

# Lecture-12 : Number System

## Important Vocabulary

Word	Meaning	Word	Meaning
Sum/Total	যোগফল	Consecutive	ধারাবাহিক
Difference	বিয়োগফল	Composite number	যৌগিক
Product	গুণফল	Multiple	গুণিতক
Quotient	ভাগফল	Divisor	ভাজক
Dividend	ভাজ্য	Least number	সর্বাপেক্ষা ছোট সংখ্যা
Reminder	ভাগশেষ	Perfect square	পূর্ণ বর্গ
Prime Number	মৌলিক সংখ্যা	Co-prime	সহ মৌলিক
Factor	উৎপাদক	Consecutive	ধারাবাহিক
Could be	হতে পারে	Composite number	যৌগিক
Preceding one	পূর্ববর্তী	Multiple	গুণিতক
Resultant	প্রাপ্ত ফলাফল	Except	ব্যতীত
Treble	তিন গুণ	Least number	সর্বাপেক্ষা ছোট সংখ্যা
Add	যোগ করা	Consecutive	ধারাবাহিক
Subtract	বিয়োগ করা	Composite number	যৌগিক
Multiply	গুণ করা	Multiple	গুণিতক
Divide	ভাগ করা		

### Teacher's Work

### Number (MCQ)

### Foundation Related

- P and Q are two positive integers such that  $PQ = 64$ . What of the following cannot be the value of  $P + Q$ ? [BB-AD: 20-10-23]  
a) 16      b) 20      c) 35      d) 65
- If  $p$  and  $q$  are positive integers with  $pq = 36$ , then  $\frac{p}{q}$  cannot be- [বাখারবাদ প্যাস এ.এম (জেনারেল) পরীক্ষা - ২০১৭]  
a)  $\frac{1}{4}$       b)  $\frac{4}{9}$       c)  $\frac{1}{2}$       d) None
- Define  $x^*$  by the equation  $x^* = \pi/x$ . Then  $((-\pi)^*)^* =$   
A)  $-1/\pi$       B)  $-1/2$       C)  $-\pi$       D)  $1/\pi$
- The difference between the local value and the face value of 7 in the numeral 32675149 is -  
a) 5149      b) 64851      c) 69993      d) 75142

### Odd & Even number

- If  $n$  is an even integer, which of the following must be an odd integer? [BB-Officer-2019]  
a)  $n^2 - n$       b)  $n + 2$       c)  $3n - 1$       d)  $3n^2$

- If  $x$  is an odd negative integer and  $y$  is an even positive integer, the  $xy$  must be which of the following? [DBBL (AO)-2017]

- a) odd & positive      b) odd & negative  
c) even & negative      d) even & positive

- In each expression below,  $N$  represents a negative integer, which expression could have a negative value? [Agrani Bank (Cash)-2017 + BB (Officer)-2018]

- a)  $N^2$       b)  $6 - N$       c)  $-N$       d)  $6 + N$

- If  $m$  is an even integer and  $n$  is an odd integer and both are positive numbers, which of the following must be even?

- A)  $m^2 + n^2$       B)  $mn + n^2$       C)  $m^3 + n^2$       D)  $mn + m^2$

### Prime Number/Prime factor

- The sum of first five prime number is: [Janata Bank (EO)-2017]

- a) 11      b) 18      c) 26      d) 28

- Which of the following cannot be a sum of two prime integers? [IBBL (PO)-2017]

- a) 7      b) 19      c) 23      d) 31

### Co-Prime numbers

- The pair of co-prime numbers is- [Rupali Bank (Cash)-2018]

- a) 2, 3      b) 2, 4      c) 2, 6      d) 2, 110

### Finding the number

- If a two-digit positive integer has its digits reversed, the resulting integer differs from the original by 27. By how much do the two digits differ? [BKB-(SO)-2017]

- a) 3      b) 4      c) 5      d) 6

- A number when divided by a divisor leaves a remainder of 24. When twice the original number is divided by the same divisor, the remainder is 11. What is the value of the divisor?

- a) 13      b) 35      c) 37      d) 59

### Related questions

- If the number  $481*673$  is completely divisible by 9 then the smallest whole number in place of \* is: [Agrani Bank (SO)-2017]

- a) 2      b) 5      c) 6      d) 7

- How many integers from 1 to 1000 are divisible by 30 but not by 16? [Agrani Bank (Cash)-2017 + BB (Officer)-2018 + Sonali Bank (SO)-2018 + Rupali Bank (Officer)-2019]

- a) 29      b) 31      c) 32      d) 38

Miscellaneous

16. When each of the three numbers is divided by 5, the remainders are, 0, 3 and 4 respectively. Which of the following could be the sum of the three numbers? [BGDCL (AM)-2017]  
 a) 52      b) 53      c) 54      d) 55
17. In a two-digit number the unit's digit exceeds its ten's digit by 2 and the product of the given number and the sum of its digits is equal to 144. The number is [BGDCL (Accounts)-2017]  
 a) 46      b) 42      c) 13      d) 24
18. For which of the following values of  $n$  is  $\frac{100+n}{n}$  not an integer? [National Bank PO-2017]  
 a) 1      b) 2      c) 3      d) 4
19. The unit digit in the product  $(784 \times 618 \times 917 \times 463)$  is:  
 A) 2      B) 3      C) 4      D) 5
20. Find the number of zeroes in  $10 \times 20 \times 30 \times \dots \times 1000$ . [Combined Senior Officer: 2022]  
 a) 100      b) 124      c) 130      d) 154
21. Which of the following is perfect square number? [TBA - MBA: 54th Intake]  
 a. 314723      b. 204314      c. 322624      d. 5382532
22. What is the least number by which 2450 must be multiplied in order to get a perfect square? [Southeast Bank : PO - 2018]  
 a. 2      b. 3      c. 4      d. 8
23. What is the smallest number to be subtracted from 549162 in order to make it a perfect square? [Janata Bank - 2019]  
 a. 28      b. 36      c. 62      d. 81
24. If  $m$  and  $n$  are whole numbers such that  $m^n = 121$ , what is the value of  $(m-1)^{n+1}$ ? [BD Gas Fields - AM Exam - 2021]  
 a) 1000      b) 676      c) 121      d) 10
25. What is the greatest prime factor of  $2^{100} - 2^{96}$ ?  
 a) 2      b) 31      c) 5      d) 7

Illustrative

Number (MCQ)

01. In a 4-digit number, the sum of the first two digits is equal to that of last two digits. The sum of the first and last digits is equal to third digit. Finally, the sum of the second and fourth digits is twice the sum of the other two digits. What is the third digit of the number?

Sol. Let the 1st, 2nd, 3rd and 4th digits be  $a$ ,  $b$ ,  $c$  and  $d$  respectively.

Then,

$$a + b = c + d$$

$$a + d = c$$

$$b + d = 2(a+c)$$

from eqn. (i) and (ii),

$$a + b = a + 2d$$

$$\Rightarrow b = 2d$$

And eqn (iii);

$$2d + d = 2(a + a + d)$$

$$\Rightarrow 3d = 2(2a + d) \Rightarrow d = 4a$$

$$\text{Or, } a = \frac{d}{4};$$

$\Rightarrow$  Now, from eqn. (ii),

$$A + d = \frac{d}{4} + d = \frac{5d}{4} = c$$

$$\text{Or, } c = \frac{5}{4}d$$

The value of  $d$  can be either 4 or 8.

If  $d = 4$ , then  $c = 5$

If  $d = 8$ , then  $c = 10$

But the value of  $c$  should be less than 10

Hence, value of  $c$  would be 5 Ans:

02. The sum of the digits of two-digit number is 10, while when the digits are reversed, the number decrease by 54. Find the changed number.

Sol. Let number be  $(10x + y)$

According to question,

$$(10x + y) - (10y + x) = 54$$

$$10x - 10y + y - x = 54$$

$$\text{Or, } 9x - 9y = 54$$

$$\text{Or, } x - y = 6 \dots\dots\dots(i)$$

Sum of digits,

$$(x + y) = 10 \dots\dots\dots(ii)$$

$$(i) - (ii)$$

$$\text{So, } x - y - x - y = 6 - 10$$

$$\text{Or, } -2y = -4$$

$$\text{Or, } y = 2 \text{ and, } x = 8$$

Then, the required number is

$$= (10y + x) = 10 \times 2 + 8 = 28 \text{ Ans:}$$

03. If the sum of two numbers is 3 and the sum of their squares is 12 then their product is equal to-

Sol.  $a + b = 3 \dots\dots\dots(i)$

( $a$  and  $b$  are two numbers)

$$a^2 + b^2 = 12$$

On squaring equation (i)

$$(a + b)^2 = 3^2$$

$$a^2 + b^2 + 2ab = 9$$

$$12 + 2ab = 9$$

$$2ab = -3$$

$$Ab = -\frac{3}{2} \text{ Ans:}$$

04. A number consists of two digits such that the digit in the ten's place is less by 2 than the digit in the unit place. Three times the number added to  $\frac{6}{7}$  times the number obtained by reversing digits equals 108. The sum of digits in the number is-

Sol. Let the unit digit

$$\text{Ten digit} = x - 2$$

$\therefore$  Number :

$$= 10(x-2) + x = 10x - 20 + x = 11x - 20$$

New number obtained after reversing the digits

$$= 10x + x - 2 = 11x - 2$$

According to the question,

$$3(11x - 20) + \frac{6}{7}(11x - 2) = 108$$

$$7(11x - 20) + 2(11x - 2) = 36 \times 7$$

$$77x - 140 + 22x - 4 = 252$$

$$99x = 252 + 144$$

$$x = \frac{396}{99} = 4$$

Number:

$$= 11x - 20 = 11 \times 4 - 20 = 24$$

$$\text{Sum of digit} = 2 + 4 = 6 \text{ (Ans)}$$

05. If  $\frac{3}{4}$  of a number is 7 more than  $\frac{1}{6}$  of the number then

$\frac{5}{3}$  of the number is:

Sol. Let the number be x

$\Rightarrow$  According to the question,

$$\Rightarrow \frac{3x}{4} - \frac{x}{6} = 7$$

$$\Rightarrow \frac{9x - 2x}{12} = 7$$

$$\Rightarrow 7x = 7 \times 12$$

$$\Rightarrow x = 12$$

$$\text{Then } \frac{5}{3} \text{ of the number will be} = \frac{x \times 5}{3} = \frac{12 \times 5}{3} = 20 \text{ Ans:}$$

06. What should be the maximum value of q in the following equation?

Sol.  $5P9 - 7Q2 + 9R6 = 823$

$$\Rightarrow (500 + 10P + 9) - (700 + 10Q + 2) + (900 + 10R + 6) = 823$$

$$\Rightarrow (500 + 900 - 700) + 10(P + R - Q) + (9 + 6 - 2) = 823$$

$$\Rightarrow 700 + 10(P + R - Q) = 810$$

$$\Rightarrow 700 + 10(P + R - Q) = 700 + 110$$

$$\Rightarrow 10(P + R - Q) = 110$$

$$\Rightarrow P + R - Q = 11$$

$$\Rightarrow Q = (P + R - 11)$$

To get maximum value of Q we take  $P = 9$  and  $R = 9$

$$\text{This gives } Q = (9 + 9 - 11) = 7$$

Hence, the maximum value of Q is 7 (Ans)

07. The largest natural number, which exactly divides the product of any four consecutive natural numbers, is:

Sol. Let  $P = n(n+1)(n+2)(n+3)$

Then,  $n = 1$  gives

$$P = (1 \times 2 \times 3 \times 4) = 24$$

Hence, the required number is 24

08. When the square of any odd number, greater than 1 is divided by 8, it always leaves remainder:

Sol. Let the number be  $N = 2x + 1$

$$N^2 = (2x + 1)^2$$

$$= 4x^2 + 1 + 4x = 4x(x + 1) + 1$$

Clearly,  $4x(x + 1)$  is always divisible by 8 since one of  $x$  and  $(x + 1)$  is even which when multiplied by 4, always divisible by 8.

Hence, required remainder = 1 (Ans)

### Home Practice

### Number (MCQ)

01. The number 3 divides 'a' with a result of 'b' and remainder of 2. The number 3 divides 'b' with result of 2 and 'a' remainder of 1. What is the value of 'a'? [21 Based Combined Officer General: 2024]

- a) 13      b) 17      c) 23      d) 21

02. Find the largest number of 5-digits which, when divided by 16, 24, 30 or 36, leaves the same remainder 10 each case. [BB-AD: 20-10-23]

- a) 99370      b) 99360  
c) 99350      d) 99340

03. A, B, C, D and E are five consecutive numbers in increasing order of size. Deleting one of the five number sum of the remaining numbers in the set is by 20%. Which one of the following numbers was deleted? [21 Based Combined SO: 10-11-23]

- a) B      b) A      c) D      d) C

04. Find the number of zeroes in  $10 \times 20 \times 30 \times \dots \times 1000$  [20 Based Combined SO: 20-01-2023]

- a) 100      b) 124      c) 130      d) 154

05. A train 240 m long passes a pole in 24 seconds. How long will it take to pass a platform 650 m long? [20 Based Combined SO: 20-01-2023]

- a) 80 Sec      b) 89 Sec  
c) 90 Sec      d) 95 Sec

06. A 3-digit number  $4a3$  is added to another 3 digit number  $948$  to give a 4-digit number  $13b7$ , which is divisible by 11. Then  $(a + b) = ?$  [BD Gas Fields - AM Exam - 2017]

- a) 10      b) 11      c) 12      d) 15

07. When each of the three numbers is divided by 5, the remainders are 0, 3 and 4 respectively. Which of the following could be the sum of the three numbers? [বাংলাবাদের গ্যাস এ.এম (জেনারেল) পরীক্ষা - ২০১৭]

- a) 52      b) 53      c) 54      d) None

08. If  $\frac{x}{y} = 0$ , which of the following must be true?

[বাখারবাদ গ্যাস এ.এম (জেনারেল) পরীক্ষা - ২০১৭]

- a)  $xy > 0$                       b)  $(x - y) > 0$   
c)  $(x + y) > 0$                       d) None

09. If  $m$  is an even integer and  $n$  is an integer (either odd or even), then which of the following will always be even? [বাখারবাদ গ্যাস এ.এম (জেনারেল) পরীক্ষা - ২০১৭]

- I.  $m^2 + n^2 + n$   
II.  $(m - n)(n + 1)$   
III.  $m^2 - n^2 + 1$

- a) Only I                      b) Only II  
c) Only III                      d) both I and II

10. If  $p$  and  $q$  are positive integers with  $pq = 36$ , then  $\frac{p}{q}$  cannot be- [BPEX - AM Exam - 2017]

- a)  $\frac{1}{4}$                       b)  $\frac{4}{9}$                       c)  $\frac{1}{2}$                       d) None

11. If 3 less than twice the number is equal to 2 more than 3 times the number, then 5 more than 5 times the number is- [BPEX - AM Exam - 2017]

- a) -30                      b) -15                      c) 0                      d) None

12. On dividing a number by 5, we get 3 as remainder. What will the remainder when the square of this number is divided by 5? [BPEX - AM Exam - 2017, BD Gas Fields - AM Exam - 2021]

- a) 0                      b) 1                      c) 2                      d) 4

13. If  $x$  is the difference of two consecutive even numbers, which of the following numbers is a divisor of  $x$ ? [BD Gas Fields - AM Exam - 2021]

- a) 4                      b) 7                      c) 8                      d) 9

14. Which of the following numbers is divisible by 3? [BD Gas Fields - AM Exam - 2021]

- a) 177                      b) 200                      c) 331                      d) 457

15. For which of the following values  $x$  is  $\frac{x+78}{x}$  an integer? [BD Gas Fields - AM Exam - 2021]

- a) 9                      b) 10                      c) 11                      d) 13

16. Two fifth of one fourth of three-seventh of a number is 15. What is the half of the number?

[কর্ণফুলী গ্যাস এ.এম (সাধারণ) পরীক্ষা - ২০২১]

- a) 57                      b) 175                      c) 157                      d) 350

17. Of the following. Which is the closest to  $(6.01 \times 501) \div (25.05 \times 19.97)$ ? [BPEX - AM Exam - 2023]

- a) 6                      b) 8                      c) 10                      d) 15

18.  $x, y$  are positive integers. When  $x$  is divided by  $y$ , the remainder is 5. If  $\frac{x}{y} = 5.20$ , what is the value of  $x$ ? [BPEX - AM Exam - 2023]

- a) 130                      b) 155                      c) 330                      d) 425

19. The subtraction between the largest and the smallest number of 4 digits comprised of 0, 1, 2 and 3 is: [IFIC Bank (MTO)-2017]

- a) 3147                      b) 2287                      c) 2987                      d) 2187

20. If  $n$  is even, which of the following cannot be odd? [UCBL-(PO)-2017]

- a)  $n + 3$                       b)  $3n + 1$                       c)  $n^2 - 1$                       d)  $2(n+3)$

21. If positive integers  $x$  and  $y$  are not both odd, which of the following must be even? [SIBL (PO)-2017]

- a)  $xy$                       b)  $x + y$   
c)  $x - y$                       d)  $2(x+y) - 1$

22. If  $n-5$  is an even Integer, what is the next large consecutive even integer? [BD House Building FC (SO)-2017]

- a)  $n-7$                       b)  $n-3$                       c)  $n-4$                       d)  $n-2$

23. If  $x$  and  $y$  are integers and  $x = 32y + 15$ , which of the following must be an odd integer? [FSIB Pro (off)-14]

- a)  $xy$                       b)  $x + y$                       c)  $x + 2y$                       d) None

24. If  $n$  is even, which of the following cannot be odd? [Exim Bank (TO)-2020]

- a)  $n + 3$                       b)  $3(n + 1)$                       c)  $2(n+3)$                       d)  $n^2 - 1$

25. If  $m$  and  $n$  are negative integers, which of the following must be true [DBBL (PO)-2017]

- a)  $m+n < 0$                       b)  $m-n < 0$                       c)  $mn < 0$                       d) None

26. If  $x^2$  is odd, then  $x^2 - x$  is must be- [Jamuna Bank (PO)-17]

- a) odd                      b) even                      c) negative                      d) positive

27. The sum of three consecutive even integers is equal to 4 times the smallest. What is the smallest integer? [Exim Bank-(TAO)-2018]

- a) 2                      b) 6                      c) 8                      d) 10

28. The smallest prime number is [Pubali Bank Lt (SO)-2017]

- a) 0                      b) 1                      c) 2                      d) 3

29. Which one of the following is not prime number? [Janata Bank (EO)-2017]

- a) 31                      b) 61                      c) 91                      d) 71

30. One third the sum of 13 and a certain number is the same as one more than twice the number. Find out the number. [BB Ass: Director-2012]

- a) 6                      b) 2                      c) 5                      d) 3

31. The sum of two numbers is 22. Five times one number is equal to 6 times the other. The bigger of the two numbers is: [BDB Ltd Execu, Officer-2014]

- a) 10                      b) 12                      c) 15                      d) 16

32. The sum of three integers is 40. The largest integer is 3 times the middle integer, and the smallest integer is 23 less than the largest integer. What is the product of the three integers? [SIBL (PO)-2017 + Nation Bank- (PO)-2017]

- a) 1104                      b) 972                      c) 672                      d) 294

33. If three consecutive even numbers, the sum of the 1<sup>st</sup> and 2<sup>nd</sup> is 166, the sum of the 2<sup>nd</sup> and 3<sup>rd</sup> is 170 and the sum of the 3<sup>rd</sup> and twice of 1<sup>st</sup> is 250. The second number is- [Exim Bank (TO)-2020]  
a) 78      b) 82      c) 86      d) None
34. A two-digit number has 3 in its unit digit. The sum of its digits is one seventh of the number itself. What is the number? [PKB (SEO)-2018]  
a) 70      b) 73      c) 63      d) 83
35. What least number must be added to 105, so that the sum is completely divisible by 23? [Pubali Bank JO-2013]  
a) 10      b) 18      c) 21      d) 25
36. When a number is divided by 5, The remainder is 3. If the square of this number is divided by 5 then what is the remainder? [BKB (officer)-2017; Bapex (AM)-17]  
a) 5      b) 4      c) 7      d) 1
37. When a certain number is divided by 7. The remainder is 0. If the remainder is not 0 when the number is divided by 14. Then the remainder must be? [PKB-(EO-cash)-2018]  
a) 7      b) 5      c) 3      d) 8
38. The average of first five multiples of 3? [Uttara Bank (PO)-2017]  
a) 3      b) 9      c) 12      d) 15
39. What is the remainder when  $6^3$  is divided by 8? [Trust Bank. MTO-2015]  
a) 5      b) 3      c) 2      d) 0
40. The sum of a number and its reciprocal is one-eighth of 34. What is the product of the number and its square root- [Pubali Bank Ltd (SO)-2017 + Uttara Bank (PO)-2017]  
a) 8      b) 27      c) 32      d) None
41. If a number is divisible by 102 then it is also divisible by which of the following numbers-  
a) 2      b) 3      c) 17      d) all
42. The smallest 5-digit number exactly divisible by 41 is- [Janata Bank Assistant Executive Officer (Teller)-2015]  
a) 10004      b) 10025      c) 10041      d) 10045
43. If x and y are consecutive positive integers, which of the following must be an even integer? [PKB (EO Cash)-2018 + PKB (SEO)-2018]  
a) x      b) y      c)  $\frac{xy}{2}$       d) xy
44. A number is doubled and 9 is added. If the resultant is trebled, it becomes 75. What is the number? [Rupali Bank Ltd. (SO)-2013 + BDHBL FC (OF)-2017]  
a) 8      b) 6      c) 3.5      d) None
45. In a family, the father took  $\frac{1}{4}$  of the cake and had 3 times as much as each of the other members had. The total number of family members is. [Bangladesh Bank Officer General: 2022]  
a) 3      b) 7      c) 10      d) 12
46. A natural number when increased by 12, equals 160 times reciprocal. The number is- [Bangladesh Bank Officer General: 2022]  
a) 16      b) 8      c) 6      d) 18
47. There are two numbers where the sum of twice the first number and thrice the second number is 100. And the sum of thrice the first number and twice the second number is 120, Which is the largest number? [BB- Officer General: 2022]  
a) 32      b) 12      c) 35      d) 14

## Lecture-13 : Number System (Written)

### Teacher's Work

### Number

#### Finding Number

01. There are two numbers. 1st Number is 12 more than the 2nd number. The average of the two numbers is 19. If 2 is added in both numbers, find the ratio of the numbers. [BB (AD- Research)-2019]

#### Consecutive numbers

02. The sum of 11 consecutive integers is 88. What is the largest of these integers? [Janata & Sonali Bank (SO-IT)-2018]
03. A, B, C, D, E are 5 consecutive numbers in increasing order, deleting one of them from the set decreased the sum of the remaining numbers by 20% of the sum of 5. Which one of the number is deleted from the set? [BB (off)-2015 + BB (AD)-2014 & PKB (SEO)-2018]

### Single equation related

04. Of the three numbers, second is twice the first and is also thrice the third. If the average of the three numbers is 44, then what will be the largest number? [BB(Officer)-2015]

### Double equation related

05. One fifth of a number is equal to  $\frac{5}{8}$  of another number. If 35 added to the first number, it becomes four times of the second number.. The second number is ? [toppr.com]
06. If 5 is added to the sum of two digits of a number consisting of two digits, the sum will be three times the digits of the tenth place. Moreover, if the place of the digits are interchanged, the number thus found will be 9 less the original number. Find the number. [HBFCL (SO)-2015 + Union Bank (SO)-2014 + Rupali Bank Ltd. (SO)-2013 + Dhka Bank (MTO)-2016]

07. In a three digit number unit digit is double of tenth digit and 1.5 times of hundred digit. Sum of three digit is 13. Find the number-[Rupali Bank (SO)-2019]
08. In a three digit number the number in unit place is 75% of tenth digit number, the tenth digit number is greater than hundred digit by 1 & their sum will be 15, find out the number? [Uttara Bank (Cash)-2018]
09. If 50 is subtracted from two-third of a number, the result is equal to sum of 40 and one-fourth of that number. What is the number? [examveda.com]
10. The difference between a two digit number and the number obtained by interchanging the digit is 36. What is the difference between the sum and the difference of the digits of the number if the ratio between the digits of the number is 1:2? [PKB (EO)-2014]
11. In a two digit number, if the number is divided by the product its digits, then the quotient is 2. If 27 is added with the number then the places of the digits are interchanged. What is the number?
12. In a list of numbers, each number after the first is exactly one-third the number immediately preceding it. If the seventh number in the list is 2, what is the fourth number in the list?
13. The digit in the unit's place of a number is equal to the digit in the ten's place of half of that number and the digit in the ten's place of that number is less than the digit in unit's place of half of the number by 1. If the sum of the digits of the number is 7, then what is the number?

### Illustrative

### Number (Written)

01. a, b, c, d, e are 5 consecutive numbers in increasing order, deleting one of them from the set decreased the sum of the remaining numbers by 20% of the sum of 5. Which one of the number is deleted from the set? [BB AD 14, BB AD 12, PKB SEO 18 + Pubali SO-2013]

Sol. Let,  $a = x$

As the series is increasing in order of size.

So,  $b = x+1$ ,  $c = x+2$ ,  $d = x+3$ ,  $e = x+4$ .

Therefore, the sum of five consecutive numbers is

$$x+x+1+x+2+x+3+x+4 = 5x+10$$

Again, let the deleted number is Y.

According to question,

$$(5x + 10) - (5x + 10 - Y) = 20\% \text{ of } (5x + 10)$$

$$\text{Or, } 5x + 10 - 5x - 10 + Y = (5x + 10) \times \frac{20}{100}$$

$$\text{Or, } Y = (5x + 10) \times \frac{1}{5} \therefore Y = x + 2$$

So, the deleted number is  $(x+2)$ ,

This is the value of "c". (Ans.)

02. If the sum of five consecutive integers is S, what is the largest of those integers in terms of S? [BB (Officer)-2015 + PKB (EO)-2018]

Sol. Let, five consecutive integers are  $x, x + 1, x+2, x+3, x+4$ .

According to the question,

$$x+x+1+x+2+x+3+x+4=s$$

$$\text{Or, } 5x+10=s$$

$$\text{Or, } 5x=s-10$$

$$\text{Or, } x = \frac{s-10}{5}$$

$$\text{Or, } x+4 = \frac{s-10}{5} + 4$$

[Add 4 on both sides]

$$\text{Or, } x+4 = \frac{s-10+20}{5}$$

$$\text{Or, } x+4 = \frac{s+10}{5}$$

$$\therefore x + 4 = \frac{s}{5} + 2$$

$$\therefore \text{The largest integer} = \frac{s}{5} + 2 \text{ (Ans.)}$$

03. A two digit number is four times the sum of the two digits. If the digits are reversed, the number so obtained is 18 more than the original number. What is the original number? [Sonali SO-2018]

Sol. Let, the unit digit be y and tenth digit be x.

$$\therefore \text{The number} = (10x + y).$$

1st condition,

$$10x + y = 4(x+y)$$

$$\text{Or, } 10x + y = 4x + 4y$$

$$\text{Or, } 10x - 4x = 4y - y$$

$$\text{Or, } 6x = 3y \therefore y = 2x \dots (i)$$

2nd condition,

$$(10y+x) - (10x+y) = 18$$

$$\text{Or, } 10y+x-10x-y=18$$

$$\text{Or, } 9y-9x=18$$

$$\text{Or, } y-x = 2 \text{ [Dividing both sides by 9]}$$

$$\text{Or, } 2x-x = 2 \text{ [From equation (i)]} \therefore x = 2$$

Putting the value of x in equation (i)

$$\therefore y = 2 \times 2 = 4$$

$$\therefore \text{The number} = 10 \times 2 + 4 = 24 \text{ (Ans.)}$$

04. In a two digit number, the digit in the unit's place is more than twice the digit in ten's place by 1. If the digits in the unit's place and ten's place are interchanged, difference between the newly formed number and the original number is less than the original number by 1. What is the original number?

**Sol.** Let the ten's digit be  $x$ .

So, unit's digit =  $2x + 1$ .

$\therefore$  The original number =  $10x + (2x+1) = 12x+1$

$\therefore$  Reverse of the number =  $10(2x+1) + x = 21x+10$

Difference of reverse number and original number =

$$(21x+10)-(12x+1) = 9x+9$$

According to the question,

$$(12x+1)-1 = (9x+9)$$

$$\text{Or, } 12x-9x = 9$$

$$\text{Or, } 3x=9$$

$$\therefore x = 3$$

$\therefore$  Ten's digit = 3 and unit's digit =  $(2 \times 3 + 1) = 7$ .

$\therefore$  The original number =  $(12 \times 3 + 1) = 37$ . (Ans.)

- 05.** The difference between a two digit number and the number obtained by interchanging the digits is 36. What is the difference between the sum and the difference of the digits of the number if the ratio between digits of the number is 1:2? [PKB EO-2014]

**Sol.** Given that, Unit digit: Tenth digit = 1:2

Let, unit digit =  $x$  and tenth digit =  $2x$

So, the number =  $10 \times 2x + x = 21x$

After interchanging the digits, the number =  $10 \times x + 2x = 12x$

According to the question,

$$21x - 12x = 36 \quad \text{Or, } 9x = 36 \quad \therefore x = 4$$

$\therefore$  Required difference =  $(2x+x) - (2x-x) = 3x - x = 2x = 2 \times 4 = 8$  (Ans.)

- 06.** In a three digit number the number in unit place is 75% of tenth digit number, the tenth digit number is greater than hundred digit by 1 & their sum will be 15, find out the number? [Uttara Cash-2018]

**Sol.** Let, tenth digit be  $4x$

So, unit digit = 75% of  $4x = 3x$  and hundred digit =  $4x-1$

$\therefore$  The number =  $100(4x-1) + 10 \times 4x + 3x =$

$$400x - 100 + 40x + 3x = 443x - 100$$

According to the question,

$$4x + 4x - 1 = 15$$

$$\text{Or, } 8x = 15 + 1 = 16 \quad \therefore x = 2$$

$\therefore$  The number =  $443 \times 2 - 100 = 786$  (Ans.)

- 07.** There are two numbers. 1st number is 12 more than the 2<sup>nd</sup> number. The average of the two numbers is 19. If 2 is added in both numbers, find the ratio of the numbers. [BB AD (Reserch)-2019]

**Sol.** Let, the 2nd number be  $x$  and 1st number =  $x+12$

As per question,

$$\frac{x+x+12}{2} = 19$$

$$\text{Or, } 2x + 12 = 38 \quad \text{Or, } 2x = 38 - 12 = 26$$

$$\therefore x = 13$$

2nd number = 13 and 1st number =  $13 + 12 = 25$

$\therefore$  Required ratio =  $(25+2) : (13+2) = 27 : 15 = 9:5$

(Ans.)

### Home Practice

### Number (Written)

- 01.** The difference between two numbers is 5 and the difference of their squares is 65. What is the larger number? [BB(Offi)-2015] 9
- 02.** The sum of square of two numbers is 80 and the square of their difference is 16. Determine the product of the two numbers? [Shahjalal Islami Bank (TSO)-2007] 32
- 03.** An old man distributed all gold coins he had to his two sons into two different numbers such that the difference between the squares of the two numbers is 36 times the difference between the two numbers. How many coins did the old man have? [Shahajalal Islam Bank (TO Cash)-2013] 36
- 04.** If the sum of five consecutive integers is  $S$ , what is the largest of those integers in terms of  $S$ ? [BB (Offi)-2015]  $\frac{S}{5} + 2$
- 05.**  $R$  is a list of 15 consecutive integers, and  $T$  is a list of 21 consecutive integers. The median of the integers in list  $R$  is equal to the least integer in list  $T$ . If the two lists are combined into one list of 36 integers, how many different integers are on the combined list? [Dhaka Bank (MTO)-2021] 28
- 06.** The average of three numbers is 135. The largest number is 180 and the difference of the other number is 25. What would be the smallest number? [BKB (CO)-2013] 100
- 07.** Of two numbers the sum of thrice of first and double of the second is 59. Again, the difference of second number from the double of first is 9. Find the two numbers? 11 & 13

08. There are two numbers such that the sum of twice the first and thrice the second is 39, while the sum of thrice the first and twice the second is 36. The largest of the two is [Janata Bank (EO)-2017] 9
09. Sum of the digits of a number consisting of two digits is 9; If the number obtained by interchanging the places of the digits is less by 45 than the given number, what is the number? 72
10. Sum of the digits of a number consisting of two digits is 7. If the places of the digits are interchanged, the number so formed is 9 more than the given number. Find the number. 34
11. A two digit number is four times the sum of the two digits. If the digits are reversed, the number so obtained is 18 more than the original number. What is the original number? [Sonal Bank (SO)-2018] 24
12. The digit of the unit's place of a number consisting of two digits is 1 more than three times the digit of tens place. But if the places of the digits are interchanged, the number thus found will be equal to eight times the sum of the digits. What is the number? 27
13. The digit of unit place of a number consisting two digits is 2 more than the digit of its tens place. If the places of the digits are interchanged, the number thus formed will be less by 6 than twice the given number. Find the number. 24
14. If a number of two digits is divided by the product of its digits the quotient is 3. When 18 is added to the number, the digits of the number change their places. Find the number. [Shahajala Islami Bank (Cash)-2013] 24
15. The sum of the digits of a two-digit number is subtracted from the number. How many such two-digit numbers can be formed so that the digit in the unit place of the resulting number is 6? [Bangladesh Bank (Officer)-2018] 10
16. The digits of a three-digit number add up to 18. If the tens digit is twice the hundreds digit and the hundreds digit is  $\frac{1}{3}$  the units digit, what is the number [Dhaka Bank (TACO)-2021] 369
17. A number consists of 3 digits whose sum is 10. The middle digit is equal to the sum of the other two and the number will be increased by 99 if these two digits are reversed. What is the number? [DBBL (AO)-2017] 253
18. In a three-digit number, the digit in the units place is four times the digit in the hundreds place. If the digit in the units place and the tens place are interchanged, the new number so formed is 18 more than the original number. If the digit in the hundreds place is one-third of the digit in the tens place, then what is 25% of the original number? 67
19. A number when divided successively by 4 and 5 leaves remainders 1 and 4 respectively. When it is successively divided by 5 and 4, what will be the respective remainders? [Janata Bank (EO)-2017] 2 and 3
20. When a number is successively divided by 7, 5, and 4, it leaves remainders of 4, 2 and 3 respectively. What will be the respective remainders when the smallest such number is successively divided by 8, 5 and 6? 3, 0, 3
21. The difference between two numbers is 5 and the difference of their squares is 65. What is the larger number? 9
22. The sum of three numbers is 264. If the first number be twice the second and third number be one-third of the first, then the second number is- 72
23. In a two digit number digit is known that its units digits by 2 and that the product of the given number and the sum of its digits is equal to 144, then are reversed, the number is? 24
24. A two digit number is such that the product of the digits is 8. When 18 is added to the number, then the digits are reversed, the number is- 24
25. A two digit number becomes five-sixth of itself when its digits are reversed. The two digits differ by one. What is the number? 54
26. The sum of the two digits of a number is 8. If the number is subtracted from the number obtained by reversing its digits, the result is 54. Find the number. 17
27. In a division problem, the divisor is 7 times of quotient and 5 times of remainder. If the dividend is 6 times of remainder, then the quotient is equal to- 1
28. A two digit number is six times the sum of the two digits. If the digits are reversed, the number so obtained is 9 less than the original number. What is the original number [Rupali Bank (Off)-2019] 54

# Lecture-15 : Exponent & Logarithm (MCQ & Written)

## Important Formula of Exponent

- |   |  |
|---|--|
| 1. $(a^m)^n = a^{mn}$                         | 8. $\sqrt[q]{a} = a^{\frac{1}{q}}$                               |
| 2. $a^m \cdot a^n = a^{m+n}$                  | 9. $a^{-n} = \frac{1}{a^n}$                                      |
| 3. $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$ | 10. $(ab)^m = a^m b^m$   |
| 4. $a^0 = 1$                                  | 11. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$               |
| 5. $\sqrt{a} = a^{\frac{1}{2}}$               | 12. $\left(\frac{m}{n}\right)^{-p} = \left(\frac{n}{m}\right)^p$ |
| 6. $\sqrt[3]{a} = a^{\frac{1}{3}}$            |  |
| 7. $\sqrt[3]{a^2} = a^{\frac{2}{3}}$          |  |

### Teacher's Work

### Exponent (MCQ)

- $(10)^2$  is how many times of  $(0.01)^3$ ? [BB-AD: 2022]  
a)  $10^5$     b)  $10^6$     c)  $10^7$     d)  $10^8$
- $2^{12} + 2^{12} + 2^{12} + 12^{12} = ?$  [BB- Officer General: 2022]  
a)  $2^{12}$     b)  $2^{14}$     c)  $2^{16}$     d)  $2^{4x}$
- If  $\left(7^{\frac{3}{4}}\right)^n = 7$ , what is the value of  $n$ ?  
a)  $\frac{1}{3}$     b)  $\frac{2}{3}$     c)  $\frac{4}{3}$     d)  $\frac{5}{3}$
- If  $y = 4^{10} + 4^{11} + 4^{12} + 4^{13}$ , then  $y$  is divisible by which number?  
a) 12    b) 13    c) 17    d) 19
- The value of  $(9 \times 10^7)(9 \times 10^8)$  is closest to which of the following?  
a)  $10^{16}$     b)  $10^{17}$     c)  $10^{56}$     d)  $10^{57}$
- If  $x$  and  $y$  are positive integers, and  $x^3 y^4 = 2,000$ , which of the following is the value of  $xy$ ?  
a) 2    b) 4    c) 8    d) 10
- Which of the following is equal to  $(2^k)(5^{k-1})$ ?  
a)  $2(10^{k-1})$     b)  $5(10^{k-1})$   
c)  $10^k$     d)  $2(10^k)$     e)  $10^{2k-1}$
- If  $\sqrt{x} = 25$ , then  $x^3 - x^2 =$   
a)  $5^{12}$     b)  $5^{20}$     c)  $6(5^5)$   
d)  $5^4(5^2 + 1)$     e)  $5^8(5^4 - 1)$
- If  $5^x - 5^{x-3} = (124)(5^y)$ , what is  $y$  in terms of  $x$ ?  
a)  $x$     b)  $x - 6$     c)  $x - 3$   
d)  $2x + 3$     e)  $2x + 6$

- If  $3^{6x} = 8,100$ , what is the value of  $(3^{x-1})^3$ ?  
a) 90    b) 30    c) 10  
d)  $\frac{10}{3}$     e)  $\frac{10}{9}$
- If  $\frac{0.0015 \times 10^m}{0.03 \times 10^k} = 5 \times 10^7$ , then  $m - k =$   
a) 9    b) 8  
c) 7    d) 6    e) 5
- $\frac{4.8(10^9)}{1.6(10^3)} =$   
a)  $30(10^5)$     b)  $[3(10)]^6$   
c)  $30^5$     d)  $30(10^6)$     e)  $3(10)^{12}$
- What is the least number of digits (including repetitions) needed to express  $10^{100}$  in decimal notation?  
a) 4    b) 100    c) 101    d) 1,000
- If  $x^{-7/2} = \frac{1}{128}$  then the value of  $X$  is? [Sonali Bank Officer (General)-2018]  
a) 8    b) -4    c) 4    d) 2
- If  $a^x = b$ ,  $b^y = c$  and,  $c^z = a$  then the value of  $xyz$  is [PKB-(EO Cash)- 2018] + [Janata (Cash)-2020]  
a) 0    b) 1    c)  $\frac{1}{abc}$     d)  $abc$
- What is the value of  $\sqrt{10 + \sqrt{25 + \sqrt{108 + \sqrt{154 + \sqrt{225}}}}}$  [21 Base Combined SO: 10-11-23]  
a) 6    b) 4  
c) 2    d) 1
- Which of the following is the value of the expression  $\sqrt{2\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}}$ ? [Uttara Bank: 2021]  
a)  $2^{\frac{29}{31}}$     b)  $2^{\frac{99}{8}}$     c)  $2^{\frac{31}{32}}$     d)  $2^{\frac{31}{2}}$
- The expression  $\sqrt{2\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}}$  extends to an infinite number of roots. Which of the following choices most closely approximate the value of this expression? [IBA, MBA 56 intec]  
a)  $\sqrt{3}$     b) 2    c)  $1 + \sqrt{3}$     d)  $1 + \sqrt{2}$
- Find the value of:  $\sqrt{6 + \sqrt{6 + \sqrt{6 + \dots}}} = ?$   
a)  $\sqrt{3}$     b) 3    c)  $1 + \sqrt{3}$     d) 2

### Important Formula: Logarithm

01.  $\log_a(MN) = \log_a M + \log_a N$

02.  $\log_a \frac{M}{N} = \log_a M - \log_a N$

03.  $\log_a + \log_b + \log_c = \log(abc)$

04.  $\log_a M^n = n \log_a M$

05.  $\log_a 1 = 0$

06.  $\log_a a = 1$

07.  $\log_a M^n = n \log_a M$

08.  $\log_a - \log_b = \log \frac{a}{b}$

09. If  $\log_a^y = x$  then  $a^x = y$

### Teacher's Work

### Logarithm (MCQ)

20. Log 36/Log 6 \_\_\_\_? [Agrani Bank- (Cash)-2017] & [BB-officer)-2018]

- a) 5      b) 8      c) 3      d) 2

21. The value of  $\log_{\sqrt{2}} 32$  is:

- a)  $\frac{5}{2}$       b) 5      c) 10      d)  $\frac{1}{10}$

22. The value of  $\log_{10}(0.0001)$  is:

- a)  $\frac{1}{4}$       b)  $-\frac{1}{4}$       c) -4      d) 4

23. If  $\log_x \frac{1}{64} = -6$ , the  $x = ?$

- a)  $\frac{-1}{2}$       b)  $\frac{1}{2}$       c) 2      d) 3

24.  $\log_x 4 = -2$ , the  $x = ?$  [Rupali Bank Off- (Cash)-2018]

- a)  $\frac{-1}{2}$       b)  $\frac{1}{2}$       c) 2      d) 17

25. If  $\log_2 \sqrt{5} 400 = x$ , then  $x$  is equal to:

- a) 4      b) -4      c) 5      d) 8

26.  $2^{\log_2 3} + \log_2 5 = ?$

- a) 8      b) 2      c) 15      d) 10

27.  $\log_3 27 = ?$

- a) 1      b) 2      c) 3      d) 4

### Teacher's Work

### Exponent & Logarithm (Written)

28.  $\frac{4\sqrt{p} 16^{pq} \times 2^{-2}\sqrt{p}}{8\sqrt{q} \times 4^{pq} \times 2^{-3}\sqrt{q}} \times 4^{-pq} = ?$  [BB AD Written Exam 2022]

29. If  $10000 = 10^{p+r}$  and  $100 = 10^{p-r}$  then find the value of  $p$  and  $r$ ? [Bangladesh Krishi Bank (SO) Examination-2011- (Written)]

### Illustrative Questions

### Exponents (MCQ & Written)

01.  $(17)^{3.5} \times (17)^? = 17^8$

Sol.  $(17)^{3.5} \times (17)^? = 17^8$

$\Rightarrow (17)^? \frac{17^8}{17^{3.5}} \Rightarrow (17)^? = 17^{(8-3.5)} \Rightarrow (17)^? = 17^{4.5}$  Ans:

02. If  $x^{-\frac{7}{2}} = \frac{1}{128}$  then the value of  $x$  is?

Sol.  $x^{-\frac{7}{2}} = \frac{1}{128}$

$\Rightarrow x \times \frac{1}{x^{\frac{7}{2}}} = \frac{1}{2^7} \Rightarrow \frac{1}{x^{\frac{7}{2}}} = \frac{1}{2^7} \Rightarrow \frac{1}{(\sqrt{x})^7} = \frac{1}{2^7} \Rightarrow (\sqrt{x})^7 = 2^7$

$= 2^7 \Rightarrow \sqrt{x} = 2 \therefore x = 4$  Ans:

03.  $3^{20} + 3^{20} + 3^{20} = ?$

Sol.  $3^{20} + 3^{20} + 3^{20} = 3 \cdot 3^{20} = 3^{1+20} = 3^{21}$  Ans:

04.  $4^n = 64$  What is the value of  $n$ ?

Sol.  $4^n = 64 \Rightarrow 4^n = 4^3 \Rightarrow n = 3$  Ans:

05. If  $\sqrt{3^n} = 729$ , the value of  $n$  is: ( $\sqrt{3^n} = 729$  হলে,  $n$  এর মান কত?)

Sol.  $\sqrt{3^n} = 729$ , or,  $\sqrt{3^n} = 3^6$  or, or  $3^n = 3^{12}$  So  $n = 12$  Ans:

06. If  $2^{2x-1} = \frac{1}{3^{x-3}}$ , Then the value of  $x$  is.

Sol.  $2^{2x-1} = \frac{1}{8^{x-3}}$ ,  $2^{(2x-1)} = 8^{-(x-3)} \Rightarrow 2^{(2x-1)} = 2^{(9-3x)}$

$\Rightarrow 2x-1=9-3x \Rightarrow 5x=10 \therefore x=2$  Ans:

07. If  $2^n = 128$ , then  $(2^{n-1})(5^{n-2}) = ?$

Sol.  $2^n = 128$  or  $2^n = 2^7 \Rightarrow n=7$

So,  $(2^{n-1})(5^{n-2}) \Rightarrow 2^{7-1} 5^{7-2} \Rightarrow 2^6 5^5 \Rightarrow 2 \cdot 2^5 5^5$

$\Rightarrow 2(2 \times 5)^5 \Rightarrow 2(10^5)$  Ans:

08. If  $3\sqrt{x} = 2\sqrt{3}$ , what is the value of  $x$ ?

Sol.  $3\sqrt{x} = 2\sqrt{3}$  or  $\sqrt{x} = \frac{2\sqrt{3}}{3}$  or  $x = \left(\frac{2\sqrt{3}}{3}\right)^2$

$\Rightarrow x = \frac{4}{3} \therefore x = 1.33$

09. If  $4^a + 4^{a+1} = 4^{a+2} - 176$ , what is the value of  $a$ ?

Sol.  $4^a + 4^{a+1} = 4^{a+2} - 176$

Or,  $4^a + 4^a \cdot 4 = 4^a \cdot 4^2 - 176$

Or,  $4^a + 4^a \cdot 4^1 - 4^a \cdot 4^2 = -176$

Or,  $4^a(1+4-16) = -176$

Or,  $4^a(-11) = -176$  Or,  $4^a = \frac{-176}{-11}$

Or,  $4^a = 16$  Or,  $4^a = 4^2$  Or,  $a = 2$  Ans:

10. Simplify:  $\left(\frac{x^a}{x^b}\right) ab \cdot \left(\frac{x^b}{x^c}\right) bc \cdot \left(\frac{x^c}{x^a}\right) ca$

Sol.  $\left(\frac{x^a}{x^b}\right) ab \cdot \left(\frac{x^b}{x^c}\right) bc \cdot \left(\frac{x^c}{x^a}\right) ca$   
 $= (x^{a-b}) ab (x^{b-c}) bc \cdot (x^{c-a}) ca$   
 $= x \frac{a-b}{ab} \cdot x \frac{b-c}{bc} \cdot x \frac{c-a}{ca}$   
 $= x \frac{a-b}{ab} + \frac{b-c}{bc} + \frac{c-a}{ca}$   
 $= x \frac{ac-bc+ab+ac+bc-ab}{abc}$   
 $= x \frac{0}{abc} = x^0 = 1 \text{ Ans.}$

11. Simplify:  $\left(\frac{x^{p+q}}{x^{2r}}\right) \left(\frac{x^{q+r}}{x^{2p}}\right) \left(\frac{x^{r+p}}{x^{2q}}\right)$

Sol.  $\left(\frac{x^{p+q}}{x^{2r}}\right) \left(\frac{x^{q+r}}{x^{2p}}\right) \left(\frac{x^{r+p}}{x^{2q}}\right)$   
 $= (x^{p+q-2r}) (x^{q+r-2p}) (x^{r+p-2q})$   
 $= x^{p+q-2r+q+r-2p+r+p-2q}$   
 $= x^{2p+2q+2r-2r-2p-2q}$   
 $= x^0 = 1 \text{ Ans.}$

12.  $\sqrt{\frac{bc}{x} \cdot \frac{b}{x} \cdot \frac{c}{x}} \times \sqrt{\frac{ca}{x} \cdot \frac{c}{x} \cdot \frac{a}{x}} \times \sqrt{\frac{ab}{x} \cdot \frac{a}{x} \cdot \frac{b}{x}}$   
 $= \frac{\frac{b}{c} \times \frac{1}{bc} \times \frac{c}{a} \times \frac{1}{ca} \times \frac{a}{b} \times \frac{1}{ab}}{\frac{c}{b} \times \frac{1}{bc} \times \frac{a}{c} \times \frac{1}{ca} \times \frac{b}{a} \times \frac{1}{ab}}$   
 $= \frac{\frac{1}{c^2} \times \frac{1}{a^2} \times \frac{1}{b^2}}{\frac{1}{b^2} \times \frac{1}{c^2} \times \frac{1}{a^2}} = 1 \text{ Ans.}$

13.  $4^x - 3 \cdot 2^{x+2} + 2^5 = 0$

Or,  $(2^2)^x - 3 \cdot 2^x \cdot 2^2 + 2^5 = 0$

Or,  $(2^x)^2 - 12 \cdot 2^x + 32 = 0$

Or,  $y^2 - 12y + 32 = 0$  [Suppose  $2^x = y$ ]

Or,  $y^2 - 4y - 8y + 32 = 0$

Or,  $y(y-4) - 8(y-4) = 0$

Or,  $(y-4)(y-8) = 0$

$\therefore y-4 = 0$  Or,  $y-8 = 0$

Or,  $2^x - 4 = 0$  Or,  $2^x - 8 = 0$  [ $\therefore 2^x = y$ ]

Or,  $2^x = 4 = 2^2$

Or,  $2^x = 8 = 2^3$

$\therefore x = 2$

$\therefore x = 3$

$\therefore$  Value of  $x = 2, 3$  Ans.

14.  $\frac{\sqrt[3]{27 \cdot 2^n} - \sqrt[3]{64 \cdot 2^{n-2}}}{2^n - 2^{n-1}}$

Sol.  $\frac{\sqrt[3]{27 \cdot 2^n} - \sqrt[3]{64 \cdot 2^{n-2}}}{2^n - 2^{n-1}}$   
 $= \frac{\sqrt[3]{3^3 \cdot 2^n} - \sqrt[3]{4^3 \cdot 2^{n-2}}}{2^n - 2^{n-1}}$   
 $= \frac{3 \cdot 2^{n/3} - 4 \cdot 2^{(n-2)/3}}{2^n - 2^{n-1}} = \frac{3 \cdot 2^{2n} - 4 \cdot 2^n \cdot 2^{-2}}{2^n - 2^{n-1}}$   
 $= \frac{2^n \left(3 - 4 \cdot \frac{1}{2^2}\right)}{2^n \left(1 - \frac{1}{2}\right)} = \frac{3 - 4 \cdot \frac{1}{4}}{1 - \frac{1}{2}}$   
 $= \frac{3-1}{2-1} = \frac{2}{1} = 2 \times \frac{2}{1} = 4 \text{ Ans.}$

15. Simplify:  $\frac{2^{n+1} \cdot 3^{2n-m} \cdot 5^{m+n} \cdot 6^m}{6^n \cdot 10^{m+2} \cdot 15^n}$

Sol.  $\frac{2^{n+1} \cdot 3^{2n-m} \cdot 5^{m+n} \cdot 6^m}{6^n \cdot 10^{m+2} \cdot 15^n}$   
 $= \frac{2^{n+1} \cdot 3^{2n-m} \cdot 5^{m+n} \cdot (3 \times 2)^m}{(2 \times 3)^n \cdot (2 \times 5)^{m+2} \cdot (3 \times 5)^n}$   
 $= \frac{2^{n+1} \cdot 3^{2n-m} \cdot 5^{m+n} \cdot 2^m \cdot 3^m}{2^n \cdot 3^n \cdot 2^{m+2} \cdot 5^{m+2} \cdot 3^n \cdot 5^n}$   
 $= \frac{2^{n+1+m} \cdot 3^{2n-m+m} \cdot 5^{m+n}}{2^{n+m+2} \cdot 3^{n+n} \cdot 5^{m+2+n}}$   
 $= \frac{2^{m+n+1} \cdot 3^{2n} \cdot 5^{m+n}}{2^{m+n+2} \cdot 3^{2n} \cdot 5^{m+n+2}}$   
 $= 2^{m+n+1-m-n-2} \cdot 3^{2n-2n} \cdot 5^{m+n-m-n-2}$   
 $= 2^{-1} \cdot 3^0 \cdot 5^{-2}$   
 $= \frac{1}{2} \times 1 \times \frac{1}{5^2} = \frac{1}{2 \times 25} = \frac{1}{50} \text{ Ans.}$

16.  $\frac{a^2 + b^2 - a^{-2} - b^{-2}}{a^2 b^2 - a^{-2} b^{-2}} + \frac{(a-a^{-1})(b-b^{-1})}{ab + a^{-1} b^{-1}}$

Sol.  $\frac{a^2 + b^2 - a^{-2} - b^{-2}}{a^2 b^2 - a^{-2} b^{-2}} + \frac{(a-a^{-1})(b-b^{-1})}{ab + a^{-1} b^{-1}}$   
 $= \frac{a^2 + b^2 - \frac{1}{a^2} - \frac{1}{b^2}}{a^2 b^2 - \frac{1}{a^2} \times \frac{1}{b^2}} + \frac{\left(a - \frac{1}{a}\right) \left(b - \frac{1}{b}\right)}{ab + \frac{1}{a} \times \frac{1}{b}}$   
 $= \frac{\frac{a^4 b^2 + a^2 b^4 - a^2 - b^2}{a^2 b^2}}{\frac{a^4 b^4 - 1}{a^2 b^2}} + \frac{\frac{a^2 - 1}{a} \times \frac{b^2 - 1}{b}}{ab + \frac{1}{ab}}$   
 $= \frac{a^4 b^2 - a^2 + a^2 b^4 - b^2}{a^4 b^4 - 1} + \frac{(a^2 - 1)(b^2 - 1)}{ab + \frac{1}{ab}}$

$$= \frac{a^2(a^2b^2-1) + b^2(a^2b^2-1)}{(a^2b^2)^2-1^2} + \frac{a^2b^2-b^2-a^2+1}{a^2b^2+1}$$

$$= \frac{(a^2b^2-1)(a^2+b^2)}{(a^2b^2+1)(a^2b^2-1)} + \frac{a^2b^2-b^2-a^2+1}{a^2b^2+1}$$

$$= \frac{a^2+b^2+a^2b^2-a^2-b^2+1}{a^2b^2+1} = \frac{a^2b^2+1}{a^2b^2+1} = 1 \text{ Ans.}$$

17.  $2^x + 2^{1-x} = 3$

Sol.  $2^x + 2^{1-x} = 3$

Or,  $2x + \frac{2^1}{2^x} = 3$

Or,  $\frac{(2^x)^2 + 2}{2^x} = 3$

Or,  $(2^x)^2 + 2 = 3 \cdot 2^x$

Or,  $(2^x)^2 - 3 \cdot 2^x + 2 = 0$

Or,  $m^2 - 3m + 2 = 0$  [Suppose  $2^x = m$ ]

Or,  $m^2 - 2m - m + 2 = 0$

Or,  $m(m-2) - 1(m-2) = 0$

Or,  $(m-2)(m-1) = 0$

Or,  $(2^x-2)(2^x-1) = 0$  [ $\therefore m = 2^x$ ]

$2^x - 2 = 0$

Or,  $2^x - 1 = 0$

Or,  $2^x = 2$

Or,  $2^x = 1$

$\therefore x = 1$

Or,  $2^x = 2^0$

$\therefore x = 0$

$\therefore$  Value of  $x = 0$  Or  $1$ . Ans:

18.  $3^{x+5} = 3^{x+3} + \frac{8}{3}$

Sol.  $3^{x+5} = 3^{x+3} + \frac{8}{3}$

Or,  $3^x \cdot 3^5 = 3^x \cdot 3^3 + \frac{8}{3}$

Or,  $3^x \cdot 3^5 - 3^x \cdot 3^3 = \frac{8}{3}$

Or,  $3(3^x \cdot 3^5 - 3^x \cdot 3^3) = 8$

Or,  $3^x \cdot 3^6 - 3^x \cdot 3^4 = 8$

Or,  $3^x \cdot 3^4(3^2-1) = 8$

Or,  $3^x \cdot 3^4(9-1) = 8$

Or,  $3^x \cdot 3^4 \cdot 8 = 8$

Or,  $3^x = \frac{8}{3^4 \cdot 8}$  Or,  $3^x = \frac{1}{3^4}$

Or,  $3^x = 3^{-4}$

$\therefore x = -4$

$\therefore$  Value of  $x = -4$ . Ans:

19.  $3^{2x-2} - 5 \cdot 3^{x-2} - 66 = 0$

Sol.  $3^{2x-2} - 5 \cdot 3^{x-2} - 66 = 0$

Or,  $\frac{3^{2x}}{3^2} - 5 \cdot \frac{3^x}{3^2} - 66 = 0$

Or,  $\frac{3^{2x}}{9} - \frac{5 \cdot 3^x}{9} - 66 = 0$

Or,  $\frac{3^{2x} - 5 \cdot 3^x - 594}{9} = 0$

Or,  $3^{2x} - 5 \cdot 3^x - 594 = 0$

Or,  $(3^x)^2 - 5 \cdot 3^x - 594 = 0$

Or,  $m^2 - 5m - 594 = 0$  [Suppose,  $3^x = m$ ]

Or,  $m^2 - 27m + 22m - 594 = 0$

Or,  $m(m-27) + 22(m-27) = 0$

Or,  $(m-27)(m+22)$

হয়,  $m-27 = 0$

Or,  $m+22 = 0$

Or,  $m = -22$

Or,  $3^x = 27$  [ $\therefore m = 3^x$ ]

Or,  $3^x = -22$  [ $m = 3^x$ ]

Or,  $3^x = 3^3$

But  $3^x \neq -22$  কেননা,  $3^x > 0$

$\therefore x = 3$

$\therefore$  Value of  $x = 3$ . Ans:

### Illustrative

### Logarithm (MCQ & Written)

01. The value of  $\log_{\sqrt{2}} 32$  is:

Sol.  $\log_{\sqrt{2}} 32 = n$ . Then,  $(\sqrt{2})^n = 32 \Rightarrow 2^{\frac{n}{2}} = 2^5$

$\Rightarrow \frac{n}{2} = 5 \Rightarrow n = 10$  Ans:

02.  $\log_x 4 = -2$ , the  $x = ?$

Sol.  $\log_x 4 = -2$  Or,  $x^{-2} = \frac{1}{4}$  Or,  $x^{-2} = 2^{-2} \therefore x = 2$  Ans:

03. If  $\log_x y = 100$  and  $\log_2 x = 10$ , then the value of  $y$  is:

Sol.  $\log_2 x = 10 \Rightarrow x = 2^{10} \therefore \log_x y = 100 \Rightarrow y = x^{100} = (2^{10})^{100} \Rightarrow y = 2^{1000}$  Ans:

04. If  $\log_x \left(\frac{9}{16}\right) = -\frac{1}{2}$ , then  $x$  is equal to:

Sol.  $\log_x \left(\frac{9}{16}\right) = -\frac{1}{2} \Rightarrow x^{-\frac{1}{2}} = \frac{9}{16} \Rightarrow \frac{1}{\sqrt{x}} = \frac{9}{16} \Rightarrow \sqrt{x} = \frac{16}{9}$   
 $\Rightarrow x = \left(\frac{16}{9}\right)^2 \Rightarrow x = \frac{256}{81}$  Ans:

05. If  $\log_{10} 125 + \log_{10} 8 = x$ , then  $x$  is equal to:

Sol.  $\log_{10} 125 + \log_{10} 8 = x \Rightarrow \log_{10} (125 \times 8) = x \Rightarrow x = \log_{10} (1000) = \log_{10} (10)^3 = 3 \log_{10} 10 = 3$  Ans:

06. The value of  $\log_2 16$  is-

Sol:  $\log_2 16 = \log_2 2^4 = 4 \log_2 2 = 4$  Ans:

07. The value of  $\log_5 \left( \frac{125 \times 625}{25} \right)$  is equal to-

Sol:  $\log_5 \left( \frac{125 \times 625}{25} \right) = \log_5 \left( \frac{5^3 \times 5^4}{5^2} \right) = \log_5 5^5 = 5$   
 $\log_5 5 = 5$ . Ans:

08. The value of  $\log \sqrt{2} 32$  is

Sol:  $\log \sqrt{2} 32 = \log_2 2^{1/2} 2^5$   
 $= \frac{5}{\left(\frac{1}{2}\right)} = \log_2 2 = 5 \times 2 = 10$  Ans:

09. Determine the value of  $\log_3 \sqrt{2} \left( \frac{1}{18} \right)$ .

Sol:  $\log_3 \sqrt{2} \left( \frac{1}{18} \right) = \log_3 \sqrt{2} = \left[ \frac{1}{(3\sqrt{2}^2)} \right] = \log_3 \sqrt{2}$   
 $(3\sqrt{2})^{-2} = (-2) \log_3 \sqrt{2} 3\sqrt{2} = -2$ . Ans:

10. The value of  $\log_{10} (.0001)$  is-

Sol:  $\log_{10} (.0001) = \log_{10} \left( \frac{1}{10000} \right) = \log_{10} \left( \frac{1}{10^4} \right) =$   
 $\log_{10} 10^{-4} = -4 \log_{10} 10 = -4$ . Ans:

11. If  $\log_3 x = -2$ , then  $x$  is =

Sol:  $\log_3 x = -2 \Rightarrow x = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ . Ans:

12. If  $\log_x \frac{9}{16} = -\frac{1}{2}$ ,  $x$  is =

Sol:  $\log_x \frac{9}{16} = -\frac{1}{2} \Leftrightarrow x^{-\frac{1}{2}} = \frac{9}{16} \Leftrightarrow \frac{1}{\sqrt{x}} = \frac{9}{16} \Leftrightarrow \sqrt{x} =$   
 $\frac{16}{9} \Leftrightarrow x = \left( \frac{16}{9} \right)^2 = \frac{256}{81}$  Ans:

13. The value of  $\log_2 \log_2 \log_3 (\log_3 27^3)$  is-

Sol:  $\log_2 \log_2 \log_3 (\log_3 27^3)$   
 $= \log_2 \log_2 \log_3 [\log_3 (3^3)^3] = \log_2 \log_2 \log_3 [\log_3 (3)^9]$   
 $= \log_2 \log_2 \log_3 [9 \log_3 3] = \log_2 \log_2 \log_3 9$   
 $[\because \log_3 3 = 1]$   
 $= \log_2 \log_2 [\log_3 (3)^2] = \log_2 \log_2 (2 \log_3 3)$   
 $= \log_2 \log_2 2 = \log_2 1 = 0$ . Ans:

14. The value of  $\log_{10} 1\frac{1}{2} + \log_{10} 1\frac{1}{3} + \dots$  up to 198 terms is =

Sol:  $\log_{10} 1\frac{1}{2} + \log_{10} 1\frac{1}{3} + \dots$  up to 198 terms  
 $= \log_{10} \left( \frac{1}{2} \times \frac{1}{3} \times \dots \times \frac{1}{199} \right)$   
 $\log_{10} \left( \frac{3}{2} \times \frac{4}{3} \times \dots \times \frac{200}{199} \right) = \log_{10} \frac{200}{2}$   
 $= \log_{10} 100 = \log_{10} 10^2 = 2 \log_{10} 10 = 2$ . Ans:

15. If  $\log 2 = x$ ,  $\log 3 = y$  and  $\log 7 = z$  then the value of  $\log \left( 4 \cdot \sqrt[3]{63} \right)$  is-

Sol:  $\log \left( 4 \cdot \sqrt[3]{63} \right) = \log 4 + \log \left( \sqrt[3]{63} \right) = \log 4 + \log (63)^{\frac{1}{3}}$   
 $= \log (2^2) + \log (7 \times 3^2)^{\frac{1}{3}}$   
 $= 2 \log 2 + \frac{1}{3} \log 7 + \frac{2}{3} \log 3 = 2x + \frac{1}{3} z + \frac{2}{3} y$ . Ans:

16. Showed,  $5 \log \frac{2}{5} + 6 \log \frac{8}{5} + 7 \log \frac{15}{16} + \log \frac{32}{25} = 7 \log 3 - 6 \log 5$

Solution:

L.H.S =  $5 \log \frac{2}{5} + 6 \log \frac{8}{5} + 7 \log \frac{15}{16} + \log \frac{32}{25}$   
 $= \log \left( \frac{2}{5} \right)^5 + \log \left( \frac{8}{5} \right)^6 + \log \left( \frac{15}{16} \right)^7 + \log \left( \frac{32}{25} \right)$   
 $= \log \left( \frac{2}{5} \right)^5 + \log \left( \frac{2^3}{5} \right)^6 + \log \left( \frac{3 \times 5}{2^4} \right)^7 + \log \left( \frac{2^5}{5^2} \right)$   
 $= \log \frac{2^5}{5^5} + \log \frac{2^{18}}{5^6} + \log \frac{3^7 \times 5^7}{2^{28}} + \log \frac{2^5}{5^2}$   
 $= \log \left( \frac{2^5}{5^5} \times \frac{2^{18}}{5^6} \times \frac{3^7 \times 5^7}{2^{28}} \times \frac{2^5}{5^2} \right)$   
 $= \log (2^{5+18+5-28} \cdot 3^7 \cdot 5^{7-5-6-2})$   
 $= \log (2^0 \cdot 3^7 \cdot 5^{-6})$   
 $= \log \left( 1 \cdot 3^7 \cdot \frac{1}{5^6} \right) = \log \frac{3^7}{5^6}$   
 $= \log 3^7 - \log 5^6$   
 $= 7 \log 3 - 6 \log 5$   
 $=$  R.H.S

$\therefore$  L.H.S = R.H.S. (Proved)

17.  $\log_7 \left( \sqrt[5]{7} \cdot \sqrt{7} \right) - \log_3 \left( \sqrt[3]{3} \right) + \log_4 2$

Solution:

$\log_7 \left( \sqrt[5]{7} \cdot \sqrt{7} \right) - \log_3 \left( \sqrt[3]{3} \right) + \log_4 2$   
 $= \log_7 \left( 7^{\frac{1}{5}} \cdot 7^{\frac{1}{2}} \right) - \log_3 3^{\frac{1}{3}} + \log_4 \sqrt{4}$   
 $= \log_7 7^{\frac{1}{5} + \frac{1}{2}} - \log_3 3^{\frac{1}{3}} + \log_4 4^{\frac{1}{2}}$   
 $= \log_7 7^{\frac{2+5}{10}} - \frac{1}{3} \log_3 3 + \frac{1}{2} \log_4 4$   
 $= \frac{7}{10} \log_7 7 - \frac{1}{3} \log_3 3 + \frac{1}{2} \log_4 4$   
 $= \frac{7}{10} \cdot 1 - \frac{1}{3} \cdot 1 + \frac{1}{2} \cdot 1 [\because \log_a a = 1]$   
 $= \frac{7}{10} - \frac{1}{3} + \frac{1}{2}$

$$= \frac{21 - 10 + 15}{30}$$

$$= \frac{36 - 10}{30} = \frac{26}{30}$$

$$= \frac{13}{15} \text{ (Ans)}$$

18. Showed,  $\frac{1}{\log_a(abc)} + \frac{1}{\log_b d(abc)} + \frac{1}{\log_c a(abc)} = 1$

**Solution:**

Suppose,  $\log_a(abc) = x$ ,  $\log_b(abc) = y$ ,  $\log_c(abc) = z$

So,  $a^x = abc$ ,  $b^y = abc$ ,  $c^z = abc$

$$\therefore a = (abc)^{\frac{1}{x}}, b = (abc)^{\frac{1}{y}}, c = (abc)^{\frac{1}{z}}$$

$$\text{Now, } (abc)^1 = abc = (abc)^x (abc)^y (abc)^z$$

$$= (abc)^{\frac{1}{x} + \frac{1}{y} + \frac{1}{z}}$$

$$\therefore \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$$

$$\frac{1}{\log_a(abc)} + \frac{1}{\log_b d(abc)} + \frac{1}{\log_c a(abc)} = 1 \text{ (Showed)}$$

19. Prove that:  $\log_a b - \log_{ab} b = \log_a b \cdot \log_{ab} b$

**Solution:**

L.H.S  $\log_a b - \log_{ab} b$

$$= \frac{1}{\log_b a} - \frac{1}{\log_b ab} \left[ \because \log_a b = \frac{1}{\log_b a} \right]$$

$$= \frac{\log_b ab - \log_b a}{\log_b a \cdot \log_b ab}$$

$$= \frac{\log_b a + \log_b b - \log_b a}{\log_b a \cdot \log_b ab} \left[ \because \log xy = \log x + \log y \right]$$

$$= \frac{\log_b b}{\log_b a \cdot \log_b ab}$$

$$= \frac{1}{\log_b a \cdot \log_b ab} \left[ \because \log_a a = 1 \right]$$

$$= \frac{1}{\log_b a} \times \frac{1}{\log_b ab} = \log_a b \cdot \log_{ab} b \left[ \because \frac{1}{\log_x y} = \log_y x \right]$$

**(Proved)**

20. Solve,  $\log \frac{a^3 b^3}{c^3} + \log \frac{b^3 c^3}{d^3} + \log \frac{c^3 d^3}{a^3} - 3 \log b^2 c$

**Solution:**

$$\log \frac{a^3 b^3}{c^3} + \log \frac{b^3 c^3}{d^3} + \log \frac{c^3 d^3}{a^3} - 3 \log b^2 c$$

$$= \log \left( \frac{a^3 b^3}{c^3} \cdot \frac{b^3 c^3}{d^3} \cdot \frac{c^3 d^3}{a^3} \right) - \log (b^2 c)^3$$

$$= \log b^6 c^3 - \log b^6 c^3 = 0 \text{ Ans}$$

21. Solve,  $\log 5 + 16 \log \frac{16}{15} + 12 \log \frac{25}{24} + 7 \log \frac{81}{80}$ ?

**Solution:**

$$\log 5 + 16 \log \frac{16}{15} + 12 \log \frac{25}{24} + 7 \log \frac{81}{80}$$

$$= \log 5 + \log \left( \frac{2^4}{3 \times 5} \right)^{16} + \log \left( \frac{5^2}{2^3 \cdot 3} \right)^{12} + \log \left( \frac{3^4}{2^4 \cdot 5} \right)^7$$

$$= \log 5 + \log \frac{2^{64}}{3^{16} \cdot 5^{16}} + \log \frac{5^{24}}{2^{36} \cdot 3^{12}} + \log \frac{3^{28}}{2^{28} \cdot 5^7}$$

$$= \log \left( 5 \times \frac{2^{64}}{3^{16} \cdot 5^{16}} \times \frac{5^{24}}{2^{36} \cdot 3^{12}} \times \frac{3^{28}}{2^{28} \cdot 5^7} \right)$$

$$= \log \left( \frac{5^{1+24-16-7} \cdot 2^{64-36-28} \cdot 3^{28-16-12}}{1} \right)$$

$$= \log \left( \frac{5^{25} \cdot 2^{64-64} \cdot 3^{28-28}}{1} \right)$$

$$= \log (5^{25-23} \cdot 2^{64-64} \cdot 3^{28-28})$$

$$= \log (5^2 \cdot 2^0 \cdot 3^0)$$

$$= \log (5^2 \cdot 1 \cdot 1)$$

$$= \log 5^2 = 2 \log 5 \text{ (Ans.)}$$

22. Solve,  $\frac{\log \sqrt{27} + \log 8 - \log \sqrt{1000}}{\log 1.2} = ?$

**Solution:**  $\frac{\log \sqrt{27} + \log 8 - \log \sqrt{1000}}{\log 1.2}$

$$= \frac{\log(3^3)^{\frac{1}{2}} + \log 2^3 - \log(10^3)^{\frac{1}{2}}}{\log \frac{12}{10}}$$

$$= \frac{\frac{3}{2} \log 3 + 3 \log 2 - \frac{3}{2} \log (5 \times 2)}{\log \frac{12}{10}}$$

$$= \frac{\log (3 \times 4) - \log (5 \times 2)}{\log \frac{12}{10}}$$

$$= \frac{\frac{3}{2} \log 3 + 3 \log 2 - \frac{3}{2} \log 5 - \frac{3}{2} \log 2}{\log 3 + \log 4 - \log 5 - \log 2}$$

$$= \frac{\frac{3}{2} \log 3 + 3 \log 2 - \frac{3}{2} \log 2 - \frac{3}{2} \log 5}{\log 3 + \log 2^2 - \log 5 - \log 2}$$

$$= \frac{\frac{3}{2} \log 3 + \left(3 - \frac{3}{2}\right) \log 2 - \frac{3}{2} \log 5}{\log 3 + 2 \log 2 - \log 5 - \log 2}$$

$$= \frac{\frac{3}{2} \log 3 + \frac{3}{2} \log 2 - \frac{3}{2} \log 5}{\log 3 + \log 2 - \log 5}$$

$$= \frac{\frac{3}{2} (\log 3 + \log 2 - \log 5)}{(\log 3 + \log 2 - \log 5)} = \frac{3}{2} \text{ (Ans)}$$

23. Solve,  $\frac{\log \sqrt{y^3} + y \log x - \frac{y}{x} \log(xz)}{\log(xy) - \log z}$  ?

**Solution:**  $\frac{\log \sqrt{y^3} + y \log x - \frac{y}{x} \log(xz)}{\log(xy) - \log z}$

$$= \frac{\log \sqrt{3^3} + 3 \log 2 - \frac{3}{2} \log(2 \times 5)}{\log(2 \times 3) - \log 5}$$

$$= \frac{\log \sqrt{27} + \log 2^3 - \frac{3}{2} \log 10}{\log 6 - \log 5}$$

$$= \frac{\log(3^3)^{\frac{1}{2}} + \log 2^3 - \log(10^3)^{\frac{1}{2}}}{\log \frac{12}{10}}$$

$$= \frac{\log 3^{\frac{3}{2}} + \log 4^{\frac{3}{2}} - \log 10^{\frac{3}{2}}}{\log \frac{6}{5}}$$

$$= \frac{\frac{3}{2} \log 3 + \frac{3}{2} \log 4 - \frac{3}{2} \log 10}{\log \frac{6}{5}}$$

$$= \frac{\frac{3}{2} \log (3 \times 4 \div 10)}{\log \frac{6}{5}}$$

$$= \frac{\frac{3}{2} \log \left( 3 \times 4 \times \frac{1}{10} \right)}{\log \frac{6}{5}} = \frac{\frac{3}{2} \log \frac{6}{5}}{\log \frac{6}{5}} = \frac{3}{2} \text{ (Ans)}$$

24. Given that  $xy^{a-1} = p$ ,  $xy^{b-1} = q$  &  $xy^{c-1} = r$  what is the value of  $(b-c) \log_k p + (c-a) \log_k q + (a-b) \log_k r$ ?

**Solution:**

$$(b-c) \log_k p + (c-a) \log_k q + (a-b) \log_k r$$

$$= (b-c) \log_k xy^{a-1} + (c-a) \log_k xy^{b-1} + (a-b) \log_k xy^{c-1}$$

$$= (b-c) (\log_k x + \log_k y^{a-1}) + (c-a) (\log_k x + \log_k y^{b-1}) + (a-b) (\log_k x + \log_k y^{c-1})$$

$$= (b-c) \log_k x + (c-a) \log_k x + (a-b) \log_k x + (b-c) \log_k y^{a-1} + (c-a) \log_k y^{b-1} + (a-b) \log_k y^{c-1}$$

$$= (b-c + c-a + a-b) \log_k x + (b-c)(a-1) \log_k y + (c-a)(b-1) \log_k y + (a-b)(c-1) \log_k y$$

$$= 0 + (ab - ca - b + c - bc - ab - c + a + ca - bc - a + b) \log_k y$$

$$= 0 + 0 \text{ Ans: } 0$$

25. If  $a^{3-x} \cdot b^{5x} = a^{5+x} \cdot b^{3x}$  Then proved  $x \log \left( \frac{b}{a} \right) = \log a$ ?

**Solution:**

$$a^{3-x} \cdot b^{5x} = a^{5+x} \cdot b^{3x}$$

$$\text{Or, } \frac{a^{5+x}}{a^{3-x}} = \frac{b^{5x}}{b^{3x}}$$

$$\text{Or, } a^{5+x-3+x} = b^{5x-3x}$$

$$\text{Or, } a^{2+2x} = b^{2x}$$

$$\text{Or, } \log a^{2+2x} = \log b^{2x} \text{ [With log on both sides]}$$

$$\text{Or, } (2+2x) \log a = 2x \log b$$

$$\text{Or, } 2(1+x) \log a = 2x \log b$$

$$\text{Or, } (1+x) \log a = x \log b \text{ [Divide both sides by 2]}$$

$$\text{Or, } \log a = x \log a = x \log b$$

$$\text{Or, } x \log b - x \log a = x \log a$$

$$\text{Or, } x (\log b - \log a) = \log a$$

$$\text{Or, } x \log \left( \frac{b}{a} \right) = \log a$$

$$\therefore x \log \left( \frac{b}{a} \right) = \log a \text{ (Proved) } \mid$$

26. If  $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$  what is the value of  $a^a \cdot b^b \cdot c^c$ ?

**Solution:**

**Given that**

$$\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$$

$$\text{Suppose, } \frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b} = x$$

$$\text{So } \frac{\log a}{b-c} = x$$

$$\Rightarrow \log a = x(b-c) \dots \dots \dots \text{(i)}$$

$$\text{Again, } \frac{\log b}{c-a} = x$$

$$\Rightarrow \log b = x(c-a) \dots \dots \dots \text{(ii)}$$

$$\&, \frac{\log c}{a-b} = x$$

$$\Rightarrow \log c = x(a-b) \dots \dots \dots \text{(iii)}$$

$$\text{Now, } \log a^a \cdot b^b \cdot c^c = \log a^a + \log b^b + \log c^c$$

$$= a \log a + b \log b + c \log c$$

$$= a \{x(b-c)\} + b \{x(c-a)\} + c \{x(a-b)\}$$

[From (i), (ii) & (iii)]

$$= ax(b-c) + bx(c-a) + cx(a-b)$$

$$= abx - acx + bcx - abx + acx - bcx$$

$$= 0$$

$$\Rightarrow \log a^a \cdot b^b \cdot c^c = \log 1 \text{ [We know } \log 1 = 0]$$

$$\therefore a^a \cdot b^b \cdot c^c = 1 \text{ [Subtracted log from both sides]}$$

$$a^a \cdot b^b \cdot c^c = 1 \text{ Ans:}$$

27. If  $a^3 + b^3 = 0$ , Prove that  $\log(a + b) = \frac{1}{2}(\log a + \log b + \log 3)$ ; ( $a + b \neq 0$ )

**Solution:**

Given that  $a^3 + b^3 = 0$  & ( $a + b \neq 0$ )

$$\Rightarrow (a + b)(a^2 - ab + b^2) = 0$$

$$\therefore a^2 - ab + b^2 = 0 \quad [\because a + b \neq 0]$$

$$\Rightarrow a^2 - ab + b^2 + 3ab = 3ab \quad [\text{Adds } 3ab \text{ to both sides}]$$

$$\Rightarrow a^2 - 2ab + b^2 = 3ab$$

$$\Rightarrow (a + b)^2 = 3ab$$

$$\Rightarrow \log(a + b)^2 = \log(3ab) \quad [\text{With log on both sides}]$$

$$\Rightarrow 2 \log(a + b) = \log 3 + \log a + \log b$$

$$[\because \log xyz = \log x + \log y + \log z]$$

$$\therefore \log(a + b) = \frac{1}{2}(\log a + \log b + \log 3) \quad \text{Proved}$$

### Home Practice

### Exponent (MCQ)

01.  $(1000)^7 \div (10)^{18}$ ? [কর্ণফুলী গ্যাস এ.ম (সাধারণ) পরীক্ষা - ২০২১]  
a) 10      b) 100      c) 10000      d) 1000
02. If  $a^x = b$ ,  $b^y = c$ , and  $c^z = a$  then the value of  $xyz$  is-[কর্ণফুলী গ্যাস এ.ম (সাধারণ) পরীক্ষা - ২০২১]  
a) 1      b) 0      c) abc      d)  $\frac{1}{abc}$
03.  $(17)^{3.5} \times (17)^7 = 17^8$  [Trust Bank Ltd. (MTO)-2016]  
a) 2.29      b) 2.75      c) 4.25      d) 4.5
04. What is the value of  $(2^6)^{\frac{2}{3}}$ ?  
a) 16      b) 12      c) 8      d) 4
05.  $(2x^{-1})^2 \times x^{-5}$  is equal to [Sonali Bank Officer (General)-2018]  
a)  $2x^2$       b)  $4x$       c)  $4x^2$       d)  $4x^{-7}$
06.  $5^{-3} + 5^{-3} + 5^{-3} + 5^{-3} + 5^{-3} = ?$  [Agrani Bank- (SO)-2017]  
a)  $25^{-15}$       b)  $25^{-2}$       c)  $5^{-2}$       d)  $5^{-15}$
07.  $2^{30} + 2^{30} + 2^{30} + 2^{30} = ?$  [Bank Asia Pro. Off-2014]  
a)  $8^{120}$       b)  $8^{30}$       c)  $2^{32}$       d)  $2^{30}$
08.  $3^{20} + 3^{20} + 3^{20} = ?$  [MBA-2010]  
a)  $9^{20}$       b)  $9^{60}$       c)  $3^{60}$       d)  $3^{21}$
09. If  $p^x = q$ ,  $q^y = r$  and  $r^z = p$  then the value of  $xyz$  is [Sonali bank (SO-FF-quota)-2019]  
a) 1      b) -1      c) 0      d) pqr
10. If  $r$  and  $s$  are positive integers such that  $(2^r)(4^s) = 16$ , then  $2r + s = ?$  [MTB-(MT)-2017]  
a) 3      b) 4      c) 5      d) 6
11.  $4^n = 64$  হলে  $n$  এর মান নিচের কোনটি হবে? [Sonali Bank S.off- 2014]  
a) 0      b) 1      c) 3      d) 4

12. If  $\sqrt{2^n} = 64$ , what will be the value of  $n$ ? [Janata Bank (EO)-2017 (afternoon)] + [Janata Bank (AEO-Teller)-2019]  
a) 2      b) 4      c) 6      d) 12
13. If  $\sqrt{3^n} = 729$ , the value of  $n$  is: [Agrani bank-(SO)2017]  
a) 6      b) 8      c) 12      d) 10
14. If  $4^{x+1} = 32$ , then  $x = ?$  [BKB - (Cash)-2017]  
a) 2      b) 3      c)  $\frac{3}{2}$       d)  $\frac{2}{3}$
15. For what value of  $x$  is  $8^{2x-4} = 16^x$ ? [Agrani Bank -(SO)-2017]  
a) 2      b) 3      c) 4      d) 6
16. If  $16^{2x+4} = 4^{3x+3}$  then  $x = ?$  [BKB (Officer)-2017]  
a) -5      b) 1      c) 13/5      d) -1
17. If  $x = 4$ , then  $-2^2\sqrt{x} + 2$ ? [Exim Bank T. Off-2015]  
a) -14      b) -8      c) -2      d) 0
18. If  $2^n = 128$ , then  $(2^{n-1})(5^{n-2}) = ?$  [Al-Arafah IB MTO 2011]  
a)  $2(10^5)$       b)  $5(10^5)$       c)  $2(10^6)$       d)  $2(5^6)$
19. Given  $2^{x+3} = 32$ , What is the value of  $3^{x+2}$ ? [MBM 2009]  
a) 4      b) 27      c) 81      d) 125
20. If  $\left(\frac{p}{q}\right)^{n-1} = \left(\frac{q}{p}\right)^{n-3}$ , Then, the value of  $n$  is- [PKB-(EO)-19]  
a) 2      b)  $\frac{1}{2}$       c)  $\frac{7}{2}$       d) 1
21.  $\left(\frac{3}{4}\right)^5 \times \left(\frac{3}{4}\right)^{-5}$  এর মান কোনটি হবে?  
a) 1      b) 2      c) 0      d) 6
22. If  $3\sqrt{x} = 2\sqrt{3}$ , what is the value of  $x$ ? [BKB (SO)-2017]  
a) 3      b) 1.33      c) 2      d)  $3\sqrt{2}$
23.  $(0.04)^{-1.5} = ?$  [Janata bank (Eo)-2017 (morning)]  
a) 25      b) 250      c) 125      d) 625
24. For what value of  $x$  is  $8^{2x-4} = 16^x$ ? [PKB-(EO Cash)-2018] + [Combi-4 Banks (Off)-2019]  
a) 2      b) 3      c) 4      d) 6
25. If  $32^{a+b} = 16^{a+2b}$ , then  $a = ?$  [Exim Bank 2004]  
a) b      b)  $b+2$       c)  $2b$       d)  $3b$
26. If  $a$  and  $b$  are positive real numbers, then  $(a^0 - 3b^0)^5 = ?$  [BKB- (SO)-2017]  
a) 0      b) 1      c) -32      d) -1
27. If  $3^x - 3^{x-1} = 18$ , the value of  $x^x$  is: [Bank Asia Pro. Off-2014]  
a) 3      b) 27      c) 8      d) 216

28.  $\sqrt[4]{x} \times x^4 = ?$  [Sonali Bank S.off-2014]  
 a)  $\sqrt{x}$     b)  $x$     c)  $\frac{1}{x^4}$     d) 1
29. Which one is true for the mathematical expression  $\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}$ ? [Bank Asia Ltd. R.P.Off-2015]  
 a)  $\frac{3}{2}$     b)  $\frac{2}{3}$     c) 1    d)  $2^{30}$
30. What is the following is equal to  $3.14 \times 10^6$ ? [Janata Bank (EO)-2017 (afternoon)]  
 a) 314    b) 3140    c) 3140000    d) None
31.  $(256)^{0.16} \times (16)^{0.18}$   
 a) 4    b) 8    c) 12    d) 16
32. What is the remainder when  $3^{24}$  is divided by 5?  
 [Bank Asia (MTO)-2017]  
 a) 0    b) 1    c) 2    d) 3
33. If  $1^0 = 1$ , then  $100^0$  is: [EMBA DU-12]  
 a) 50    b) 25    c) 10    d) 1
34.  $17^3 + 17^4 = ?$  [National Bank-(PO)-2017]  
 a)  $17^7$     b)  $17^{12}$     c)  $17^3(18)$     d)  $2(17^4) + 17$
35.  $50^7 \times 20^7$  is  $10^x$  times larger than  $1 \times 10^8$ , where x is:  
 [Agrani Bank-(SO)-2017]  
 a) 13    b) 6    c) 21    d) 29
36.  $3^x + 3^x + 3^x = ?$  [BKB-(SO)-2017]  
 a)  $9x$     b)  $27x^2$     c)  $3^{x+1}$     d)  $3x^3$
37. If  $n = (33)^{43} + (43)^{33}$  what is the units digit of n?  
 [Bkb-(SO)-2017]  
 a) 0    b) 2    c) 4    d) 6
38. If  $8^{2x+3} = 2^{3x+6}$  then x =? [Al-Arafah bank 06 Commercial Bank 06]  
 a) -1    b) 1    c) 1/2    d) 2
39. Which of the following is not equal to 0.0675? [BB-Officer General: 2022]  
 a)  $67.5 \times 10^{-2}$     b)  $6.75 \times 10^{-2}$   
 c)  $0.675 \times 10^{-1}$     d)  $0.00757 \times 10^2$

### Home Practice

### Logarithm (MCQ)

01.  $3\log 2 + \log 5 = ?$   
 a)  $\log 20$     b)  $\log 40$     c)  $\log 60$     d)  $\log 13$
02.  $\log_a 2 + \log_a 4 + \log_a 8 = ?$   
 a)  $\log_a 12$     b)  $\log_a 1$     c)  $\log_a 64$     d)  $\log_a 32$
03.  $\log_a 7 + \log_a \sqrt{7} + \log_a \sqrt{7} = ?$   
 a)  $\log_a 10$     b)  $\log_a 40$     c)  $\log_a 49$     d)  $\log_a 35$
04.  $\frac{\log 36}{\log 6} = ?$  [Agrai Bank-(Cash)-2017] & [BB-(officer)-2018]  
 a) 5    b) 8    c) 3    d) 2

05.  $\log_2 32 = ?$   
 a)  $\frac{1}{25}$     b) -5    c)  $\frac{1}{5}$     d)  $-\frac{1}{5}$
06.  $\log_3 9 = ?$   
 a) 2    b) -2    c) 3    d) -3
07. If  $\log_x 9 = -2$  then the value of x = ?  
 a) 3    b) 2    c)  $\frac{1}{3}$     d)  $-\frac{1}{3}$
08. If  $\text{Log}_x 8 = -2$  then the value of x = ?  
 a) 2    b)  $\sqrt{2}$     c)  $2\sqrt{2}$     d) 4
09. If  $\text{Log}_x \frac{1}{16} = -6$  then the value of x = ?  
 a) 2    b) 3    c) 4    d) 5
10. If  $\log_x 2\sqrt[6]{16} = -\frac{1}{2}$  then the value of base :  
 [Combined 5 banks (off:-2018)]  
 a)  $\frac{16}{9}$     b)  $\frac{9}{16}$     c)  $\frac{256}{81}$     d)  $\frac{81}{256}$
11. If  $\frac{1}{5} \log_x (2187\sqrt{3}) = 1$  then the value of x = ?  
 a)  $3\sqrt{3}$     b) 3    c)  $\sqrt{3}$     d)  $\frac{1}{\sqrt{3}}$
12.  $\text{Log}_b a^2 \cdot \log_b b^2 \cdot \log_a c^2 = ?$   
 a) 1    b) 2    c) 6    d) 8
13.  $\log_5 (x^2 + x) - \log_5 (x + 1) = 3$  find the value of  
 [Microcredit Regulatory Authority (AD)-2021]  
 a) 130    b) 125    c) 145    d) 135
14.  $\log 5 \sqrt{5\sqrt{5}\sqrt{5}} \dots \infty = x = ?$   
 a) 3    b)  $\sqrt{5}$     c) 1    d)  $\sqrt{3}$
15. What is the sum of the first 10 terms of following series :  
 $\log 2 + \log 4 + \log 8 + \dots$   
 a)  $45 \log 11$     b)  $55 \log 2$   
 c)  $36 \log 11$     d)  $66 \log 11$
16. If  $\log_{2\sqrt{5}} 400 = x$ , then x = ?  
 a) 7    b) 4    c) 5    d) 3
17. If  $\log_x y = 100$  and  $\log_2 x = 10$ , then the value of  
 is:  
 a)  $2^{10}$     b)  $2^{100}$     c)  $2^{1000}$     d)  $2^{10000}$
18. If  $\log_{10} 125 + \log_{10} 8 = x$ , then x is equal to:  
 a) 1/4    b) 064    c) -3    d) 3

## Home Practice Square Roots and Cube Roots

- Evaluate :  $\sqrt{248} + \sqrt{51} \sqrt{169}$ . 16
- Find the value of  $\sqrt[3]{1\frac{9}{16}}$ .  $\frac{1}{4}$
- What will come in place of question mark in each of the following questions? 2
- Find the value of  $\sqrt{\frac{0.289}{0.00121}}$ .  $\frac{170}{11}$
- If  $x = 1 + \sqrt{2}$  and  $y = 1 - \sqrt{2}$ , find the value of  $(x^2 + y^2)$ . 6
- Find the least square number which is exactly divisible by 10, 12, 15 and 18. 900
- Find the sum :  $3 + \frac{1}{\sqrt{3}} + \frac{1}{3 + \sqrt{3}} - \frac{1}{3 - \sqrt{3}}$ . 3
- By what least number 4320 be multiplied to obtain a number which is a perfect cube? 50
- Simplify:  $\frac{112}{\sqrt{196}} \times \frac{\sqrt{576}}{(12)} \times \frac{\sqrt{256}}{8}$ .
- If  $\sqrt{2} = 1.4142$ , find the value of  $\frac{\sqrt{2}}{(2 + \sqrt{2})}$ .

## Home Practice Exponent & Logarithm (Written)

- Prove that,  $\log_{\sqrt{a}} b \times \log_{\sqrt{b}} c \times \log_{\sqrt{c}} a = 8$
- If  $x = \log_{2a} a$ ,  $y = \log_{3a} 2a$  &  $z = \log_{4a} 3a$  then  $xyz + 1 = ?$  2yz
- Simplify:  $\frac{3 \cdot 2^n - 4 \cdot 2^{n-2}}{2^n - 2^{n-1}} \div \frac{3 \cdot 2^{n+1} + 2^n}{2^{n+2} - 2^{n-1}}$  2
- $\frac{2^{n+4} - 2(2^n)}{2(2^{n+3})}$ , when simplified is=?  $\frac{7}{8}$
- The value of  $\frac{2^{3x+4} + 8x + 1}{8^{x+1} - 2^{3x+2}}$  is: 6
- $\frac{1}{1 + x^{(b-a)} + x^{(c-a)}} + \frac{1}{1 + x^{(a-b)} + x^{(c-b)}} + \frac{1}{1 + x^{(b-c)} + x^{(a-c)}} = ?$  1
- If  $a + b + c = 0$ , then what is the value of  $\frac{1}{x^a + x^{-b} + 1} + \frac{1}{x^b + x^{-c} + 1} + \frac{1}{x^c + x^{-a} + 1}$ ? 1
- The value of  $(x^{b+c})^{b-c} \times (x^{c+a})^{c-a} \times (x^{a+b})^{a-b}$  is? ( $x \neq 0$ ) 1
- The value of  $\left(x \frac{b+a}{c-a}\right)^{\frac{1}{a-b}} \left(x \frac{c+a}{a-b}\right)^{\frac{1}{b-c}} \left(x \frac{a+b}{b-c}\right)^{\frac{1}{c-a}}$  is=? 1
- If  $2^a + 3^b = 17$  and  $2^{a+2} - 3^{b+1} = 5$ , then determine the values of a and b. [SJB TSO 07] 2

- If  $3^a - 5^b =$  and  $3^{a+2} - 5^{b+1} = 56$ , then determine the values of a & b? [ICB AP 08] 2
- Given  $2^x = 8^{y+1}$  and  $9^y = 3^{x-9}$ , then value of  $x+y$  is = ? 27
- If  $5^{(x+3)} = 25^{(3x-4)}$ , then the value of x is=?  $\frac{11}{5}$
- If  $a = xy^{p-1}$ ,  $b = xy^{q-1}$ ,  $c = xy^{r-1}$  and  $p+q+r = 3$  then prove that  $a^{q-r} \cdot b^{r-p} \cdot c^{p-q} = 1$  [Agrani Bank Officer (Cash)-2018-(Written) Rupali Bank-(Cash)-2018-(Written)]
- $\sqrt[3]{8x^2 \sqrt{32x} \sqrt{4x^2}} = 4$  then the value of x=? [Premier bank Ltd. (MTO)-2012 (Written)] 1
- If  $5^{\log_{10} x} + x^{\log_{10} 5} = 50$  then value of x=? 100
- If  $2^n = 128$  and  $y = 5$ , then  $\frac{(2^{n-1})(5^{n-2})}{\sqrt{y}-1} = ?$  [Uttara bank Ltd. (PO)-2009 (Written)] 100000
- Simplify:  $\frac{2^{x+4} - 4 \cdot 2x + 1}{2^{x+1} \div 2}$  4
- If  $x = a^{q+r} b^p$ ,  $y = a^{r+q} b^q$ ,  $z = a^{p+q} b^r$ , then show that  $x^{q-r} \cdot y^{r-p} \cdot z^{p-q} = 1$ .
- If  $p = xy^{a-1}$ ,  $q = xy^{b-1}$ ,  $r = xy^{c-1}$  then  $\left(\frac{p}{q}\right)^c \times \left(\frac{q}{r}\right)^a \times \left(\frac{r}{p}\right)^b = ?$  1

- Prove that,  $\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} = 1$
- Prove that,  $\left(\frac{x^a}{x^b}\right)^{a+b} \cdot \left(\frac{x^b}{x^c}\right)^{b+c} \left(\frac{x^c}{x^a}\right)^{c+a} = 1$
- Prove that,  $\left(\frac{x^p}{x^q}\right)^{p+q+r} \cdot \left(\frac{x^q}{x^p}\right)^{q+r-p} \cdot \left(\frac{x^r}{x^p}\right)^{r+p-q} = 1$
- Prove that,  $\frac{2^{2p+1} \cdot 3^{2p+q} \cdot 5^{p+q} \cdot 6^p}{3^{p-2} \cdot 6^{2p+2} \cdot 10^p \cdot 15^q} = \frac{1}{2}$
- Solve:  $4^x - 3(2^{x+2}) + 2^5 = 0$  3
- $3(9^x - 4 \cdot 3^{x-1}) + 1 = 0$  -1
- If  $2^x = \sqrt{1024}$ , what is the value of x? 5
- Find the value of x;  $\sqrt[3]{8x^2 \sqrt{32x} \sqrt{4x^2}} = 4$ ? 1
- Simplify:  $\sqrt[bc]{\frac{b}{x} \frac{c}{c}} \times \sqrt[ca]{\frac{c}{x} \frac{a}{a}} \times \sqrt[ab]{\frac{a}{x} \frac{b}{b}}$  1
- Solve:  $3 \cdot 27^x = 9^{x+4}$  7
- Simplify the following algebraic expression 9/254