BCS Math written

LECTURE-1,2

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46th BCS written

Math 41+ 45

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বীজ গাণিতিক সূত্রাবলী

জ্ঞান বিষয়

① $a^2+b^2 = (a+b)^2 - 2ab$
$= (a-b)^2 + 2ab$

② $a^3+b^3 = (a+b)^3 - 3ab(a+b)$

③ $a^3-b^3 = (a-b)^3 + 3ab(a-b)$

④ $a^2-b^2 = (a+b)(a-b)$

⑤ $4ab = (a+b)^2 - (a-b)^2$

⑥ $2(a^2+b^2) = (a+b)^2 + (a-b)^2$

যদি $b^2 - 2\sqrt{6}b + 1 = 0$ হয়, তবে $b^5 + \frac{1}{b^5} = ?$ [45thBCS]

$\Rightarrow b^2 + 1 = 2\sqrt{6}b$

$\Rightarrow b + \frac{1}{b} = 2\sqrt{6}$

$\left(b^2 + \frac{1}{b^2}\right)\left(b^3 + \frac{1}{b^3}\right)$

$= b^5 + \frac{1}{b} + b + \frac{1}{b^5}$

$= \left(b^5 + \frac{1}{b^5}\right) + \left(b + \frac{1}{b}\right)$

$= \left(b^2 + \frac{1}{b^2}\right)\left(b^3 + \frac{1}{b^3}\right) - \left(b + \frac{1}{b}\right)$

$= \left\{\left(b + \frac{1}{b}\right)^2 - 2.b.\frac{1}{b}\right\}\left\{\left(b + \frac{1}{b}\right)^3 - 3.b.\frac{1}{b}\left(b + \frac{1}{b}\right)\right\} - \left(b + \frac{1}{b}\right)$

$= \left\{(2\sqrt{6})^2 - 2\right\}\left\{(2\sqrt{6})^3 - 3\times 2\sqrt{6}\right\} - 2\sqrt{6}$

$= 22 \times (48\sqrt{6} - 6\sqrt{6}) - 2\sqrt{6}$

$= 22 \times 42\sqrt{6} - 2\sqrt{6}$

$x + \frac{1}{x} = 3$ হলে, $x^9 + \frac{1}{x^9} = ?$

[43rd BCS]

$$x^9 + \frac{1}{x^9} = \left(x^6 + \frac{1}{x^6}\right)\left(x^3 + \frac{1}{x^3}\right) - \left(x^3 + \frac{1}{x^3}\right)$$

$$= \left\{(x^3)^2 + \left(\frac{1}{x^3}\right)^2\right\}\left(x^3 + \frac{1}{x^3}\right) - \left(x^3 + \frac{1}{x^3}\right)$$

$$= \left\{\left(x^3 + \frac{1}{x^3}\right)^2 - 2 \cdot x^3 \cdot \frac{1}{x^3}\right\}\left(x^3 + \frac{1}{x^3}\right) - \left(x^3 + \frac{1}{x^3}\right)$$

$$= \left(18^2 - 2\right) \times 18 - 18$$

$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3 \cdot x \cdot \frac{1}{x}\left(x + \frac{1}{x}\right)$$

$$= 3^3 - 3 \times 3$$

$$= 27 - 9$$

$$= 18$$

$$a^3 + b^3 = (a+b)^3 - 3ab(a+b)$$

$a = \sqrt{5} + \sqrt{3}$ হলে, $\frac{a^2+2}{2a} = ?$

[43rd BCS]

$a = \sqrt{5} + \sqrt{3}$

$\therefore \frac{1}{a} = \frac{1}{\sqrt{5}+\sqrt{3}}$

$= \frac{(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$

$= \frac{(\sqrt{5}-\sqrt{3})}{5-3}$

$= \frac{\sqrt{5}-\sqrt{3}}{2}$

$a^2 - b^2 = (a+b)(a-b)$

$\frac{a^2+1}{2a} = \frac{a}{2} + \frac{1}{a}$

$= \frac{\sqrt{5}+\sqrt{3}}{2} + \frac{\sqrt{5}-\sqrt{3}}{2}$

$= \frac{\sqrt{5}+\sqrt{3}+\sqrt{5}-\sqrt{3}}{2}$

$= \frac{2\sqrt{5}}{2}$

$= \sqrt{5}^2$

$x + \frac{1}{x} = 5$ হলে, $x^4 + \frac{1}{x^4} = ?$ [41st BCS]

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$x^2 + \frac{1}{x^2} = 7$ হয়, তবে $\frac{x^6+1}{x^3}$=? [40th BCS]

$$x^2 + \frac{1}{x^2} = 7 \longrightarrow \frac{x^6+1}{x^3}$$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2 - 2 \cdot x \cdot \frac{1}{x} = 7$$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2 = 9$$

$$\therefore \boxed{x+\frac{1}{x} = \pm 3}$$

$$= x^3 + \frac{1}{x^3}$$

$$= \left(x+\frac{1}{x}\right)^3 - 3 \cdot x \cdot \frac{1}{x}\left(x+\frac{1}{x}\right)$$

$$= (\pm 3)^3 - 3 \times (\pm 3)$$

$$= \pm 27 \pm 9$$

$$= \pm 18$$

$x - \frac{1}{x} = \sqrt{3}$ হলে, $x^6 + \frac{1}{x^6} = ?$ [38th BCS]

$$x^6 + \frac{1}{x^6} = (x^3)^2 + \left(\frac{1}{x^3}\right)^2$$

$$= \left\{\left(x^3 + \frac{1}{x^3}\right)^2 - 2 \cdot x^3 \cdot \frac{1}{x^3}\right\}$$

$$= (4\sqrt{7})^2 - 2$$

$$= 16 \times 7 - 2 = 110$$

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$$x^3 + \frac{1}{x^3}$$

$$= \left(x + \frac{1}{x}\right)^3 - 3 \cdot x \cdot \frac{1}{x}\left(x + \frac{1}{x}\right)$$

$$= (\sqrt{7})^3 - 3\sqrt{7}$$

$$= 7\sqrt{7} - 3\sqrt{7} = 4\sqrt{7}$$

$x - \frac{1}{x} = \sqrt{3}$ ⟶ $\boxed{x + \frac{1}{x} = ?}$

$\therefore \left(x + \frac{1}{x}\right)^2 = \left(x - \frac{1}{x}\right)^2 + 4 \cdot x \cdot \frac{1}{x}$

$= \left(x - \frac{1}{x}\right)^2 + 4$

$= \left(\sqrt{3}\right)^2 + 4$

$= 7$

$\rightarrow (a+b)^2 = (a-b)^2 + 4ab$

$\rightarrow (a-b)^2 = (a+b)^2 - 4ab$

$2x^2 - 3x = 2$ হলে, $x^3 - \frac{1}{x^3} = ?$ [37th BCS]

$2x^2 - 2 = 3x$

$\Rightarrow \frac{2x^2 - 2}{x} = 3$

$\Rightarrow 2x - \frac{2}{x} = 3$

$\Rightarrow 2\left(x - \frac{1}{x}\right) = 3$

$x - \frac{1}{x} = \frac{3}{2}$

$a^3 - b^3$

$= (a-b)^3 + 3ab(a-b)$

$x + \frac{1}{x} = 3$ হলে, $x^4 + x^3 + x^2 + \frac{1}{x^2} + \frac{1}{x^3} + \frac{1}{x^4} = ?$ [36th BCS]

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$x + \frac{1}{x} = 3$

$$x^4 + x^3 + x^2 + \frac{1}{x^2} + \frac{1}{x^3} + \frac{1}{x^4}$$

$$= \left(x^4 + \frac{1}{x^4}\right) + \left(x^3 + \frac{1}{x^3}\right) + \left(x^2 + \frac{1}{x^2}\right)$$

$$= \left\{(x^2)^2 + \left(\frac{1}{x^2}\right)^2\right\} + \left(x^3 + \frac{1}{x^3}\right) + \left(x^2 + \frac{1}{x^2}\right)$$

$$= \left\{\left(x^2 + \frac{1}{x^2}\right)^2 - 2 \cdot x^2 \cdot \frac{1}{x^2}\right\} + \left\{\left(x + \frac{1}{x}\right)^3 - 3 \cdot x \cdot \frac{1}{x}\left(x + \frac{1}{x}\right)\right\}$$

$$= \left(7^2 - 2\right) + \left(3^3 - 3 \times 3\right) + \left(3^2 - 2\right) + \left\{\left(x + \frac{1}{x}\right)^2 - 2 \cdot x \cdot \frac{1}{x}\right\}$$

$$= 47 + 18 + 7$$

$$= 72$$

$y = \sqrt{2} + \sqrt{3}$ হলে, $\left(y^2 + \frac{1}{y^2}\right)\left(y^3 - \frac{1}{y^3}\right) = ?$ [35th BCS]

যদি $x = b + c - a, y = c + a - b$ এবং $z = a + b - c$ হয়, তবে দেখান যে, $x^3 + y^3 + z^3 - 3xyz = 4(a^3 + b^3 + c^3 - 3abc)$

[34th ,21st BCS]

$$\text{***}\quad a^3+b^3+c^3-3abc=\frac{1}{2}(a+b+c)\{(a-b)^2+(b-c)^2+(c-a)^2\}$$

ให้ $x = b+c-a$

$y = c+a-b$

$z = a+b-c$

$x+y+z = a+b+c$

$x-y = (b+c-a)-(c+a-b)$

$= b+c-a-c-a+b$

$= -2a+2b$

$= -2(a-b)$

จงพิสูจน์ว่า

$x^3+y^3+z^3-3xyz = 4(a^3+b^3+c^3-3abc)$

$y-z = -2(b-c)$

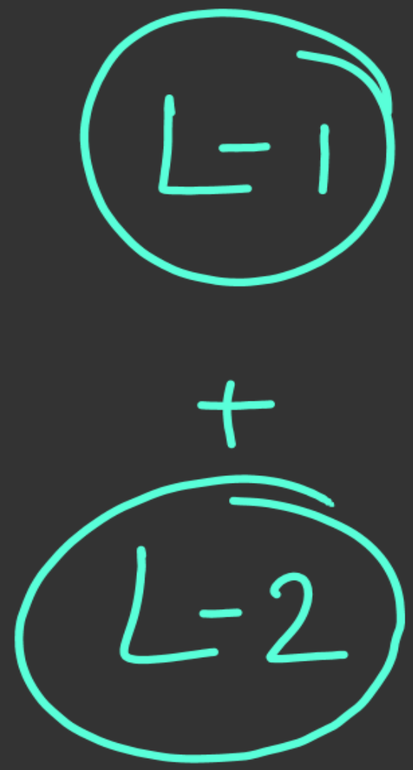
$z-x = -2(a-c)$

L.S. $= x^3+y^3+z^3-3xyz$

$= \frac{1}{2}(x+y+z)\{(x-y)^2+(y-z)^2+(z-x)^2\}$

$= \frac{1}{2}(a+b+c)\{4(a-b)^2+4(b-c)^2+4(c-a)^2\}$

$= 4\times(a^3+b^3+c^3-3abc)$


L-1
+
L-2

দুটি রাশির বর্গের অন্তরফলরূপে প্রকাশ করুনঃ

I. $2p^2 - 15p - 27$ [34th BCS]

II. $(3x + 5y)(7x - 5y)$ [27th BCS]

$2x = \frac{2}{x} + 3$ হলে প্রমাণ করুন যে, $8x^3 = \frac{8}{x^3} + 63$ [33rd BCS]

যদি $\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}=\frac{3}{abc}$ হলে, প্রমাণ করুন যে, $ab+bc+ca=0$,
অথবা

$a=b=c$ [33rd BCS]

$x^2 - \sqrt{3}x + 1 = 0$, হলে, $\frac{x^6+1}{x^3} = ?$ [32nd BCS]

$(p+q)^2 = \sqrt[3]{27}$ এবং $p^2 = \sqrt{16} + q^2$ হলে, $p^3q + pq^3 = ?$ [32nd BCS]

$2x + \frac{2}{x} = 3$ হলে, $x^4 + \frac{1}{x^4} = ?$ [23rd BCS]

$x - y = 7, xy = 15$ হলে, $(x^2 + y^2)(x^3 - y^3) = ?$ [17th BCS]

$\frac{x}{y}+\frac{y}{x}=4$ হলে, $\frac{x^2}{y^2}+\frac{y^2}{x^2}=?$

$x + \frac{1}{x} = 2$ হলে, $\frac{x}{x^2+x-1} = ?$

$x = a + \frac{1}{a}, y = a - \frac{1}{a}$ হলে $x^4 + y^4 - 2x^2y^2 = ?$

যদি $a + b = \sqrt{3}$ এবং $a - b = \sqrt{2}$ হলে প্রমাণ কর যে, $8ab(a^2 + b^2) = 5$

$a + b + c = 10$ এবং $a^2 + b^2 + c^2 = 38$ হলে
$(a - b)^2 + (b - c)^2 + (c - a)^2$ এর মান কত?

যদি $(x+\frac{1}{x})^2 = 3$ হয়, তবে $x^3 + \frac{1}{x^3}$=?

$a + b = c$ হলে , দেখাও যে, $a^3 + b^3 + 3abc = c^3$

যদি $a + b = m, a^2 + b^2 = n$ এবং $a^3 + b^3 = p^3$ হয়, তবে দেখান যে,
$m^3 + 2p^3 = 3mn$

Practice

1. $x - \frac{1}{x} = 4$ হলে,

a) $x^4 + \frac{1}{x^4} = ?$

b) $x^2 + \frac{1}{x^2} = ?$

2. $a + \frac{1}{a} = 2$ হলে, দেখান যে, $a^2 + \frac{1}{a^2} = a^4 + \frac{1}{a^4}$

3. $x = 3 + \frac{1}{x}$,হলে প্রমাণ করুন যে, $x^4 = 119 - \frac{1}{x^4}$

4. যদি $\sqrt{x} + \frac{1}{\sqrt{x}} = a$ হয় ,তবে $x^2 + \frac{1}{x^2} = ?$

5. $a+\frac{1}{a}=2$ হলে, $\left(a^2+\frac{1}{a^2}\right)\left(a^3+\frac{1}{a^3}\right)=?$

6. $a=\sqrt{6}+\sqrt{5}$ হলে, $\frac{a^6-1}{a^3}=?$

⊞ $a^2+1=4a$ হলে, নির্ণয় করুন ▷ $\frac{a+1}{\sqrt{a}}$, $a^3+\frac{1}{a^3}$, $a^3-\frac{1}{a^3}$,
$a^4+\frac{1}{a^4}$, $a^5+\frac{1}{a^5}$, $a^6+\frac{1}{a^6}$,
$a^7+\frac{1}{a^7}$, $\frac{a^{18}+1}{a^9}=?$

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