.87
+ 1.1	86.43	− 1.1	13.57
+ 1.2	88.49	− 1.2	11.51
+ 1.3	90.32	− 1.3	9.68
+ 1.4	91.92	− 1.4	8.08
+ 1.5	93.32	− 1.5	6.68
+ 1.6	94.52	− 1.6	5.48
+ 1.7	95.54	− 1.7	4.46
+ 1.8	96.41	− 1.8	3.59
+ 1.9	97.13	− 1.9	2.87
+ 2.0	97.72	− 2.0	2.28
+ 2.1	98.21	− 2.1	1.79
+ 2.2	98.61	− 2.2	1.39
+ 2.3	98.93	− 2.3	1.07
+ 2.4	99.18	− 2.4	0.82
+ 2.5	99.38	− 2.5	0.62
+ 2.6	99.53	− 2.6	0.47
+ 2.7	99.65	− 2.7	0.35
+ 2.8	99.74	− 2.8	0.26
+ 2.9	99.81	− 2.9	0.19
+ 3.0	99.87	− 3.0	0.13

Step 1. Determine the standard deviation (σ) appropriate to the critical path, for the network, using the relation.

$$\sigma = \sqrt{\text{sum of variances along critical path}}$$

or
$$\sigma = \sqrt{\Sigma\sigma_{ij}^2} \qquad \text{...(7.5)}$$

where σ_{ij}^2 = variance for the activity i—j along the critical path

$$= \left(\frac{t_P^{ij} - t_O^{ij}}{6}\right)^2 \qquad \text{...(7.6)}$$

Step 2. Knowing the scheduled time of completion (t_S) and earliest expected time of completion (T_E), find the time distance $T_S - T_E$ and express it in terms of *probability factor Z by the relation :*

$$Z = \frac{T_S - T_E}{\sigma} \qquad \text{...(7.7)}$$

or
$$Z = \frac{T_S - T_E}{\sqrt{\Sigma\sigma_{ij}^2}} \qquad \text{...[7.7 (a)]}$$

The probability factor (Z) is the same as *normal deviate* of Table 7.6.

The probability factor (Z) can be positive, zero or negative.

When Z is *positive* (*i.e.* T_S to the right of T_E), the chances of completing the project in time are *more* than 50%.

When Z is *zero* (*i.e.* T_S coinciding with T_E), the chances of completing the project in time is fifty-fifty.

When Z is *negative* (*i.e.* T_S to the left of T_E), the chances of completing the project in time is *less* than 50%.

Step 3. Find % probability with respect to the normal deviate Z from Table 7.6.

Example 7.5. *A project is expected to take 15 months along the critical path, having a standard deviation of 3 months. What is the probability of completing the project within (a) 15 months, (b) 18 months and (c) 12 months ?*

Solution. The probability factor is given by

$$Z = \frac{T_S - T_E}{\sigma} \qquad \text{...(7.7)}$$

where T_E = 15 months and σ = 3 months (*a*) Given : $T_S = 15$ months

$$\therefore \qquad Z = \frac{15 - 15}{3} = 0$$

From Table 7.7, for $Z = 0$, probability = **50%**

(*b*) Given : $T_S = 18$ months

$$\therefore \qquad Z = \frac{18 - 15}{3} = 1$$

From Table 7.7, for $Z = 1$, probability = **84.13%**

(*c*) Given : $T_S = 12$ months

$$Z = \frac{12 - 15}{3} = -1.$$

From Table 7.6, for $Z = -1$, probability = **15.87%**

Example 7.6. *PERT calculations yield a project length of 50 weeks, with a variance of 16. Within how many weeks would you expect the project to be completed with probability of (a) 95% (b) 75% (c) 40% ?*

Solution. Standard deviation $\sigma = \sqrt{16} = 4$.

(*a*) For 95% probability $Z \approx 1.65$ (Table 7.6)

$$\text{Now} \quad \frac{T_S - T_E}{\sigma} = Z$$

$$\begin{aligned} \therefore \qquad T_S &= \sigma Z + T_E \\ &= 4 \times 1.65 + 50 \\ &= \mathbf{56.6} \approx 57 \text{ weeks.} \end{aligned}$$

(*b*) For 75% probability, $Z = 0.69$

$$\begin{aligned} \therefore \qquad T_S &= \sigma Z + T_E \\ &= 4 \times 0.69 + 50 \\ &= \mathbf{52.76} \approx 53 \text{ weeks.} \end{aligned}$$

(*c*) For 40% probability, $Z \approx -0.25$

$$\begin{aligned} \therefore \qquad T_S &= \sigma Z + T_E \\ &= .4\,(-0.25) + 50 \\ &= \mathbf{49 \text{ weeks.}} \end{aligned}$$

Example 7.7. *For the network shown in Fig. 7.13, the time estimates (in days) each for activity are mentioned. Determine the probability of completing the project in 35 days.*

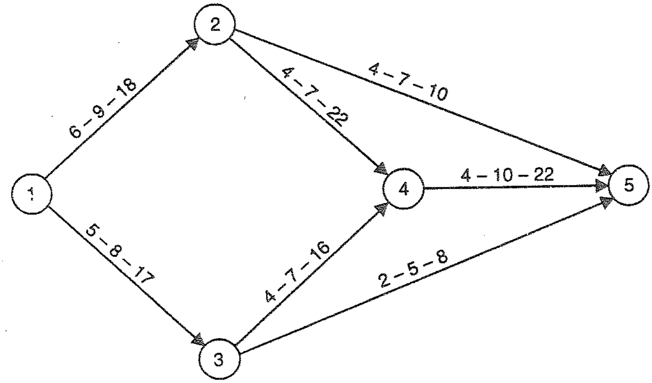


FIG. 7.13

Solution. Let us first determine the expected time of completion (t_E) and variance σ^2 for each activity using the relations

$$t_E = \frac{t_O + 4t_L + t_P}{6}$$

and $$\sigma^2 = \left(\frac{t_P - t_O}{6}\right)^2$$

The computation are arranged in Table 7.7.

Table 7.7

Activity	t_O	t_L	t_P	t_E	σ^2
1—2	6	9	18	10	4
1—3	5	8	17	9	4
2—4	4	7	22	9	9
3—4	4	7	16	8	4
4—5	4	10	22	11	9
2—5	4	7	10	7	1
3—5	2	5	8	5	1

The computation for T_E and T_L and slack for various events are arranged in Table 7.8. For the last event, T_L is taken equal to its T_E. From Table 7.8, we find that slack is minimum at events 1, 2, 4 and 5. Hence path 1—2—4—5 is the critical path. The critical path is marked by thick lines on Fig. 7.14.

Table 7.8

Event No.	Earliest expected time ↓				Latest occurrence time ↑				Slack S
	Predecessor event (i)	t_E^{ij}	T_E^j	T_E	Successor event (j)	t_E^{ij}	T_L^i	T_L	
1	—	—	0	0	2 3	10 9	<u>0</u> 2	0	0
2	1	10	<u>10</u>	10	4 5	9 7	<u>10</u> 23	10	0
3	1	9	<u>9</u>	9	4 5	8 5	<u>11</u> 25	11	2
4	2 3	9 8	<u>19</u> 17	19	5	11	<u>19</u>	19	0
5	2 3	7 5	17 14	30			30	30	0

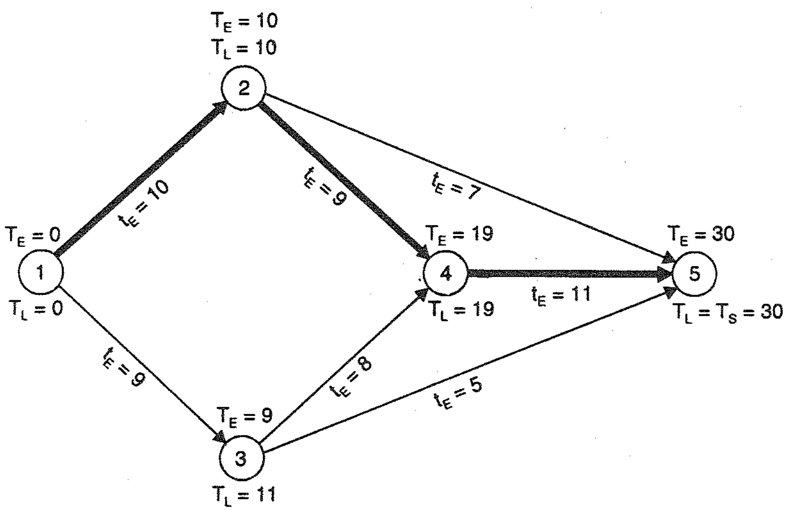


FIG. 7.14

Now deviation along the critical path is given by

$$\sigma = \sqrt{\Sigma\sigma_{ij}^2}$$

where $\Sigma\sigma_{ij}^2$ = sum of variances along critical path

$$= \sigma^2_{1-2} + \sigma^2_{2-4} + \sigma^2_{4-5}$$

$$= 4 + 9 + 9 = 22$$

$$\therefore \qquad \sigma = \sqrt{22} = 4.69$$

$$\therefore \qquad Z = \frac{T_S - T_E}{\sigma}, \text{ where } T_S = 35 \text{ days (given)}$$

$$= \frac{35 - 30}{4.69} = 1.066$$

From Table 7.6, for $Z = 1.066$, $P_r \approx$ **85.7%**.

Hence there is 85.7% probability of completion of the project in 35 days.

Example 7.8. *Fig. 7.15 shows the network for a construction project, with the three time estimates of each activity marked. Determine :*

(a) Critical path and its standard deviation.

(b) Probability of completion of project in 40 days.

(c) Time duration that will provide 95% probability of its completion in time.

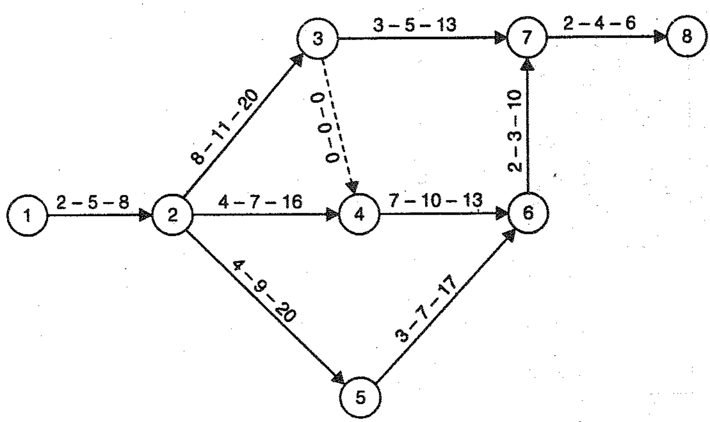


FIG. 7.15

Solution. The computations of t_E and σ^2 are arranged in Table 7.9. The computation for T_E, T_L and S for each event are given in Table 7.10. For the last event, T_L is taken equal to its T_E. From Table 7.10, we find that slack is minimum at events 1, 2, 3, 4, 6, 7 and 8. Hence critical path is along 1—2—3—4—6—7—8. The value of T_E and T_L for each event, along with the critical path is marked on Fig. 7.16.

Table 7.9

Activity	t_O	t_L	t_P	t_E	σ^2
1—2	2	5	8	5	1
2—3	8	11	20	12	4
3—4	0	0	0	0	0
2—4	4	7	16	8	4
2—5	4	9	20	10	7.11
4—6	7	10	13	10	1
5—6	3	7	17	8	5.44
3—7	3	5	13	6	2.78
6—7	2	3	10	4	1.77
7—8	2	4	6	4	0.44

Table 7.10

Event No.	*Earliest expected time* ↓				*Latest occurrence time* ↑				*S*
	Predecessor event (i)	t_E^{ij}	T_E^j	T_E	*Successor event (j)*	t_E^{ij}	T_L^i	T_L	
1	—	—	0	0	2	5	0	0	0
2	1	5	5	5	3	12	5	5	0
					4	8	9		
					5	10	9		
3	2	12	17	17	7	6	25	17	0
					4	0	17		
4	2	8	13	17	6	10	17	17	0
	3	0	17						
5	2	10	15	15	6	8	19	19	4
6	4	10	27	27	7	4	27	27	0
	5	8	23						
7	3	6	23	31	8	4	31	31	0
	6	4	31						
8	7	4	35	35	—	—	35	35	0

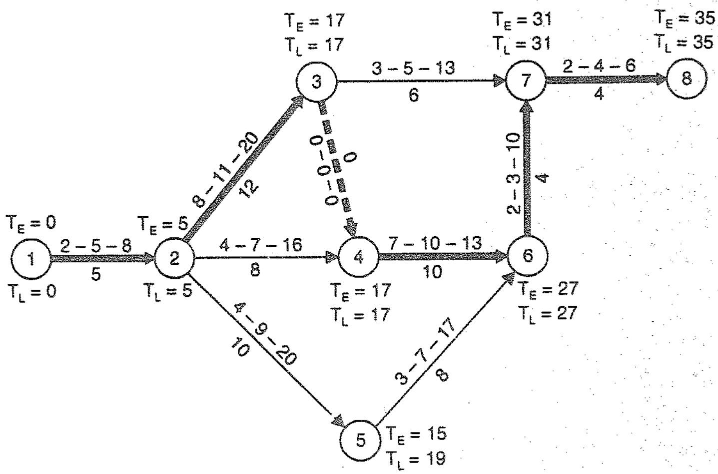


FIG. 7.16

Standard deviation along critical path

$$= \sqrt{\Sigma(\sigma_{ij})^2}$$

where $\Sigma(\sigma_{ij})^2 = 1 + 4 + 0 + 7.11 + 1.77 + 0.44 = 14.32$

$\therefore \quad \sigma = \sqrt{14.32} = \mathbf{3.78}.$

Now $\quad Z = \dfrac{T_S - T_E}{\sigma}$

(*a*) When $T_S = 40$ days

$$Z = \frac{40 - 36}{3.75} = 1.07.$$

Hence from Table 7.6, Probability = **85.7%.**

(*b*) For $P_r = 95\%$, we have $Z \approx 1.65$.

$\therefore \quad T_S = \sigma Z + T_E$

$= (3.78 \times 1.65) + 35 = \mathbf{41.2}$ days.

PROBLEMS

1. Explain the following terms : (*i*) Latest allowable occurrence time, (*ii*) earliest expected time, (*iii*) slack, (*iv*) critical path. What does a negative stack indicate ?

2. For the network shown in Fig. 7.17 (on next page), determine the slack for various events, if the scheduled date of completion of the project is 36 days. Present the computations in tabular form.

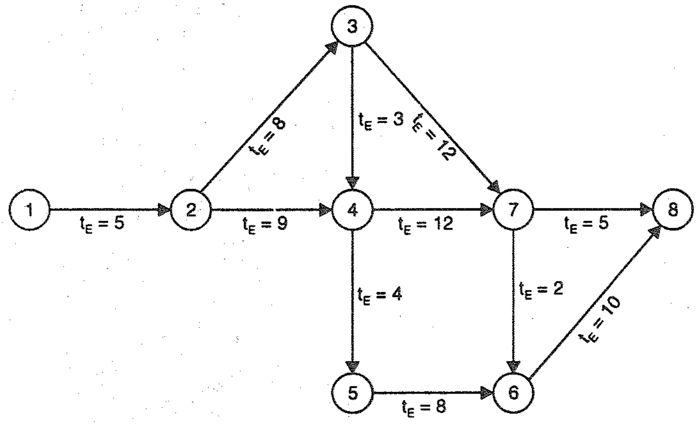


FIG. 7.17

3. For the network of problem 2, determine the critical path.
4. Explain how do you determine the probability of meeting the scheduled date of completion of a project.
5. If the expected time along the critical path of a project is 27 weeks and the standard deviation along it is 6 weeks, determine the probability of completing the project within (*a*) 21 weeks, (*b*) 24 weeks, (*c*) 36 weeks.
6. On a network, PERT calculations yield a project length of 60 days with a variance of 9 days. Estimate the number of days required to complete the project with a probability of 98%.
7. A construction company has an opportunity to submit a bid for the construction of a new apartment building. From the specification provided by the developer, the PERT network along with the three time estimate (in week) for each activity are shown in Fig. 7.18.

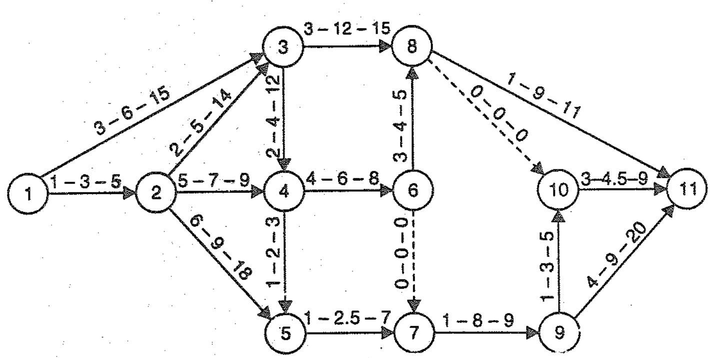


FIG. 7.18

Determine :

(*a*) Critical path and its standard deviation.

(*b*) Probability of completing the work in 38 weeks.

(*c*) Completion time duration for which the company should bid to provide 95% probability of completing the project in time.

8

CPM : Network Analysis

8.1. INTRODUCTION

The Critical Path Method, commonly abbreviated as CPM was discovered independently of PERT, by Du Pont and Spery Rand Corporation in 1957, for applications to industrial situations like construction, manufacturing, maintenance etc. Since then, it has found wide acceptance by construction industry with application to bridges, dams, tunnels, buildings, highways, power plants etc. Following are the examples from fairly diverse field where application of CPM can be made :

1. Building a new bridge across river Ganges.
2. Constructing a multi-storeyed building.
3. Extension of a factory building.
4. Shifting a manufacturing unit to another site.
5. Overhauling of a diesel engine.
6. Manufacture of a new car.

The above mentioned projects are more or less *unique* in nature. Either the projects are repetitive in nature such as 'overhaul of a diesel engine', or of such a nature that similar projects have been executed previously. However, take the example of building a new bridge across river Ganges : neither in the past nor in future, this project is likely to be undertaken in the same form. But activities required to complete the above project can be listed easily and earlier experience and knowledge can be used for gaining information regarding execution, duration, resources, cost etc. of these activities. Hence realistic and more accurate information can be obtained and the results so available will be more reliable than in PERT.

CPM networks are generally used for repetitive type projects, or for these projects for which fairly accurate estimate of time for completion of each activity can be made, and for which cost estimations can be made with fair degree of accuracy. However, it is not suitable for research and development projects.

PERT and CPM *differ* on the following points :

(*i*) In CPM, time estimates for completion of activities are with fair degree of accuracy, while in PERT system, time estimates are not so accurate and definite.

(*ii*) In CPM, cost optimisation is given prime importance. The time duration for completion depends on this cost optimisation. The cost is not directly proportional to time. The cost is minimum corresponding to a certain *optimum time duration* and the cost *increases* if the time duration is either *increased* or *decreased*. On the other hand, in PERT, it is assumed that cost varies directly with time. Attention is, therefore, paid to minimise the time so that minimum cost results. Thus, in CPM *cost* is the direct controlling factor while in PERT, time is controlling factor.

As the name suggests, the, *critical path* in the CPM method plays an important role in planning and scheduling. A *critical path* is the timewise *longest path* in a network. A similar term was used in PERT also. However, in PERT, a critical path is the path that joins the *critical events* ; this path was determined on the basis of *slack* at each event. In CPM, however, a *critical path* is the one which passes through *critical activities*. This critical path is determined on the basis of minimum *float* for each activity. In other words, *in PERT the critical path is determined on the event-oriented slack philosophy while in CPM, the critical path is determined on activity-oriented float philosophy.*

8.2. CPM : PROCESS

CPM has several levels of application in project management, namely : *planning, scheduling* and *controlling.*

Planning

Planning is the most important of project management, in which the logical sequence in which the jobs or activities must be performed, is formalised. The logic should be reviewed with correctness. It should be ascertained that all the activities are shown, and the scope of the project has been interpreted correctly, and also that the resources that are required for performing each job are applied. Resources can be time, money, manpower, equipment and facilities.

Scheduling

Scheduling is the determination of time required for execution of each operation and the time order in which each operation has to be carried out, to meet the plan objectives. Scheduling has to be done not only in respect of operations but also in respect of *resources*. The resources, in general, include *time, space, equipment, material* and *effort*. More specifically, scheduling is the mechanical process of formalising the planned functions, assigning the starting and completion dates to each part (or activity) of the project in such a manner that the whole project proceeds in a logical sequence and in an orderly and systematic manner.

Controlling

Controlling is the process in which difference or deviations between the plan and actual performances are reviewed after the project has started. The analysis and correction to these deviations form the basic aspect of control. Replanning and rescheduling is done to compensate for the deviations. In CPM, controlling is required not only in respect of physical progress of work, but also in respect of cost.

The complete *process* of CPM application can be summarised in the following major *steps* :

1. DESCRIBE	:	project in terms of dependencies among activities, *i.e.* plan the project.
2. DETERMINE	:	the schedule of the activities.
3. PREDICT	:	those activities which control significant target dates of the project.
4. ANALYSE	:	the schedule developed.
5. REPLAN	:	the project if analysis so indicates.
6. ALLOCATE	:	resources to project in an efficient manner for the schedule developed.
or MAKE	:	schedule changes required by resources limitations.
7. DEVELOP	:	time-cost relationship for activities and optimise total project cost by selecting suitable project duration.

Fig. 8.1 shows the CPM process diagrammatically.

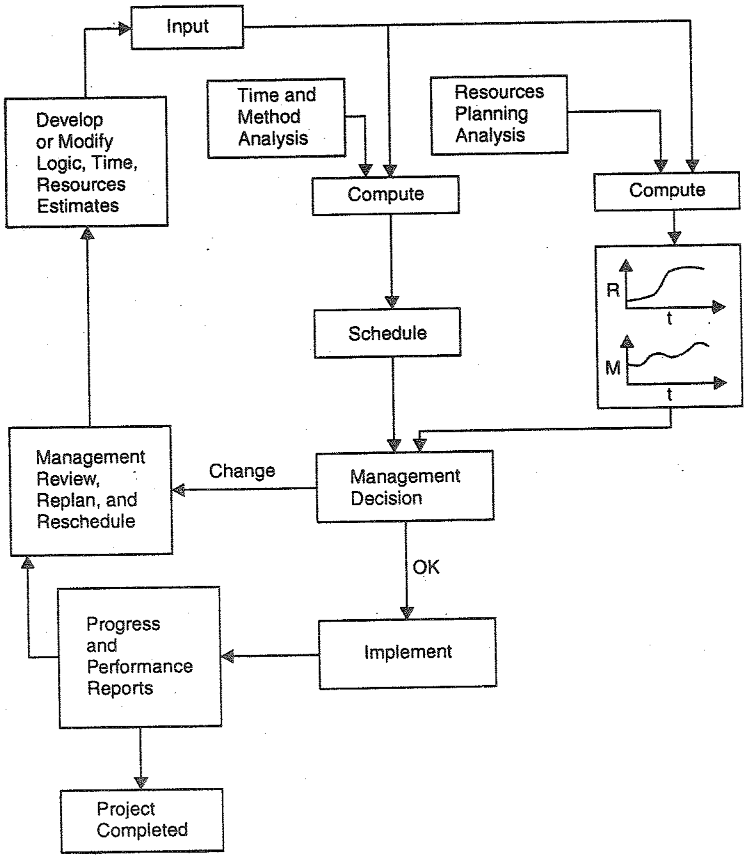


FIG. 8.1. CPM PROCESS.

8.3. CPM : NETWORKS

CPM networks are generally *activity-oriented* while the PERT networks are *event-oriented*. Essentially, this means that in activity oriented networks, the arrows representing activities or jobs are lebelled with some description of activity, while in event-oriented networks, events are the object of interest and are appropriately described. *Activities* are usually operations which take time to carry out, and on which resources are expended. The junctions between activities are termed as *events*. An event is a point in time, a milestone representing the beginning or the completion of some activity or group of activities. In CPM network, each activity is represented by an arrow, and the sequence in which the activities are performed is shown by the sequence of the arrows. The activities are suitably lebelled. However, both events and activities can be lebelled if so desired, though the tendency seems to be towards activity levelling, since it is the *activity* with which engineers are most directly concerned. Engineers design activities, not events. When a project falls behind schedule, it is the critical activities which receive managerial attention.

In CPM network planning, a tentative list of activities, that must be performed in order to accomplish the project objectives, is prepared. These activities are then interconnected to show how they must be performed with respect to one another. The placement of an activity arrow in the network, with respect to other activities of the project, is done by answering the following three basic questions :

1. What activity or activities must immediately precede this activity ?
2. What activity or activities cannot be started until after the completion of this activity ?
3. What activity or activities can be performed concurrently with this activity ?

The process of drawing a network consists of the following :

(*a*) listing the activities of a project

(*b*) answering the three questions with regard to each activity

(*c*) drawing the network that represents on paper answer to these three questions. The interdependencies can be represented by suitable use of dummies.

The elements of network have been discussed in detail in chapter 3, giving the network rules. However, for illustration purposes, let us take the project of purchasing a new heavy duty lathe and disposing the old lathe. The project consists of the following activities :

(*i*) Await delivery of lathe,

(*ii*) Remove existing lathe,

(*iii*) Install power supply,

(*iv*) Install lathe,

(*v*) Connect to power,

(*vi*) Test and

(*vii*) Dispose of existing lathe.

Fig. 8.2 shows the network for the project.

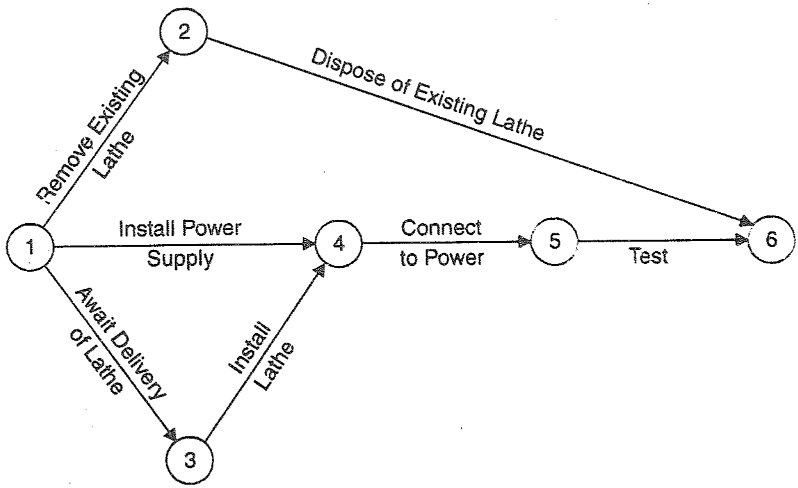


FIG. 8.2

Numbering the events

Though CPM networks are activity-oriented, the events constitute important control points. The events should, therefore, be so numbered that they reflect the logical sequence of the activities. This can be best done by following Fulkerson's rule, discussed in § 3.8.

8.4. ACTIVITY TIME ESTIMATE

After finalising the network, the next step is to estimate the time required for the completion of each activity of the network. Two approaches may be used for the assessment of duration of activity completion : (*a*) *probabilistic approach* and (*b*) *deterministic approach*. The first approach (*i.e.* the probabilistic approach) is followed by PERT planners, in which three time estimates are made for each activity : the optimistic time (t_O), the likely time (t_L) and the pessimistic time (t_P). In the second approach (deterministic approach), we may assume that we know enough about each job or operation, so that a single time estimate of their duration is sufficiently accurate to give reasonable results. This approach is followed in the CPM networks. No uncertainties are taken into consideration.

The two approaches are represented in Fig. 8.3. In Fig. 8.3 (*a*), the time estimates of the activity has greater range and hence greater uncertainty. In Fig. 8.3 (*b*), however, the range of variation is very narrow, and we approach towards a more deterministic model.

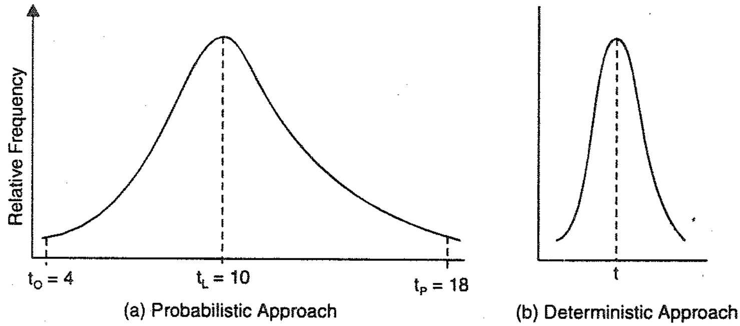


FIG. 8.3. ACTIVITY TIME ESTIMATE.

In PERT, we calculated the expected time t_E for each activity, from the three time estimates. This expected time had the same degree of uncertainty, reflected in the range of the time estimates. In CPM, however, no calculations are required. The estimated time, represented simply by t, is directly used for network analysis. The activity duration t is directly written near the arrow representing that activity.

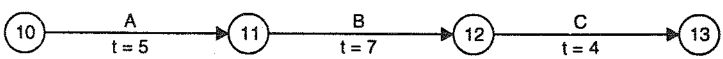


FIG. 8.4

For example, the time of completion (t) of activity A (or 10—11) in Fig. 8.4 is 5 units, while for those of activity B (or 11—12) and activity C (or 12—13), are 7 and 4 units respectively. The time of completion for any activity $i - j$ is denoted by symbol t^{ij}, in the place of symbol t_E^{ij}, used in PERT.

8.5. EARLIEST EVENT TIME

After having determined or estimated the time duration (t) for an activity, let us now determine the time of occurrence of an event at the head of the activity. The *earliest occurrence time* or *earliest event time* (T_E) is the earliest time at which an event can occur. It is the time by which all the activities discharging into the event under consideration are completed. The term is analogous to the *earliest expected time* used in the PERT analysis, except that the degree of uncertainty inherent in the word 'expected,' is not there.

In a CPM network, the time of completion of each activity (t^{ij}) is known. Hence the earliest occurrence time can be easily calculated. For example, event 10 for Fig. 8.4 is the beginning of the project, and hence it can be assumed to occur at zero time. Event 2 is at the end of activity A for which completion duration (t) is 5 days. Hence event 11 occurs at 0 + 5 = 5 days. Similarly, event 12 will occur at 5 + 7 = 12 days and end event 13 will occur at 12 + 4 = 16 days. These event times (t_E) are entered at the top of the node, as shown in Fig. 8.5.

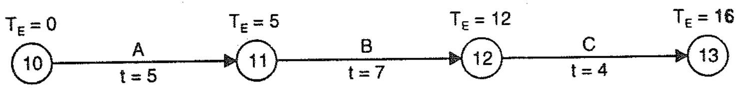


FIG. 8.5

From the above discussions, one can formulate the following expression for T_E of any event j

$$T_E^j = T_E^i + t^{ij} \qquad ...(8.1)$$

where T_E^i = earliest occurrence time for the tail event

T_E^j = earliest occurrence time for the head event

ij = activity under consideration

t^{ij} = time of completion of activity ij

Let us now take the case of a network shown in Fig. 8.6.

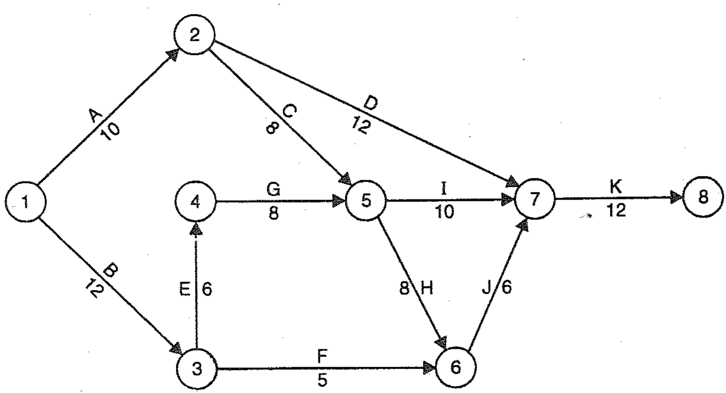


FIG. 8.6

In this network, any event, say No. 5 can be reached by two paths : Path A through 1—2—5 and path B through 1—3—4—5.

Along path 1—2—5, T_E for event 5 = 10 + 8 = 18

Along path 1—3—4—5, T_E for event 5 = 12 + 6 + 8 = 26.

From the above, we note that T_E for event 5 are different for the two activity paths. *However, no event can be considered to have reached until all activities leading to the event are completed.* Hence event 5 cannot be considered to have occurred until all the activities along both the paths are complete. Thus, T_E will be the greater of the two, and its value will be 26.

From the above discussions, the earliest occurrence time or earliest event time (T_E^j) for any event j can be found from the following expression.

$$T_E^j = (T_E^i + t^{ij})_{max} \qquad \text{...(8.2)}$$

Based on Eqns. 8.1 and 8.2, the earliest occurrence times for various events of network shown in Fig. 8.6 will be as under.

Event 1 : Since it does not have any predecessor event,

$$T_E^1 = 0$$

Event 2 : $T_E^2 = T_E^1 + t^{1-2}$

$= 0 + 10 = 10$

Event 3 : $T_E^3 = T_E^1 + t^{1-3}$

$= 0 + 12 = 12$

Event 4 : $T_E^4 = T_E^3 + t^{3-4}$

$= 12 + 6 = 18$

Event 5 : It has two predecessor events, 2 and 4. For each of these,

$$T_E^5 = T_E^2 + t^{2-5} = 10 + 8 = 18$$

and $T_E^5 = T_E^4 + t^{4-5} = 18 + 8 = 26$

Hence $T_E^5 = 26.$

Event 6 : It has two predecessor events 5 and 3

$\therefore$ $T_E^6 = T_E^5 + t^{5-6} = 26 + 8 = 34$

and $T_E^6 = T_E^3 + t^{3-6} = 12 + 5 = 17$

$\therefore$ $T_E^6 = 34$

Event 7 : It has three predecessor events : 2, 5 and 6

$\therefore \qquad T_E^7 = T_E^2 + t^{2-7} = 10 + 12 = 22$

$\qquad T_E^7 = T_E^5 + t^{5-7} = 26 + 10 = 36$

and $\qquad T_E^7 = T_E^6 + t^{6-7} = 34 + 6 = 40$

Hence $\qquad T_E^7 = 40$

Event 8 : It has only one predecessor event, 7.

$$\because \qquad T_E^8 = T_E^7 + t^{7-8}$$
$$= 40 + 12 = 52.$$

These computations may also be conveniently done by entering T_E^j values for each node, obtained from different paths, in squares, and cancelling each except the one giving highest value of T_E. *Forward pass* is used, going from one node to the other, as illustrated in Fig. 8.7, for the network of Fig. 8.6. It should be noted

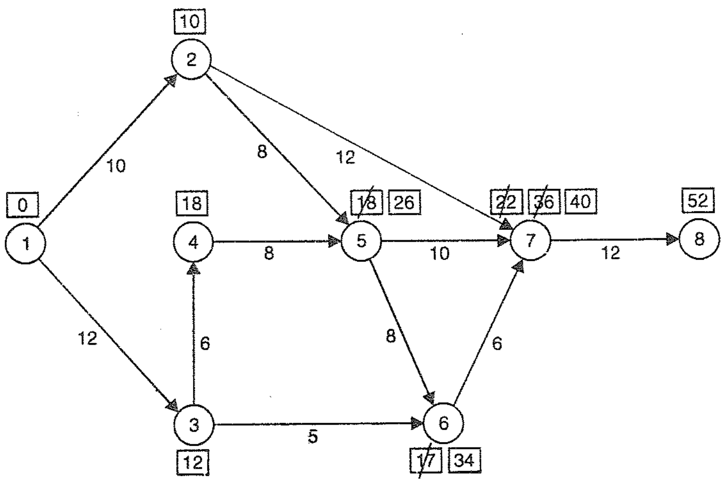


FIG. 8.7

that if the node has only one predecessor event, only one square will be there. If it has 3 predecessor events (such as for event 7), it will have three values of T_E entered in three squares made over the node ; only that square will be retained which will have highest value of T_E. The method is not very suitable for large networks, where no space is available for making so many squares.

For large networks, computations are usually done in a tabular form. This is illustrated in Table 8.1, in which events are tabulated, starting with the end event, for convenience in computations. Thus, event 8 is entered first in the successor event column. It's predecessor event (7) is entered in the predecessor event column. The next *lower event* 7 is then entered in the successor event column. It has three predecessor events (6, 5, 2) which are entered in the predecessor event column in the decreasing order of their numerical value. This procedure is followed till all the events are entered. The last successor event is 2 which has predecessor event 1. The initial event is not entered in the successor event column since it does not have any predecessor event.

Then the activity time (t^{ij}) is entered for each activity i—j joining the predecessor event i and successor event j. The column for T_E^j is then filled, starting from the bottom, and using the relations

$$T_E^j = T_E^i + t^{ij}$$

and $$T_E^j = (T_E + t^{ij})_{max}.$$

When more than one value of T_E^j is obtained for an event, the highest value is underlined for forward use.

Table 8.1

Computations for Earliest Event Time (T_E^j)

Successor event j	*Predecessor event i*	*Activity i—j*	t^{ij}	T_E^j
8	7	7—8	12	$\underline{52}$
7	6	6—7	6	$\underline{40}$
	5	5—7	10	36
	2	2—7	12	22
6	5	5—6	8	$\underline{34}$
	3	3—6	5	27
5	4	4—5	8	$\underline{26}$
	2	2—5	8	18
4	3	3—4	6	$\underline{18}$
3	1	1—3	12	$\underline{12}$
2	1	1—2	10	

8.6. LATEST ALLOWABLE OCCURRENCE TIME

The *latest allowable occurrence time* or the *latest event time* is the latest time by which an event must occur to keep the project on schedule. It is denoted by symbol T_L. If the *scheduled completion time* (T_S) of the project is given, the latest *event time* of the end event will be equal to T_S. If the scheduled completion time is not specified, then T_L is taken equal to the earliest event time T_E.

The latest event time for an activity is computed by starting from the tail event and using the *backward pass*. For illustration, let us again consider the network of Fig. 8.4, for which calculated values of T_E for each event are shown in Fig. 8.5. The *latest event time* for end event $(T_L^{13}) = T_E^{13} = 16$.

The activity 12—13 takes 4 days. Hence event 12 cannot occur later than $16 - 4 = 12$ days. Thus, $T_L^{12} = 12$.

Similarly, for event 11,

$$T_L^{11} = T_L^{12} - t^{11-12} = 12 - 7 = 5$$

and

$$T_L^{10} = T_L^{11} - t^{10-11} = 5 - 5 = 0.$$

The values of T_L so obtained are entered either at the top or at the bottom of each node as shown in Fig. 8.8.

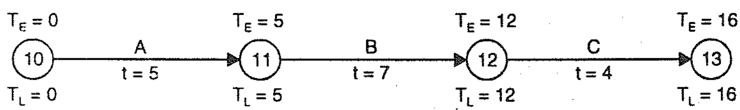


FIG. 8.8

From the above discussion, one can formulate the following expression for T_L^i for a predecessor event i, from the known value T_L^j of a successor event :

$$T_L^i = T_L^j - t^{ij} \qquad ...(8.3)$$

Let us now take the case of another network of Fig. 8.6. The values of T_E for each event are known. The project completion time is taken equal to T_E, since T_S is not given. Hence $T_L = T_E = 52$, for the last event. For this network, some events have more than one successor events.

Event 7. $T_L^7 = T_L^8 - t^{7-8}$

$= 52 - 12 = 40$

Event 6. $T_L^6 = T_L^7 - t^{6-7}$

$= 40 - 6 = 34$

Event 5. It has two successor events : 7 and 6

$\therefore \qquad T_L^5 = T_L^7 - t^{6-7} = 40 - 10 = 30$

Also, $\qquad T_L^5 = T_L^6 - t^{5-6} = 34 - 8 = 26$

Out of these, the *minimum value* (*i.e.* 26) will be the appropriate value of T_L^5.

Event 4. $T_L^4 = T_L^5 - t^{4-5}$

$= 26 - 8 = 18$

Event 3. It has two successor events : 6 and 4

$\therefore \qquad T_L^3 = T_L^6 - t^{3-6} = 34 - 5 = 29$

and $\qquad T_L^3 = T_L^4 - t^{3-4} = 18 - 6 = 12$

$\therefore \qquad T_L^3 = 12$ (minimum)

Event 2. This has also two successor events : 7 and 5

$\therefore \qquad T_L^2 = T_L^7 - t^{2-7} = 40 - 12 = 28$

and $\qquad T_L^2 = T_L^5 - t^{2-5} = 26 - 8 = 18$

$\therefore \qquad T_L^2 = 18.$

Event 1. This has also two successor events : 3 and 2

$\therefore \qquad T_L^1 = T_L^3 - t^{1-3} = 12 - 12 = 0$

and $\qquad T_L^1 = T_L^2 - t^{1-2} = 18 - 10 = 8$

$\therefore \qquad T_L^1 = 0$

Thus, we have the following expression for T_L^i

$$T_L^i = (T_L^j - t^{ij})_{min}. \qquad ...(8.4)$$

These computations may also be conveniently done by entering T_L^i values for each node, obtained from different paths, in triangles and cancelling each except the one giving the least value of T_L. *Backward pass* is used, going from one node to the other as illustrated in Fig. 8.9. It should be noted that if an event has only one successor event, only one triangle will be there. If it has 2 successor events it will have two values of T_L entered in two triangles made over or below the node ; only that triangle will be retained which will have the lowest value of T_L. The method is not very suitable for large networks ; space limitations are there for making so many triangles.

For large networks, computations are usually done in a tabular form. This is illustrated in Table 8.2. The first two columns

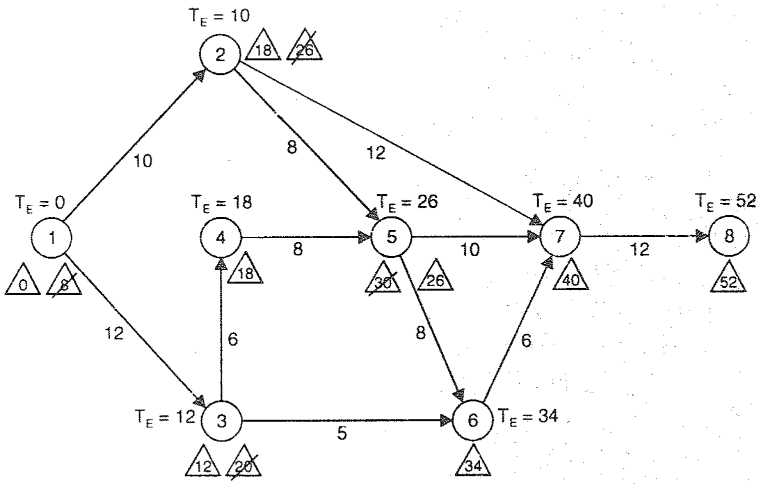


FIG. 8.9

Table 8.2
Computation of Latest Event Occurrence Time (T_L)

Predecessor event i	*Successor event j*	*Activity i—j*	t^{ij}	T_L^i
7	8	7—8	12	40
6	7	6—7	6	34
5	7	5—7	10	30
	6	5—6	8	26
4	5	4—5	8	18
3	6	3—6	5	29
	4	3—4	6	12
2	7	2—7	12	28
	5	2—5	8	18
1	3	1—3	12	0
	2	1—2	10	8

are for the predecessor events (i) and successor events (j) respectively. Here also, the tabulation is done with the highest numbered events, decreasing downwards in numerical order. Column 3 is for the activity i—j while column 4 is for the corresponding completion time t^{ij} of the activity. The computation T_L^i is started in the *backward direction*, starting with the highest number predecessor event. The following equations are used :

$$T_L^i = T_L^i - t^{ij} \text{ (for single path)}$$

and $$T_L^i = (T_L^j - t^{ij})_{min} \text{ for multiple paths.}$$

The computations are continued till the first event is reached. For multiple paths, the minimum value of T_L^i is underlined, for further use, as the appropriate value of the latest event time.

8.7. COMBINED TABULAR COMPUTATIONS FOR T_E AND T_L

The *earliest event time* (T_E) is computed using *forward pass,* as illustrated in Table 8.1, while the *latest allowable event time* (T_L) is computed using backward pass, as illustrated in Table 8.2. Since both T_L and T_E are to be calculated for each event, their computations are done in a combined tabular form illustrated in Table 8.3.

For illustration, we will take the same network of Fig. 8.6. Column 1 of Table 8.3 gives the event number, starting with the initial event and proceeding in the direction of increasing numbers of the events. Column 2 gives the predecessor events while column 6 gives the successor events to the events of column 1. These columns are completed first, using the network. An event under consideration (column 1) may have one or more than one predecessor events (column 2), and one or more than one predecessor events (column 6). A horizontal line is drawn after entering all the predecessor events and successor events to every event of column 1.

Then, computations are done for the *earliest event time* (T_E) in columns 3, 4 and 5. Column 3 is for the activity time t^{ij} where j is the event under consideration (column 1) and i is the predecessor event (column 2). T_E^j is computed from the relation

$$T_E^j = T_E^i + t^{ij}$$

Where there are more than one predecessor events, several values of T_E^j are obtained, which are entered in column 4. The maximum value of T_E^j is underlined. This underlined value is the appropriate value of the earliest event time for the event under consideration (column 1), and is entered as T_E in column 5. For the computation of T_E, we thus use the *forward pass*, starting from the initial event and proceeding in the downward direction (↓) in the Table.

Then we compute the *latest event (occurrence) time* of the same events under consideration (column 1), in columns 7, 8 and 9. Column 7 is the activity time t^{ij} where i is the event under consideration and j is the successor event (column 6). T_L^i is computed from the relation

$$T_L^i = T_L^j - t^{ij}$$

Table 8.3
Computations of T_E and T_L

Event No.	*Earliest event time (↓)*				*Latest event time (↓)*			
	Predecessor event (i)	t^{ij}	T_E^j	T_E	Successor event (j)	t^{ij}	T_L^i	T_L
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	—	—	0	0	2 3	10 12	8 $\underline{0}$	0
2	1	10	$\underline{10}$	10	5 7	8 12	$\underline{18}$ 28	18
3	1	12	12	12	4 6	6 5	$\underline{12}$ 29	12
4	3	6	$\underline{18}$	18	5	8	$\underline{18}$	18
5	2 4	8 8	18 $\underline{26}$	26	6 7	8 10	$\underline{26}$ 30	26
6	3 5	5 8	17 $\underline{34}$	34	7	7	$\underline{34}$	34
7	2 5 6	12 10 6	22 36 $\underline{40}$	40	8	12	$\underline{40}$	40
8	7	12	$\underline{52}$	52	—		$\underline{52}$	52

Computations are done by backward pass, starting with the end event and proceeding upwards (↑) in the Table. If T_S is not given, T_L of the last event is taken equal to its T_E. Where there are more than one successor events, several values of T_L^i are obtained, which are entered in column 8. The minimum value of T_L^i is underlined. This underlined value is the appropriate value of the *latest event time* for the event under consideration, and is entered as T_L in column 9.

Thus, for each of the activities of column 1, T_E is given in column 5 while T_L is given in column 9.

8.8. START AND FINISH TIMES OF ACTIVITY

So far we have discussed two event times : the *earliest event time* (T_E) and the *latest allowable occurrence time* (T_L). Since CPM networks are activity oriented, the following activity times are useful for network computations :

(*i*) Earliest start time

(*ii*) Earliest finish time

(*iii*) Latest start time

(*iv*) Latest finish time.

1. Earliest Start Time (EST)

The *earliest start time* for an activity is the earliest time by which it can commence. This is naturally equal to the earliest event time associated with the tail of the activity arrow. It is abbreviated by EST.

Thus, EST = Earliest event time at its tail.

If the activity is denoted by i—j, and if the earliest event time at its tail is T_E^i, we have

$$\text{EST} = T_E^i \qquad ...(8.5)$$

2. Earliest Finish Time (EFT)

If an activity proceeds from its early start time and takes the estimated duration for completion, then it will have an early finish. Hence *earliest finish time* (EFT) for an activity is defined as the earliest time by which it can be finished. This is evidently equal to the earliest start time plus estimated duration of the activity :

$$\text{EFT} = \text{earliest start time} + \text{activity duration}$$

or

$$\text{EFT} = T_E^i + t^{ij} \qquad ....(8.6)$$

3. Latest Start Time (LST)

Latest start time for an activity is the *latest time* by which an activity can be started without delaying the completion of the project. For 'no delay' condition to be fulfilled it should be naturally equal to the latest finish time (LFT) minus the activity duration.

$$\therefore \quad \text{LST} = \text{LFT} - \text{Activity duration.}$$

Since the Latest Finish Time (LFT) is equal to T_L^j (See Eqn. 8.8), we have

$$\text{LST} = T_L^j - t^{ij} \quad \ldots(8.7)$$

4. Latest Finish Time (LFT)

The latest finish time for an activity is the latest time by which an activity can be finished without delaying the completion of the project. Naturally, the latest finish time for an activity will be equal to the latest allowable occurrence time for the event at the head of the arrow. Hence

$$\text{LFT} = \text{Latest event time at the head of activity arrow}$$

or $$\text{LFT} = T_L^j \quad \ldots(8.8)$$

From the above definitions, we note that out of the *four* times of an activity, the earliest start time and the latest finish time are equal to the earliest event time (T_E^i) at its tail and the latest event time (T_L^j) at its head respectively, the values of which are already available. From these, the earliest finish time and the latest start time can be found by simply adding activity time to T_E^i and by subtracting activity time from T_L^j respectively.

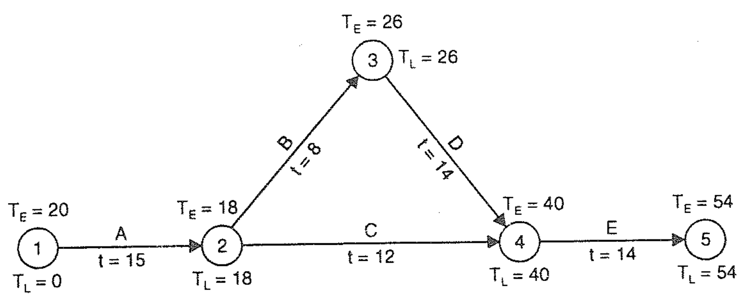


FIG. 8.10

For example, consider the network of Fig. 8.8. For activity 11-12, we have

$$\text{EST} = T_E^i = T_E^{11} = 5$$
$$\text{EFT} = \text{EST} + t^{11-12} = 5 + 7 = 12$$
$$\text{LST} = T_L^j - t^{ij} = T_L^{12} - t^{11-12} = 12 - 7 = 5$$
$$\text{LFT} = T_L^j = T_L^{12} = 12.$$

Here we find that EST and LST are the same ; also the EFT and LFT are also the same.

Again consider a *partial network* situation shown in Fig. 8.10.

Consider activity 2—4

$$\text{EST} = T_E^j = T_E^2 = 18$$
$$\text{EFT} = T_E^i + i^{ij} = 18 + 12 = 30$$
$$\text{LST} = T_L^j - t^{ij} = T_L^4 - t^{2-4} = 40 - 12 = 28$$
$$\text{LFT} = T_L^j = T_L^4 = 40.$$

The above *four activity times* are generally written on the activity arrow, along with activity *name* and *activity time* (t^{ij}), as illustrated in Fig. 8.11. The activity name [*i.e.* C etc.], EST and EFT are written on the top of the arrow, while the activity duration (t^{ij}), LST and LFT are written below the arrow.

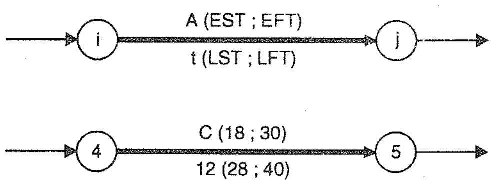


FIG. 8.11

8.9. FLOAT

The term *float* is associated with the activity times. It is analogous to the term *slack* which was associated with the event times. Just as the *slack* denotes the flexibility range within which an event can occur (*i.e.* it is the difference between the earliest event time and latest event occurrence time), *float* denotes the range within which an activity start time or its finish time may fluctuate without affecting the completion of the project.

Floats are of the following types :

(*i*) Total float

(*ii*) Free float

(*iii*) Independent float

(*iv*) Interfering float.

1. Total Float

Total float is the time span by which the starting (or finishing) of an activity can be delayed without delaying the completion of the project. In certain activities, it will be found that there is a difference between maximum time available and the actual time required to perform the activity. *This difference is known as the total float.*

Effectively, it is an inbuilt reserve of a resource (time) which is available for use in certain tail event and head event. Now, maximum time available to perform events can therefore be considered as setting the limits.

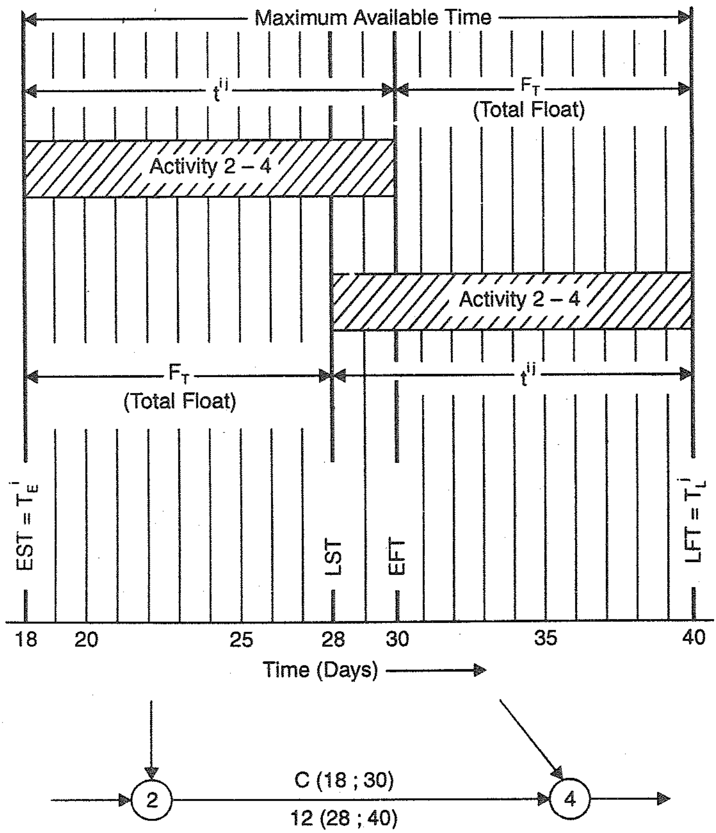


FIG. 8.12

Consider an activity i—j. The time duration available for this activity is equal to the difference between its earliest start time (T_E^i) and the latest finish time (T_L^j) :

$\therefore$ Maximum time available = $T_L^j - T_E^i$

activity time required = t^{ij}

$\therefore$ Total float (F_T) = time available – time required

or $$F_T = (T_L^j - T_E^i) - t^{ij} \quad ...(8.9)$$

or $$F_T = T_L^j - (T_E^i + t^{ij}) \quad ...[8.9\ (a)]$$

Also $$F_T = (T_L^j - t^{ij}) - T_E^i \quad ...[8.9\ (b)]$$

Now from Eqn. 8.7, $T_E^j - t^{ij}$ = LST

and from Eqn. 8.5, T_E^i = EST

$$\therefore \quad F_T = \text{LST} - \text{EST} \quad ...(8.10)$$

Similarly $$F_T = \text{LFT} - \text{EFT} \quad ...[8.10\ (a)]$$

Eqn. 8.10 is directly used for computing the total float of an activity i—j.

For example, for the activity 2—4 of Fig. 8.10, the *total float*

$$F_T = \text{LST} - \text{EST}$$
$$= 28 - 18 = 10$$

This means that even if the starting of this activity is delayed by 10 days, the project completion will not be delayed.

Total float of an activity ij (2—4) is denoted diagrammatically in Fig. 8.12.

It should be clearly noted that the total float for each activity is a measure of its particular relationship to all other activities in the project, since the earliest start time ties in all preceding activities and the latest finish time ties in all succeeding activities.

2. Free Float

Free float is that portion of positive total float that can be used by an activity without delaying any succeeding activity (or without affecting the total float of the succeeding activity). The concept of *free float* is based on the possibility that all the events occur at their earliest times (*i.e.* all activities start as early as possible).

To get a clear concept of the free float, consider activity i-j and its successor activity j-k. Events i and j has earliest occurrence times as T_E^i and T_E^j. Earliest start time for activity ij will be T_E^i, while EST for j-k will be T_E^j. However, if t^{ij} is the activity time, activity i-j will be complete by $(T_E^i + t^{ij})$ time, while activity j-k cannot start because its EST (T_E^j) is greater than $(T_E^i + t^{ij})$. The difference between the two is the free float for i-j.

$$\therefore \quad F_F \text{ for } i\text{-}j = T_E^j - (T_E^i + t^{ij}) \qquad \text{...(8.11)}$$

But $(T_E^i + t^{ij})$ is the early finish time (EFT) of the activity i-j, while T_E^j is the early start time for activity j-k.

$$\therefore \quad F_F \text{ for } ij = \text{EST for successor activity} - \text{EFT of present activity.}$$

or
$$F_F \text{ for } ij = T_E^j - \text{EFT} \qquad \text{...(8.12)}$$

Hence free float for an activity i-j is the difference between its earliest finish time and the earliest start time of its successor activity. It can be found by taking the difference between the head event time (T_E^j) and early finish time of the activity i-j.

Again from Eqn. 8.11, F_F for i-j

$$= T_E^j - T_E^i - t^{ij} \qquad \text{...(8.11)}$$

In the above equation T_E^j is the EST for successor activity, T_E^i is the EST of present activity (i-j) and t^{ij} is the activity duration.

Hence, *free float is the excess of the available time over the required time when the activity, as well as its successor activity start as early as possible.*

Again, from Eqn. 8.9 (a),

$$F_T = (T_L^j - T_E^i) - t^{ij}$$

$$\therefore \quad T_E^i + t^{ij} = T_L^j - F_T$$

Substituting in Eqn. 8.11,

$$F_F = T_E^j - (T_L^j - F_T)$$

or
$$F_F = F_T - (T_L^j - T_E^j) = F_T - S_j \qquad \text{...(8.13)}$$

Equation 8.13 gives another method of calculating the free float. *It is the difference of the total float and the head event slack.* If head slack is zero, free float will be equal to the total float.

All the three methods (*i.e.* Eqn. 8.11, 8.12 and 8.13) of calculating *free float* of an activity i-j is shown diagrammatically in Fig. 8.13. Activity 21-24 has activity duration of 12 days, with tail and head event times marked. Free float so marked satisfy both

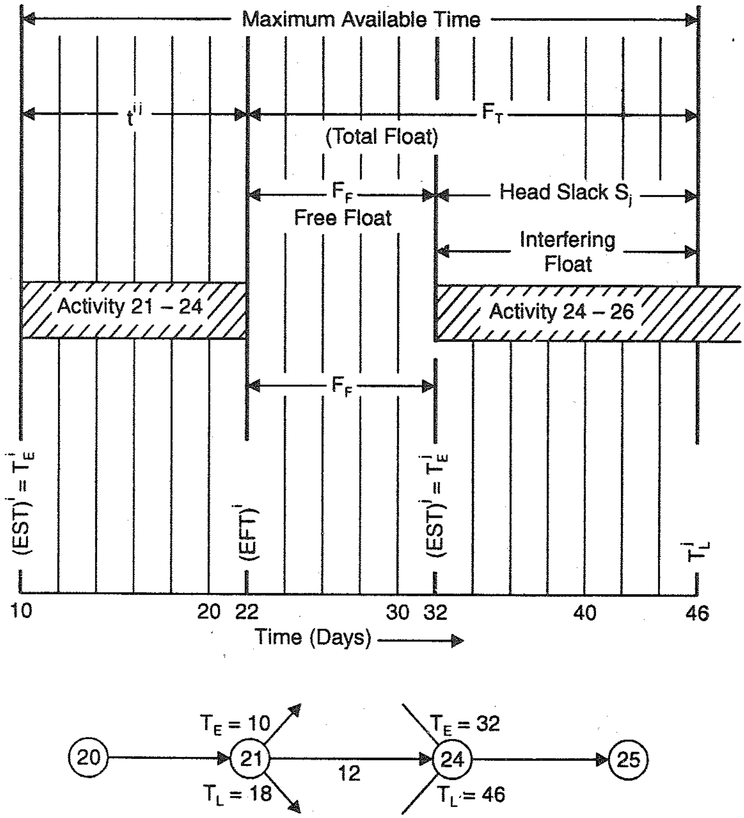


FIG. 8.13

Eqn. 8.12 as well as Eqn. 8.13. For the activity 21-35, free float is given by

$$F_F = T_E^j - T_E^i - t^{ij} = 32 - 10 - 12 = 10 \text{ days} \quad ...(i)$$

$$F_F = T_E^j - \text{EFT} = 32 - 22 = 10 \text{ days} \quad ...(ii)$$

Also, $\quad F_F = F_T - S_j$

where $F_T = (T_L^j - T_E^i) - t^{ij} = (46 - 10) - 12 = 24$ days.

$$S_j = T_L^j - T_E^j = 46 - 32 = 14$$

$$\therefore \quad F_F = 24 - 14 = 10 \text{ days.} \quad ...(iii)$$

3. Independent Float

Independent float gives us an idea about the *excess* time that exists if the preceding activity ends as *late* as possible and the succeeding activity starts as *early* as possible. *The independent float is, therefore, defined as the excess of minimum available time over the required activity duration.*

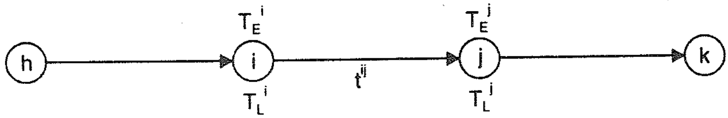


FIG. 8.14

To get the concept of independent float, consider activity *i-j* of Fig. 8.14, having a predecessor activity *h-i* and successor activity *j-k*. The *latest time* by which the predecessor activity *h-i* finishes is T_L^i, while the *earliest time* by which the successor activity *j-k* can start is T_E^j. Hence, under this condition, *minimum time available* for activity $i\text{-}j = T_E^j - T_L^i$. Thus, by definition, independent float (F_{ID}) is given by

$$F_{ID} = \text{minimum available time} - \text{activity time}$$

or $$F_{ID} = (T_E^j - T_L^i) - t^{ij} \quad \text{...(8.14)}$$

It should be noted that *independent float is a part of the free float*. To show this, consider Eqn. 8.11 for free float :

$$F_F = T_E^j - T_E^i - t^{ij} \quad \text{...(Eqn. 8.11)}$$

Substituting the value of $T_E^j - t^{ij}$ from Eqn. 8.14,

i.e. $$T_E^j - t^{ij} = F_{ID} + T_L^i, \text{ we get}$$

$$F_F = (F_{ID} + T_L^i) - T_E^i = F_{ID} + (T_L^i - T_E^i)$$

or $$F_{ID} = F_F - (T_L^i - T_E^i)$$

But $T_L^i - T_E^i$ = tail event slack = S_i

$$\therefore \quad F_{ID} = F_F - S_i \quad \text{...(8.15)}$$

Thus, independent float is equal to the free float minus tail event slack. If the tail slack is zero, free float and independent float are equal. It is to be noted that if a negative value of independent float is obtained, then independent float is taken as zero.

The two methods of calculating independent float (by Eqn. 8.14 and 8.15) are shown diagrammatically in Fig. 8.15.

In Fig. 8.15, activity 21-24 (*i-j* activity) has 12 days duration, with tail and head event times marked. Independent float so marked

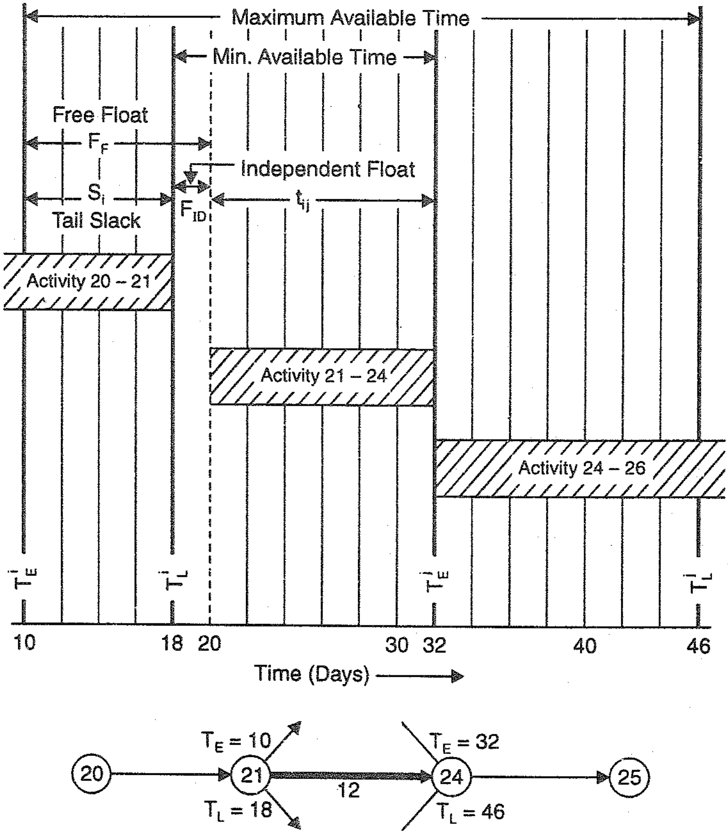


FIG. 8.15

satisfy both Eqn. 8.14 as well as Eqn. 8.15. For the activity 21-24, independent float is given by

$$F_{ID} = (T_E^j - T_L^i) - t^{ij} \qquad \text{...(Eqn. 8.14)}$$

$$= (32 - 18) - 12 = 2 \text{ days} \qquad \text{...}(i)$$

also, $$F_{ID} = F_F - S_i \qquad \text{...(Eqn. 8.15)}$$

where free float $= T_E^j - T_E^i - t^{ij} = 32 - 10 - 12 = 10$

and $S_i = T_L^i - T_E^i = 18 - 10 = 8$

$$\therefore \qquad F_{ID} = 10 - 8 = 2 \text{ days.} \qquad \text{...}(ii)$$

4. Interfering Float

Interfering float (F_{IT}) is just another name given to the head event slack (S_j), specially in CPM networks which are activity oriented. Interfering float is the potential downstream interference of any activity, and is equal to the *difference between total float and the free float.*

Thus, $$F_{IT} = F_T - F_F \qquad ...(8.16)$$

But $$F_T = (T_L^j - T_E^i) - t^{ij}$$

$$F_F = (T_E^j - T_E^i) - t^{ij}$$

$\therefore$ $$F_{IT} = (T_L^j - T_E^i - t^{ij}) - (T_E^j - T_E^i - t^{ij})$$

or $$F_{IT} = (T_L^j - T_E^j) = S_j \qquad ...(8.17)$$

Thus, interfering float is equal to the head event slack. This definition of interfering float has been incorporated in Fig. 8.13.

Summary of Floats

Let us summarise by defining the various types of floats as under :

1. ***Total float*** of an activity is the *excess* of the *'minimum available time'* over *activity time*. Thus

$$F_T = (T_L^j - T_E^i) - t^{ij} \qquad ...(8.9)$$

2. ***Free float*** of an activity is the *excess* of *'available time'* over the *activity time, when all jobs start as early as possible.* Thus

$$F_T = (T_E^j - T_E^i) - t^{ij} \qquad ...(8.11)$$

3. ***Independent float*** of an activity is the *excess* of *'minimum available time'* over the *activity time*. Thus

$$F_{ID} = (T_E^j - T_L^i) - t^{ij}$$

4. ***Interfering float*** of an activity is the *difference* between *total float* and *free float*. It is thus equal to the head event slack. Thus,

$$F_{IN} = F_T - F_F = S_j.$$

Example. *Let us now take the example of network shown in Fig. 8.6, for which the event times have been already computed in Table 8.4. The network is reproduced in Fig. 8.16, with the event times marked.*

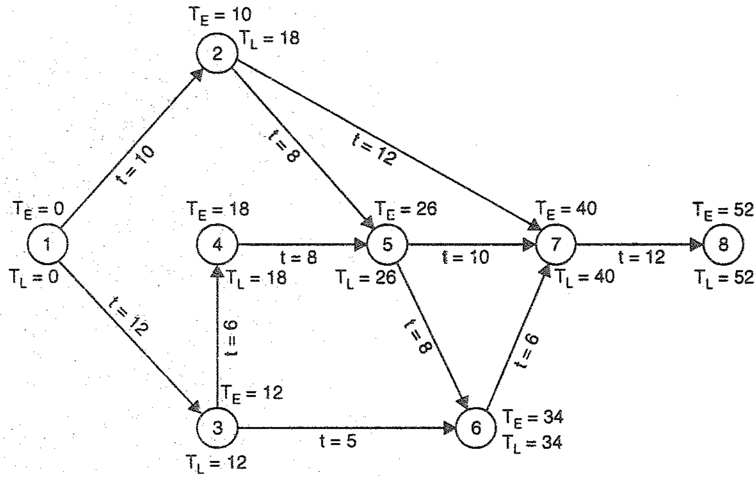


FIG. 8.16

Solution. The values of the activity times, total float, free float and independent float are tabulated in Table 8.4.

Table 8.4

Activity (i—j)	*Duration t^{ij}*	*Earliest*		*Latest*		*Total float F_T*	*Free float F_F*	*Independent float F_{ID}*
		Start time (EST)	*Finish time (EFT)*	*Start time (LST)*	*Finish time (LFT)*			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1—2	10	0	10	8	18	8	0	0
1—3	12	0	12	0	12	0	0	0
2—5	8	10	18	18	26	8	8	0
2—7	12	10	22	28	40	18	18	10
3—4	6	12	18	12	18	0	0	0
3—6	5	12	17	29	34	17	17	17
4—5	8	18	26	18	26	0	0	0
5—6	8	26	34	26	34	0	0	0
5—7	10	26	36	30	40	4	4	4
6—7	6	34	40	34	40	0	0	0
7—8	12	40	52	40	52	0	0	0

Table 8.4 has 9 columns, out of which column 1 gives the activity $(i\text{-}j)$ while column 2 gives the activity duration (t^{ij}). While filling column 1, it is advisable to start with the lowest number event (initial event) and proceed in the next high numerical order completing all the activities originating from that event before going to the next higher event. This will facilitate in filling column 3 for early start time (EST) which is equal to T_E^i. Column 4 for early finish time (EFT) is the sum of column (2) and column (3). Then column (6) for latest finish time (LFT) should be filled, which is equal to T_L^j. Column 5 for latest start time (LST) is then obtained by subtracting column (2) from column (6).

After having entered all the four activity times, float calculations are done. Though various equations for the three types of floats have been suggested in this article, the one relating to the activity times and activity duration should be used, so that data from columns (2) to (6) can be directly used.

Column 7 for *total float* is obtained *either* by finding difference between columns (5) and (3), *or* by finding the difference between columns (6) and (4) ; both ways indentical results should be obtained.

Column (8) is for *free float*, which is obtained by finding the difference between EST of succeeding activity and the EFT of the activity under consideration. For example, free float for 1-2 = EST of successor activity (2-5) or (2-7) minus EFT of (1-2) = 10 – 10 = 0. Similarly, F_F for 1-3 = EST of (3-4) or (3-5) minus EFT of (1-3) = 12 – 12 = 0. It should be noted that F_F cannot be greater than F_T.

Column (9) is for *independent float*, which is obtained by subtracting the tail event slack from the free float. Tail event slack can be easily taken from Fig. 8.16, while free float is already available in column (8). For example, for activity 2-5, tail event slack is 8 while free float is also 8 ; hence independent float for activity 2-5 will be zero. However, for activity 2-7, free float is 18 while tail event slack = (18 – 10) = 8. Hence independent float = 18 – 8 = 10.

8.10. CRITICAL ACTIVITIES AND CRITICAL PATH

Out of the various types of floats, *total float* is the most useful. Since the total float is the difference between maximum available time and the activity duration, there are three possibilities :

(*i*) It may have a *negative value,* if the time availability is less than activity duration.

(*ii*) It may have a *zero value* of the time availability is equal to the activity duration.

(*iii*) It may have a *positive value* if *time availability* exceeds the activity duration.

This information about the degree of the total float is very useful as regards the *criticality of the activity*. There may be various sequences of activities, leading the initial event to the final event. These activities can be *classed* on the basis of the degree of the float as under :

(*a*) ***Super Critical Activity***	:	When the float of the activity is negative ; such activity demands very special attention and action.
(*b*) ***Critical Activity***	:	When float is zero : such activity demands above normal attention with no freedom of action.
(*c*) ***Sub-Critical Activity***	:	When the float is positive, demanding normal attention, but allowing some freedom of action.

Negative float results when the activity completion time is more than the available time. Such negative float, though possible, indicates an abnormal situation requiring a decision on how to compress the network, *i.e.* an attempt is to be made by employing more resources so as to make total float zero or positive from the original negative value. Naturally, compression of a network would mean additional cost. This concept of time-cost analysis will be discussed in the next chapter.

The *critical path*, as already defined, is the longest path through the network and time along this path gives the project duration. Critical path joins those activities which are *critical.* Critical path can be easily determined with the help of *total float calculations*. The activities on the critical path are those activities that have total float equal to zero. The activities that control the project duration are the ones that have zero total float and form continuous chain (or path) starting at the first node and ending with the last node.

The critical path for the network of Fig. 8.6 is shown by thick lines in Fig. 8.16, along with the activities start and finish times marked.

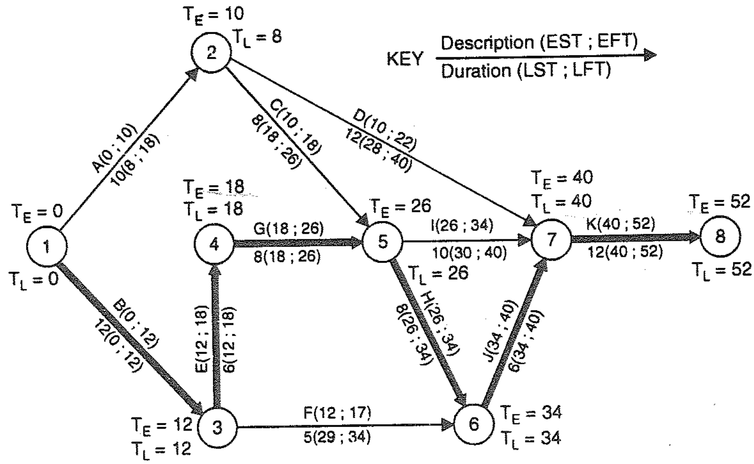


FIG. 8.17. CRITICAL PATH.

From the above, the following points are noteworthy.

1. Critical path starts from the initial event and ends at the end event of the network. All events and activities lying along the critical path are *critical* for the completion of the activities. Thus events 1, 3, 4, 5, 6, 7 and 8 are critical, and activities 1-3, 3-4, 4-5, 5-6, 6-7 and 7-8 are critical.

2. Critical path passes through those events where *slack is* zero. Although, this is *necessary condition* but it is not *sufficient*. This is evident from the fact that though events 3 and 6 are critical but activity 3-6 (connecting events 3 and 6) is *not* critical. In order to identify the critical path, the *float concept* is more useful since it provides both the necessary and sufficient condition for the activity to be critical. *This explains the basic difference in approach for determining critical path via event-oriented slack philosophy (used in PERT) and activity oriented float philosophy.*

3. There can be more than one critical path in a network, and depending upon the total float value, *degree of criticality* can be assigned to a particular path.

4. Non-critical activities have flexibilities in their start time or finish time.

8.11. ILLUSTRATIVE EXAMPLES

Example 8.1. *A building project consists of 10 activities, represented by the network shown below in Fig. 8.18.*

The normal durations required to perform various activities of the above project are given in Table 8.5 below. Compute : (a) event times, (b) activity times and (c) total float. Also, determine the critical path.

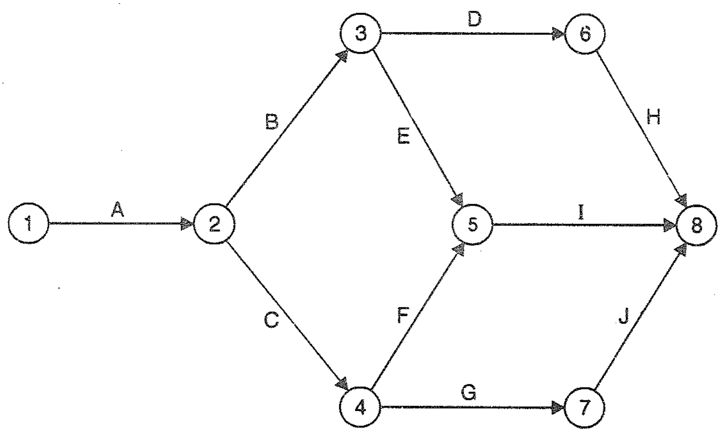


FIG. 8.18

Table 8.5

Activity	*Estimated duration*	*Activity*	*Estimated duration*
A	5	*F*	2
B	2	*G*	3
C	6	*H*	8
D	4	*I*	7
E	4	*J*	2

Solution.

1. Computation of event times

The *earliest event times* (T_E) and latest event occurrence times (T_L) are computed in a tabular form, shown below in Table 8.6.

Table 8.6
Computation of Event Times

Event No.	Earliest event time (↓)				Latest event time (↑)			
	Predecessor event (i)	t^{ij}	T_E^j	T_E	Successor event (j)	t^{ij}	T_L^i	T_L
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	—	—	—	0	2	5	0	0
2	1	5	5	5	3 4	2 6	6 5	5
3	2	2	7	7	5 6	4 4	9 8	8
4	2	6	11	11	5 7	2 3	11 15	11
5	3 4	4 2	11 13	13	8	7	13	13
6	3	4	11	11	8	8	12	12
7	4	3	14	14	8	2	18	18
8	5 6 7	7 8 2	20 19 16	20	—	—		20

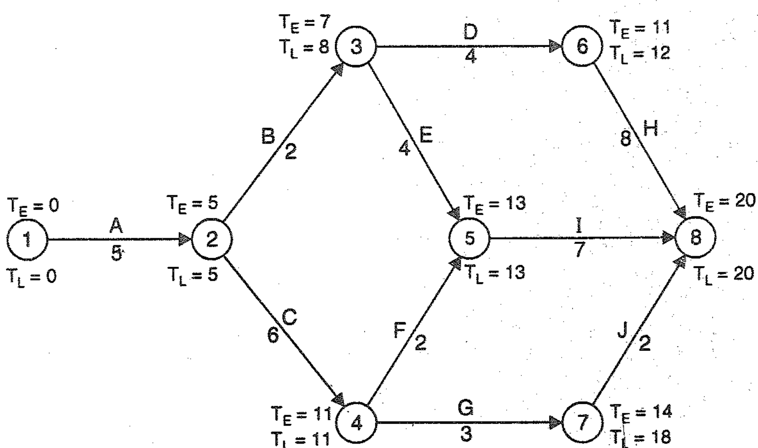


FIG. 8.19

The network, marked with the event times is shown in Fig. 8.19.

2. Computation of activity times and the total float

The earliest start time (EST), earliest finish time (EFT), latest start time (LST) and latest finish time (LFT) of each activity, along with the total float are computed in Table 8.7. The EST of each activity is equal T_E^i, while EFT is equal to $T_E^i + t^{ij}$. Similarly, LFT is equal to T_L^j while LST is equal to $T_L^j - t^{ij}$, as explained earlier. Finally, the total float of each activity is equal to either LST—EST or LFT—EFT.

Table 8.7
Computation of Activity Time and Total Float

Activity (i—j)	*Duration t^{ij}*	*Earliest*		*Latest*		*Total Float F_T*
		Start time (EST)	*Finish time EFT*	*Start time (LST)*	*Finish time LFT*	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1—2	5	0	5	0	5	0
2—3	2	5	7	6	8	1
2—4	6	5	11	5	11	0
3—5	4	7	11	9	13	2
3—6	4	7	11	8	12	1
4—5	2	11	13	11	13	0
4—7	3	11	14	15	18	4
5—8	7	13	20	13	20	0
6—8	8	11	19	12	20	1
7—8	2	14	16	18	20	4

3. Location of critical path

The activities for which total float is zero, are the critical activities, and these are 1-2, 2-4, 4-5 and 5-8. The critical path is therefore along these activities, starting from event 1 and ending at event 8. The critical path is shown with thick lines in Fig. 8.20, along with the activity times marked on each activity.

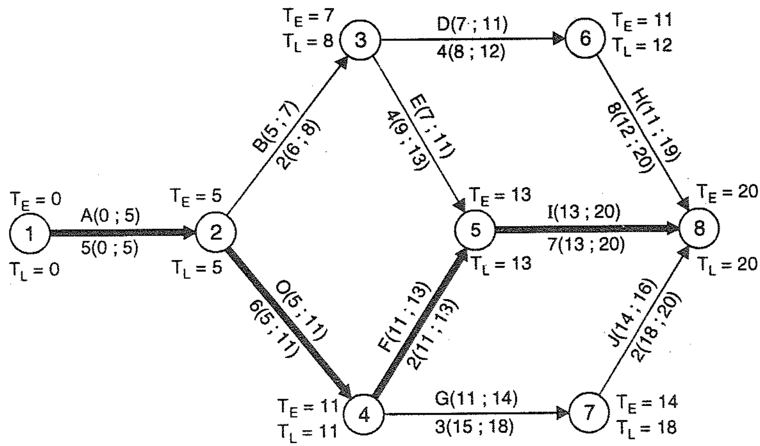


FIG. 8.20. CRITICAL PATH.

Example 8.2. *The network for a certain project is shown in Fig. 8.21, along with the estimated time of completion of each activity marked. Compute the activity times, and total float, free float and independent float for each activity. Locate the critical path on the network.*

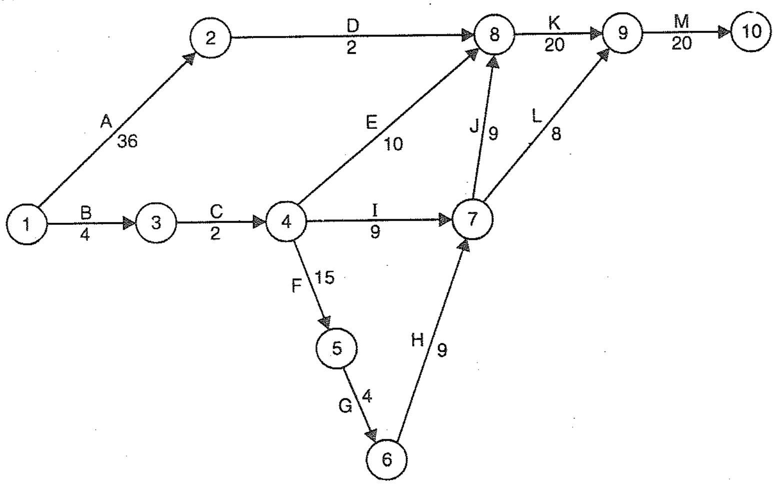


FIG. 8.21

Solution.

1. Computation of event times

The earliest event time (T_E) and latest event occurrence times (T_L) for each activity are shown tabulated in Table 8.8.

Table 8.8
Computation of Event Times

Event No.	*Earliest event time* (↓)				*Latest event time* (↑)			
	Predecessor event (i)	t^{ij}	T_E^j	T_E	*Successor event (j)*	t^{ij}	T_L^i	T_L
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	—	—	—	0	2	36	5	0
					3	4	0	
2	1	36	36	36	8	2	41	41
3	1	4	4	4	4	2	4	4
4	3	2	6	6	5	15	6	6
					7	9	25	
					8	10	33	
5	4	15	21	21	6	4	21	21
6	5	4	25	25	7	9	25	25
7	4	9	15	34	8	9	34	34
	6	9	34		9	8	55	
8	2	2	38	43	9	20	43	43
	4	10	16					
	7	9	43					
9	7	8	42	63	10	20	63	63
	8	20	63					
10	9	20	83	83	—			83

2. Activity Times

The earliest start time (EST), earliest finish time (EFT), latest start time (LST) and latest finish time (LFT) for each activity are computed in tabular form as shown in Table 8.9. EST for each activity is equal to T_E^i, while EFT is equal to $T_E^i + t^{ij}$. Similarly, LFT of each activity is equal to T_L^j while LST is equal to $T_L^j - t^{ij}$.

Table 8.9
Computation of Activity Times and Floats

Activity *(i-j)*	*Duration* (t^{ij})	*Earliest*		*Latest*		*Float*		
		Start time EST	*Finish time EFT*	*Start time LST*	*Finish time LFT*	*Total* F_T	*Free* F_F	*Inde-pendent* F_{ID}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1—2	36	0	36	5	41	5	0	0
1—3	4	0	4	0	4	0	0	0
2—8	2	36	38	41	43	5	5	0
3—4	2	4	6	4	6	0	0	0
4—5	15	6	21	6	21	0	0	0
4—7	9	6	15	25	34	19	19	19
4—8	10	6	16	33	43	27	27	27
5—6	4	21	25	21	25	0	0	0
6—7	9	25	34	25	34	0	0	0
7—8	9	34	43	34	43	0	0	0
7—9	8	34	42	55	63	21	21	21
8—9	20	43	63	43	63	0	0	0
9—10	20	63	83	63	83	0	0	0

3. Calculation of Floats

The total float for each activity is computed taking the difference of LST and EST ; it is shown computed in column 7. The free float is computed by taking the difference of EST of successor activity and EFT of the activity under consideration. It is shown computed in column 8. The independent float is computed by subtracting tail event slack from the free float ; it is shown tabulated in column 9.

4. Critical Path

Activities 1—3, 3—4, 4—5, 5—6, 6—7, 7—8, 8—9 and 9—10, having total float equal to zero, are critical. Hence the path 1—3—4—5—6—7—8—9—10 is the critical path, as marked by thick lines in Fig. 8.22.

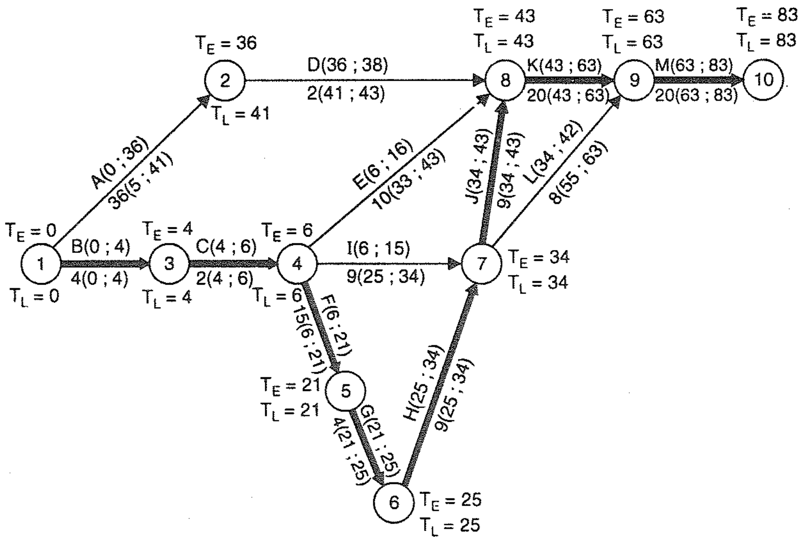


FIG. 8.22

PROBLEMS

1. Define 'earliest event time' and 'latest occurrence event time'. How are these determined ? Explain the tabular form for determining these.
2. What do you understand by 'earliest start time' and 'latest start time' of an activity ? How are these determined ?
3. Define 'latest start time' and 'latest finish time'. How are these determined ?
4. What do you understand by total float ? How is it determined ? What is its importance in network planning ?
5. Differentiate clearly between 'total float', 'free float' and 'independent float'.
6. Define 'free float'. What is its importance ? How is it determined ?
7. What do you understand by independent float ? Show that it can be determined by subtracting the tail event slack from the free float ?
8. Explain the tabular form of doing computations for CPM network elements.
9. What do you understand by critical path ? How is it determined ?
10. The network of a certain project is shown in Fig. 8.23, with the estimated durations of various activities. Determine the following :

    (a) Earliest event time and latest event time.

    (b) Earliest and latest start and finish times of each activity.

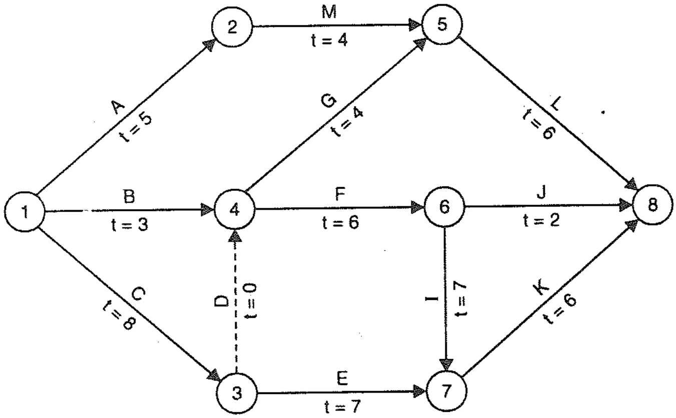


FIG. 8.23

(*c*) Total and free floats for each activity.

(*d*) Critical path for the network.

11. The network shown in Fig. 8.24 has the estimated duration for each activity marked. Determine the total float for each activity and establish the critical path. Also determine free float and independent float for each activity.

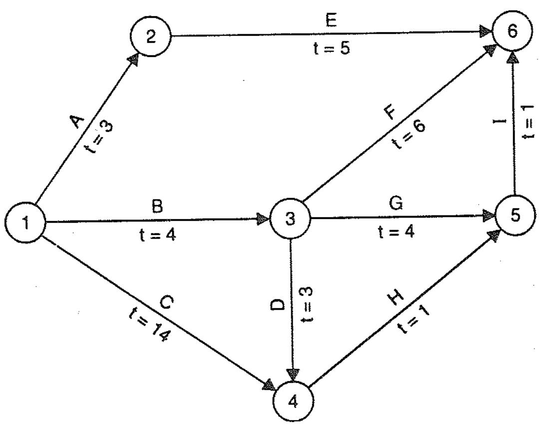


FIG. 8.24

9

CPM : Cost Model

9.1. INTRODUCTION

In CPM, time is related to cost and the object is to develop an optimum time-cost relationship. Many times it becomes necessary to complete the project earlier than the normal time (latest allowable time). In such situations, the cost of expediting the operations or activities has to be considered. *CPM makes use of the cost estimate along with the time estimate and provides a schedule for completing the activities at the minimum total cost.*

The ultimate object of the network techniques is not only to bring improvement in planning, scheduling and control of project but also to assess the possibility of arriving at a feasible and desirable time-cost relationship. The policy of every organisation is to reduce the target time so that the time saved may be utilised for additional production or otherwise. The overall project duration can be reduced by reducing the duration of only the critical activities in the project network. The durations of such activities may be reduced in two ways :

(*a*) by deploying more resources for the early completion of such activities.

(*b*) by relaxing the technical specifications for such activities.

The latter method mostly depends on engineering considerations, and is not being discussed here. In the whole of CPM Cost Model, we will be assuming that project duration is reduced by deploying more resources on critical activities.

For a given project, there is a certain range of time during which the project may be completed depending upon the resources employed on various critical operations. If the duration is made larger, cost will be reduced. On the other hand, reducing the project duration would increase the cost. A decision on this will depend on whether the committment of additional resources and expenses is worthwhile. *The optimum duration will be one which gives the most economic cost for completing the project.*

In CPM, there are two time and cost estimates for each activity : '*normal estimate*' and '*crash estimate*'. In the normal estimate, the emphasis is on *cost* with time being associated with minimum cost. The '*crash*' estimate involves the absolute minimum time required for the job and the cost necessary to achieve it. Here the emphasis is on '*time*'.

9.2. PROJECT COST

For any project, the relationship between total cost and overall duration is shown in the Fig. 9.1. It is clear from the figure that

(*a*) if a project goes on indefinitely, the cost will increase,

(*b*) similarly, cost will increase if project is expedited, and

(*c*) cost is minimum at some optimum project duration.

Our main concern, naturally, is to find the project duration which will keep the total project cost at a minimum.

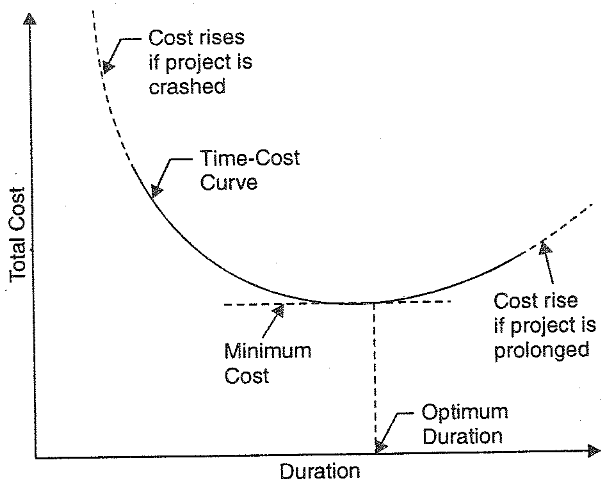


FIG. 9.1. VARIATION OF TOTAL PROJECT COST WITH DURATION.

Cost is considered to be a common parameter of the resources expenditure on a project. In other words, the application and use of man, money, machines, materials and time for the performance of various activities are all related to this common measure of cost.

Total Project Cost is the sum of two separate costs :

(*a*) *the direct cost* for accomplishing the work, and

(*b*) *the indirect cost* related to the control or duration of that work, financial overhead, lost production, and the like.

The components of the total cost are depicted in Fig. 9.2.

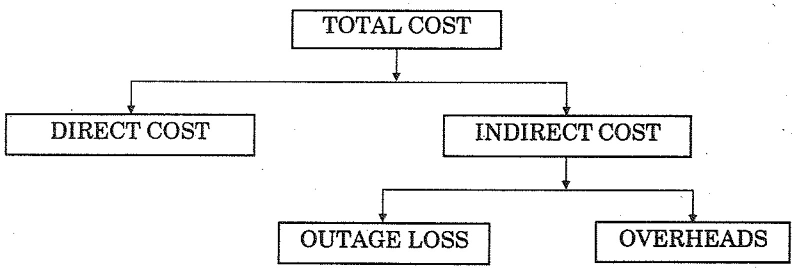
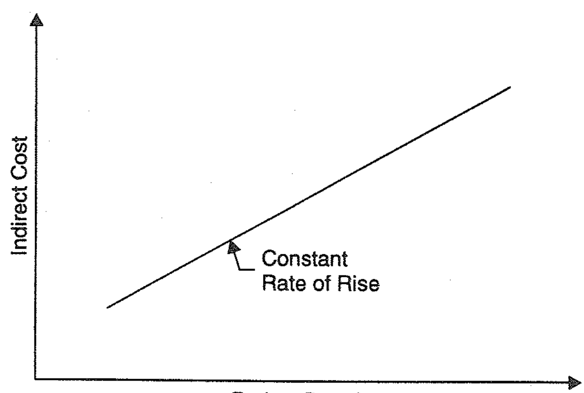


FIG. 9.2. COMPONENTS OF PROJECT COST.

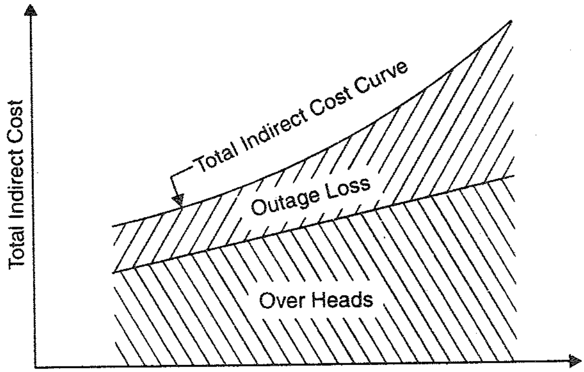
9.3. INDIRECT PROJECT COST

Indirect costs on a project are those expenditures which cannot be apportioned or clearly allocated to the individual activities of a project, but are assessed as a whole. The indirect cost includes the expenditure related to administrative and establishment charges, overhead, supervision, expenditure on a central store organisation, loss of revenue, lost profit, penalty etc. etc.

Indirect cost rises with increased duration. For any project, the relationship between Indirect Cost and Project Duration is shown in Fig. 9.3 (*a*) considering only overhead and supervision. It



(a) Indirect Cost (Supervisory & Overheads)



(b) Indirect Cost (Overhead + Outage Loss)

FIG. 9.3. VARIATION OF INDIRECT COSTS WITH PROJECT DURATION.

would be represented by a straight line, with a slope equal to daily overhead.

But when there is a loss in profits, due to inability to meet demand or due to some penalty due to delay, a corresponding cost increase must be added to the cost of overheads, producing the curve as shown in Fig. 9.3 (*b*). Such a loss is called the *outage loss*.

The total *indirect cost curve* will thus be curved.

9.4. DIRECT PROJECT COST

Direct project costs are those expenditures which are directly chargeable to and can be identified specifically with the activities of the project. These include labour cost, material cost, equipment cost etc.

For direct cost versus time relationship, consider the excavation of a trench for laying a pipe-line. It may be assumed that the nature of this job is such that only one man at a time would be able to work on it, taking a total of 40 manhours of work. The various ways of completing the work is illustrated in Table 9.1 assuming that the labour charges are at the rates of Rs. 12, 16 and 20 for the first, second and the third shifts respectively.

Table 9.1

S. No.	*Man power employed*	*Shift system*	*Time to complete*	*Charges*			*Total*
				Ist shift @ Rs. 12	*IInd shift @ Rs. 16*	*IIIrd shift @ Rs. 20*	
1.	One man	Single shift	5 days	60	—	—	60
2.	Two men	Double shift system	3 days (Three first shifts and 2 second shifts)	36	32	—	68
3.	Three men	Three shift system	2 days (Two first shifts, two second shifts and one third shift)	24	32	20	76

Fig. 9.4 shows the variation of direct cost with time for the case of Table 9.1.

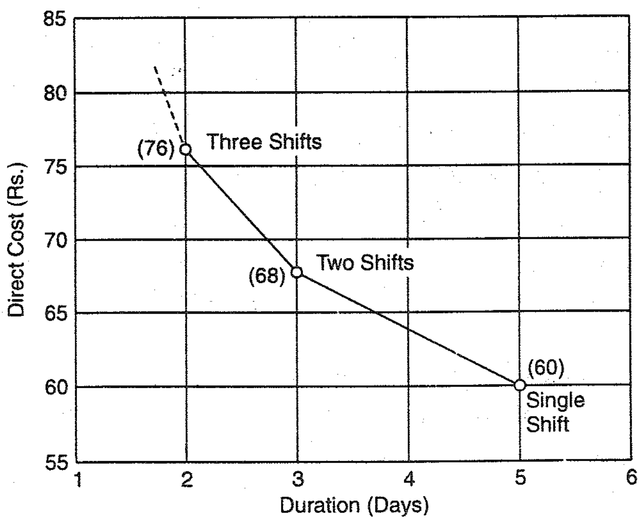


FIG. 9.4. VARIATION OF DIRECT COST WITH TIME.

The *direct cost curve*, having many segments, thus falls with increase in duration. However, the total *indirect cost curve* [Fig. 9.3 (*b*)] rises with increase in duration.

Fig. 9.5 shows a generalised curve between direct cost and project duration. The project has the *highest cost* corresponding to

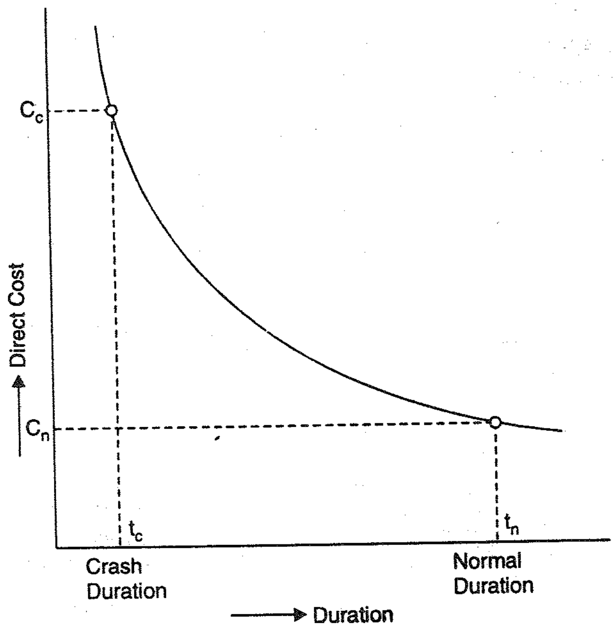


FIG. 9.5. GENERALISED DIRECT COST-TIME CURVE.

the *crash duration*, and has *normal cost* corresponding to the *normal duration*. Thus, we have two types of *costs* and two types of *times* defined below :

Normal time (t_n). Normal time is the standard time that an estimator would usually allow for an activity.

Crash time (t_c). Crash time is the *minimum possible* time in which an activity can be completed, by employing extra resources. Crash time is that time, beyond which the activity cannot be shortened by any amount of increase in resources.

Normal cost (C_n). This is direct cost required to complete the activity in normal time duration.

Crash cost (C_c). This is the direct cost corresponding to the completion of the activity within crash time.

9.5. SLOPE OF DIRECT COST CURVE

The direct cost curve is generally a curve, as shown in Fig. 9.5. However, this curve can be approximated by straight line or more than one straight line, depending upon the flatness of the curve. For example, the flat curve of Fig. 9.6 (a) can be approximated by a single straight line, while the curve of Fig. 9.6 (*b*) can be approximated by three straight lines.

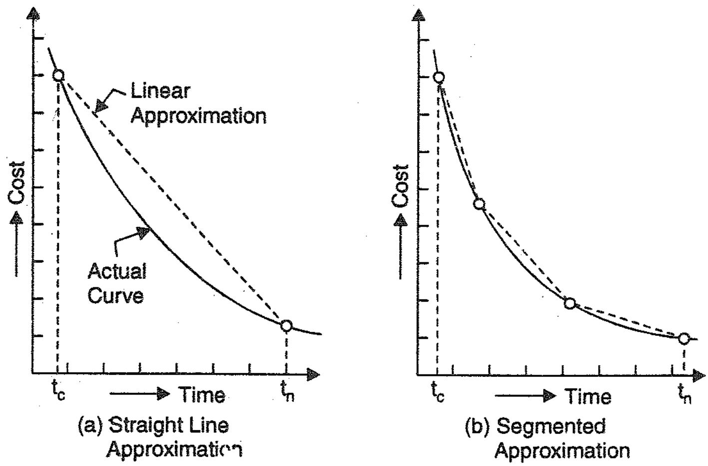


FIG. 9.6. DIRECT COST CURVE APPROXIMATION.

The straight line or segmented approximation of the direct cost curve is helpful in carrying out the project cost analysis. In such analysis, the *cost slope* is used.

Cost Slope

The cost slope is the slope of the direct cost curve, approximated as straight line. It is defined as follows :

$$\text{Cost slope} = \frac{\text{crash cost} - \text{normal cost}}{\text{normal time} - \text{crash time}}$$

or

$$CS = \frac{C_c - C_n}{t_n - t_c} = \frac{\Delta C}{\Delta t} \qquad \text{...(9.1)}$$

where CS = cost slope

ΔC = increase in cost

Δt = decrease in time.

If the cost curve is approximated to a single straight line, it will have one cost slope. If, however, the cost curve has segmented approximation, it will have more than one cost slope. The single or multiple cost slopes will depend upon the non-linearity of the direct cost curve. The segmented approximation of cost curve, having multiple cost slopes, is more accurate but calculations are more involved. Generally, single cost slope is assumed.

9.6. TOTAL PROJECT COST AND OPTIMUM DURATION

The total project cost is the sum of the direct costs and indirect costs. Indirect cost curve is shown in Fig. 9.3 (*a*) which includes supervisory and over-head costs. However, if outage losses are also there, the curve for indirect costs will be as shown in Fig. 9.3 (*b*).

Fig. 9.7 shows the indirect cost curve, direct cost curve and the corresponding total curve.

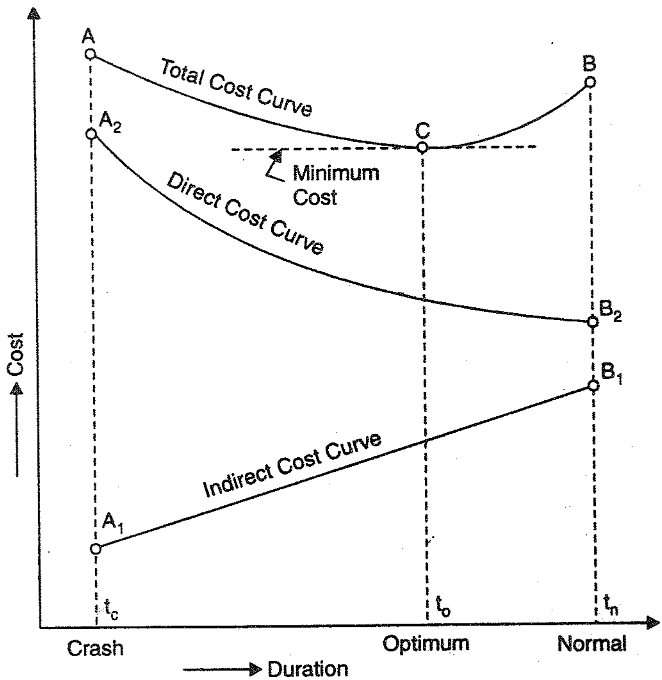


FIG. 9.7. TOTAL COST CURVE.

From the total cost curve *ACB* of Fig. 9.7, we find that the *minimum total cost* is obtained at some duration known as the *optimum duration*. The corresponding cost is known as the *minimum cost*. If the project duration is increased, total cost will increase, while if project duration is decreased to the crash value, project cost will be the highest.

If the direct cost curve is approximated to one or more straight lines, the corresponding total cost curve will be as shown in Fig. 9.8.

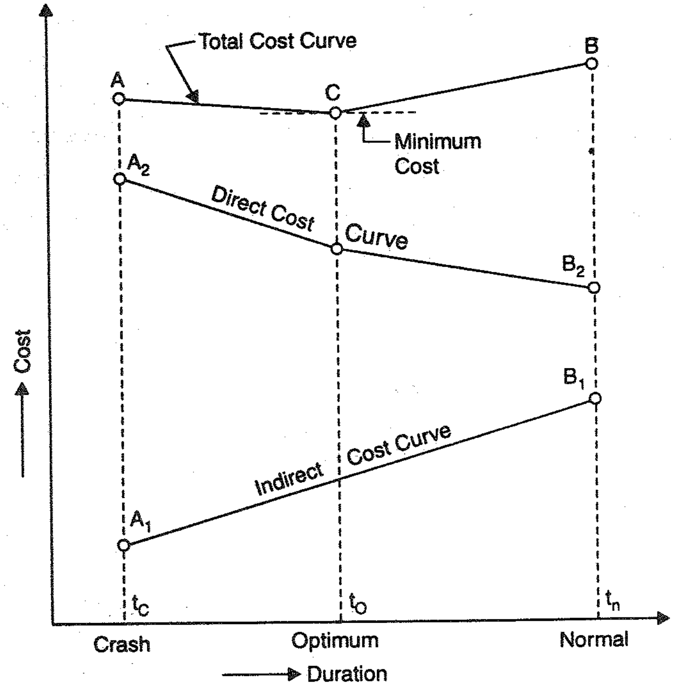


FIG. 9.8. TOTAL COST CURVE, LINEAR APPROXIMATION.

9.7. CONTRACTING THE NETWORK FOR COST OPTIMIZATION

After having established the *critical path* in a network (chapter 8) corresponding to the given normal durations of the activities, the next question that remains to be examined is 'what will be the cost structure of the project if some or all of the activities are crashed ?' For answering this question, we must have the normal direct cost data for each activity if it is to be completed in normal time duration and also the crashed direct cost data if that activity is crashed or hastened. For this data, the cost slope for each activity can be determined, *assuming* straight lines for the direct cost curve of each activity. The indirect cost rate per day should also be known, so that the total cost of the project can be found by adding the direct and indirect costs.

The *normal time* that the project will take for its completion will be the *sum of the normal time durations of each activity along the critical path*. Similarly, the *minimum time* that the project will take for its completion will be the sum of the *crashed time duration of each activity along the critical path*. If all the activities (critical as well as non-critical) are crashed, the cost will be very high without any additional advantage over and above the one obtained by crashing only the critical activities. The non-critical activities need not be speeded up, since their crashing is not going to decrease the project duration further.

However, it may happen that certain non-critical activities may become *critical* in the process of crashing the critical activities. It is, therefore, essential to proceed step by step in crashing one critical activity at a time and examining whether any other non-critical activity has also become critical in that process or not. For this, it is better to start with crashing *first* that critical activity which has the lowest cost slope. Then we take another critical activity which is having next higher cost slope. While crashing an activity fully (*i.e.* by Δt duration), it should be examined whether this crashing affects any other non-critical activity or not. For example, suppose activities A, B and D are critical, while activity C is not critical (Fig. 9.9).

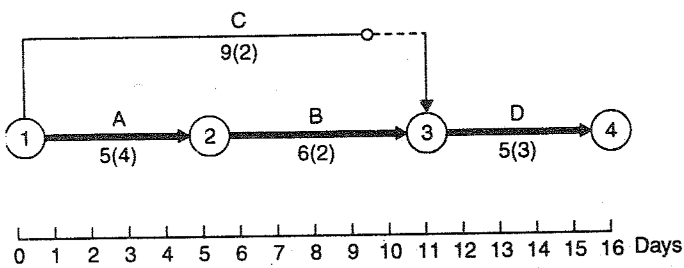


FIG. 9.9. ORIGINAL NETWORK BEFORE CRASHING.

The normal time for each activity are given below the activity arrow while the crash times are also given in the bracket. Now if critical activity B is fully crashed from 6 days (normal time) to 2 days (crashed time) such that Δt = 4 days, activity C will become critical. Hence activity B is first crashed by 2 days only, and extra cost is found. Then activity B is crashed by 2 days along with crashing activity C also by 2 days. The extra cost so involved will be equal to the combined cost slope of the activities B and C multiplied by 2. For doing this, it is better to draw the time-scaled version of the network shown in Fig. 9.9 based on the assumption that all the activities start at their earliest start times. The corresponding network, after crashing activity B partly by 2 days only is shown in

Fig. 9.10 (a) in which all the activities are critical, while Fig. 9.9 (b) shows the network after further crashing B by Δt = 2 days, and crashing C also by Δt = 2 days.

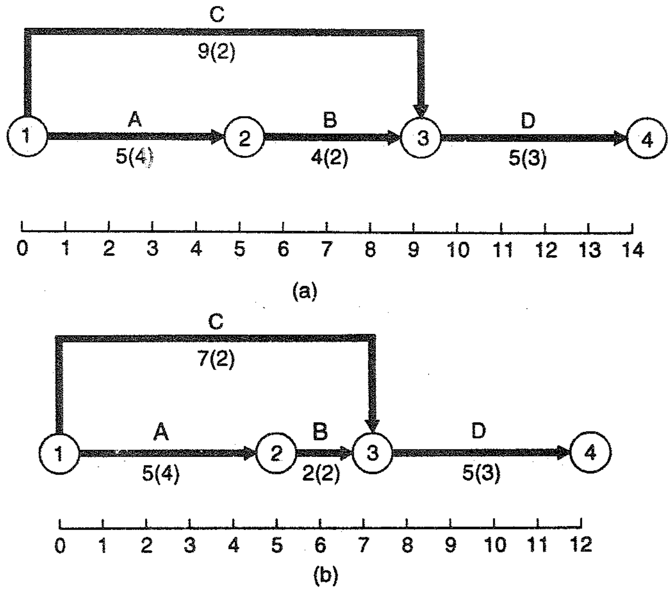


FIG. 9.10. CRASHING ACTIVITY B IN STEPS.

During each step of crashing procedure, the direct cost is calculated ; this direct cost will naturally be higher than the one corresponding to the normal time duration. However, the indirect cost will *decrease*, because the project duration has been decreased.

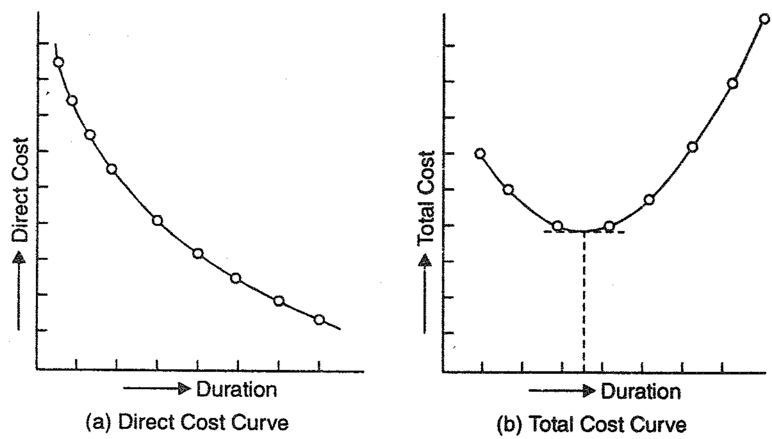


FIG. 9.11. TIME COST RELATIONS.

Hence the total cost is found in each step. A curve is then plotted between the total cost and time. It will be seen that while the direct cost goes no increasing as we reduce the duration, the total cost may decrease till an optimum duration is reached when the cost has a minimum value. If the time duration decreases further, the total cost may increase again, as shown in Fig. 9.11 (*b*). *Thus, it may not always be profitable to crash all the critical activities to their fullest duration.*

9.8. STEPS IN TIME COST OPTIMIZATION

The time-cost optimization is done in the following steps :

1. ESTABLISH	:	direct cost-time relationships for various activities of the project, by analysing past cost records.
2. DETERMINE	:	cost slopes for various activities and arrange them in the ascending order of cost slope.
3. COMPUTE	:	direct cost for the network with normal duration of activities.
4. CRASH	:	the activities in the critical path as per ranking, *i.e.* starting with the critical activity having the lowest slope.
5. CONTINUE	:	crashing the critical activities in the ascending order of the slope.
6. CRASH	:	parallel non-critical activities which have become critical by the reduction of critical path duration due to crashing in steps 4 and 5.
7. CONTINUE	:	crashing process through steps 4 to 6, till a stage is reached beyond which no further crashing is possible.
8. FIND	:	total cost of project at every stage by adding indirect costs to the direct costs determind above.
9. PLOT	:	total cost-duration curve.

10. PICK UP : the optimum duration corresponding to which least total project cost is obtained.

The process of cost optimization is illustrated with the following examples.

9.9. ILLUSTRATIVE EXAMPLES

Example 9.1. *Table 9.2 gives the information about various activities of network shown in Fig. 9.12.*

Table 9.2

Activity	*Normal duration (days)*	*Normal cost (Rs.)*	*Crash duration (days)*	*Crash cost (Rs.)*
1—2	*9*	*8000*	*6*	*9500*
2—3	*5*	*5000*	*3*	*5500*

The project overhead costs are @ Rs. 300.0 per day. Determine (a) direct cost-duration relationship, (b) total cost-duration relationship and the corresponding least cost plan (network).

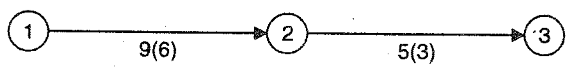


FIG. 9.12

Solution.

Step 1. *Cost-slopes.*

Fig. 9.12 shows the network, with the normal duration of each activity entered below its activity arrow, while the crash duration entered in the bracket.

The cost slope for each activity will be as under :

Table 9.3

Activity (i—j)	*ΔC (Rs.)*	*Δt (days)*	*Cost slope $= \frac{\Delta C}{\Delta t}$ (Rs. / days)*
1—2	1500	3	500
2—3	500	2	250

Step 2. *Normal duration direct cost.*

The normal duration for the project = 9 + 5 = 14 days.

$\therefore$ Normal duration cost = 8000 + 5000 = 13000.

Step 3. Activity 2-3 has the least slope. Let us therefore crash it first, though in serial-activities network, any activity can be crashed first, or even all the activities can be crashed simultaneously to their corresponding crash durations. Duration by which activity 2-3 can be crashed = 2 days.

Extra cost of crashing activity = 2-3 by 2 days

$= 250 \times 2 = 500$

Project duration $= 9 + 3 = 12$ days.

$\therefore$ Direct cost for 12 days project duration

$= 13000 + 500 = 13500.$

Step 4. After having fully crashed activity 2-3, let us crash activity 1-2 from its normal duration of 9 days to its crash duration of 6 days.

$\Delta t = 9 - 6 = 3$ days.

Extra cost of crashing $= 3 \times 500 = 1500$

Project duration $= 6 + 3 = 9$ days.

$\therefore$ Direct cost for 9 days project duration $= 13500 + 1500$

$= 15000$

The corresponding network with all the activities crashed is shown in Fig. 9.13.

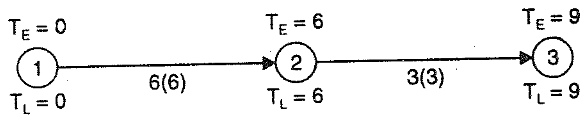


FIG. 9.13

Step 5. ***Total cost of project.***

The total cost of the project, for any duration, is obtained by adding the indirect costs overheads to the corresponding direct costs. The values are tabulated in Table 9.4.

Table 9.4

Duration → *(days)*	*14 Normal*	*12*	*9*
Direct cost	13000	13500	15000
Indirect cost	4200	3600	2700
Total cost	17200	17100	17700

Step 6. ***Cost duration on curves.***

Fig. 9.14 shows the cost-time curves for direct cost, indirect cost and total cost. From Table 9.3 as well as Fig. 9.14, it is evident that total cost is *minimum* for a project duration of 12 days. Thus the *optimum duration* of the project is **12 days** and *minimum cost* corresponding to it is **Rs. 17100.**

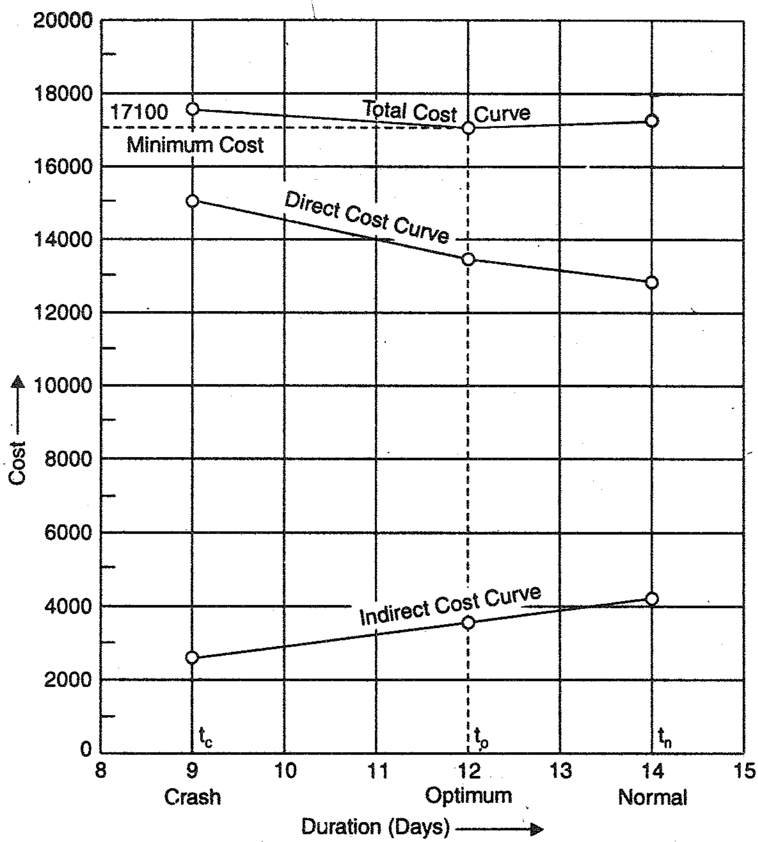


FIG. 9.14. COST-DURATION CURVES.

Example 9.2. *If in example 9.1, in addition to the overheads there is an outage loss of Rs.100 per day upto and including 11th day and Rs. 200 per day thereafter, find the total cost-duration relationship.*

Solution. The indirect costs consist of cost of overheads and the outage loss. Hence the indirect cost curve will not be a single straight line, but will be segmented. In order to get the total cost, we should have the data of the direct costs for all durations. In Table

9.4, the direct cost data is available for 14, 12 and 9 days durations only. The direct costs for intermediate durations (*i.e.*, 13, 11 and 10 days) can be calculated as under :

(*i*) ***For 13 days duration***

Activity 2-3 crashed by $\Delta t = 1$ day.

$\therefore$ Cost of activity 1-2 for 9 days duration = 8000

Cost of activity 2-3 for 3 days duration

$$= 5000 + 1 \times 250$$
$$= 5250$$

$\therefore$ Total direct cost

$$= 8000 + 5250$$
$$= 13250.$$

(*ii*) ***For 11 days duration***

Activity 2-3 is fully crashed to its duration of 3 days, while activity 1-2 has been crashed from its 9 days duration to 8 days duration.

Cost of activity 2-3 = 5500

Cost of activity 1-2 for 8 days duration

$$= 8000 + 1 \times 500$$
$$= 8500$$

$\therefore$ Total direct cost = 5500 + 8500 = 14000.

(*iii*) ***For 10 days duration***

Activity 1-2 further crashed by 1 day, at an extra cost of $1 \times 500 = 500$.

$\therefore$ Total direct cost = 14000 + 500 = 14500.

The total cost cannot be found by adding the indirect costs to the direct cost, as shown in Table 9.5

Table 9.5

Duration (days) →	*14 Normal*	*13*	*12*	*11*	*10*	*9 (Fully crashed)*
Direct cost	13000	13250	13500	14000	14500	15000
Overheads	4200	3900	3600	3300	3000	2700
Outage loss	1700	1500	1300	1100	1000	900
Total cost	18900	18650	18400	18400	18500	18600

The time-cost relationships are shown in Fig. 9.15. From Fig. 9.15, it is clear that the minimum total cost is obtained corresponding to the project duration of 11 days or 12 days. Hence the *optimum* project duration is **11 days** and the corresponding *minimum cost* is **Rs. 18400.**

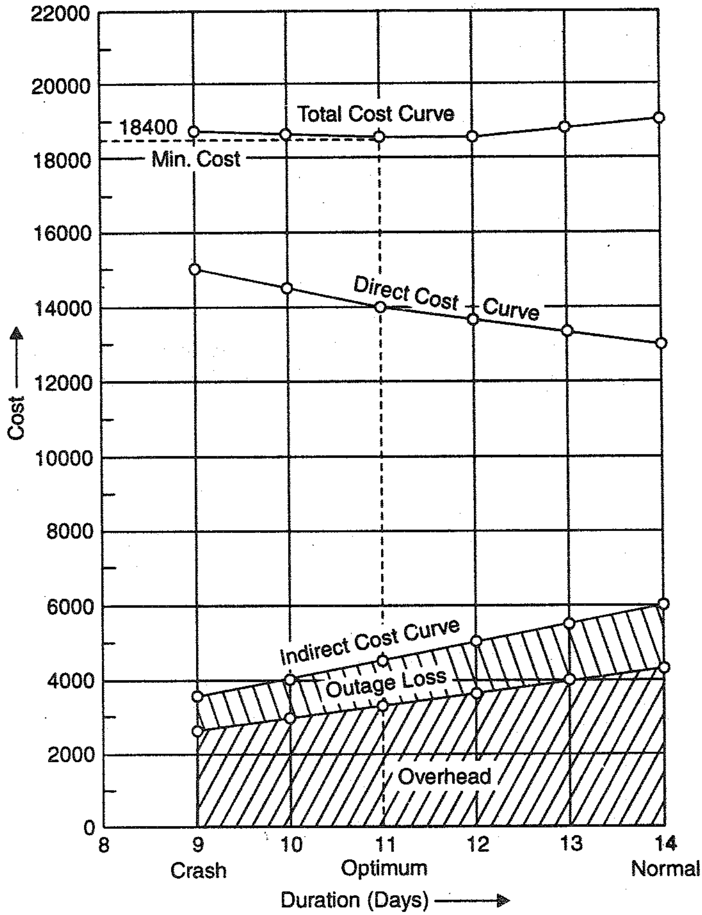


FIG. 9.15

Example 9.3. *Table 9.6 gives the data about durations and costs if various activities of the network shown in Fig. 9.16.*

Table 9.6

Activity	*Normal duration (weeks)*	*Normal cost (Rs.)*	*Crash duration (weeks)*	*Crash cost (Rs.)*
1—2	*4*	*4000*	*2*	*12000*
2—3	*5*	*3000*	*2*	*7500*
2—4	*7*	*3600*	*5*	*6000*
3—4	*4*	*5000*	*2*	*10000*

The project overhead costs are Rs. 2000 per week. Find the optimum duration and the cost associated with it. Also, draw the least cost network.

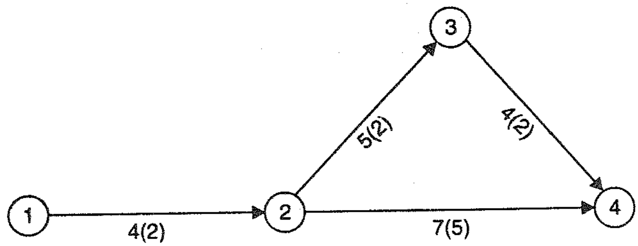


FIG. 9.16

Solution.

Step 1. ***Time-scaled version of network***

Fig. 9.17 (*a*) shows the original network while Fig. 9.17 (*b*) shows the time-scaled version of the network drawn on the assumption that all the activities start at their earliest times. The normal completion time of each activity is mentioned below its arrow, while the corresponding crash duration is mentioned in the bracket. The dotted portion of any activity arrow denotes the total float of that activity. The *critical path* is along the activities 1-2, 2-3, 3-4 (shown by thick lines). The critical activities are shown along the horizontal straight path in the time-scaled version of the network.

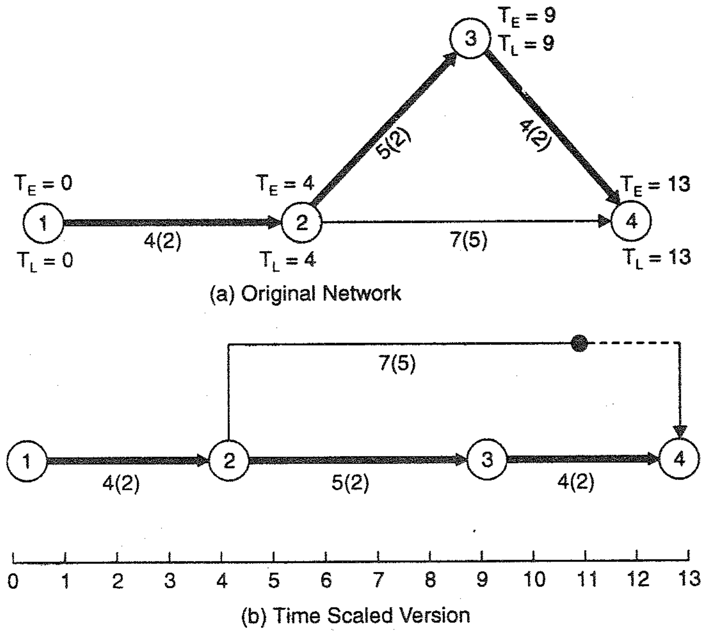


FIG. 9.17

Step 2. ***Cost slopes***

The cost slopes of various activities will be as shown in Table 9.7 :

Table 9.7

Activities	ΔC *(Rs.)*	Δt *(weeks)*	*Cost slope Rs./week*
1—2	12000 – 4000 = 8000	4 – 2 = 2	4000
2—3	7500 – 3000 = 4500	5 – 2 = 3	1500
2—4	6000 – 3600 = 2400	7 – 5 = 2	1200
3—4	10000 – 5000 = 5000	4 – 2 = 2	2500

Step 3. ***Direct cost of normal duration project***

The normal duration of the project is the sum of the normal durations of each activity on the critical path. (It is not the sum of normal durations of all the activities).

$\therefore$ Normal duration of the project

$$= 4 + 5 + 4 = 13 \text{ weeks.}$$

Direct cost of the project will be equal to the sum of the normal cost of *all* the activities.

$$\therefore \text{Direct cost} = 4000 + 3000 + 3600 + 5000$$
$$= 15600.$$

Step 4. *First stage crashing*

While crashing the activities, we shall first select that critical activity which has the minimum cost slope. For the present case, critical activity 2-4 has the minimum cost slop of 1500 per week. Let us crash this first. Its crash period is 2 weeks, *i.e.*, $\Delta t = 5 - 2 = 3$ weeks. However, crashing it by 3 weeks will affect non-critical activity 2-4, which has a float of only 2 weeks. Hence let us restrict the crashing of 2-3 by 2 weeks only, in the *first stage*. New duration of the project = 13 – 2 = 11 weeks.

Extra cost of crashing activity 2-3 by 2 weeks

$$= 2 \times 1500 = 3000$$

$\therefore$ Direct cost of project of 11 weeks duration

$$= 15600 + 3000 = 18600.$$

The time-scaled version of the network, after first stage crashing, for the project of 11 weeks duration is shown in Fig. 9.18.

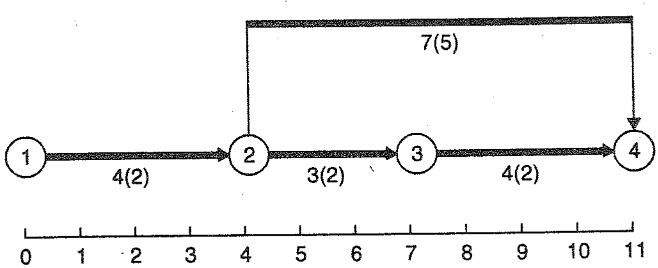


FIG. 9.18. NETWORK FOR 11 WEEKS DURATION.

Step 5. *Second stage crashing*

From Fig. 9.18, it is clear that activity 2-4, lying on the parallel path, has also become *critical*, though activity 2-3 has still 1 week crashing left. However, activity 2-3 cannot be crashed along, unless 2-4 is also crashed. Let us therefore crash 2-3 for 1 week, and also crash 2-4 simultaneously for 1 week. However, this combined crashing will be useful *only* if the *combined* cost slope of these two activities is *less* than the cost slope of any of the remaining critical activity or the combined cost slopes of critical activities on parallel path.

For the present case, further crashing can be done with three alternatives :

(*i*) Crashing activities 2-3 and 2-4 simultaneously, having a combined cost slope of 1500 + 1200 = 2700 per week.

(*ii*) Crashing activities 3-4 and 2-4 simultaneously, having a combined cost slope of 2500 + 1200 = 3700 per week.

(*iii*) Crashing activity 1-2 alone, having a cost slope of Rs. 4500 per week.

Out of these, the first alternative has the minimum cost slope.

Thus, the extra cost of crashing 2-3 and 2-4 by 1 week

$$= 2700 \times 1 = 2700$$

$\therefore$ Direct cost of project for 10 weeks duration

$$= 18600 + 2700 = 21300.$$

In this step, activity 2-3 has been crashed to its fullest extent. Fig. 9.19 shows the time-scaled version of the network, for 10 weeks duration.

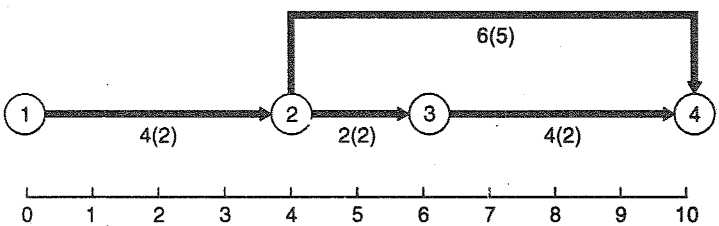


FIG. 9.19. NETWORK FOR 10 WEEKS DURATION.

Step 6. *Third stage crashing*

The remaining activities to be crashed (either fully or partly) are 1-2, 2-4 and 3-4. Out of these, activities 2-4 and 3-4 are to be crashed jointly, with a combined cost slope of 2500 + 1200 = 3700. The cost slope of activity 1-2 is 4000, which is higher. Hence activities 2-4 and 3-4 will be crashed *first*. Activity 2-4 has a crashing period of 6 – 5 = 1 week left. Hence only 1 week crashing will be done in this step, leading to a project duration of 9 weeks.

Cost of crashing 2-4 and 3-4 by 1 week.

$$= 3700 \times 1 = 3700.$$

$\therefore$ Direct cost of project for 9 weeks duration

$$= 21300 + 3700 = 25000.$$

The time scaled version of the network for 9 weeks duration is shown in Fig. 9.20. Upto this stage, we have crashed activity 2-4 also to its fullest extent.

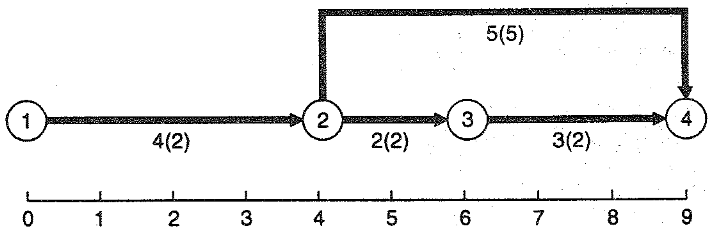


FIG. 9.20. NETWORK FOR 9 WEEKS DURATION.

Step 7. *Fourth stage crashing*

Out of remaining critical activities (*i.e.* 1-2 and 3-4), activity 3-4 cannot be further crashed to its fullest crash period (of 2 weeks), since it will affect activity 2-4 which has already been fully crashed.

Hence activity 1-2 is the only remaining activity to be crashed. The period by which it can be crashed is = 4 – 2 = 2 weeks, reducing the project duration to 9 – 2 = 7 weeks.

Extra cost of crashing 1-2 by 2 weeks = 2 × 4000 = 8000

$\therefore$ Direct cost of project of 7 weeks duration

$$= 25000 + 8000 = 33000.$$

The time-scale version of the network of the project of 7 weeks duration is shown in Fig. 9.21.

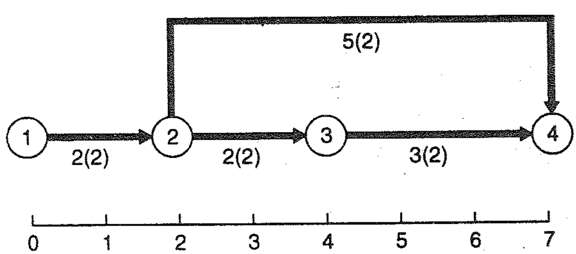


FIG. 9.21. NETWORK FOR 7 WEEKS DURATION.

It should be noted that the project duration has been crashed from its original duration of 13 weeks to the present duration of

7 weeks ; thus the period by which it has been crashed = Δt = 13 – 7 = 6 weeks only, as against the total available crashing period of 13 – 6 = 7 weeks along the original critical path.

Step 8. *Total cost of the project*

The total cost of the project is computed by adding the direct cost and the indirect cost, as illustrated in Table 9.8. The indirect cost is at the rate of Rs. 2000 per week.

Table 9.8. Total Costs

Project Duration (weeks)	*13 (normal)*	*11*	*10*	*9*	*7*
Direct cost (Rs.)	15600	18600	21200	25000	31000
Indirect cost (Rs.)	26000	22000	20000	18000	14000
Total cost (Rs.)	41600	40600	41300	43000	47000

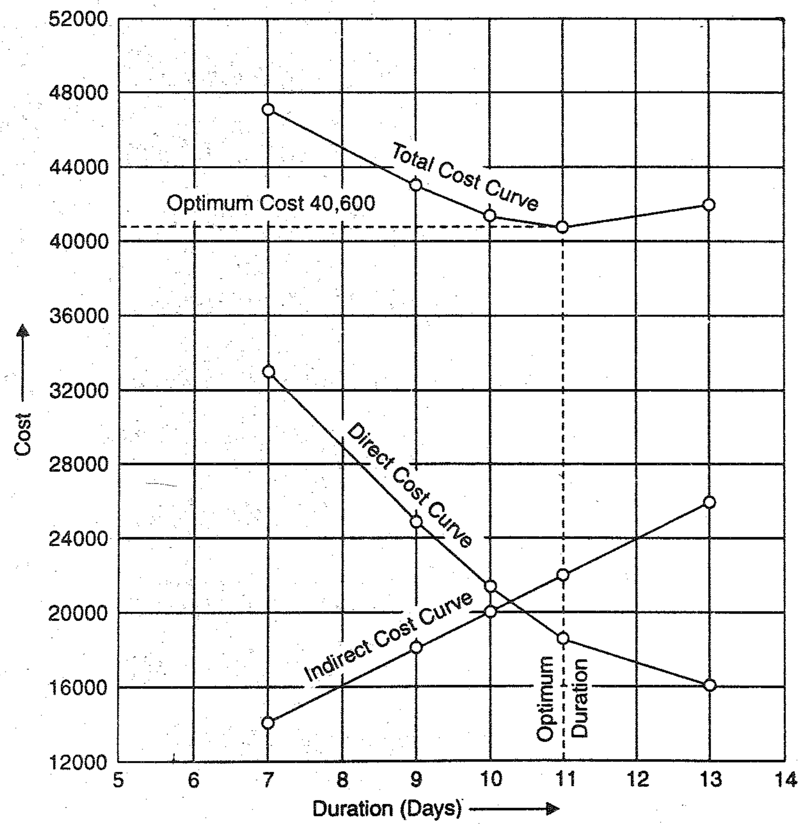


FIG. 9.22. DURATION-COST CURVES.

The direct cost curve, indirect cost curve and total cost curve are shown in Fig. 9.22. From Table 9.8 as well as Fig. 9.22, it is clear that the optimum project duration is **11 weeks** and the optimum cost corresponding to this period is Rs. **40600.** The time scaled network for this project duration is shown in Fig. 9.18.

PROBLEMS

1. Define the terms 'direct cost', 'indirect cost' and 'outage loss'.
2. What do you understand by 'cost-slope' ? How do you determine it ?
3. Define 'normal project time', 'normal cost', 'crash time' and 'crash cost'.
4. Draw a typical cost-duration curve and show on it optimum duration and minimum project cost.
5. Explain the method of times-cost optimization of project network.
6. Fig. 9.23 show the network for a project, the data for the duration and costs of each activity are given in Table 9.9.

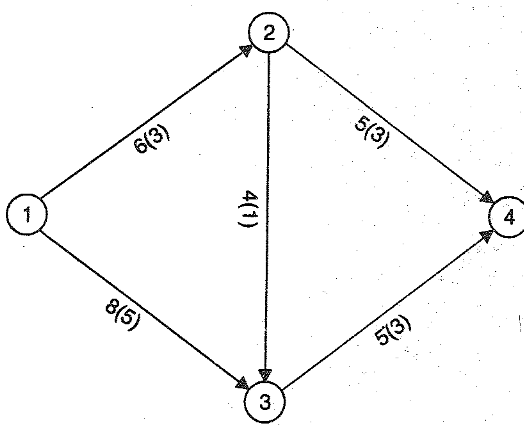


FIG. 9.23

Table 9.9

Activity	*Normal duration (weeks)*	*Normal cost (Rs.)*	*Crash duration (weeks)*	*Crash cost (Rs.)*
1—2	6	7000	3	14500
1—3	8	4000	5	8500
2—3	4	6000	1	9000
2—4	5	8000	3	15000
3—4	5	5000	3	11000

The direct cost of the project is Rs. 3000 per week. Determine the optimum duration of the project and the corresponding minimum cost. Draw the time scaled version of the network at each stage of crashing.

10

CPM : Updating

10.1. INTRODUCTION

Controlling is complementary to the planning. Once the scheduled plan has been prepared and execution commenced, control over the progress of work has to be exercised in order to complete the work by the stipulated date. Control involves comparing at regular intervals, the actual achievement with the original plans and then taking any necessary corrective action to bring things back on schedule. Therefore, *Controlling* requires an upward flow of information through a suitably designed reporting system. The information so fed is analysed and the project plan is brought upto date with necessary variations to keep performance as per the schedule.

10.2. UPDATING : PROCESS

During the process of implementing the plan according to the network, we may come across one or more of the following possibilities :

1. that some or all activities are progressing according to schedule ;
2. that some or all activities are ahead of schedule ; and
3. that some or all activities are behind schedule.

If all activities are progressing according to the schedule, there is no need for updating the network but this is seldom the case.

Therefore, based on the progress of the work and the revised durations of unfinished activities due to delays, the network diagram has to be redrawn and this process is known as *Updating*.

As stated previously, critical path method is a numerical technique which project management can use as an aid in planning, scheduling and controlling any type of project. The calculations made previously were based on the assumption that the planner has taken an entirely new project in hand and tail event of the project has been taken as the base for the calculations. The networks previously developed can also be used to aid planners or managers in decision-making, after the commencement of the project *i.e.* when project is already in progress.

When the project is partially completed and is at an intermediate stage, it may be possible that :

1. the time durations originally assigned for some activities were erroneous and

2. the planner may himself feel it desirable as a result of experience or he may be enriched with additional information, to reconsider and re-estimate duration times of activities not yet being performed. Now, new information and considerations can be placed on the original network and fresh calculations are made for controlling the project.

The process of replanning and rescheduling based on the results which serve a guidance for decision by performing calculations made by taking into consideration the new knowledge and latest information at an intermediate stage of the project thus modifying the original network, is known as the process of Updating.

10.3. DATA REQUIRED FOR UPDATING

The following information is necessary to update the plan at an intermediate stage of execution of a project :

1. original network ;
2. original network calculation chart ;
3. stage at which updating is being done *i.e.*, a point in time of updating ;
4. execution position of the project at that stage and
5. new information and knowledge which will affect the duration time of the activities to be performed.

10.4. STEPS IN THE PROCESS OF UPDATING

1. DESCRIBE	:	the point in time at which updating is to be done according to the original plan.
2. RECORD	:	what has happened actually till the updating point.

3. SUMMARISE : the knowledge attained in the tabulated form as given below :

Table 10.1. Updating

Activity	*Whether completed or not*		*If in progress, additional time required for the completion*	*Completion required for activities yet to begin*
	Yes/No	*If yes, time taken for completion*		

4. PLACE : the information contained in the updating table on to the original network. This is done by :

1. assigning the time of update as the earliest occurrence time for the tail event of the project ;
2. alloting a zero time duration for all activities which have been completed ;
3. entering the remaining estimated durations of those activities which are in progress ; and
4. entering the estimated durations based on new knowledge of activities which are still to be commenced.

5. PERFORM calculations of earliest occurrence time and latest occurrence time and mark these on the network known as updated network.

The updating cycle is shown in Fig. 10.1.

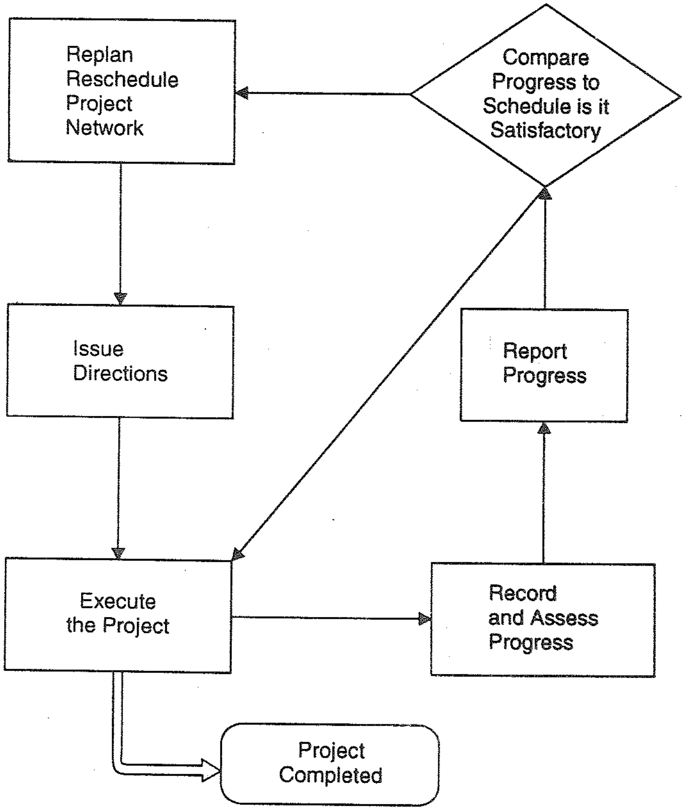


FIG. 10.1. UPDATING CYCLE.

10.5. WHEN TO UPDATE

The following points must be kept in view while deciding the time of Updating :

1. for shorter duration projects, the updating must be done frequently by taking into account the latest position of the execution of the project.

2. for large duration projects, the process of updating must be increased as the project is progressing toward completion. Duration of project goes on decreasing as project progresses, and behaving more or less like a small duration project.

3. whenever there is major change in the duration of any of the activity the updating is to be done.

4. updating is essential if there is change in the estimated duration of any activity falling on the critical path. If the duration of a critical activity increases, remedial measures are necessary and if the activity duration decreases, this may allow changes in the project plan which were not possible previously.

After updating there may be some changes in the completion time of the project. If updating time comes out to be more, the planner has two options :

1. He may ask the executing authorities to perform the operations on the critical path faster than previously estimated. Such execution will require the arrangement of more resources *i.e.*, man-power and material etc., and

2. He may redraw some portion of the network containing those activities which have still not commenced. Such alteration means change in the company's policy of execution which leads to revised inter-dependence of operations.

From the above it is clear that CPM is not only useful in the planning stage of a project but also aids the decision-maker during execution and to some extent in controlling the completion of project on schedule.

10.6. ILLUSTRATIVE EXAMPLES

Example 10.1. *Fig. 10.2 shows the network of a project which is to be updated at the end of 12 days. The following conditions exist at the time of updating :*

1. Activity 1-4 was completed as originally planned.

2. Activity 1-3 was executed more rapidly then originally scheduled, and it took 8 days for its completion.

3. Activity 3-4 commenced following the completion of activity 1-3 and was finished at the end of 11th day.

4. Activity 4-5 was commenced following the completion of activity 3-4 (i.e., at the end of 11th day), and still requires 6 more days for its completion.

5. Completion of activity 1-2 was delayed drastically, and it still requires 10 more days for its completion.

6. Activity 2-7 will commence following the completion of activity 1-2 and will require 9 days for its completion instead of 6 days originally estimated.

7. The time required to perform activity 5-8 has been revised, based on the experience on the project, gained to this point. It now requires 10 days in the place of 6 days originally estimated.

8. No other activities have been started, and the original time estimates for these activities still appear to be accurate.

Update the network, and determine the revised critical path.

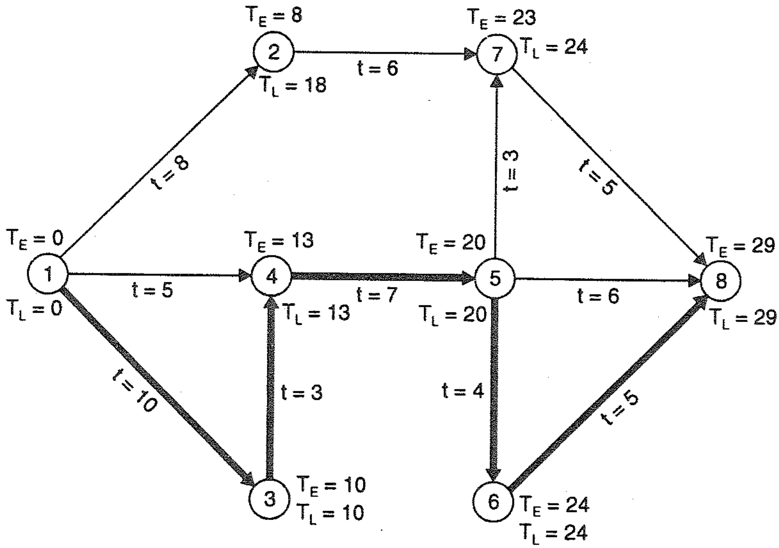


FIG. 10.2

Solution. Fig. 10.2 shows the original network, with T_E and T_L marked. The critical path, shown by dark lines is along activities 1-3, 3-4, 4-5, 4-6 and 6-8.

Table 10.2 gives the details of execution of the various activities at the end of 12 days.

The updated network can now be drawn on the basis of data of columns (1), (2), (4) and (5) of the above table. For those activities, which have already been completed, completion time t is taken to be zero, since they require zero time after the 12th day. Also the earliest event time (T_E) and leasted occurrence time (T_L) of each event is computed *with reference* to the original starting date of the project. This can be best achieved by taking T_E for event 1 as equal to 12.

Table 10.2
Review after 12 days

Activity	*Whether completed or not*		*Additional time required for activities in progress (days)*	*Completion time required for activities yet to begin (days)*
	Yes / No	*If yes, time taken (days)*		
(1)	*(2)*	*(3)*	*(4)*	*(5)*
1—2	No	—	10	—
1—3	Yes	8	—	—
1—4	Yes	5	—	—
2—7	No	—	—	9
3—4	Yes	3	—	—
4—5	No	—	6	—
5—6	No	—	—	4
5—7	No	—	—	3
5—8	No	—	—	10
6—8	No	—	—	5
7—8	No	—	—	5

After having determined the updated T_E for each event, corresponding T_L can be computed by the *backward pass*. The updated network is shown in Fig. 10.3. The critical path of the

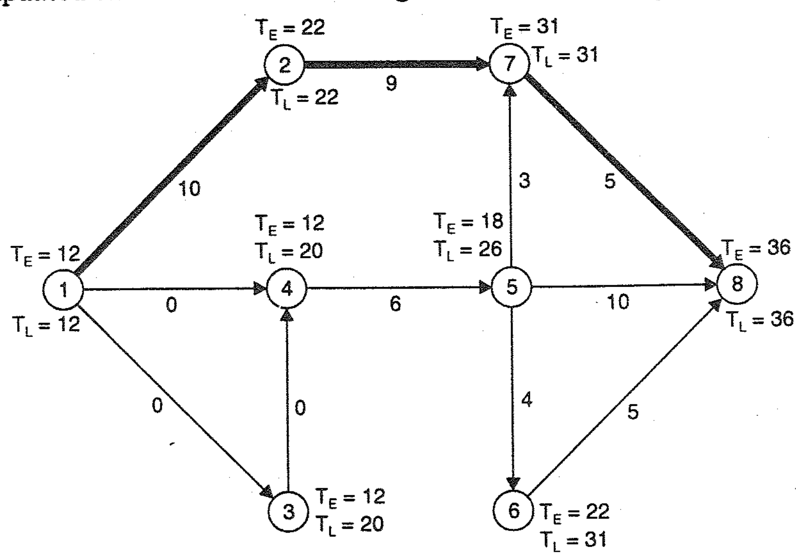


FIG. 10.3. UPDATED NETWORK.

updated network has now changed ; it is now along activities 1-2, 2-7, 7-8, shown by dark lines. According to the updated network, the project will take a total time of 36 days, instead to 29 days originally planned. On the day of updating, the remaining duration of project = 36 – 13 = 24 days.

PROBLEMS

1. What do you understand by updating ? Why is it essential ?
2. Illustrate the method of updating a network during its execution period.
3. A network for a project is shown in Fig. 10.4. The network is to be updated after 10 days of its execution. The following conditions exist at the end of 10 days :
   (*i*) Activity 1-2, 1-3 and 1-4 have been completed as originally scheduled.
   (*ii*) Activity 4-5 is in progress and will require 6 more days for its completion.
   (*iii*) Activity 4-6 is in progress and will require 6 more days for its completion.
   (*iv*) Activity 3-6 is in progress and will be completed in one day.
   (*v*) Other activities have not been commenced and their original predicted durations will hold good, except for activity 5-7 which will require only three days instead of 5 days originally planned.

   Update the network and determine the critical path of the updated network. What is the total increase in the project duration ?

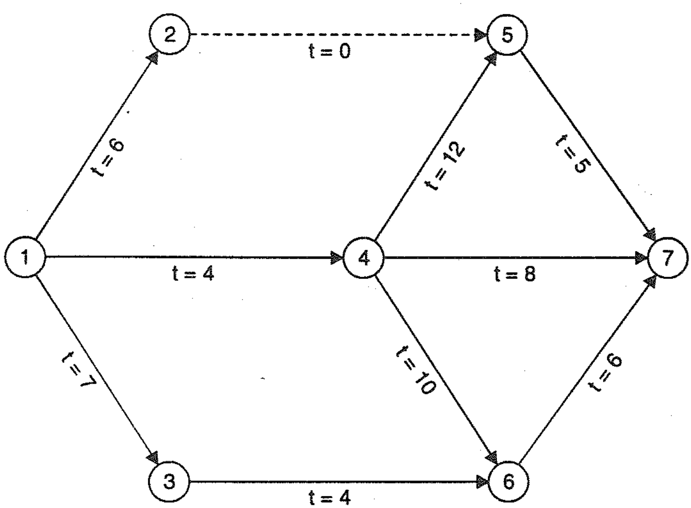


FIG. 10.4

11

Resources Allocation

11.1. INTRODUCTION

A resource is a physical variable, such as men, materials, machines, space, money that is required for completing various activities/jobs of a project. The network analysis for PERT/CPM, carried out so far is valid only if the availability of resources is liberal or unlimited. But all the necessary resources are not available in unlimited quantities. Availability of some of the resources may be restricted. Availability of manpower (supervisory staff, technical and specialist personnel, skilled and unskilled labour etc.) and materials etc. may be restricted. Availability of funds, credits, capital investment and heavy equipment may be restricted. In certain cases, there may be space limitations, which prevent more than one or two technicians working simultaneously. Supervisory, technical and skilled manpower, space and equipment are usually the most important resources that need be allocated carefully. The various activities of the project are to be scheduled in such a way that the demand of various resources is more or less uniform all along the project duration. Large fluctuations in their demand may cause problems in the project execution.

11.2. RESOURCES USAGE PROFILES : HISTOGRAMS

For a given network, the requirements of various resources are determined, using the early start schedule of each activity. In a network, various activities are involved, and each activity requires some resources to perform it. There may be activities which are to be performed simultaneously and may require common resources. The requirements of resources to execute these simultaneous activities may exceed the available resources. However, at some other

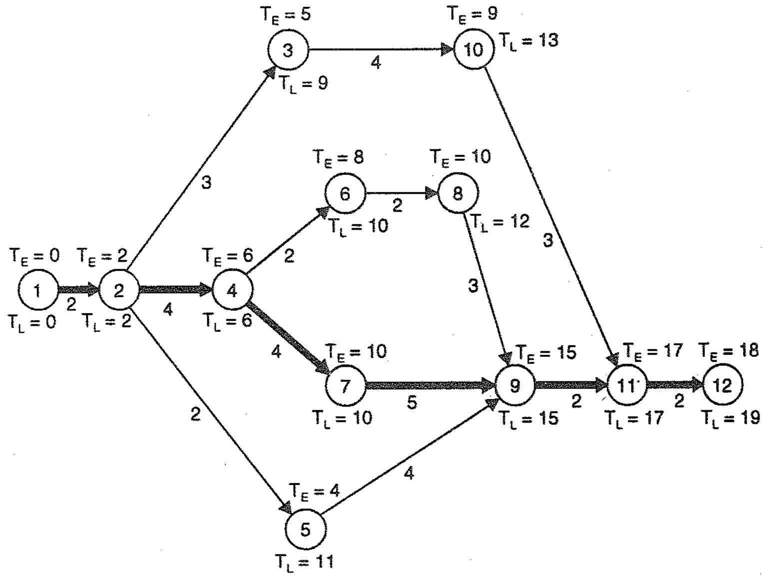


FIG. 11.1

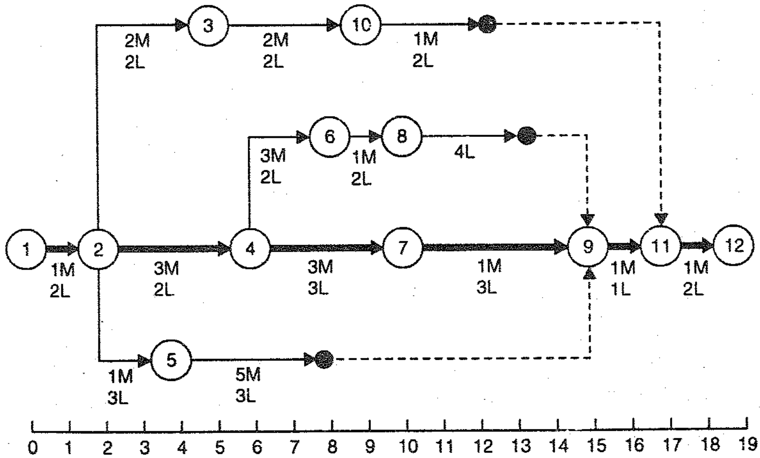
period of the execution of the same project, there may be very few activities which may require these resources. Hence the requirement of a particular type of resource may not be uniform during the project duration. This can be best known by plotting the *resources usage profiles* or *histograms*.

Consider a network shown in Fig. 11.1, having 14 activities. The duration of each activity is marked under its activity arrow. The early event times and late event times are marked near each event circle. The critical path is along activities 1-2, 2-4, 4-7, 7-9, 9-11 and 11-12, shown by thick lines. Table 11.1 shows the requirements of masons (marked by *M*) and labourers (marked by *L*) for each activity. Let us analyse the project from resources requirements point of view.

Table 11.1
Resources Requirements

Activity	*Duration*	*Masons (M)*	*Labourers (L)*
1—2	2	1	2
2—3	3	2	2
2—4	4	3	2
2—5	2	1	3
3—10	4	2	2
4—6	2	3	2
4—7	4	3	3
5—9	4	5	3
6—8	2	1	2
7—9	5	1	3
8—9	3	—	4
9—11	2	1	1
10—11	3	1	2
11—11	2	1	2

Fig. 11.2 shows the time scaled version of the network, assuming early start times for each activity. The activities along the

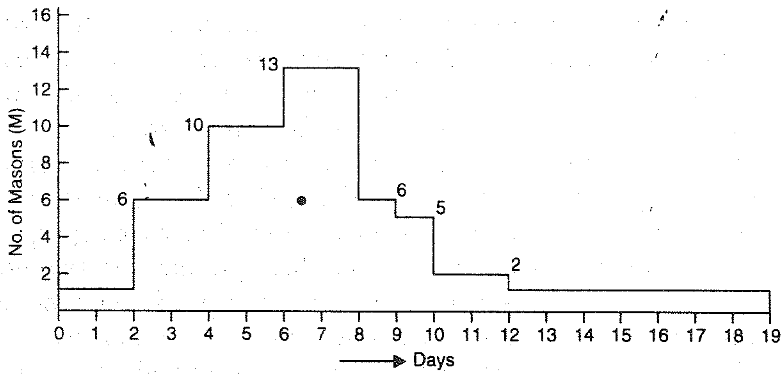


M	1	1	6	6	10	10	13	13	6	5	2	2	1	1	1	1	1	1	1
L	2	2	7	7	7	7	10	10	7	7	9	9	7	3	3	1	1	2	2

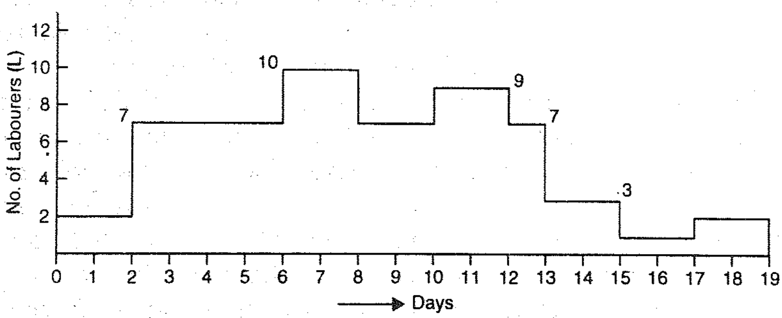
FIG. 11.2

critical path have been arranged along horizontal line. The dotted lines show the total float of each activity. The requirements of masons (M) and labourers (L) for each activity is marked under the activity arrow. The table below the time scale shows total requirements of masons and labourers each day.

Fig. 11.3 (a) and (b) show the variation in the requirements of masons and labourers respectively, with time. These diagrams are known as *resources usage profiles* or *histograms*.



(a) Requirements of Masons



(b) Requirements of Labourers

FIG. 11.3. RESOURCES USAGE PROFILES.

From Fig. 11.2 as well as 11.3, it is clear that the requirements of masons and labourers is not uniform along the project duration.

The demand of the masons is as high as 13 on 7th and 8th days, while it is as low as 1 in the beginning and end of the project. Similarly, the demand of the labourers is as high as 10 on 7th and 8th days, while it is as low as 2 in the beginning and the end of the project. This shows the great variation in the resources (*i.e.* manpower) requirements. If 13 masons are employed to meet the peak demand, they will sit idle during the non-peak periods. This will be highly uneconomical unless we employ them on temporary basis only as per actual requirements each day. However, skilled persons such as masons, foreman etc. are required to be employed on the permanent basis. Therefore, the planning should be done in such a manner that resources are utilized in a more or less uniform manner. This can be achieved by the following two approaches :

(*a*) Resources smoothing

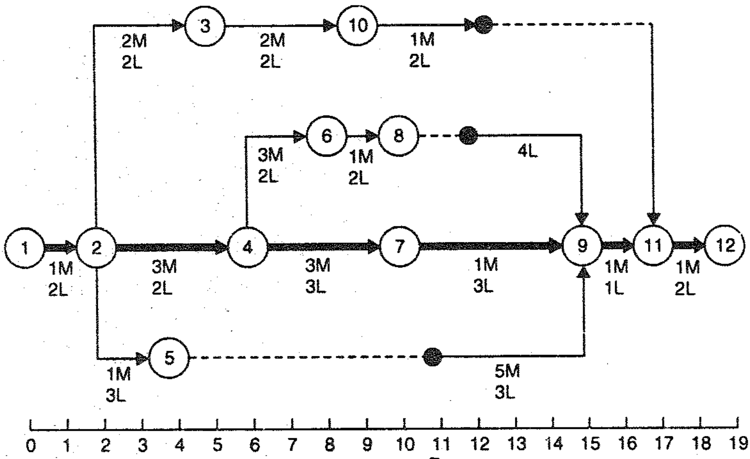
(*b*) Resources levelling.

The above nomenclature for the two approaches to solve the resources allocation problem has not been standardized so far with the result that some people use them interchangely. In the *first approach*, known as *resources smoothing method*, the total project duration is not changed, but some of the activities start times are shifted by their available floats so that a uniform demand for the resources is generated. However, the resources are considered to be unlimited. In the *second approach*, known as *resources levelling*, the activity start times are so re-scheduled that the peak demand for a particular resource does not cross the available limit of the resources. Thus, the resources are considered to be *limited*. In rescheduling the activities, the floats are first used, but if it does not give the desirable results, the total project duration may be changed.

11.3. RESOURCES SMOOTHING

This is the first approach of solving the resources allocation problem, in which the resources are considered to be unlimited. The original project duration (*i.e.* duration along the critical path) is however maintained. The start times of some of the activities are so shifted within their available floats that uniform demand is created for the resources.

To illustrate the procedure, let us consider the network of Fig. 11.2. We find that the peak requirements of masons are there on 7th and 8th day. Also, the requirements of masons on 5th and 6th day is high. Also, the requirements of mason on 11th day and



M	1	1	6	6	5	5	8	8	6	5	2	7	6	6	6	1	1	1	1
L	2	2	7	7	4	4	7	7	7	7	5	8	10	10	10	1	1	2	2

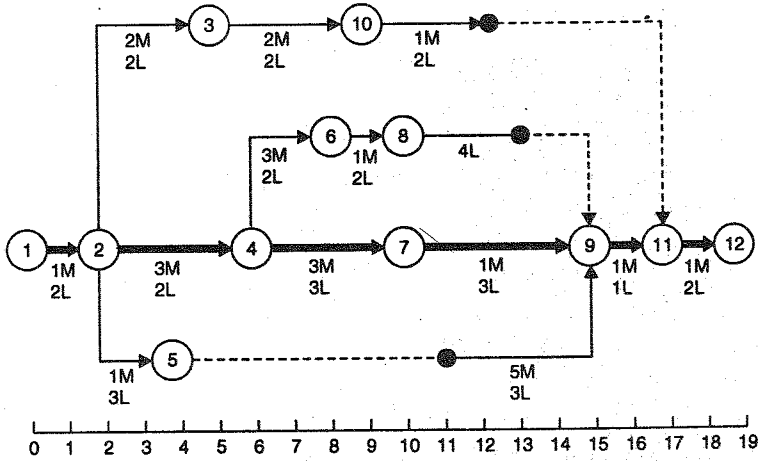
FIG. 11.4

onwards is very low. By inspection, we find the activities 1-5 and 5-9, have a total float of 7 days. Hence the start time of activity 5-9 can be shifted by 7 days. This will give encouraging results since this activity requires 5 masons. As a first trial therefore, let us shift activity 5-9 by 7 days, so that it starts on 12th day instead of 5th day. Fig. 11.4 shows revised network, along with the modified resources accumulation table.

From Fig. 11.4, we find that the peak demand for masons has decreased from 13 (for 7th and 8th day) to 8 (for 7th and 8th day). Also, the demand of masons has decreased from 10 to 12.

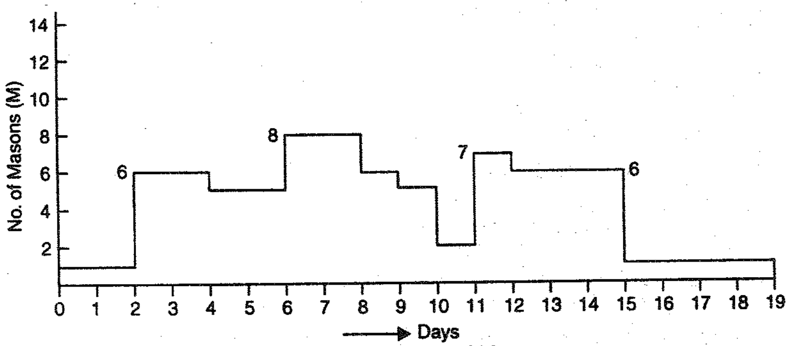
In the second trial, we can shift activity 8-9 by its total float period of 2 days. This will result in smoothing the labour requirements.

From Fig. 11.5, we observe that the demand of labourers has been decreased from 12 to 10. Fig. 11.6 shows the corresponding histograms for the masons and the labourers. Thus, following this procedure, it is always possible to smoothen the resources requirements, without affecting the project duration.

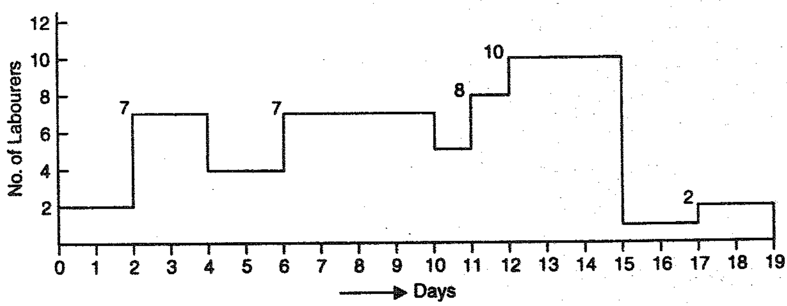


M	1	1	6	6	5	5	8	8	6	5	2	7	6	6	6	1	1	1	1
L	2	2	7	7	4	4	7	7	7	7	9	12	10	6	6	1	1	2	2

FIG. 11.5



(a) Requirements of Masons



(b) Requirements of Labourers

FIG. 11.6

11.4. RESOURCES LEVELLING

In the resources levelling process, the activities are so rescheduled that the maximum or peak resources requirement does not cross the limit of available resources. The available resources should, however, not be less than the maximum number or quantity required for any activity of the project. In rescheduling, the available floats are first used. If by doing so, the resources demand is more than the available resources, the duration of some of the activities is *increased* so that the resources requirements for these activities is decreased. Thus in the sources levelling process, the project duration, initially planned, might be changed.

PROBLEMS

1. Discuss in brief the resources allocation problem. What are the methods of solving the problem ?
2. With the help of an illustrative example, explain the resources smoothing method.

Answers

CHAPTER 2

4. (*a*) Bar chart shown in Fig. 2.13.

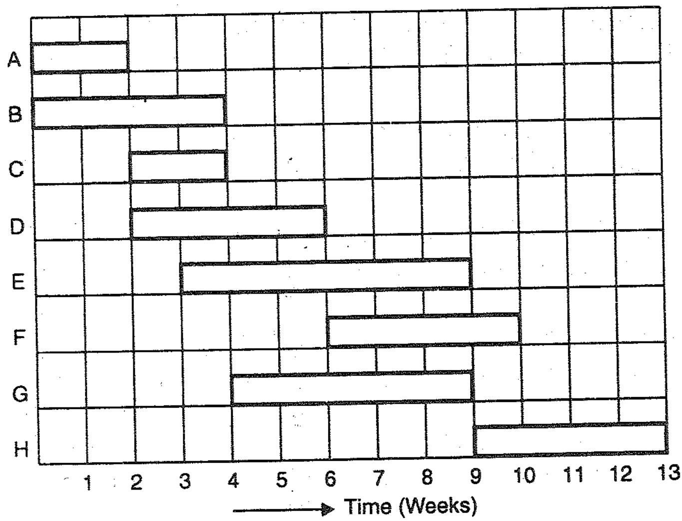


FIG. 2.13

(*b*) 13 weeks,

(*c*) 2 weeks.

CHAPTER 3

4. Events : *b, c, g, i*.

Activities : *a, d, e, f, h, j*.

10.

Event No.	*Immediate predecessor*	*Predecessor*	*Immediate successor*	*Successor*
1	—	—	2, 3	2, 3, 4, 5, 6, 7
2	1	1	4	4, 6, 7
3	1	1	4, 5	4, 5, 6, 7
4	2, 3	2, 3, 1	6, 7	6, 7
5	3	3, 1	6	6, 7
6	4, 5	4, 5, 3, 2, 1	7	7
7	4, 6	4, 6, 5, 3, 2, 1	—	—

11. Network shown in Fig. 3.47.

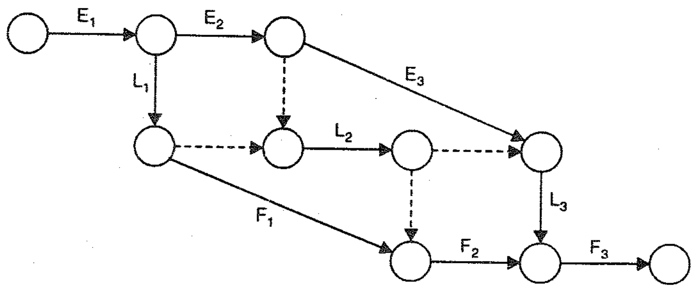


FIG. 3.47

12. Fig. 3.48.

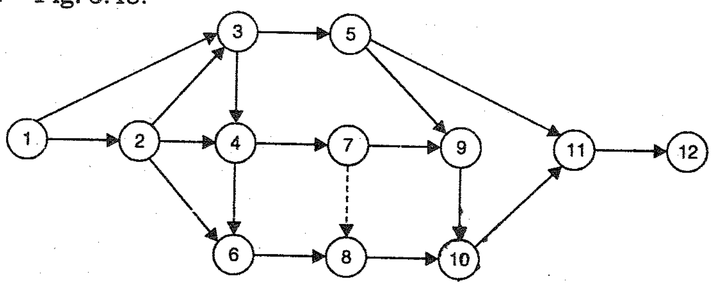


FIG. 3.48

13. Fig. 3.49.

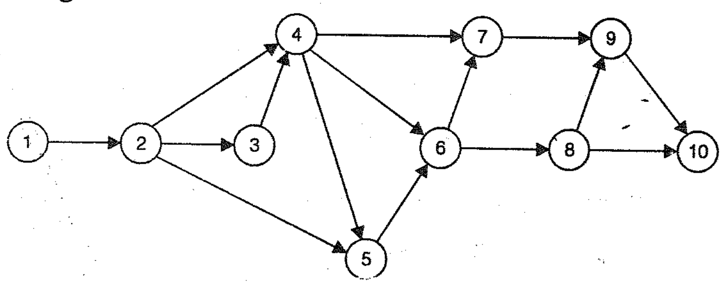


FIG. 3.49

14. Network shown in Fig. 3.50.

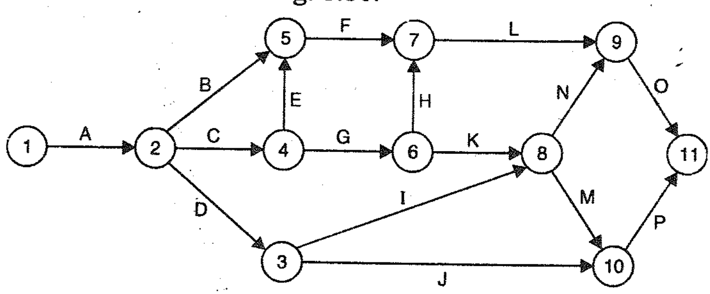


FIG. 3.50

15. Network shown in Fig. 3.51.

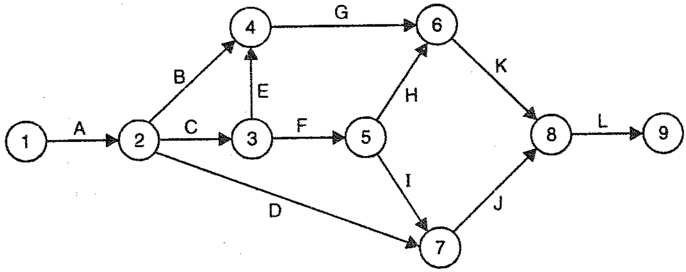


FIG. 3.51

16. Network shown in Fig. 3.52.

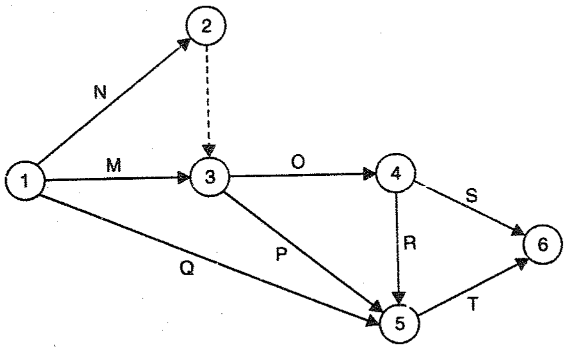


FIG. 3.52

17. Network shown in Fig. 3.53.

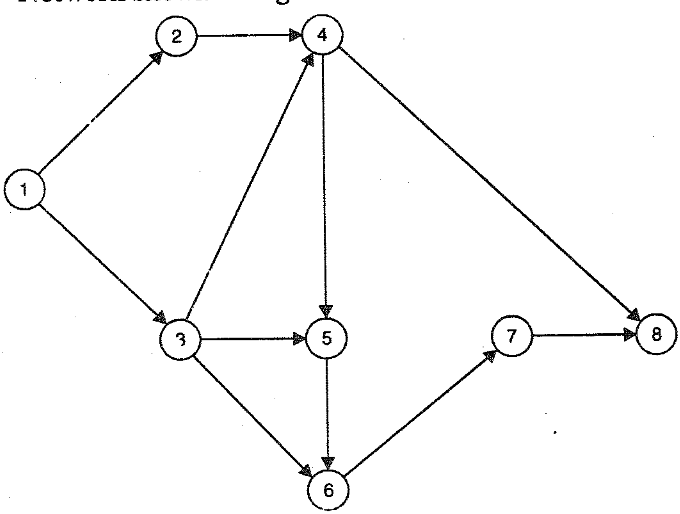


FIG. 3.53

CHAPTER 4

6. Network shown in Fig. 4.13.

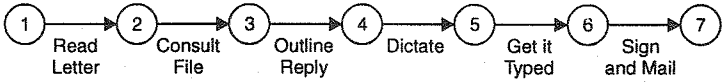


FIG. 4.13

7. Network shown in Fig. 4.14.

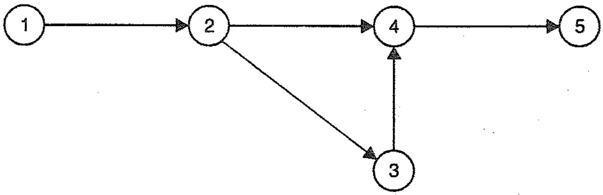


FIG. 4.14

Activity No.	*Description*
1—2	Forecasting and assessment of unit sales
2—4	Pricing sales
2—3	Preparing production schedule
3—4	Costing the production
4—5	Preparing the budget.

8. Network shown in Fig. 4.15.

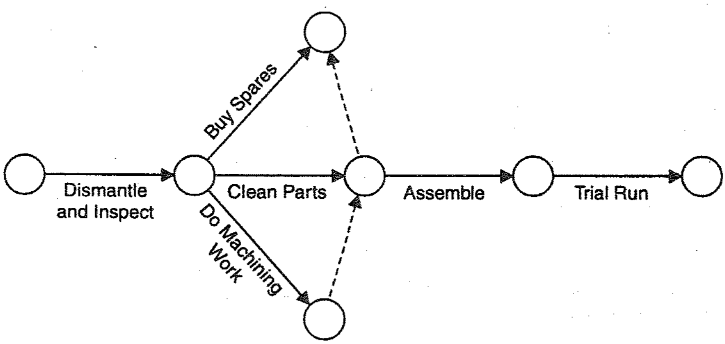


FIG. 4.15

CHAPTER 5

7.

Activity	t_E	σ^2
A	12	0.444
B	8.5	1.35
C	9.5	0.694

Activity *A* has more reliable time estimate.

8. $\Sigma t_E = 20.0$; $\sigma = 1.93$.

9.

Path	*Activities*	Σt_E
A	1—2—3—5—7	26
B	1—2—4—5—7	27
C	1—2—3—6—7	27
D	1—3—5—7	19
E	1—3—6—7	20

Critical paths : (*i*) 1—2—4—5—7 and
(*ii*) 1—2—3—6—7

CHAPTER 6

4. T_E and T_L as shown in Table 6.4 (shown on page 242).

5. (*a*) Expected time of completion for each activity as under :

Activity	T_E	*Activity*	T_E
1—2	10	5—6	3
1—3	9	5—7	3
2—4	9	5—10	12
2—6	7	6—8	6
3—4	8	7—9	10
3—7	5	8—10	9
4—5	11	9—10	8

(*b*) T_E and T_L as tabulated in Table 6.5 shown on page 243.

Table 6.4
(Problem 4)

Event No.	Earliest event time (↓)				Latest occurrence time (↑)			
	Predecessor event (*i*)	t_E^{ij}	T_E^j	T_E	Successor event (*j*)	t_E^{ij}	T_L^i	T_L
1	—			0	2 3	5 6	4 $\underline{0}$	0
2	1	5	$\underline{5}$	5	5 7	4 6	$\underline{9}$ 18	9
3	1	6	$\underline{6}$	6	4	2	$\underline{6}$	6
4	3	2	$\underline{8}$	8	5 6 8	5 7 6	$\underline{8}$ 14 19	8
5	2 4	4 5	9 $\underline{13}$	13	6	8	$\underline{13}$	13
6	4 5	7 8	15 $\underline{21}$	21	7 9	3 3	$\underline{21}$ 26	21
7	2 6	6 3	11 $\underline{24}$	24	9	5	$\underline{24}$	24
8	4	6	$\underline{14}$	14	9	4	$\underline{25}$	25
9	6 7 8	3 5 4	24 $\underline{29}$ 18	29				29

Table 6.5
(Problem 5)

Event No.	Earliest event time (↓)				Latest event time (↑)			
	Predecessor event (i)	t_E^{ij}	T_E^i	t_E	Successor event (j)	t_E^{ij}	T_L^i	T_L
1	—			0	2 3	10 9	$\underline{0}$ 2	0
2	1	10	$\underline{10}$	10	4 6	9 7	$\underline{10}$ 23	10
3	1	9	$\underline{9}$	9	4 7	8 5	$\underline{11}$ 28	11
4	2 3	9 8	$\underline{19}$ 17	19	5	11	$\underline{19}$	19
5	4	11	$\underline{30}$	30	6 7 10	3 3 12	33 $\underline{30}$ 39	30
6		7 3	17 $\underline{33}$	33	8	6	36	36
7	5	5 3	14 $\underline{33}$	33	9	10	$\underline{33}$	33
8	6	6	$\underline{39}$	39	10	9	$\underline{42}$	42
9	7	10	$\underline{43}$	43	10	8	$\underline{43}$	43
10	6 8 9	12 9 8	42 48 $\underline{51}$	51	—			51

CHAPTER 7

2. Results as tabulated in Table 7.11.

Table 7.11

Event No.	Earliest Event Time (↓)				Latest Event Time (↑)				Slack S
	Predecessor event (i)	t_E^{ij}	T_E^i	T_E	Successor event (j)	t_E^{ij}	T_L^i	T_L	
1	—	—	—	0	2	5	−2	−2	−2
2	1	5	<u>5</u>	5	3 4	8 9	<u>3</u> 5	3	−2
3	2	8	<u>13</u>	13	4 7	3 12	<u>11</u> 19	11	−2
4	2 3	9 3	14 <u>16</u>	16	5 7	4 12	<u>14</u> 19	14	−2
5	4	4	<u>20</u>	20	6	8	<u>18</u>	18	−2
6	5	8	<u>28</u>	28	7 8	2 10	29 <u>26</u>	26	−2
7	3 4 6	12 12 2	25 28 <u>30</u>	30	8	5	<u>31</u>	31	+1
8	6 7	10 5	<u>38</u> 35	<u>38</u>	—	—	36	36	−2

3. Critical path as shown in Fig. 7.19.
Critical path 1—2—3—4—5—6—8.

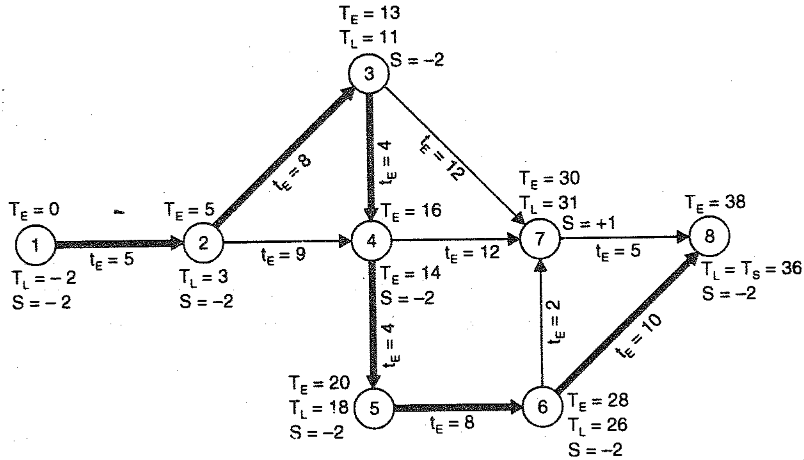


FIG. 7.19

5. (*a*) 15.9%

(*b*) 30.9%

(*c*) 93.3%.

6. T_S = 66 days.

7. (*a*) t_B, T_E, T_L and S as marked in Fig. 7.20.

Critical path 1—2—3—4—6—7—8—11

σ along critical path = 4.07

(*b*) P_r = 59.7

(*c*) T_S = 43.7 weeks.

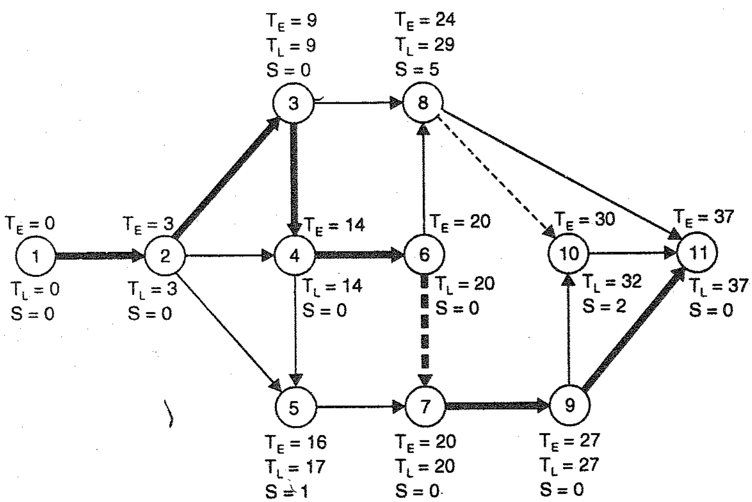


FIG. 7.20

CHAPTER 8

10. (*a*) T_E and T_L as marked in Fig. 8.25

(*b*) and (*c*) ; Table 8.11

(*d*) Critical path marked in Fig. 8.25.

Table 8.11

Activity	*Duration*	*Early*		*Late*		*Float*	
		Start time EST	*Finish time EFT*	*Start time LST*	*Finish time LFT*	*Total (F_T)*	*Free (F_F)*
1—2	5	0	5	12	17	12	0
1—3	8	0	8	0	8	0	0
1—4	3	0	3	5	8	5	5
2—5	4	5	9	17	21	12	3
3—4	0	8	8	8	8	0	0
3—7	7	8	15	14	21	6	6
4—5	4	8	12	17	21	9	0
4—6	6	8	14	8	14	0	0
5—8	6	12	18	21	27	9	9
6—7	7	14	21	14	21	0	0
6—8	2	14	16	25	27	11	11
7—8	6	21	27	21	27	0	0

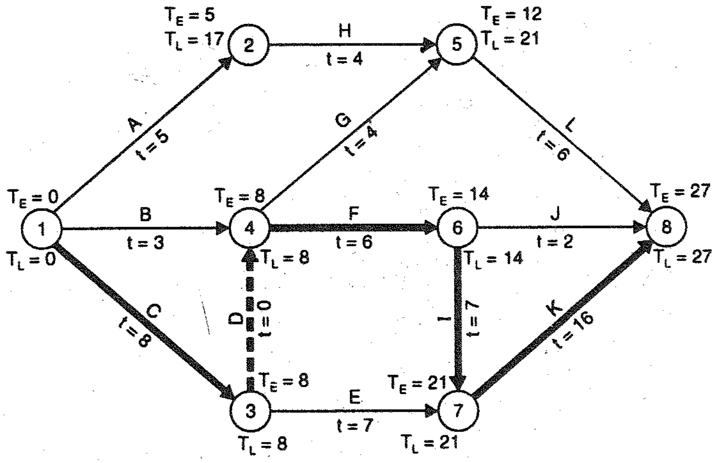


FIG. 8.25

11. Table 8.12.

Table 8.12

Activity	*Duration*	*Early*		*Late*		*Float*		
		Start (EST)	*Finish (EFT)*	*Start (LST)*	*Finish (LFT)*	*Total (F_T)*	*Free (F_F)*	*Independent (F_{ID})*
1—2	3	0	3	8	11	8	0	0
1—3	4	0	4	6	10	6	0	0
1—4	14	0	14	0	14	0	0	0
2—6	5	3	8	11	16	8	8	0
3—4	3	4	7	11	14	7	7	1
3—5	4	4	8	11	15	7	7	1
3—6	6	4	10	10	16	6	6	0
4—5	1	14	15	14	15	0	0	0
5—6	1	15	16	15	16	0	0	0

Critical path as marked in Fig. 8.26.

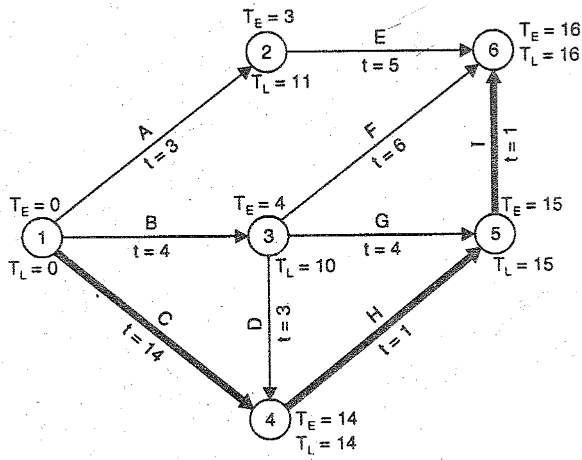


FIG. 8.26

CHAPTER 9

6. Time Scaled version of the network, at various stages of crashing is shown in Fig. 9.24 on page 249. The variation of cost with various durations of the project is summarised in Table 9.9.

Table 9.9

Duration weeks	15	13	12	11	9	8
Direct cost	30000	32000	34500	37500	45500	52000
Indirect cost	45000	39000	36000	33000	27000	24000
Total cost	75000	71000	70500	70500	72500	76000

Optimum duration = 11 weeks

Minimum cost = Rs. 70500

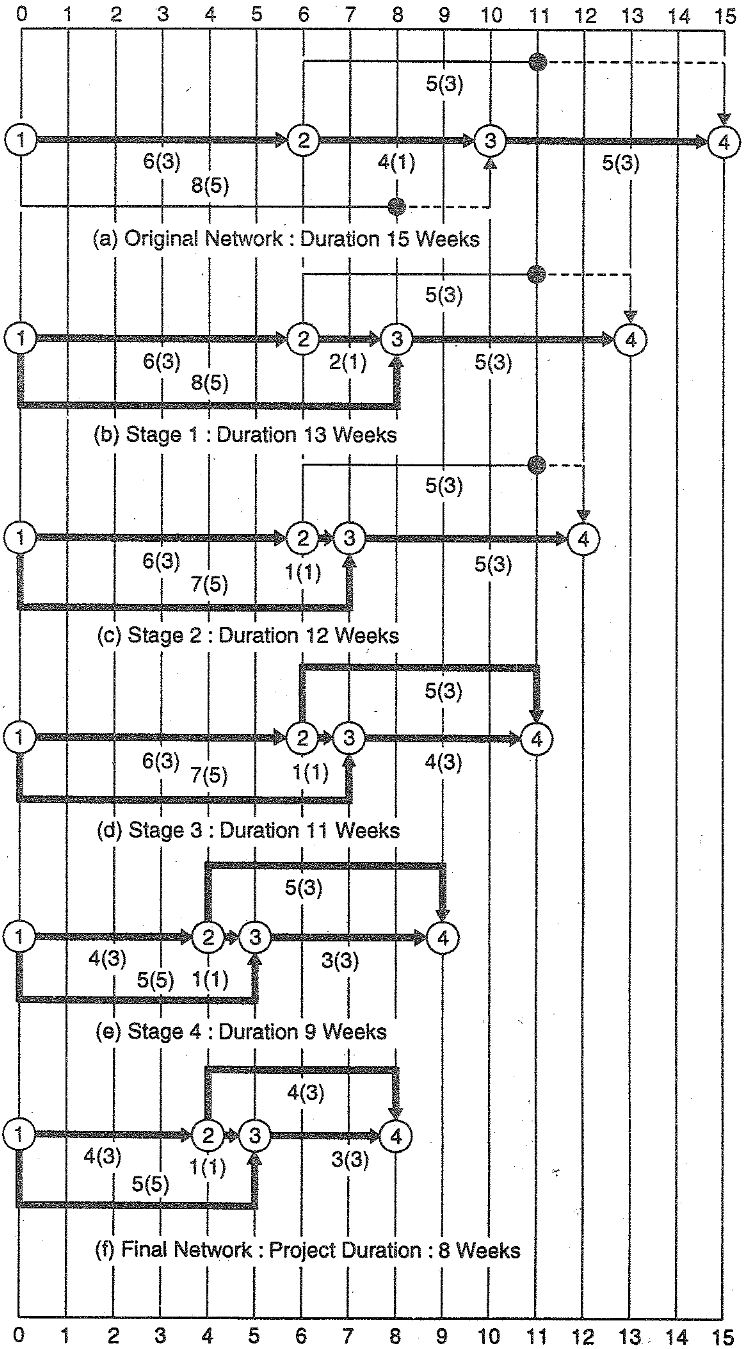

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
5(3)
6(3)
8(5)
4(1)
5(3)
(a) Original Network : Duration 15 Weeks
5(3)
6(3)
8(5)
2(1)
5(3)
(b) Stage 1 : Duration 13 Weeks
5(3)
6(3)
7(5)
1(1)
5(3)
(c) Stage 2 : Duration 12 Weeks
5(3)
6(3)
7(5)
1(1)
4(3)
(d) Stage 3 : Duration 11 Weeks
5(3)
4(3)
5(5)
1(1)
3(3)
(e) Stage 4 : Duration 9 Weeks
4(3)
4(3)
1(1)
5(5)
3(3)
(f) Final Network : Project Duration : 8 Weeks
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

FIG. 9.24

CHAPTER 10

3. Updated network shown in Fig. 10.5.
Increase in project duration = 1 day.

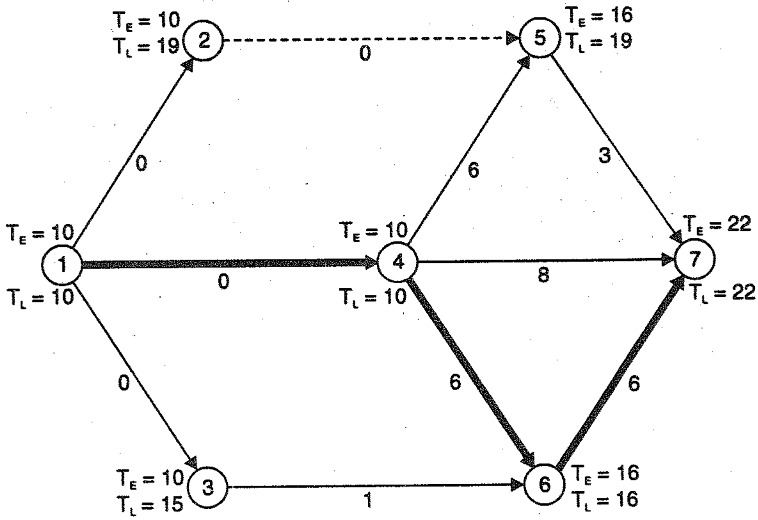


FIG. 10.5

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