৫০তম বিসিএস প্রিলি

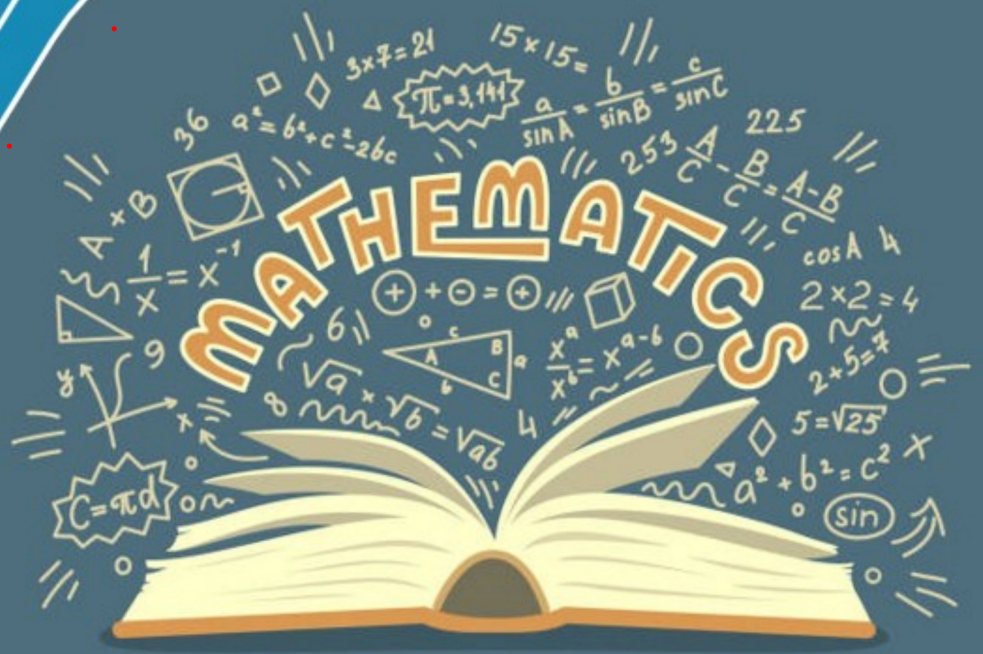
Pioneer Batch

গাণিতিক যুক্তি

লেকচার: ০৬

টপিক:

- সূচক ও লগারিদম



উত্তরণ
ক্যারিয়ার এন্ড স্কিলস একাডেমি

সূচক

ক্রমিক	সূচক সম্পর্কিত সূত্রাবলি	উদাহরণ
০১	$a^m \times a^n = a^{m+n}$, $a^m \times a^n \times a^p = a^{m+n+p}$ $a^m \times a^{-n} = a^{m+(-n)} = a^{m-n}\ ;\ a \neq 0$	$x^2 \times x^3 = x^{2+3} = x^5$ $x^2 \times x^3 \times x^4 = x^{2+3+4} = x^9$ $x^6 \times x^{-4} = x^{6+(-4)} = x^2$
০২	$a^m \div a^n = a^{m-n}$	$x^{10} \div x^3 = x^{10-3} = x^7$
০৩	$a^0 = 1,\ a \neq 0$	$9^0 = 1;\ (1000909)^0 = 1;\ (0.1)^0 = 1$
০৪	$(a^m)^n = a^{mn} = (a^n)^m$	$(x^2)^5 = (x^5)^2 = x^{5\times2} = x^{10}$
০৫	$\sqrt[n]{a^m} = a^{\frac{m}{n}}$	$\sqrt[3]{x^2} = x^{\frac{2}{3}};\ \sqrt{p^8} = p^{\frac{8}{2}} = p^4$

i) Base

ii) Power

x^y → power (ঘাত)

→ base (ভিত্তি)

x^2 / x^3

y^2 / y^3

$81 \rightarrow 3^4$

$\rightarrow 9^2$

$\sqrt[n]{a^m} = (a^m)^{1/n} = a^{\frac{m}{n}}$

$\sqrt[2]{x} = (x)^{1/2}$

$\sqrt[3]{x} = (x)^{1/3}$

ক্রমিক	সূচক সম্পর্কিত সূত্রাবলি	উদাহরণ
০৬	$a^{-m} = \frac{1}{a^m}$,	$a^{-2} = \frac{1}{a^2}$
	$\left(\frac{a}{b}\right)^{-m} = \frac{1}{\left(\frac{a}{b}\right)^m} = \left(\frac{b}{a}\right)^m ; a, b \neq 0$	$\left(\frac{2}{5}\right)^{-3} = \frac{1}{\left(\frac{2}{5}\right)^3} = \left(\frac{5}{2}\right)^3$
০৭	$(ab)^m = a^m b^m$	$(ab)^5 = a^5 . b^5$
	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} ; b \neq 0$	$\left(\frac{a}{b}\right)^5 = \frac{a^5}{b^5}$
০৮	$a^x = a^y$ হলে, $x = y$	$a^m = a^n$ হলে, $m = n$
	$a^x = b^x$ হলে, $a = b$	$p^3 = q^3$ হলে, $p = q$

(i)

$a^x = a^y$ → base → সমান, ঘাত → অবশ্যই

⇒ $x = y$

(ii)

$a^x = b^x$ → base → ভিন্ন, ঘাত → সমান

⇒ $a = b$

➲ $\frac{1}{2} \times 2^{x-3} + 1 = 5$ হলে x এর মান কত? [৪৬তম বিসিএস]

(ক) 3 (খ) 4 (গ) 5 (ঘ) 6

$\Rightarrow \frac{1}{2} \times 2^{x-3} = 5 - 1 = 4$

$\Rightarrow 2^{x-3} = 8 \Rightarrow 2^{x-3} = 2^3$

$\Rightarrow x - 3 = 3$

$\therefore x = 6$

➲ $2^{x+7} = 4^{x+2}$ হলে x এর মান কত? [৪৫তম বিসিএস]

(ক) 2 (খ) 3 (গ) 4 (ঘ) 6

$\Rightarrow 2^{x+7} = 4^{x+2}$

$\Rightarrow 2^{x+7} = (2^2)^{x+2}$

$\Rightarrow 2^{x+7} = 2^{2x+4}$

$\therefore x+7 = 2x+4$

$\therefore x = 3$

➲ যদি $\sqrt[4]{x^3} = 2$ হয়, তাহলে $x^{\frac{3}{2}} = ?$ [৪৪তম বিসিএস]

(ক) 8 (খ) 16 (গ) 4 (ঘ) 6

$\frac{3}{4} \times 2 = \frac{3}{2}$

$\Rightarrow \sqrt[4]{x^3} = 2$

$\Rightarrow (x^3)^{1/4} = 2$

$\Rightarrow x^{3/4} = 2$

$\Rightarrow (x^{3/4})^2 = (2)^2$

$\Rightarrow x^{3/2} = 4$

❑ $\sqrt[4]{x} = 0.1$ হলে, x = কত?

(a) 0.1

(b) 0.01

(c) 0.001

(d) 0.0001

$(x)^{1/4} = 0.1$

$\Rightarrow (x^{1/4})^4 = (0.1)^4$

$\therefore x = (0.1)^4$

$= \left(\frac{1}{10}\right)^4 = \frac{1}{10000}$

➲ $\left[\left\{1-\left(1-\frac{1}{p}\right)\right\}^{-1} \div \left(1-\frac{1}{p}\right)^{-1}\right] =$ কত?

(ক) 1 (খ) -1 (গ) $\frac{1}{p}$ (ঘ) $(p-1)$

$\Rightarrow \left[\left\{1-\left(1-\frac{1}{p}\right)\right\}^{-1} \div \left(1-\frac{1}{p}\right)^{-1}\right]$

$\Rightarrow \left[\left\{1-1+\frac{1}{p}\right\}^{-1} \div \left(\frac{p-1}{p}\right)^{-1}\right]$

$\Rightarrow \left[\left(\frac{1}{p}\right)^{-1} \div \left(\frac{p-1}{p}\right)^{-1}\right]$

$\Rightarrow \left[p \div \frac{p}{p-1}\right]$

$\Rightarrow p \times \frac{p-1}{p}$

$= p-1$

➲ $\frac{5^{n+2}+35\times5^{n-1}}{4\times5^n}$ এর মান কত? [৩৪তম বিসিএস]

(a) 4 (b) 8 (c) 5 (d) 7

$5^{n+2} = 5^n \times 5^2$

$(a)^m \times (a)^n = a^{m+n}$

$\Rightarrow \frac{5^{n+2}+35\times5^{n-1}}{4\times5^n}$

$5^n \times 5^{-1}$

$\Rightarrow \frac{25\times5^n+35\times5^n\times5^{-1}}{4\times5^n}$

$\frac{1}{5}$

$\Rightarrow \frac{25\times5^n+7\times5^n}{4\times5^n}$

$\Rightarrow \frac{5^n(25+7)}{4\times5^n}$

$\Rightarrow \frac{32}{4} = 8$

Easy

➲ $\frac{3^{m+1}}{(3^m)^{m-1}} \div \frac{9^{m+1}}{(3^{m-1})^{m+1}}$ এর মান কত?

(ক) 0 (খ) 1 (গ) $\frac{1}{9}$ (ঘ) $\frac{1}{3}$

$(3^2)^{m+1} = 3^{2m+2}$

3^m 3m

$\Rightarrow \left(\frac{3^{m+1}}{3^{m^2-m}}\right) \div \left(\frac{3^{2m+2}}{3^{m^2-1}}\right)$

$\Rightarrow 3^{m+1-m^2+m} \div 3^{2m+2-m^2+1}$

$\Rightarrow \left(3^{2m-m^2+1}\right) \div \left(3^{2m-m^2+3}\right)$

$\Rightarrow 3^{2m-m^2+1-2m+m^2-3}$

$\Rightarrow 3^{-2}$

$= \frac{1}{3^2} = \frac{1}{9}$

➲ $\sqrt{x^{-1}.y}\sqrt{y^{-1}.z}\sqrt{z^{-1}.x}$ এর মান কত?

(ক) 0 (খ) 1 (গ) xyz (ঘ) $\sqrt{xyz}$

$\Rightarrow \sqrt{x^{-1}.y}\ \sqrt{y^{-1}.z}\ \sqrt{z^{-1}.x}$

$\Rightarrow \sqrt{\frac{y}{x}} \times \sqrt{\frac{z}{y}} \times \sqrt{\frac{x}{z}}$

$\Rightarrow \sqrt{\frac{y}{x} \times \frac{z}{y} \times \frac{x}{z}} = 1$

৩ $x^{x\sqrt{x}} = (x\sqrt{x})^{x}$ হলে, x এর মান কত? [৪০তম বিসিএস]

(ক) $\frac{3}{2}$ (খ) $\frac{4}{5}$ (গ) $\frac{9}{4}$ (ঘ) $\frac{2}{3}$

2 min

$\Rightarrow x^{x\sqrt{x}} = (x\sqrt{x})^{x}$

$\Rightarrow (x^{x})^{\sqrt{x}} = (x^{1}\cdot x^{1/2})^{x}$

$\Rightarrow (x^{x})^{\sqrt{x}} = (x^{1+\frac{1}{2}})^{x}$

$x^{x}(\sqrt{x})^{x}$

$\Rightarrow \frac{(x^{x})^{\sqrt{x}}}{(x^{x})^{\sqrt{x}}} = \frac{(x^{3/2})^{x}}{(x^{x})^{3/2}}$

$\Rightarrow \sqrt{x} = 3/2$

$\therefore x = 9/4$

Class 8, 9, 10

মাধ্যমিক দক্ষতা

[৩৯তম বিসিএস]

২. $125(\sqrt{5})^{2x} = 1$ হলে x এর মান কত?

(ক) 3 (খ) −3 (গ) 7 (ঘ) 9

$(\quad)^0 = 1$

$\Rightarrow 125(\sqrt{5})^{2x} = 1$

$125 = 5^3$, $\sqrt{5} = 5^{1/2}$

$\Rightarrow 5^3(\sqrt{5})^{2x} = 1$

$\Rightarrow 5^3 \times (5^{1/2})^{2x} = 1$

$\Rightarrow 5^3 \times 5^x = 1$

$\Rightarrow 5^{x+3} = 5^0$

$\Rightarrow x+3 = 0$

$\Rightarrow x = -3$

➲ $3^{mx-1} = 3a^{mx-2}$ হলে, x এর মান কত?

(a) $\frac{2}{m}$ (b) $2m$ (c) $\frac{m}{2}$ (d) 1

$\Rightarrow 3^{mx-1} = 3 \cdot a^{mx-2}$

$\Rightarrow \frac{3^{mx-1}}{3^1} = a^{mx-2}$

$\Rightarrow 3^{mx-1-1} = a^{mx-2}$

$\Rightarrow 3^{mx-2} = a^{mx-2}$

$\Rightarrow \frac{3^{mx-2}}{a^{mx-2}} = 1$

$\Rightarrow \left(\frac{3}{a}\right)^{mx-2} = 1 = \left(\frac{3}{a}\right)^0$

$a = 3$

$mx - 2 = 0$

$\therefore x = \frac{2}{m}$

❑ $xy=yx$ এবং $x=2y$ $(x\neq0, y\neq0)$ হলে, (x, y) = কত?

(a) (8, 4)

(b) (6, 3)

(c) (2, 1)

(d) (4, 2)

$x^y = y^x$

$x=2y$

$(x,y)=?$

$x=2\times0=0$

$x=2\times2=4$

$(x,y)=(0,0),(4,2)$

$x^y = y^x$

$\Rightarrow \{(x)^y\}^{1/y} = (y^x)^{1/y}$

$\Rightarrow x = y^{\frac{x}{y}}$

$\Rightarrow x = y^{\frac{2y}{y}}$

$\Rightarrow 2y = y$

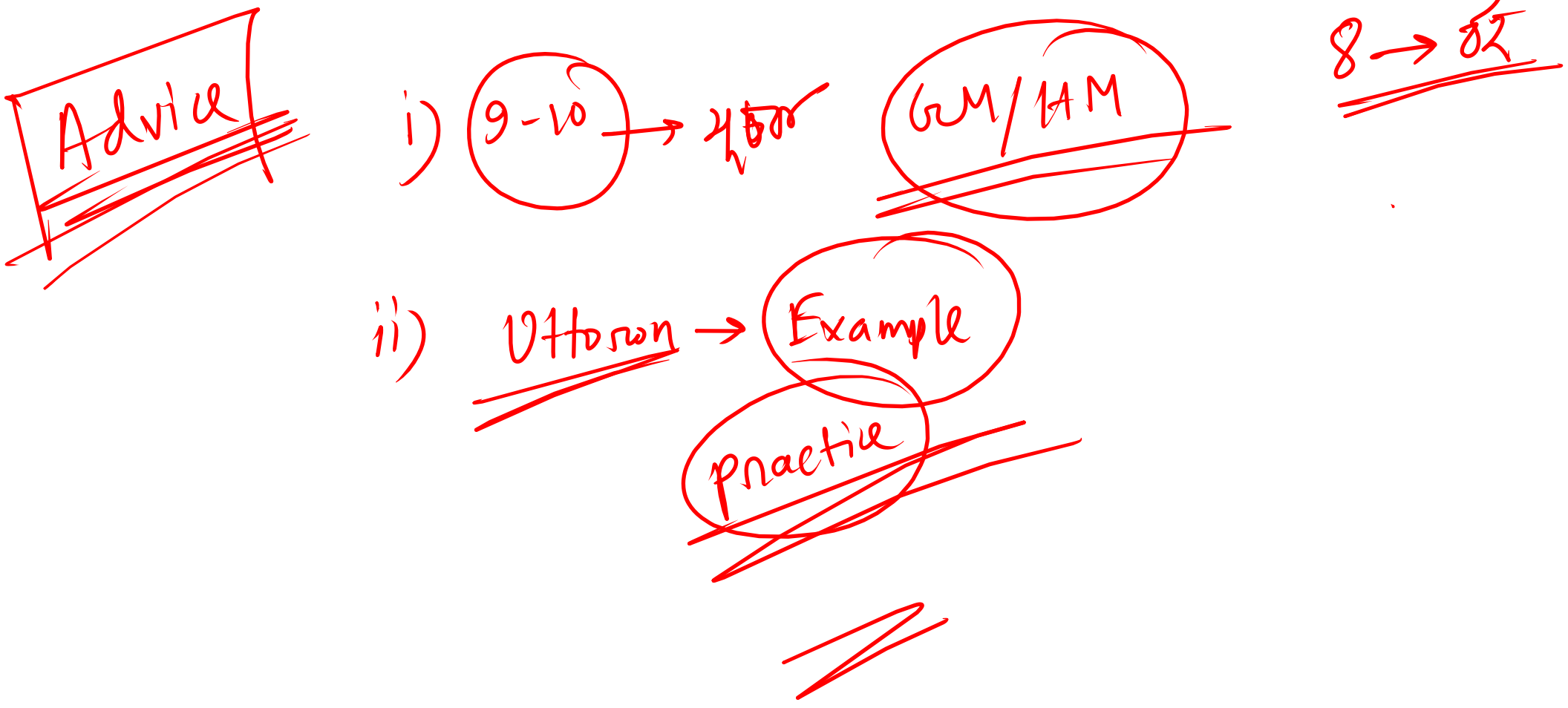
$\Rightarrow 2y = y^2 \Rightarrow y^2-2y=0$

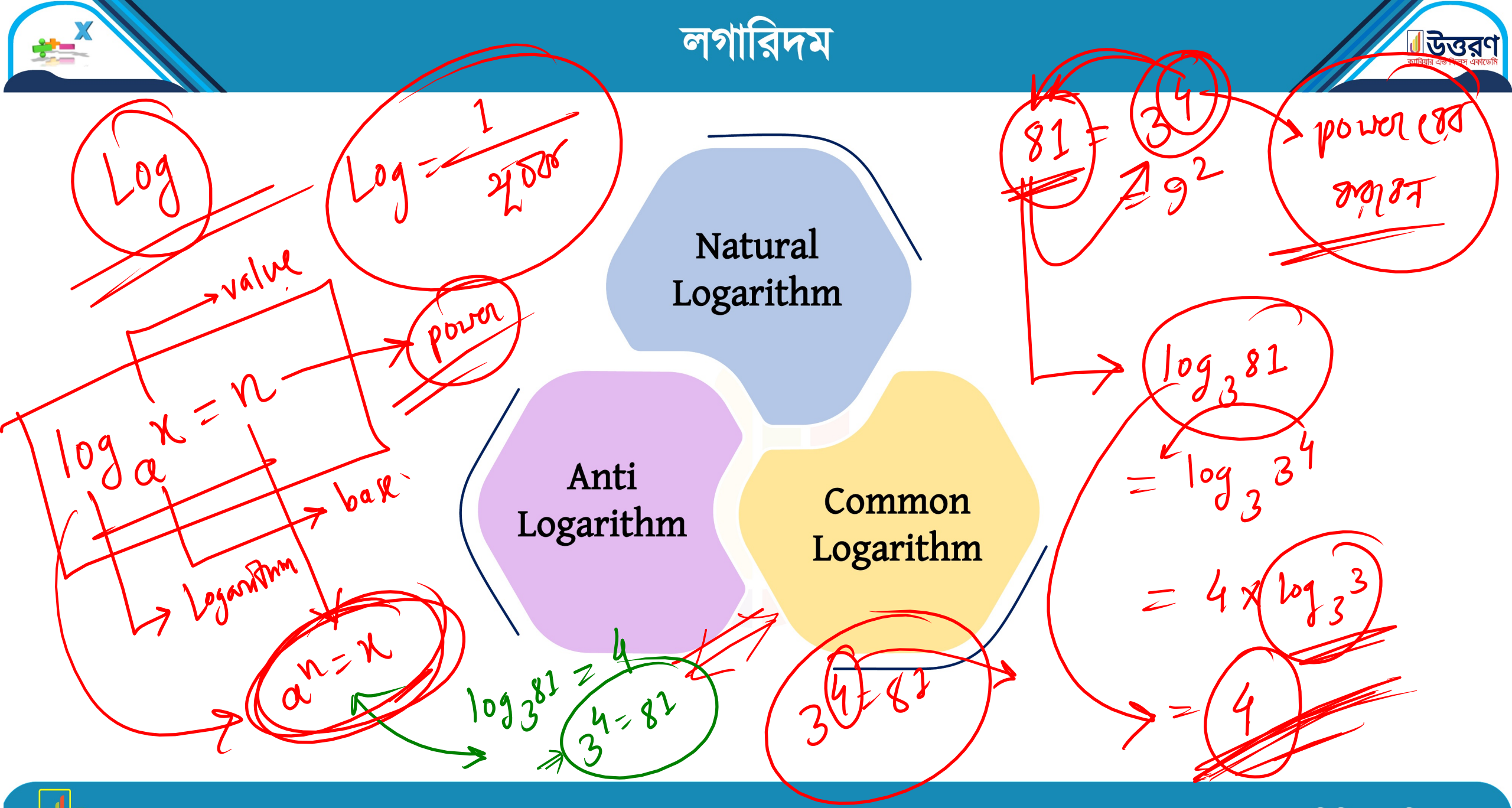
$\therefore y(y-2)=0$

$y-2=0$

$y=2$

$y\neq0$


Advice
i) 9-10 → 4 hour
GM/AM
8 → 12
ii) Utorrent → Example
practice


Log
Log = 1
power
value
$\log_a x = n$
base
Logarithm
$a^n = x$
Natural Logarithm
Anti Logarithm
Common Logarithm
$\log_3 81 = 4$
$3^4 = 81$
$3^4 = 81$
$81 = 3^4$
$= 9^2$
power এর সমান
$\log_3 81$
$= \log_3 3^4$
$= 4 \times \log_3 3$
$= 4$

$\log_{10}$ $\log_5$

Natural Logarithm

$\log e$ ⇒ natural

$x = x^{-1}$ বিপরীত ধর্ম

$\pi = 3.1416$

$\log_e$

$\ln$

$\ln = \log_e$

স্বাভাবিক লগ

Common

$\log_{10}$

$\log_{10} a$

$e = 2.7182818 \ldots$

Anti-log

$m = \log_{10} 5$

$\log a$

$\log_{10} a$

⇒ Anti log m = Anti $\log_{10} 5$

$m = \log_{10} 5$

base = 10

$\Rightarrow 10^{m} = 10^{\log_{10} 5} = 5$

$\therefore 10^{m} = 5$

{ Antilog করা মানে হল log এর যে base থাকে এখানে সেই base টাকে অন্যপাশে power হিসেবে থাকবে }

$a^{\log_a b} = b$

$\therefore$ ধরি, $\log_a b = x$

$\Rightarrow a^{x} = b$

$\therefore a^{\log_a b} = b$

ক্রমিক	লগের সূত্রাবলি	উদাহরণ
০১	$\log_a(xyz) = \log_a x + \log_a y + \log_a z$	$\log_2(2 \times 3 \times 7) = \log_2 2 + \log_2 3 + \log_2 7$
০২	$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$	$\log_2 \frac{7}{5} = \log_2 7 - \log_2 5$
০৩	$\log_a x^m = m \log_a x$	$\log_a 10^7 = 7 \log_a 10$
০৪	$\log_a \sqrt[n]{x} = \frac{1}{n} \log_a x$	$\log_a \sqrt[3]{7} = \log_a 7^{\frac{1}{3}} = \frac{1}{3} \log_a 7$
০৫	$\log_a b \times \log_b a = 1$	$\log_{10} 7 \times \log_7 10 = 1$
০৬	$\log_a b = \frac{1}{\log_b a}$	$\log_{10} 11 = \frac{1}{\log_{11} 10}$

(*) $\log_a b \times \log_b a = 1$

$\log_{10} 10 = 1$

base = value

answer = 1

$\log_a a = 1$

$\log_a b \times \log_b a = 1$

$\therefore \log_a b = \frac{1}{\log_b a}$

ధరిస్తే, $\log_a b = x$

$\Rightarrow a^x = b$

$\Rightarrow \log_b a^x = \log_b b$

$\Rightarrow x \log_b a = 1$

$\Rightarrow \log_b a = \frac{1}{x}$

$\Rightarrow \log_b a = \frac{1}{\log_a b}$

লগারিদম

ক্রমিক	লগের সূত্রাবলি	উদাহরণ
০৭	$\log_a 1 = 0$	$\log_{10} 1 = 0, \log_{100} 1 = 0, \log_{99} 1 = 0$
০৮	$\log_a a = 1$	$\log_{10} 10 = 1, \log_{19} 19 = 1$
০৯	$\log_a m = x$ হলে, $a^x = m$	$\log_{10} 100 = 2$ হলে, $10^2 = 100$
১০	$a^{\log_a b} = b$	$2^{\log_2 20} = 20$
১১	$\log_a m = \log_b m \times \log_a b = \frac{\log_b m}{\log_b a}$	$\log_{10} 7 = \log_3 7 \times \log_{10} 3 = \frac{\log_3 7}{\log_3 10}$
১২	$\log_a b = \frac{\log_{10} b}{\log_{10} a} = \frac{1}{\log_b a}$	$\log_5 6 = \frac{\log_{10} 6}{\log_{10} 5} = \frac{1}{\log_6 5}$

(*) $\log_a b \times \log_b m = \log_a m$

ft), $\log_a b = x$

$\Rightarrow a^x = b$

$\log_b m = y$

$\Rightarrow b^y = m$

$\Rightarrow (a^x)^y = m$

$\Rightarrow a^{xy} = m$

$\Rightarrow \log_a (a^{xy}) = \log_a m$

$\Rightarrow xy \times \log_a a = \log_a m$

$\Rightarrow xy = \log_a m$

$\therefore \log_a b \times \log_b m = \log_a m$

❸ $\log_{\sqrt{8}} x = 3\frac{1}{3}$ হলে x এর মান কত? [৪৬তম বিসিএস]

(ক) 32 (খ) 8 (গ) 3 (ঘ) $\sqrt{8}$

$\log_{\sqrt{8}} x = 3\frac{1}{3}$

$\Rightarrow \log_{\sqrt{8}} x = \frac{10}{3}$

$\Rightarrow (\sqrt{8})^{10/3} = x$

$\Rightarrow x = (\sqrt{8})^{10/3}$

$= (8^{1/2})^{10/3}$

$= 8^{\frac{1}{2}\times\frac{10}{3}}$

$= 8^{5/3}$

$= (2^3)^{5/3}$

$2^5 = 32$

$b^0 x = 10$

➲ যদি $\log\left(\frac{a}{b}\right) + \log\left(\frac{b}{a}\right) = \log(a+b)$ হয়, তবে –

[৪৫তম বিসিএস]

(ক) $a + b = 1$ (খ) $a - b = 1$ (গ) $a = b$ (ঘ) $a^2 - b^2 = 1$

$\Rightarrow \log\left(\frac{a}{b}\right) + \log\left(\frac{b}{a}\right) = \log(a+b)$

$\Rightarrow \log\left(\frac{a}{b} \times \frac{b}{a}\right) = \log(a+b)$

$\Rightarrow \log 1 = \log(a+b)$

$\Rightarrow$

0

$a + b = 1$

$\log_{10} x = \log_{10} y$

$\Rightarrow x = y$

➲ যদি $\log_{10} x = -1$ হয়, তাহলে নিচের কোনটি x এর মান? [৪৪তম বিসিএস]

(ক) 0.1 (খ) 0.01 (গ) $\frac{1}{10000}$ (ঘ) 0.001

$\Rightarrow$

$10^{-1} = x$

$\therefore x = 0.1$

➲ $2^{\log_2 3+\log_2 5}$ এর মান কত? [৪৩তম বিসিএস]

(ক) 8 (খ) 2 (গ) 15 (ঘ) 10

$\Rightarrow 2^{(\log_2 3 + \log_2 5)}$

$\Rightarrow 2^{\log_2(5\times3)}$

$\Rightarrow 2^{\log_2 15} = 15$

$a^{\log_a b} = b$

➲ $\log_x \frac{1}{9} = -2$ হলে x এর মান কোনটি? [৪২তম বিসিএস]

(ক) 3 (খ) 2 (গ) $\frac{1}{3}$ (ঘ) $-\frac{1}{3}$

$\Rightarrow \log_x \frac{1}{9} = -2$

$\therefore x^{-2} = \frac{1}{9} \Rightarrow \frac{1}{x^2} = \frac{1}{9}$

$\therefore x^2 = 9 \quad x = \pm 3$

$\log_a x = n$

$\Rightarrow a^n = x$

❷ $\log_2 \log_{\sqrt{e}} e^2 = ?$ [৪১তম বিসিএস]

(ক) -2 (খ) -1 (গ) 1 (ঘ) 2

$\Rightarrow \log_2 \log_{\sqrt{e}} e^2 \Rightarrow \log_2 \{\log_{\sqrt{e}} (e^2)\}$

$\Rightarrow \log_2 \{4 \log_{\sqrt{e}} (\sqrt{e})^4\}$

$\Rightarrow \log_2 \{4 (\log_{\sqrt{e}} \sqrt{e})\}$

$= \log_2 4$

$= \log_2 2^2$

$= 2 \times \log_2 2 = 2 \times 1 = 2$

❷ কোন শর্তে $\log_a 1 = 0$?

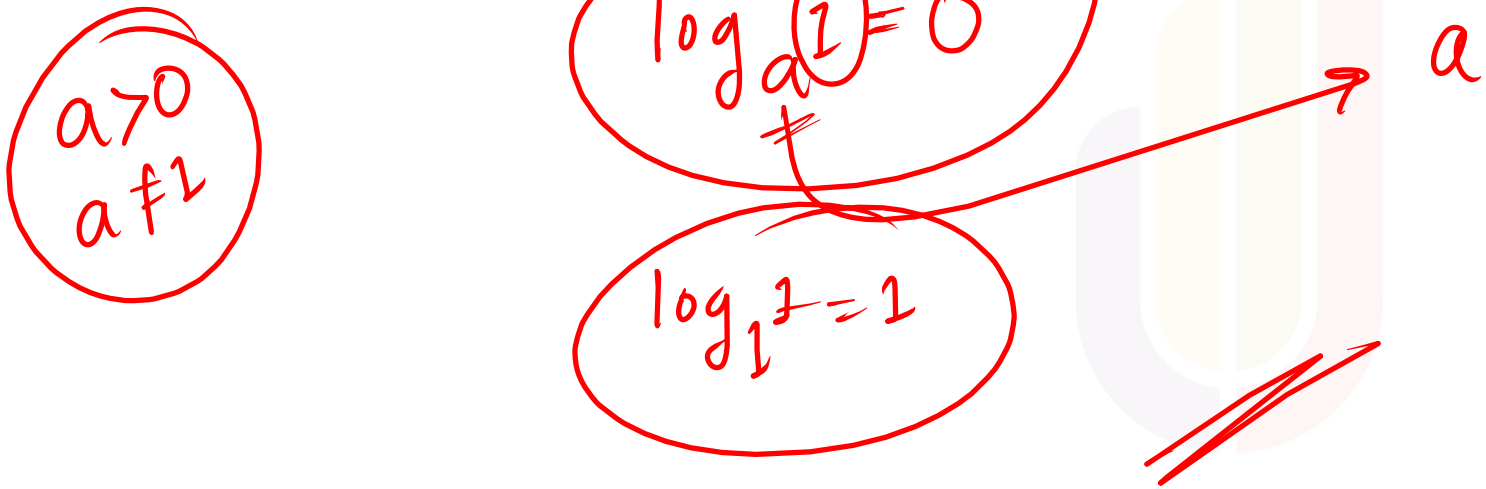
[৪০তম বিসিএস]

(ক) $a > 0, a \neq 1$

(খ) $a \neq 0, a > 1$

(গ) $a > 0, a = 1$

(ঘ) $a \neq 1, a < 0$



log base ধনাত্মক negative হতে পারবে না।

(*) $\log_x\left(\frac{1}{8}\right) = -2$ হলে, $x = ?$ (38th BCS)

$\Rightarrow \log_x\left(\frac{1}{8}\right) = -2$, $x = 2$.

$\Rightarrow x^{-2} = \frac{1}{8}$

$\therefore \frac{1}{x^2} = \frac{1}{8}$ $\therefore x^2 = 8$

$\therefore x = \sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$

(37th BCS)

(*) $\log_x\left(\frac{3}{2}\right) = -\frac{1}{2}$, $x = ?$

$\Rightarrow$ $x^{-1/2} = \frac{3}{2}$

$\Rightarrow$ $\frac{1}{\sqrt{x}} = \frac{3}{2}$

$\Rightarrow$ $\sqrt{x} = \frac{2}{3}$

$\therefore$ $x = \frac{4}{9}$

➲ $\log_{2.5} 6.25$ এর মান কোনটি?

(ক) 1 (খ) 3 (গ) 2 (ঘ) 4

$\Rightarrow \log_{2.5}(6.25) = ?$

$\Rightarrow \log_{2.5}(2.5)^2 = 2 \times 1$

$= 2$

➲ $\log_a p \times \log_p q \times \log_q r \times \log_r b = ?$

(ক) $\log_a b$ (খ) $\log_a p$ (গ) $pqrb$ (ঘ) $\log_r b$

$\Rightarrow \log_a q \times \log_q r \times \log_r b$

$\Rightarrow \log_a r \times \log_r b$

$\Rightarrow \log_a b$

$\log_a b \times \log_b m = \log_a m$

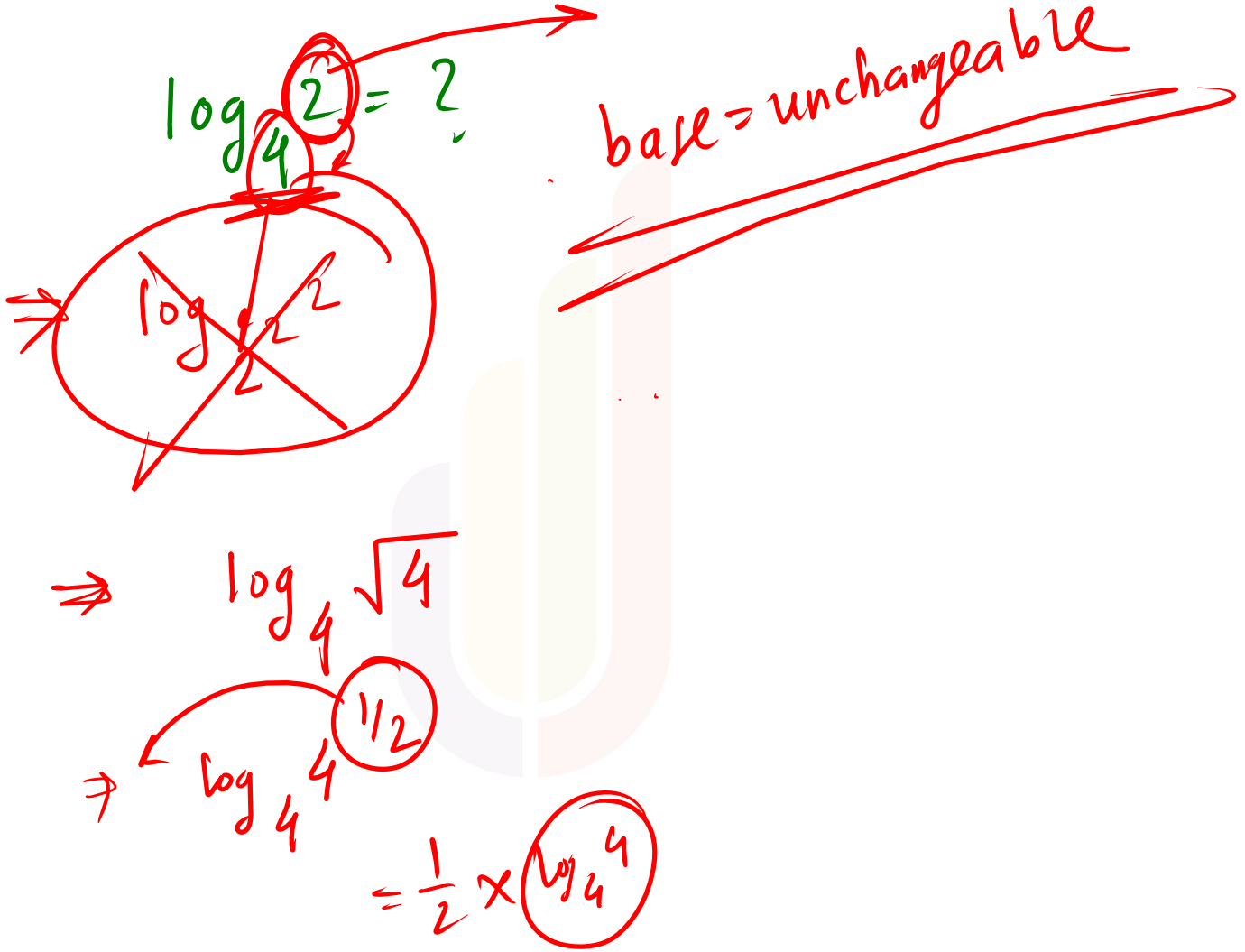
❑ $\log_4 2 =$ কত?

(a) -3

(b) 2

(c) $\frac{1}{2}$

(d) 3



Logarithm

i) অধ্যায়ের সমস্যা বের করে সমাধান করবো

ii) 9-10 → GM, HM → Class-8

iii) Uttoron → guide

10-11/12
15

3-P
practice
practice
practice

Best of Luck

Thank you

BCS কঠিন নয়;
প্রস্তুতি যদি গোছানো হয়