

BCS Career SPARK

Engr. ALIF EMRAN (B.Sc in EEE, BUET)

ASP, 43rd BCS POLICE(recommended)

Former Assistant Engineer, DESCO (power division)

AD, BANGLADESH BANK (recommended)

Assistant Engineer ,Bangladesh Railway(Gazetted)

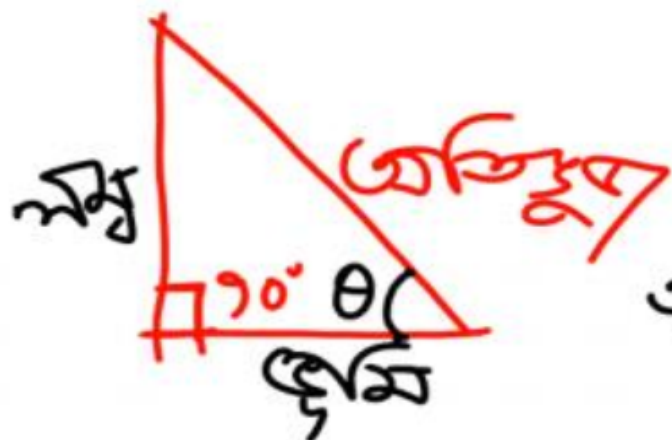
প্রিত্যক্যক্রমিক - → Prelim
অফিসক্রমিক - → Prelim
Written ✓
5
(written)
BCS CAREER
SPARK



BCS CAREER
SPARK

Trigonometry

Basic



* नामावली सिद्ध

$$\textcircled{i} \sin \theta = \frac{\text{अध}}{\text{कस}} \rightarrow \text{cosec } \theta = \frac{1}{\sin \theta}$$

$$\textcircled{ii} \cos \theta = \frac{\text{अस}}{\text{कस}} \rightarrow \sec \theta = \frac{1}{\cos \theta}$$

$$\textcircled{iii} \tan \theta = \frac{\text{अध}}{\text{अस}} \rightarrow \cot \theta = \frac{1}{\tan \theta}$$

$$\textcircled{i} \sin^2 \theta + \cos^2 \theta = 1$$

$$\textcircled{ii} \sec^2 \theta - \tan^2 \theta = 1$$

$$\textcircled{iii} \text{cosec}^2 \theta - \cot^2 \theta = 1$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$



BCS CAREER
SPARK

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \text{cosec}\theta + \cot\theta = \sqrt{\frac{\sec\theta+1}{\sec\theta-1}}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$L.S. = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$$

$$= \sqrt{\frac{(1+\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)}}$$

[$\sqrt{1+\cos\theta}$ द्वारा गुणा करने पर
 $\sqrt{1-\cos\theta}$ द्वारा भाग देने पर]

$$= \frac{1+\cos\theta}{\sqrt{(1-\cos^2\theta)}}$$

$$= \frac{1+\cos\theta}{\sqrt{\sin^2\theta}} = \frac{1+\cos\theta}{\sin\theta} = \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = \text{cosec}\theta + \cot\theta = M.S.$$



BCS CAREER
SPARK

$$R.S = \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}}$$

$$= \sqrt{\frac{(\sec\theta + 1)^2}{(\sec\theta - 1)(\sec\theta + 1)}}$$

$$= \frac{\sec\theta + 1}{\sqrt{\sec^2\theta - 1}}$$

$$= \frac{\sec\theta + 1}{\sqrt{\tan^2\theta}}$$

$$= \frac{\sec\theta + 1}{\tan\theta}$$

$$= \frac{\sec\theta}{\tan\theta} + \frac{1}{\tan\theta}$$

$$= \frac{\left(\frac{1}{\cos\theta}\right)}{\left(\frac{\sin\theta}{\cos\theta}\right)} + \cot\theta$$

$$= \frac{1}{\sin\theta} + \cot\theta$$

$$= \csc\theta + \cot\theta$$

$$= M.S$$

$$\textcircled{*} \sec^2\theta - \tan^2\theta = 1$$

$$\Rightarrow \boxed{\sec^2\theta - 1 = \tan^2\theta}$$

$$* \sec^4 \theta - \sec^2 \theta = \tan^4 \theta + \tan^2 \theta$$

$$\text{L.S.} = \sec^4 \theta - \sec^2 \theta$$

$$= (\sec^2 \theta)^2 - \sec^2 \theta$$

$$= (1 + \tan^2 \theta)^2 - (1 + \tan^2 \theta)$$

$$= 1 + 2\tan^2 \theta + (\tan^2 \theta)^2 - 1 - \tan^2 \theta$$

$$= \tan^4 \theta + \tan^2 \theta$$

$$= \text{R.S.}$$

$$\textcircled{A} \sec^2 \theta - \tan^2 \theta = 1$$

$$\boxed{\sec^2 \theta = 1 + \tan^2 \theta}$$



$$\bullet \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \tan\theta + \sec\theta$$

$$\sec^2\theta - \tan^2\theta = 1$$

$$L.S. = \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$$

$$= \frac{\tan\theta + \sec\theta - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\tan\theta + \sec\theta) - (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\tan\theta + \sec\theta)(1 - \sec\theta + \tan\theta)}{(1 - \sec\theta + \tan\theta)}$$

$$= \tan\theta + \sec\theta$$

$$= R.S$$



* $P = \sin \theta$, $Q = \cos \theta$ এবং $PQ = \frac{1}{2}$ হলে, $P+Q = ?$

$$PQ = \frac{1}{2}$$

$$\Rightarrow \sin \theta \cos \theta = \frac{1}{2}$$

$$\Rightarrow 2 \sin \theta \cos \theta = 1$$

$$\Rightarrow \sin 2\theta = 1$$

$$\Rightarrow \sin 2\theta = \sin 90^\circ$$

$$\Rightarrow 2\theta = 90^\circ$$

$$\Rightarrow \theta = 45^\circ$$

$$2 \sin A \cos A = \sin 2A$$

$$\therefore P+Q = \sin \theta + \cos \theta$$
$$= \sin 45^\circ + \cos 45^\circ$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$$

$$= \frac{1+1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \frac{\sqrt{2} \times \sqrt{2}}{\sqrt{2}} = \sqrt{2}$$



• $\sin^4 A + \sin^2 A = 1$ হলে প্রমাণ কর যে, $\tan^4 A - \tan^2 A = 1$

$$\Rightarrow \sin^4 A = 1 - \sin^2 A$$

$$\Rightarrow \sin^4 A = \cos^2 A$$

$$\Rightarrow \frac{\sin^4 A}{\cos^4 A} = \frac{\cos^2 A}{\cos^4 A}$$

$$\Rightarrow \tan^4 A = \frac{1}{\cos^2 A}$$

$$\Rightarrow \tan^4 A = \sec^2 A$$

$$\Rightarrow \tan^4 A = 1 + \tan^2 A$$

$$\Rightarrow \tan^4 A - \tan^2 A = 1 \quad \checkmark$$

প্রমাণ

$$\sin^2 A + \cos^2 A = 1$$

$$\cos^2 A = 1 - \sin^2 A$$

$$\sec^2 A - \tan^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$



যদি $\operatorname{cosec} A = \frac{a}{b}$ হয়, যেখানে $a > b > 0$, তবে প্রমাণ কর যে, $\tan A = \frac{\pm b}{\sqrt{a^2 - b^2}}$

$$\Rightarrow \operatorname{Cosec}^2 A = \frac{a^2}{b^2}$$

$$\Rightarrow 1 + \operatorname{Cot}^2 A = \frac{a^2}{b^2}$$

$$\Rightarrow \operatorname{Cot}^2 A = \frac{a^2}{b^2} - 1$$

$$\Rightarrow \operatorname{Cot}^2 A = \frac{a^2 - b^2}{b^2}$$

$$\Rightarrow \frac{1}{\tan^2 A} = \frac{a^2 - b^2}{b^2}$$

$$\therefore \tan^2 A = \frac{b^2}{a^2 - b^2}$$

$$\tan A = \frac{\pm b}{\sqrt{a^2 - b^2}}$$

$$\operatorname{Cosec}^2 A - \operatorname{Cot}^2 A = 1$$

$$\operatorname{Cosec}^2 A = 1 + \operatorname{Cot}^2 A$$



• $\tan\theta = \frac{x}{y}$, $x \neq y$ হলে, $\frac{x\sin\theta + y\cos\theta}{x\sin\theta - y\cos\theta}$ এর মান নির্ণয় কর

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{x}{y}$$

$$\Rightarrow \frac{x\sin\theta}{y\cos\theta} = \frac{x^2}{y^2} \quad \left[\text{উভয় পক্ষকে } \frac{x}{y} \text{ দ্বারা গুণ করি} \right]$$

$$\Rightarrow \frac{x\sin\theta + y\cos\theta}{x\sin\theta - y\cos\theta} = \frac{x^2 + y^2}{x^2 - y^2} \quad (\text{যেখানে } - \text{স্বাক্ষরিত})$$

(Ans)



• $x = \tan\theta + \sec\theta$ হলে প্রমাণ করুন যে, $\sin\theta = \frac{x^2-1}{x^2+1}$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \sin^2\theta$$

এখন, $\tan\theta + \sec\theta = x$

$$\Rightarrow \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta} = x$$

$$\Rightarrow \frac{\sin\theta + 1}{\cos\theta} = x$$

$$\Rightarrow \frac{(\sin\theta + 1)^2}{\cos^2\theta} = x^2$$

$$\Rightarrow \frac{(\sin\theta + 1)^2}{1 - \sin^2\theta} = x^2$$

$$\Rightarrow \frac{(1 + \sin\theta)^2}{(1 + \sin\theta)(1 - \sin\theta)} = x^2$$

$$\Rightarrow \frac{1 + \sin\theta}{1 - \sin\theta} = x^2$$

$$\Rightarrow \frac{(1 + \sin\theta) + (1 - \sin\theta)}{(1 + \sin\theta) - (1 - \sin\theta)} = \frac{x^2 + 1}{x^2 - 1}$$

$$\Rightarrow \frac{2}{2\sin\theta} = \frac{x^2 + 1}{x^2 - 1}$$

$$\Rightarrow \frac{1}{\sin\theta} = \frac{x^2 + 1}{x^2 - 1}$$

$$\sin\theta = \frac{x^2 - 1}{x^2 + 1}$$

[যেহেতু-সিদ্ধান্ত]
=



BCS CAREER
SPARK

• $a \cos \theta - b \sin \theta = c$ হলে, প্রমাণ কর যে, $a \sin \theta + b \cos \theta = \pm \sqrt{a^2 + b^2 - c^2}$ $\neq \frac{a}{c} \neq$

ধরি, $a \sin \theta + b \cos \theta = x$ (ii)

(i)² + (ii)² : $(a \cos \theta - b \sin \theta)^2 + (a \sin \theta + b \cos \theta)^2 = c^2 + x^2$

$\Rightarrow a^2 \cos^2 \theta - 2ab \cos \theta \sin \theta + b^2 \sin^2 \theta + a^2 \sin^2 \theta + 2ab \sin \theta \cos \theta + b^2 \cos^2 \theta = c^2 + x^2$

$\Rightarrow a^2 (\cos^2 \theta + \sin^2 \theta) + b^2 (\sin^2 \theta + \cos^2 \theta) = c^2 + x^2$

$\Rightarrow a^2 \cdot 1 + b^2 \cdot 1 = c^2 + x^2$

$\Rightarrow x^2 = a^2 + b^2 - c^2$
 $x = \pm \sqrt{a^2 + b^2 - c^2}$

$\therefore a \sin \theta + b \cos \theta = \pm \sqrt{a^2 + b^2 - c^2}$



• $\frac{\sin\theta}{x} = \frac{\cos\theta}{y}$ হলে, প্রমাণ করুন যে, $\sin\theta - \cos\theta = \frac{x-y}{\sqrt{x^2+y^2}}$

$$\Rightarrow y \sin\theta = x \cos\theta$$

$$\Rightarrow y^2 \sin^2\theta = x^2 \cos^2\theta$$

$$\Rightarrow y^2 (1 - \cos^2\theta) = x^2 \cos^2\theta$$

$$\Rightarrow y^2 - y^2 \cos^2\theta = x^2 \cos^2\theta$$

$$\Rightarrow y^2 = x^2 \cos^2\theta + y^2 \cos^2\theta$$

$$\Rightarrow y^2 = (\cos^2\theta) \cdot (x^2 + y^2)$$

$$\therefore \cos^2\theta = \frac{y^2}{x^2 + y^2}$$

$$\cos\theta = \frac{y}{\sqrt{x^2 + y^2}}$$

$$\Rightarrow y^2 \sin^2\theta = x^2 (1 - \sin^2\theta)$$

$$\Rightarrow y^2 \sin^2\theta = x^2 - x^2 \sin^2\theta$$

$$\Rightarrow (\sin^2\theta)(y^2 + x^2) = x^2$$

$$\Rightarrow \sin^2\theta = \frac{x^2}{x^2 + y^2}$$

$$\Rightarrow \sin\theta = \frac{x}{\sqrt{x^2 + y^2}}$$



$$L.S. = \sin \theta - \cos \theta$$

$$= \frac{x}{\sqrt{x^2 + y^2}} - \frac{y}{\sqrt{x^2 + y^2}}$$

$$= \frac{x - y}{\sqrt{x^2 + y^2}}$$

$$= R.S.$$

• $\text{Cot}A - \text{Cosec}A = \frac{1}{p}$ হলে, প্রমাণ করুন $\text{Sec}A = \frac{p^2+1}{p^2-1}$

$$\Rightarrow \frac{\text{Cos}A}{\text{Sin}A} - \frac{1}{\text{Sin}A} = \frac{1}{p}$$

$$\Rightarrow \frac{\text{Cos}A - 1}{\text{Sin}A} = \frac{1}{p}$$

$$\Rightarrow \frac{(\text{Cos}A - 1)^2}{\text{Sin}^2A} = \frac{1}{p^2}$$

$$\Rightarrow \frac{\text{Sin}^2A}{(1 - \text{Cos}A)^2} = \frac{p^2}{1}$$

$$\Rightarrow \frac{1 - \text{Cos}^2A}{(1 - \text{Cos}A)^2} = \frac{p^2}{1}$$

$$\Rightarrow \frac{(1 - \text{Cos}A)(1 + \text{Cos}A)}{(1 - \text{Cos}A)^2} = \frac{p^2}{1}$$

$$\Rightarrow \frac{1 + \text{Cos}A}{1 - \text{Cos}A} = \frac{p^2}{1}$$

$$\Rightarrow \frac{1 + \text{Cos}A + 1 - \text{Cos}A}{1 + \text{Cos}A - 1 + \text{Cos}A} = \frac{p^2 + 1}{p^2 - 1}$$

[যেহেতু-বিয়োজন]

$$\Rightarrow \frac{2}{2\text{Cos}A} = \frac{p^2+1}{p^2-1}$$

$$\Rightarrow \frac{1}{\text{Cos}A} = \frac{p^2+1}{p^2-1}$$

$$\therefore \text{Sec}A = \frac{p^2+1}{p^2-1}$$

(proved)



BCS CAREER
SPARK

• $A = \cos\theta + \sin\theta$, $B = \cos\theta - \sin\theta$ দুইটি ত্রিকোণমিতিক রাশি এবং যদি $A = \sqrt{2}(A - \sin\theta)$ হয়, তবে প্রমাণ করুন যে, $B = \sqrt{2}(A - \cos\theta)$

এখানে, $A = \sqrt{2}(A - \sin\theta)$

$$\Rightarrow \cos\theta + \sin\theta = \sqrt{2}(\cos\theta + \sin\theta - \sin\theta)$$

$$\Rightarrow \cos\theta + \sin\theta = \sqrt{2}\cos\theta$$

$$\Rightarrow \sin\theta = \sqrt{2}\cos\theta - \cos\theta$$

$$\Rightarrow \sin\theta = (\sqrt{2} - 1)\cos\theta$$

$$\Rightarrow (\sqrt{2} + 1)\sin\theta = (\sqrt{2} + 1)(\sqrt{2} - 1)\cos\theta$$

$$\Rightarrow \sqrt{2}\sin\theta + \sin\theta = (2 - 1)\cos\theta$$

$$\Rightarrow \sqrt{2}\sin\theta + \sin\theta = \cos\theta$$

$$\Rightarrow \sqrt{2}\sin\theta = \cos\theta - \sin\theta$$

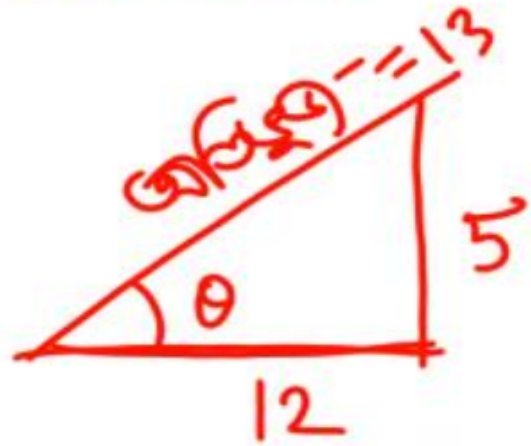
$$\Rightarrow \sqrt{2}\sin\theta = B$$

$$\Rightarrow B = \sqrt{2}(\sin\theta + \cos\theta - \cos\theta)$$

$$\therefore B = \sqrt{2}(A - \cos\theta)$$



• $\tan\theta = \frac{5}{12}$ এবং $\cos\theta$ ~~স্ব~~গাত্মক হলে, প্রমাণ কর যে, $\frac{\sin\theta + \cos(-\theta)}{\sec(-\theta) + \tan\theta} = \frac{51}{26}$

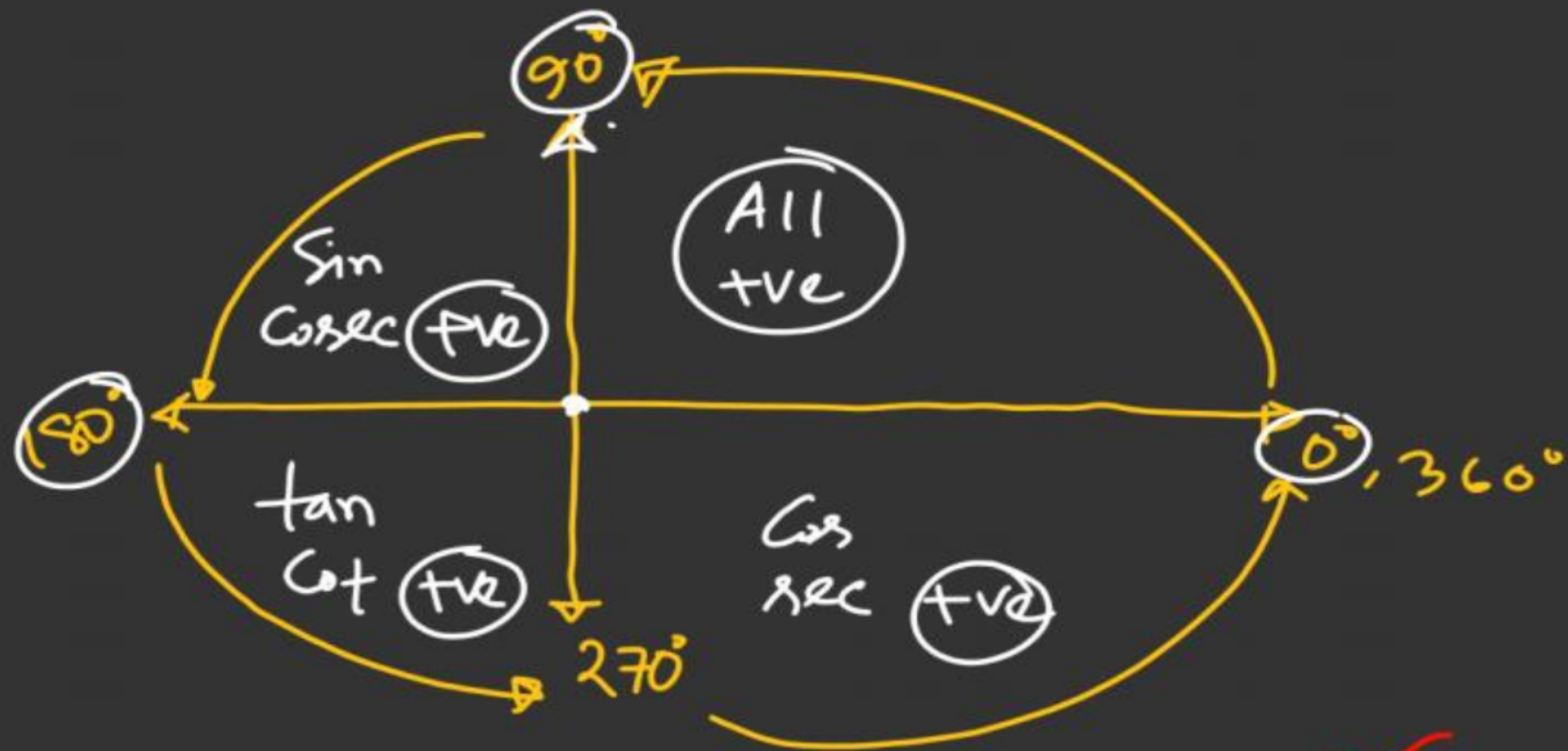


$$\tan\theta = \frac{\text{পাশ}}{\text{ভূমি}}$$

পীথাগোরাস, $[\text{অভিলম্ব}]^2 = \text{পাশ}^2 + \text{ভূমি}^2$
 $\Rightarrow [\text{অভিলম্ব}]^2 = 5^2 + 12^2$
 $= 169$
 $\therefore \text{অভিলম্ব} = 13$

$$\begin{aligned} \text{L.S.} &= \frac{\sin\theta + \cos\theta}{\sec\theta + \tan\theta} \\ &= \frac{\left(-\frac{5}{13}\right) + \left(-\frac{12}{13}\right)}{\left(-\frac{13}{12}\right) + \left(\frac{5}{12}\right)} \\ &= \frac{\left(\frac{-5-12}{13}\right)}{\left(\frac{-13+5}{12}\right)} \\ &= \left(\frac{-17}{13}\right) \times \left(\frac{12}{-8}\right) = \frac{51}{26} \end{aligned}$$





$$\left\{ \begin{array}{l} \sin(-\theta) = -\sin\theta \\ \operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta \\ \tan(-\theta) = -\tan\theta \\ \cot(-\theta) = -\cot\theta \end{array} \right.$$

$\sin, \operatorname{cosec}$



\tan, \cot



\cos, sec



$$\left\{ \begin{array}{l} \cos(-\theta) = \cos\theta \\ \operatorname{sec}(-\theta) = \operatorname{sec}\theta \end{array} \right.$$

• $\sqrt{2}\cos(A-B) = 1$, $2\sin(A+B) = \sqrt{3}$ এবং A, B সূক্ষ্ণকোণ হলে, A ও B এর মান নির্ণয় কর।

$$\Rightarrow \cos(A-B) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \underline{\cos(A-B)} = \underline{\cos 45^\circ}$$

$$\boxed{A-B = 45^\circ} \quad \text{--- (i)}$$

$$\sin(A+B) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \underline{\sin(A+B)} = \underline{\sin 60^\circ}$$

$$\boxed{A+B = 60^\circ} \quad \text{--- (ii)}$$

$$\text{(i)} + \text{(ii)}: 2A = 105^\circ$$

$$A = \frac{105}{2} = 52\frac{1}{2}$$

$$\text{(ii)} - \text{(i)}: 2B = 15^\circ$$

$$B = \frac{15}{2} = 7\frac{1}{2}$$

$$A = 52\frac{1}{2}^\circ$$

$$B = 7\frac{1}{2}^\circ$$

Ans



BCS CAREER
SPARK

	0°	30°	45°	60°	90°
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

सिद्ध करें
 $A+B=90^\circ$

$$7\sin^2\theta + 3\cos^2\theta = 4 \text{ হলে, } \tan\theta = ?$$

$$\Rightarrow 4\sin^2\theta + 3\sin^2\theta + 3\cos^2\theta = 4$$

$$\Rightarrow 4\sin^2\theta + 3(\sin^2\theta + \cos^2\theta) = 4$$

$$\Rightarrow 4\sin^2\theta + 3 \times 1 = 4$$

$$\Rightarrow 4\sin^2\theta = 1$$

$$\Rightarrow \sin^2\theta = \frac{1}{4}$$

$$\Rightarrow \frac{1}{\operatorname{cosec}^2\theta} = \frac{1}{4}$$

$$\operatorname{cosec}^2 A - \cot^2 A = 1$$

$$\Rightarrow \operatorname{cosec}^2\theta = 4$$

$$\Rightarrow 1 + \cot^2\theta = 4$$

$$\Rightarrow \cot^2\theta = 3$$

$$\cot\theta = \pm\sqrt{3}$$

$$\tan\theta = \pm\frac{1}{\sqrt{3}}$$

(Ans)



BCS CAREER
SPARK

সমাধান কর: $\frac{\cos A - \sin A}{\cos A + \sin A} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$

$\Rightarrow \frac{(\cancel{\cos A} - \cancel{\sin A}) + (\cancel{\cos A} + \cancel{\sin A})}{(\cancel{\cos A} - \cancel{\sin A}) - (\cancel{\cos A} + \cancel{\sin A})} = \frac{(\cancel{\sqrt{3}} - 1) + (\cancel{\sqrt{3}} + 1)}{(\cancel{\sqrt{3}} - 1) - (\cancel{\sqrt{3}} + 1)}$ [যোগন-বিয়োগ]

$\Rightarrow \frac{2\cos A}{(-2\sin A)} = \frac{2\sqrt{3}}{(-2)}$

$\Rightarrow -\frac{\cos A}{\sin A} = -\sqrt{3}$

$\Rightarrow \frac{\sin A}{\cos A} = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan A = \tan 30^\circ$

$A = 30^\circ$



BCS CAREER
SPARK

• সমাধান কর: $2\sin^2\theta + 3\cos\theta + 3 = 0$, θ সূক্ষ্মকোণ।

$$\Rightarrow 2(1 - \cos^2\theta) + 3\cos\theta + 3 = 0$$

$$\Rightarrow 2 - 2\cos^2\theta + 3\cos\theta + 3 = 0$$

$$\Rightarrow -2\cos^2\theta + 3\cos\theta + 5 = 0$$

$$\Rightarrow 2\cos^2\theta - 3\cos\theta - 5 = 0$$

$$\Rightarrow 2\cos^2\theta - 5\cos\theta + 2\cos\theta - 5 = 0$$

$$\Rightarrow \cos\theta(2\cos\theta - 5) + 1(2\cos\theta - 5) = 0$$

$$\Rightarrow (2\cos\theta - 5)(\cos\theta + 1) = 0$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$2\cos\theta - 5 = 0 \text{ যেহেতু,}$$

$$\cos\theta = -1$$

$$\cos\theta \neq \frac{5}{2}$$

$$\cos\theta \neq -1$$

θ সূক্ষ্মকোণ

\therefore প্রদত্ত সমীকরণের
সমাধান নেই।

$$\begin{aligned} -1 &\leq \sin\theta \leq 1 \\ -1 &\leq \cos\theta \leq 1 \end{aligned}$$



সমাধান কর; $\tan^2\theta - (1+\sqrt{3})\tan\theta + \sqrt{3} = 0$ ✓ θ নির্ণয় করো

$$\Rightarrow \tan^2\theta - \tan\theta - \sqrt{3}\tan\theta + \sqrt{3} = 0$$

$$\Rightarrow \tan\theta(\tan\theta - 1) - \sqrt{3}(\tan\theta - 1) = 0$$

$$\Rightarrow \underline{(\tan\theta - 1)} \underline{(\tan\theta - \sqrt{3})} = 0$$

$$\tan\theta - 1 = 0$$

$$\Rightarrow \tan\theta = 1$$

$$\Rightarrow \tan\theta = \tan 45^\circ$$

$$\theta = 45^\circ$$

$$\tan\theta = \sqrt{3}$$

$$= \tan\theta = \tan 60^\circ$$

$$\theta = 60^\circ$$



$0 < \theta < 2\pi$ হলে, $2(\sin\theta\cos\theta + \sqrt{3}) = \sqrt{3}\cos\theta + 4\sin\theta$ এর সমাধান নির্ণয় করুন।

$$2\sin\theta\cos\theta + 2\sqrt{3} = \sqrt{3}\cos\theta + 4\sin\theta$$

$$\Rightarrow 2\sin\theta\cos\theta - 4\sin\theta - \sqrt{3}\cos\theta + 2\sqrt{3} = 0$$

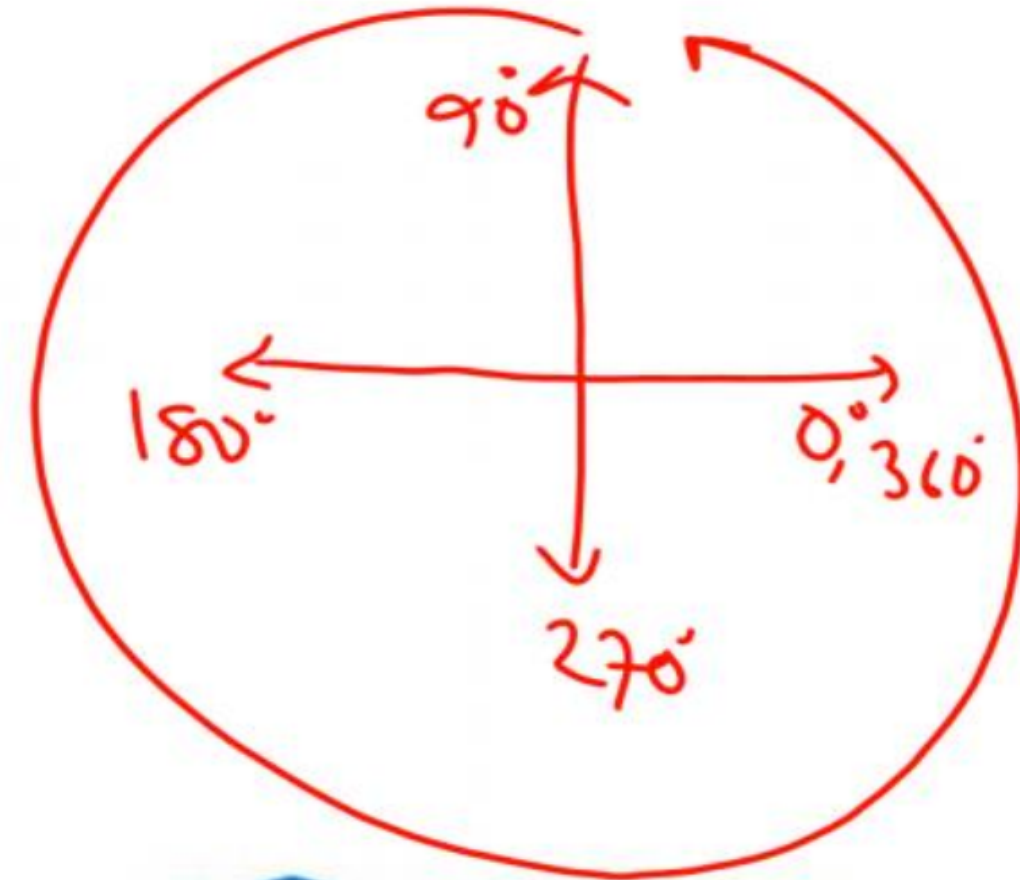
$$\Rightarrow 2\sin\theta(\cos\theta - 2) - \sqrt{3}(\cos\theta - 2) = 0$$

$$\Rightarrow (\cos\theta - 2)(2\sin\theta - \sqrt{3}) = 0$$

$$\therefore \cos\theta \neq 2$$
$$[-1 \leq \cos\theta \leq 1]$$

$$\therefore 2\sin\theta - \sqrt{3} = 0$$
$$\Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$$

π (180°)
 2π (360°)



BCS CAREER
SPARK

$$\sin \theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin \theta = \boxed{\sin 60^\circ} \text{ or } \sin(180 - 60)$$
$$\Rightarrow \sin \theta = \sin 60^\circ \text{ or, } \sin 120^\circ$$

$0 < \theta < 360^\circ$
सिनुस

$\theta = 60^\circ, 120^\circ$

Ans



द्वि → $180 - \theta$
द्वि → $(180 - \theta)$
द्वि → $(180 + \theta)$
द्वि → $(360 - \theta)$

$$\textcircled{i} \quad 30 + 120 + x = 180$$

$$\Rightarrow x = 30^\circ$$

$$\angle CAD = 30^\circ$$

$$\textcircled{ii} \quad \triangle ABC \quad \tan 60^\circ = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{AB}{BC}$$

$$\therefore BC = \frac{AB}{\sqrt{3}} \quad \text{--- (i)}$$

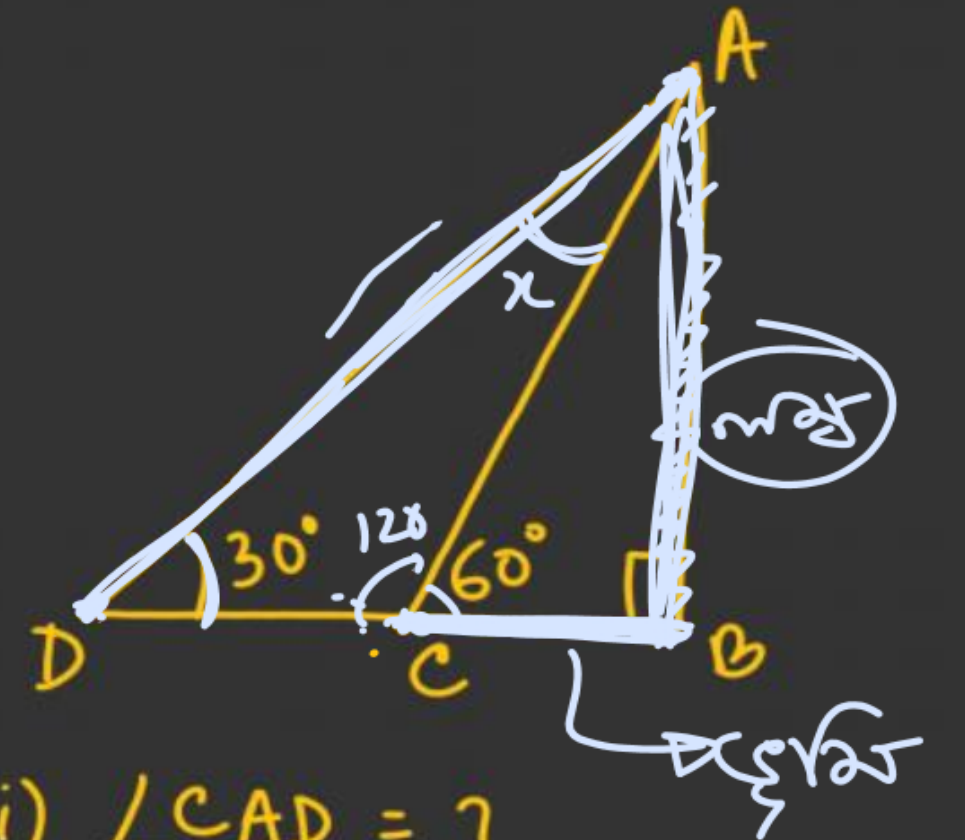
$$\triangle ABD$$

$$\sin 30 = \frac{AB}{AD}$$

$$\Rightarrow \frac{1}{2} = \frac{AB}{AD}$$

$$\therefore AD = 2AB \quad \text{--- (ii)}$$

$$\therefore \frac{BC}{AD} = \frac{\frac{AB}{\sqrt{3}}}{2AB} = \frac{AB}{\sqrt{3}} \times \frac{1}{2AB} = \frac{1}{2\sqrt{3}}$$



$$\textcircled{i} \quad \angle CAD = ?$$

~~(ii) prove that,~~

$$BC : AD = 1 : 2\sqrt{3}$$

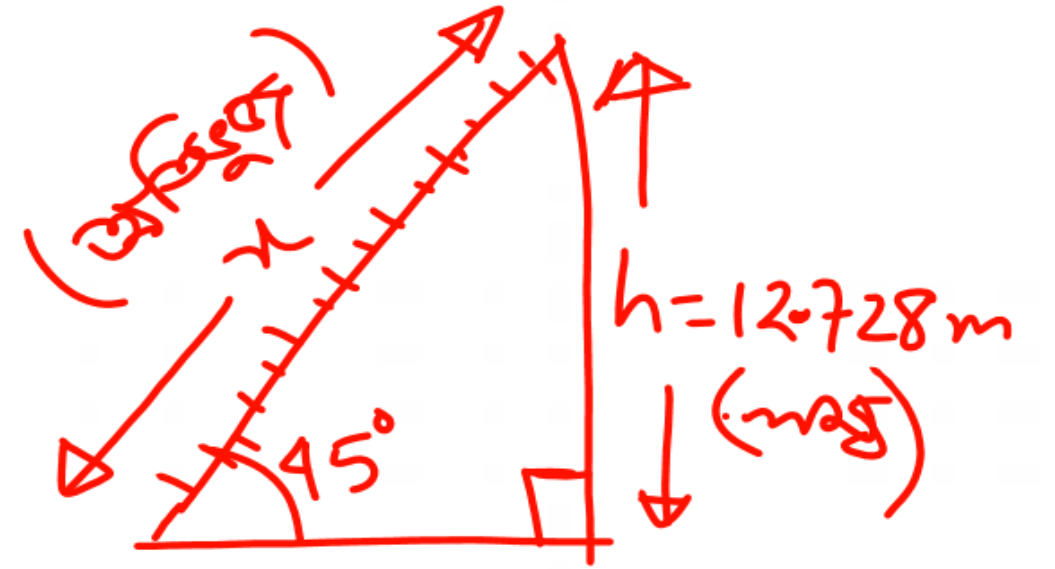
12.728 মিটার উচ্চতাবিশিষ্ট একটি দেয়ালের ছাদের সঙ্গে লাগানো মই ভূমির সাথে 45° কোণ উৎপন্ন করে। মইটির দৈর্ঘ্য কত?

সমাধান,

$$\sin 45^\circ = \frac{h}{x}$$

$$\Rightarrow x = \frac{h}{\sin 45^\circ}$$

$$\begin{aligned} \therefore x &= \frac{12.728}{\left(\frac{1}{\sqrt{2}}\right)} \\ &= (\sqrt{2}) \times 12.728 \text{ মিটার} \end{aligned}$$



একটি গাছ এমনভাবে ভেঙ্গে গেল যে, তার ভাঙ্গা অংশ দণ্ডায়মান অংশের সাথে 30° কোণ করে গাছের গোড়া থেকে 10 মিটার দূরে মাটি স্পর্শ করে। গাছটির সম্পূর্ণ দৈর্ঘ্য কত?

প্রদানে,

$$\tan 30^\circ = \frac{a}{x}$$

$$\Rightarrow x = \frac{a}{\tan 30^\circ}$$

$$\Rightarrow x = \frac{10}{\left(\frac{1}{\sqrt{3}}\right)}$$

$$x = 10\sqrt{3} \text{ মিটার}$$

অন্য,

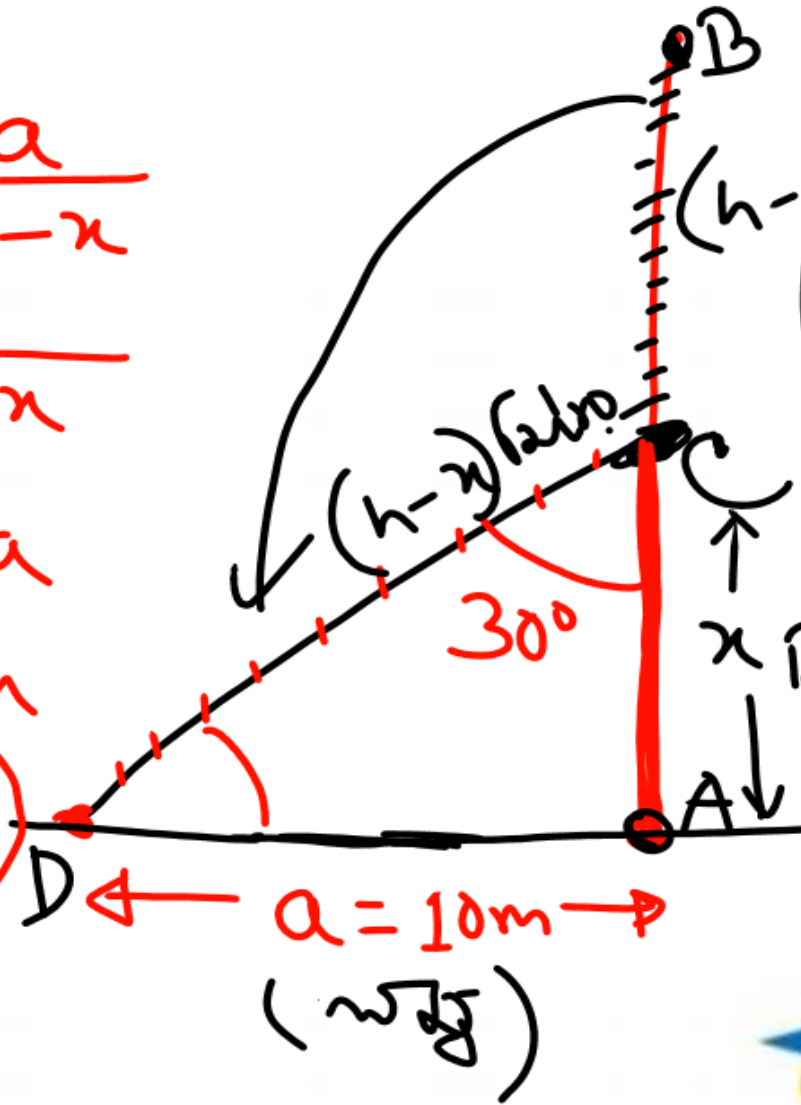
$$\sin 30^\circ = \frac{a}{h-x}$$

$$\Rightarrow \frac{1}{2} = \frac{a}{h-x}$$

$$\Rightarrow h-x = 2a$$

$$\Rightarrow h = 2a + x$$

$$h = 2 \times 10 + 10\sqrt{3}$$



$AB = h$ মিটার

$BC = CD = (h-x)$ মিটার

64 মিটার লম্বা একটি খুঁটি ভেঙে গিয়ে সম্পূর্ণ বিচ্ছিন্ন না হয়ে ভূমির সাথে 60° কোণ উৎপন্ন করে। খুঁটির ভাঙা অংশের দৈর্ঘ্য নির্ণয় করুন।

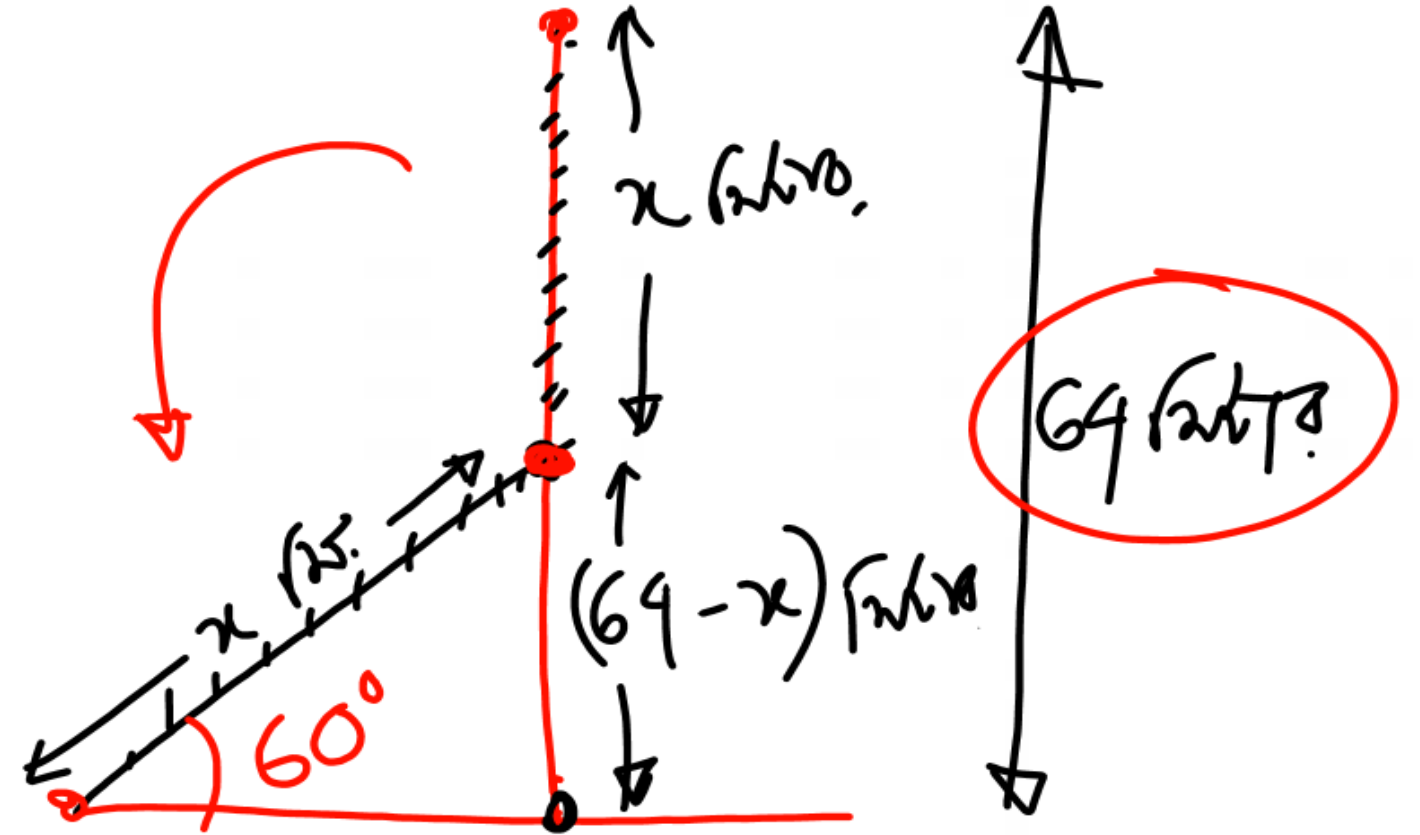
$$\sin 60^\circ = \frac{64-x}{x}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{64-x}{x}$$

$$\Rightarrow \sqrt{3}x = 2 \times 64 - 2x$$

$$\Rightarrow (\sqrt{3}+2)x = 128$$

$$\therefore x = \frac{128}{\sqrt{3}+2} = \left(\frac{128}{3.73}\right) \text{ মিটার}$$



BCS CAREER
SPARK

কোন স্থান থেকে একটি মিনারের দিকে 25 মিটার এগিয়ে এলে মিনারের শীর্ষবিন্দুর উন্নতি কোণ 30° থেকে 45° হয়। মিনারটির উচ্চতা নির্ণয় করুন

$$\boxed{\triangle ABD}: \tan 45^\circ = \frac{h}{x}$$

$$\Rightarrow 1 = \frac{h}{x}$$

$$\Rightarrow x = h \text{ — (i)}$$

$$\boxed{\triangle ABC}: \tan 30^\circ = \frac{h}{x+25}$$

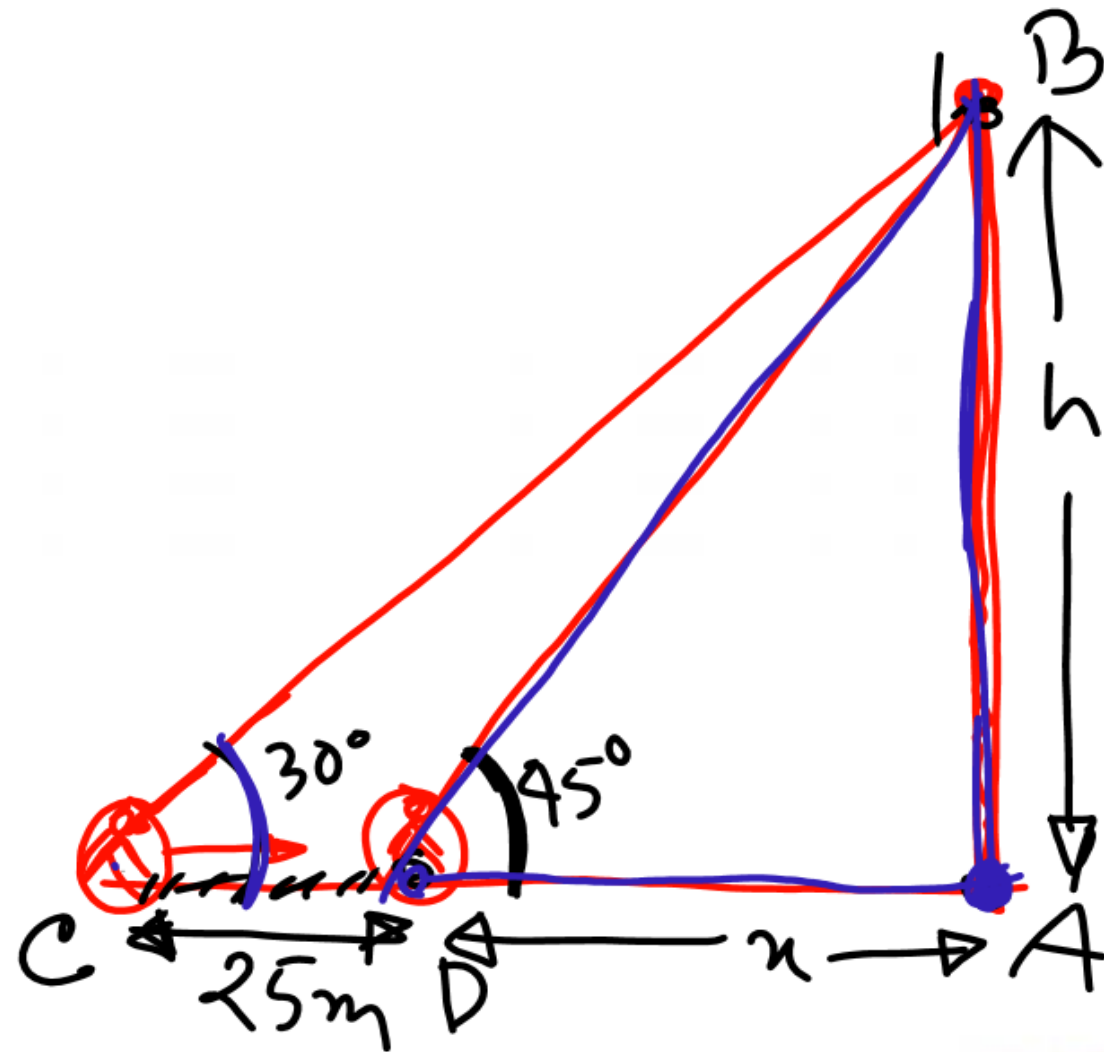
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+25}$$

$$\Rightarrow x+25 = \sqrt{3}h$$

$$\Rightarrow h+25 = \sqrt{3}h$$

$$\Rightarrow 25 = (\sqrt{3}-1)h$$

$$\therefore h = \left(\frac{25}{\sqrt{3}-1} \right) \text{ মিটার}$$





BCS CAREER
SPARK

একটি নদীর তীরে কোন এক স্থানে দাঁড়িয়ে একজন লোক দেখলো যে, ঠিক সোজাসুজি অপর তীরে অবস্থিত একটি স্তম্ভের উন্নতি কোণ 60° , ঐ স্থান থেকে 20 মিটার পিছিয়ে গিয়ে দেখলো যে, স্তম্ভটির উন্নতি কোণ 30° হয়েছে। স্তম্ভটির উচ্চতা ও নদীর বিস্তার নির্ণয় করুন।

$\triangle ABD$

$$\tan 60^\circ = \frac{h}{x}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow h = \sqrt{3}x \quad \text{--- (i)}$$

$\triangle ABC$

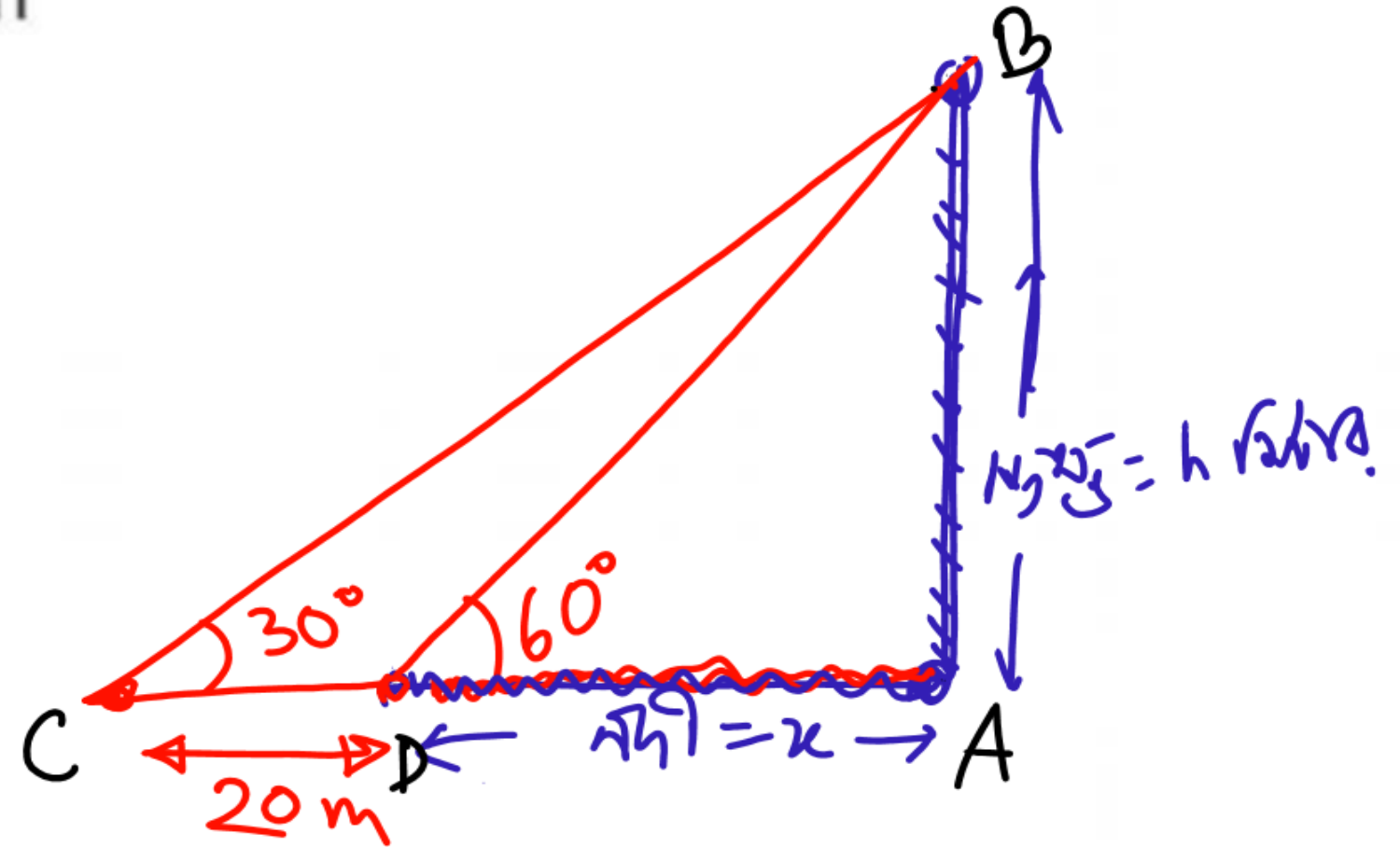
$$\tan 30^\circ = \frac{h}{x+20}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+20}$$

$$\Rightarrow x+20 = \sqrt{3}h$$

$$\Rightarrow x+20 = \sqrt{3} \cdot \sqrt{3}x$$

$$\Rightarrow x+20 = 3x$$



AD (নদী)

AB (স্তম্ভ)

$$x = 10 \text{ m}$$

$$h = 10\sqrt{3} \text{ m}$$



BCS CAREER
SPARK

- দুইটি মাইল পোস্টের মধ্যবর্তী কোনো স্থানের উপরে একটি বেলুন উড়ছে। বেলুনের স্থানে ঐ মাইল পোস্ট দুইটির অবনতি কোণ যথাক্রমে 30° ও 60° হলে, বেলুনটির উচ্চতা মিটারে নির্ণয় কর।



$$\triangle ADC \quad \tan 30 = \frac{h}{x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\Rightarrow x = \sqrt{3}h \quad \text{--- (i)}$$

$$\triangle BDC \quad \tan 60 = \frac{h}{1610 - x}$$

$$\Rightarrow \sqrt{3} = \frac{h}{1610 - x}$$

$$\Rightarrow \sqrt{3} \times 1610 - \sqrt{3}x = h$$

$$\Rightarrow \sqrt{3} \times 1610 - \sqrt{3} \times \sqrt{3}h = h$$

$$\Rightarrow \sqrt{3} \times 1610 = 4h$$

$$\therefore h = \frac{\sqrt{3} \times 1610}{4}$$

$$1 \text{ mile} = 1.61 \text{ km} \\ \approx 1610 \text{ m}$$

