# Lecture-09 : (Partnership & LCM, HCF )

### Important Vocabulary

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|---|---|
| Additional = অতিরিক্ত বা আরো | Invest = বিনিয়োগ করা |
| Afterwards = পরবর্তী | Partner = অংশীদার |
| Altogether = একত্রে | Pasture = চারণভূমি |
| Annual = বার্ষিক | Profit = মুনাফা বা সুদ |
| Approximate = প্রায় | Proportion = সমানুপাত |
| Capital = পুঁজি বা আসল | Ratio = অনুপাত |
| Charity = দান | Remainder = অবশিষ্ট |
| Contribute = অবদান রাখা | Rent = ভাড়া করা |
| Distribute = বণ্টন করা | Respective = সম্পর্কিত |
| Divided = ভাগ করে দেয়া | Rest of = বাকি |
| Duration = স্থায়িত্ব | Subscribe = চাঁদা দেয়া |
| Estimate = হিসাব করা | Thrice = তিনগুণ |
| Exceed = অতিক্রম করা | Whereas = যেখানে বা পক্ষান্তরে |
| Grazed = ঘাস খাওয়ানো | Withdraw = ফেরত নেয়া |
| Initially = প্রাথমিকভাবে | Yield = উৎপন্ন করা |

## Teacher's Work: Partnership (MCQ)

### Profit Related

**01.** **In a business A and C invested amounts in the ratio 2 : 1 whereas A and B invested amounts in the ratio 3:2. If their annual profit be Tk 157300, then B's share in the profit is-** [Janata Bank (EO)-2017 + Combined 4 Banks (Offi-Cash)-2018]

a) Tk. 24,200 b) Tk. 36,300
c) Tk. 72,000 d) Tk. 48,400

**02.** **P and Q started a business in the ratio of 2 : 3. After 2 years P left the business but Q continued. After 3 years he had a profit of Tk. 26000. What was the profit of P?** [BB-AD: 20-10-23]

a) Tk. 8000 b) Tk. 15600
c) Tk. 18000 d) No Profit

### Join After Few Days

**03.** **A and B started a partnership business investing some amount in the ratio of 3: 5. C joined them after six months with as amount equal to that of B. In what proportion should the profit at the end of the first year be distributed among A, B and C?**

a) 3 : 5 : 2 b) 3 :5 : 5
c) 6 : 10 : 5 d) Cannot be determined

### Investment Related

**04.** **Three partners shared the profit in a business in the ratio 5:7:8. They had partnered for 14 months, 8 months and 7 months respectively. What was the ratio of their investments?** [Combined 5 Banks (Cash)-2019 + Combined 8 Bank (SO)-2019]

a) 5 : 7 : 8 b) 20 : 49 : 64
c) 38 : 28 : 21 d) None of these

### Time Related

**05.** **A and B start a business with initial investment in the ratio 12:11 and their annual profit were in the ratio 4:1. If A invested the money for 11 months, B invested the money for?** [Rupali Bank (SO)-2019]

a) 2 b) 3 c) 6 d) 4

### Miscellaneous

**06.** **In a partnership, A invests $\frac{1}{6}$ of the capital for $\frac{1}{6}$ of the time, B invests $\frac{1}{3}$ of the capital for $\frac{1}{3}$ of the time and C, the rest of the capital for the whole time. Out of a profit of Tk. 4600, B's share is**

a) 803 b) 840 c) 200 d) 800

## Teacher's Work: Partnership (Written)

**07.** A and B started a business with the capital Tk. 3000 and Tk. 4000 After 8 months, a invested Tk. 2500 more in the business and 7 months after, total profit Tk. 980. Find the share of each. [BKB (Cash)-2018]

**08.** A, B and C inter into partnership. A invests 3 times as much as B invests and B invests two-third of what C invests. At the end of the year, the profit earned is Tk. 6600. What is the share of B? [Uttara Bank (PO)-2017 + Pubali Bank (Cash)-2017 + Sonali Bank (Off)-2018 + Sadharan Bima (JO)-2017]

**09.** Shakil started a business investing Tk. 25000 in 2009. In 2010, he invested an additional amount of Tk. 10000 and Raihan joined him with an amount of Tk. 35000. In 2011, Shakil invested another additional amount of Tk. 10000 and Jafor joined them with an amount of Tk. 35000. What will be Raihan's share in profit of Tk. 150000 earned at the end of 3 years from the start of the business in 2009? [BB (Cash Off)-2017]

**10.** A and B started a business with initial investments in the respective ratio of 18: 7. After four months from the start of the business, A invested Tk. 2000 more and B invested Tk. 7000 more. At the end of one year, if the profit was distributed among them in the ratio of 2:1 respectively. What was the total initial investment with which A and B started the business? [Sonali Bank (Off: FF-quota) 2019 + Pubali Bank (JO)-2019]

**11.** A, B and C started a business by investing Tk. 24000, Tk. 32000 and Tk. 18000 respectively. A and B are active partners and get 15% and 12% of total profit and remaining profit is to be distributed among them in the ratio of their investment. If C got total Tk. 65700 as profit, what was the total amount of profit? [Combined 4 Banks (Officer)-2019]

01. Find the smallest number of oranges that can be distributed completely among 4, 6, 10 or 18 children.
a) 16 b) 60 c) 240 d) 180

02. Find the minimum number of pencil that can be distributed equally among 12 & 20 people.
a) 12 b) 42 c) 45 d) 84

03. What is the greatest number that divides into 84, 144 or 18 without any reminder?
a) 18 b) 12 c) 24 d) 6

04. Find the smallest number which when divided by 3, 6, 9 gives the remainder 2, but there is no remainder when divided by 8.
a) 112 b) 152 c) 176 d) 192

05. What is the smallest number of apples that can be distributed equally among 4, 6, 9 or 15 students having a surplus of two apples each time?
a) 422 b) 362 c) 182 d) 62

06. Find the smallest number which when divided by 18 and 24 will have a remainder of 4 and 10 respectively?
a) 56 b) 52 c) 58 d) 54

07. Six bells commence tolling together and toll at intervals of 2, 4, 6, 8, 10 and 12 seconds respectively. In 30 minutes, how many times do they toll together? [Exim Bank Ltd. Cash (Offi)-2013]
a) 4 b) 10 c) 15 d) 18

08. The L.C.M. of two numbers is 48. The numbers are in the ratio 2 : 3. The sum of the numbers is- [PKB (SO)-2014 + Agrani Bank (SO)-2017]
a) 28 b) 40 c) 32 d) 64

## Partnership | Illustrative Questions

01. A, B and C started a business by investing Tk 120000, Tk. 135000 and Tk. 150000 respectively. Find the share of each, out of an annual profit of Tk. 56700.

Sol. Ratio of shares of A, B and C = Ratio of their investments
= 120000 : 135000 : 150000 = 8 : 9 : 10

$\therefore$ A's share = Tk. $\left(56700\times\frac{8}{27}\right)$ = Tk. 16800;

B's share = Tk. $\left(56700\times\frac{9}{27}\right)$ = Tk. 18900;

C's share = Tk. $\left(56700\times\frac{10}{27}\right)$ = Tk. 21000

02. A, B and C start a business each investing Tk. 20000. After 5 months A withdrew Tk. 5000, B withdrew Tk. 4000 and C invests Tk. 6000 more. At the end of the year, a total profit of Tk. 69900 was recorded. Find the share of each.

Sol. Ratio of the capitals of A, B and C
= 20000 × 5 + 15000 × 7 : 20000 × 5 + 16000 × 7 : 20000 × 5 + 26000 × 7
= 205000 : 212000 : 282000 = 205 : 212 : 282.

$\therefore$ A's share = Tk. $\left(69900\times\frac{205}{699}\right)$ = Tk. 20500;

B's share = Tk. $\left(69900\times\frac{212}{699}\right)$ = Tk 21200;

C's share = Tk. $\left(69900\times\frac{282}{699}\right)$ = Tk. 28200.

03. A, B and C enter into a partnership with capitals in the ratio $\frac{7}{2}:\frac{4}{3}:\frac{6}{5}$ = After 4 months A increases his share of capital by 50%. If at the end of the year the total profit earned is Tk 2430, find the share of each in the profit.

Sol. Ratio of capitals = $\frac{7}{2}:\frac{4}{3}:\frac{6}{5}=\left(\frac{7}{2}\times 30\right):\left(\frac{4}{3}\times 30\right):\left(\frac{6}{5}\times 30\right)$ = 105 : 40 : 36.

Let the initial capitals of A, B and C be Tk. 105 x, Tk. 40 x and Tk. 36 x respectively.
Then, ratio of profits
= [105x × 4 +(150% of 105x) × 8] : (40x ×12) : (36x × 12)
=1680 : 480 : 432=35 : 10 : 9

$\therefore$ A's share = Tk. $\left(2430\times\frac{35}{54}\right)$ = Tk. 1575; b's share = Tk. $\left(2430\times\frac{10}{54}\right)$ = Tk. 450;

C's share = Tk. $\left(2430\times\frac{9}{54}\right)$ = Tk. 405.

04. Two persons A and B take a field on rent. A put on it 21 horses for 3 months and 15 cows for 2 months; B puts 15 cows for 6 months and 40 sheep for $7\frac{1}{2}$ months. If, in one day, 3 horses eat as much as 5 cows eat and 6 cows as much as 10 sheep, what part of the rent should A pay?

Sol. 6 cows = 10 sheep $\Rightarrow$ 1 cow = $\frac{5}{3}$ sheep.

3 horses = 5 cows $\Rightarrow$ 1 horse = $\frac{5}{3}$ cows = $\left(\frac{5}{3}\times\frac{5}{3}\right)$ sheep = $\frac{25}{9}$ sheep.

$\therefore$ Ratio of shares of A and B

$=\left[\left(21\times\frac{25}{9}\times 3\right)+\left(15\times\frac{5}{3}\times 2\right)\right]:\left[\left(15\times\frac{5}{3}\times 6\right)+\left(40\frac{15}{2}\right)\right]$

$= 225 : 450 = 1 : 2.$

Hence, part of the rent paid by A $=\frac{1}{3}$.

**05. A, B and C took a house on rent for one year for Tk. 13824. They remained together for 4 months and then C left the house. After 5 more months, B also left the house. How much rent should each pay?**

**Sol.** Monthly rent = Tk. $\left(\frac{13824}{12}\right)$ = Tk. 1152.

Rent for first 4 months = Tk. (1152 × 4) = Tk. 4608.
It is to be divided equally among A, B and C.

$\therefore$ Share of each = Tk. $\left(\frac{4608}{3}\right)$ = Tk. 1536.

Rent for next 5 months = Tk. (1152 × 5) = Tk. 5760.
It is to be divided equally between A and B.

$\therefore$ Share of each = Tk. $\left(\frac{5760}{2}\right)$ = Tk. 2880.

Rent for last 3 months = Tk. (1152 × 3) = Tk. 3456.
It is to be paid by A only.
$\therefore$ Total rent paid by A = Tk. (1536 + 2880 + 3456) = Tk. 7872.
Total rent paid by B = Tk (1536 + 2880) = Tk. 4416.
Total rent paid by C = Tk. 1536.

**06. A, B and C are partners in a business. A, whose money has been used for 4 months, claims $\frac{1}{8}$ of the profit. B, whose money has been used for 6 months, claims $\frac{1}{3}$ of the profit. C had invested Tk. 1560 for 8 months. How much money did A and B contribute?**

**Sol.** Let the total profit be Tk. x.

Then, A's share = Tk. $\frac{x}{8}$; B's share = Tk. $\frac{x}{3}$;

C's share = Tk. $\left(x-\frac{x}{8}+\frac{x}{3}\right)$ = Tk. $\left(x-\frac{11x}{24}\right)$ = Tk. $\left(\frac{13x}{24}\right)$

$\therefore$ Ratio of shares of A, B and C $=\frac{x}{8}:\frac{x}{3}:\frac{13x}{24}=3:8:13.$

Suppose A invested Tk. y for 4 months and B invested Tk. z for 6 months.

Then, $\frac{y\times 4}{1560\times 8}=\frac{3}{13}\Rightarrow 52y=37440\Rightarrow y=720.$

And, $\frac{z\times 6}{1560\times 8}=\frac{8}{13}\Rightarrow 78z=99840\Rightarrow z=1280.$

Hence A's contribution = Tk. 720; B's contribution = Tk. 1280.

**07. A, B and C are partners in a business. Their shares are in the proportion of $\frac{1}{3}:\frac{1}{4}:\frac{1}{5}$. A withdraws half of his capital after 15 months and after another 15 months, a profit of Tk. 4340 is divided. The share of C is**

**Sol.** Ratio of initial investments $=\frac{1}{3}:\frac{1}{4}:\frac{1}{5}=20:15:12.$

Let their initial investments be 20x, 15x and 12x respectively.

$A : B : C = (20x \times 15 + 10x \times 15) : (15x \times 30) : (12x \times 30)$

$= 450x : 450x : 360x = 5 : 5 : 4.$

$\therefore$ C's share = Tk. $\left(4340\times\frac{4}{14}\right)$ = Tk. 1240.

**08. In partnership, A invests $\frac{1}{6}$ of the capital for $\frac{1}{6}$ of the time, B invests $\frac{1}{3}$ of the capital for $\frac{1}{3}$ of the time and C, the rest of the capital for the whole time. Out of a profit of Tk. 4600; B's share is-**

**Sol.** Suppose A invest Tk. $\frac{x}{6}$ for $\frac{y}{6}$ months.

Then, B invests Tk. $\frac{x}{3}$ for $\frac{y}{3}$ months.

C invests $\left[x-\left(\frac{x}{6}+\frac{x}{3}\right)\right]$, i.e, Tk. $\frac{x}{2}$ for y months.

$\therefore A : B : C=\left(\frac{x}{6}\times\frac{y}{6}\right):\left(\frac{x}{3}\times\frac{y}{3}\right):\left(\frac{x}{2}\times y\right)$

$=\frac{1}{36}:\frac{1}{9}:\frac{1}{2}=1:4:18.$

Hence, B's share = Tk. $\left(4600\times\frac{4}{23}\right)$ = Tk. 800.

**09. A, B and C jointly thought of engaging themselves in a business venture. It was agreed that A would invest Tk. 6500 for 6 months; B, Tk. 8400 for 5 months and C, Tk. 10,000 for 3 months. A wants to be the working member for which he was to receive 5% of the profits. The profit earned was Tk. 7400. Calculate the share of B in the profit.**

**Sol.** For managing, A receives = 5% of Tk. 7400 = Tk. 370.

Balance = Tk. (7400–370) = Tk. 7030.

Ratio of their investments = (6500 × 6) : (8400 × 5) : (1000 × 3)

= 3900 : 42000 : 30000 = 13 : 14 : 10.

$\therefore$ B's share = Tk. $\left(7030\times\frac{14}{37}\right)$ = Tk. 2660.

**10. A, B and C enter into a partnership. A contributes one-third of the capital while B contributes as much as A and C together contribute. If the profit at the end of the year amounts to Tk. 900, what would C receive?**

**Sol.** Let total capital = Tk. x. Then, A's capital = Tk. $\left(\frac{x}{3}\right)$

B's capital = (A + C)'s capital $\Rightarrow$2 (B's capital)
= (A + B + C)'s capital = Tk. x

$\Rightarrow$ B's capital = Tk. $\left(\frac{x}{2}\right)$.

C's capital = Tk. $\left[x-\left(\frac{x}{3}+\frac{x}{2}\right)\right]$ = Tk. $\frac{x}{6}$.

$\therefore$ A : B : C = $\frac{x}{3}:\frac{x}{2}:\frac{x}{6}$ = 2 : 3 : 1.

So, C's share = Tk. $\left(900\times\frac{1}{6}\right)$ = Tk. 150.

**11. A started a business with Tk. 21000 and is joined afterwards by B with Tk. 36000. After how many months did B join if the profits at the end of the year are divided equally?**

**Sol.** Suppose B joined after x months.

Then 21000 × 12 = 36000 × (12–x)

$\Rightarrow$36x = 180$\Leftrightarrow$ x = 5.

Hence, B joined after 5 months.

Then, $\frac{85000\times12}{42500\times x}=\frac{3}{1}$ or $x=\frac{85000\times12}{42500\times3}=8$.

So, B joined for 8 months.

**12. Swati and Rajni enter into a partnership with their capitals in the ratio 5 : 6. At the end of 7 months Swati withdraws her capital. If they receive the profit in the ratio of 5 : 9, find how long was Rajni's capital used.**

**Sol.** Suppose Swati invested Tk. 5x for 7 months and Rajni invested Tk. 6x for y months. Then,

$\frac{5x\times7}{6x\times y}=\frac{5}{9}\Rightarrow 30y=315\Rightarrow y=10\frac{1}{2}$.

Hence, Rajni's capital was used for $10\frac{1}{2}$ months.

| Students' Work | Partnership |
|---|---|

**01.** A, B and C started a business by investing Tk. 1,20,000, Tk. 1,35,000 and Tk. 1,50,000 respectively. Find the share of each, out of an annual profit of Tk. 56,700 [PBL (off)-2016]

**Tk. 16800, Tk.18900 and Tk. 21000]**

**02.** X, Y and Z invested Tk. 9000, Tk. 7000 and Tk. 6000 respectively in a business where profit will be distributed according to the ratio of their investment. The business made a profit of Tk. 880. If Y uses portion of his share of the profit to repay a personal loan of Tk. 230, then calculate Y's remaining profit balance. [BB (DE/CO)-2012] **50**

**03.** P and Q invested in a business. The profit earned was divided in the ratio 2:3. If P invested Tk. 40,000, the amount invested by Q is- [Combined 6 Banks & 2FIs (SO)-2019] **Tk. 60000**

**04.** A, B and C enter into a partnership. A initially invests Tk. 25 lakhs and adds another Tk. 10 lakhs after one year. B initially invests Tk. 35 lakhs and withdraws Tk. 10 lakhs after 2 years and C invests Tk. 30 lakhs. In what ratio should the profits be divided at the end of 3 years? [Meghna Bank (MTO)-2016] **19: 19 : 18**

**05.** Sumon and Jamal two friends started a business with Tk. 5000 and Tk. 4000 respectively. After 3 months Sumon added Tk. 1000 and simultaneously Dilip joined with them with Tk. 7000. What is the share of profit among them after one year if profit is Tk. 36000? [Janata Bank (IT Officer)-2016]

**Tk. 13800, Tk. 9600 and Tk. 12600**

**06.** A, B and C start a business each investing Tk. 20,000. After 5 months A withdrew 5,000, B widtdrew Tk. 4,000 and C invests Tk. 6,000 more. At the end of the year, a total profit of Tk. 69,900 was recorded. Find the share of each. [Janata Bank Ltd (AEO)-2015]

**Tk. 20500, Tk. 21200 and Tk. 28200**

**07.** A and B entered into partnership with capitals in the ratio 4:5 of the capital. After 3 months A withdrew 1/4 of his capital and B withdrew 1/5 his capital. The gain at the end of 10 months was tk. 760. Find the profit of A's share. [Al-Arafah Islami Bank (MTO)-2011]

**Tk. 330**

**08.** A.B and C started a business jointly with a total amount of Tk. 28000. A paid Tk. 4500 more than B and B paid Tk. 7000 less than C. If the company made a profit of Tk. 5600, how much profit should C receive? [Al-Arafah Islami Bank (MTO)-2017]

**Tk. 2500**

**09.** Mr. X, Mr. Y and Mr. Z started a business with a capital of Tk. 280. Mr. X Paid Tk. 45 more than the amount paid by Mr. Y and Mr. Y paid Tk. 70 less than the amount paid by Mr. Z. If they made a profit of Tk. 56, how will the profit be shared among the partners [Mercantile Bank Ltd.-2004]

**Tk. 20, tk. 11 and Tk. 25**

10. Rakib, Liton and Pranto started a business jointly with a total amount of Tk. 280. Rakib paid tk. 45 more than Liton and Liton paid Tk. 70 less than Pranto. If the company made a profit of Tk. 56, how much profit should Liton receive? [PKSF (AM)-2009]
**Tk. 11**

11. Aslam and Babul invested in a business in the ratio 3:2. Assume that 5% of total profit goes to workers' provident fund. If Aslam's share is Tk. 855000, what is the amount of total profit? [Janata Bank (AEO-RC)-2017]
**Tk. 15,00,000**

12. A, B and C are Partners. 'A' whose money has been in the business for 4 months claims 1/8 of the profits, 'B' whose money has been in the business for 6 months claims 1/3 of the profits. It 'C' had Tk. 1560 in the business for 8 months, how much money did A and B contribute to the business? [Sonali Bank (Cash)-2018]
**Tk. 720 and Tk. 1280**

13. Two friends P and Q started a business investing in the ratio of 5 : 6. R joined them after six months investing an amount equal to that of Q's. At the end of the year, 20% profit was earned which was equal to Tk. 98,000. What was the amount invested by R? [Bangladesh Bank (IT)-2016]
**Tk. 210000**

14. In a business, Piku invested Tk 6,500 for 6 months; Qazi invested Tk 8,400 for 5 months and Raj invested Tk. 10,000 for 3 months. Piku wants to be the working member, for which he will receive 5% of the profit if the total profit earned is tk 7,400. What is the share of Qazi in the profit? [Jamuna Bank (PO)-2012]
**Tk. 2660**

15. Four milkmen rented a pasture. A grazed 24 cows for 3 months, B 10 cows for 5 months, C 35 cows for 4 months and D 21 cows for 3 months. If A's share of rent is Tk. 720, find the total rent of the field? [Pubali Bank Ltd (SO-Officer)-2016]
**Tk. 3250**

## Illustrative | HCF & LCM (MCQ)

**01. Find the greatest number, which on dividing 1657 and 2037 leaves remainders 6 and 5 respectively.**

**Sol.** The number on dividing 1657 and 2037 leaves remainders 6 and 5 respectively.
Hence, make the dividend completely divisible by the divisor. This is possible, if we subtract remainder from the dividend.
Therefore,
1657–6 = 1651
2037–5 = 2032
H. C. F of 1651 and 2032 is 127.
127 is the common factor.
127 × 13 = 1651
Thus by adding 6, we get 1651 + 6 = 1657
**127 is the correct answer.**

**02. The traffice lights at three different road crossings change after every 40 sec, 72 sec and 108 sec respectively. If they all change simultaneously at 5 : 20 : 00 hours, then find the time at which they will change simultaneously.**

**Sol.** Traffic lights at three different road crossing changes after every 40 sec, 72 sec and 108 sec respectively.
Therefore, find the L. C. M of 40, 72 and 108,
L. C. M of 40, 72 and 108 = 1080
The traffic lights will change again after 1080 seconds = 18 min
The next simultaneous change takes place at 5 : 38 : 00 hrs.

**03. Find the least number which when divided by 16, 18, 20 and 2 leaves 4 as remainder in each case, but when divided by 7 leaves no. remainder:**

**Sol.** L. C. M of 16, 18, 20, 25 = 3600. Required number is of the form 3600k + 4.
Least value of k for which (3600k + 4) is divisible by 7 is k = 5
∴ Required number = (3600 × 5 + 4) = 18004.

**04. The least number which should be added to 2497 so that the sum is exactly divisible by 5, 6, 4 and 3 is:**

**Sol.** L. C. M of 5, 6, 4 and 3 = 60. On dividing 2497 by 60, the remainder is 37.
∴ Number to be added = (60–37) = 23

**05. The greates number which can divide 1356, 1868 and 2764 leaving the same remainder 12 in each case is:**

**Sol.** Required number = H. C. F of (1356–12), (1868–12) and (2764–12)
= H. C. F of 1344, 1856 and 2752 = 64

**06. The H. C. F and L. C. M of two numbers are 11 and 385 respectively. If one number lies between 75 and 125, then that number is:**

**Sol.** Product of numbers = 11 × 395 = 4235.
Let the numbers be 11a and 11b.
Then, 11a × 11b = 4235 or, ab = 35
Now, co-primes with product 35 are (1, 35) and (5, 7)
So, the numbers are (11×1, 11×35) and ( 11×11×7)
Since one number lies between 75 and 125,
The suitable pair is (55, 77).
Hence, required number = 77

**07. The sum of two numbers is 2000 and their L. C. M is 21879. The two numbers are:**

**Sol.** Let the numbers be x and (2000–x).

Then, their L.C.M = x (2000–x).

So, x (2000–x) = 21879

Or, $x^2$–2000x + 21879 = 0

Or, (x–1989) (x–11) = 0

Or, x = 1989 or, x = 11

Hence, the numbers are 1989 and 11.

**08. Three numbers which are co-prime to each other are such that the product of the first two is 551 and that of the last two is 1073. The sum of the three numbrs is:**

**Sol.** Since the numbers are co-prime they contain only 1 as the common factor

Number is common so middle number

Also, the given two products have the middle

= H. C. F of 551 and 1073 = 29

First number = $\left(\frac{551}{29}\right)$ = 19

Third number = $\left(\frac{1073}{29}\right)$ = 37

∴ Required sum = (19 + 29 + 37) = 85

**09. The product of two numbers is 4107. If the HCF of these number is 37, then the greater number is:**

**Sol.** Let the number be 37a and 37b.

Then, 37a × 37b = 4107 or, ab = 3.

Now, co-primes with product 3 are (1, 3)

So, the required numbers are (37 × 1, 37 × 3) i.e. (1, 111)

∴ Greater number = 111

**10. N is the greatest number which divides 1305, 4665 and 6905 and gives the same remainder in each case. What is the sum of the digits in N?**

**Sol.** 6905–1305 = 5600

6905–4665 = 2240

4665–1305 = 3360

Hence, the greatest number which divides 1305, 4665 and 6905 and gives the same remainder, N = HCF of 5600, 2240, 3360 = 1120

Sum of digits in N = Sum of digits in 1120 = 1+1+2+0 = 4

## Home Practice | HCF & LCM

**01. The greatest number that exactly divides 105, 1001 and 243 is-** [কর্ণফুলী গ্যাস এ.ম (সাধারণ) পরীক্ষা - ২০২১]

a) 3 b) 7 c) 11 d) 21

**02. What is the smallest number of apples that can be distributed equally (without cutting any apple) among 6, 10, 14 and 18 boys?** [BB-AD: 28-10-22]

a) 1260 b) 315 c) 360 d) 630

**03. What is the H.C.F of the numbers 36, 54 and 90?** [Pubali Bank Ltd. (SO)-2013]

a) 6 b) 9 c) 12 d) 18

**04. The highest common factor of 0 and 6 is-** [Uttara Bank (Cash)-2017]

a) 0 b) 3 c) Undefined d) 6

**05. The H.C.F of two numbers is 11 and their L.C.M is 693. If one of the numbers is 77, find the other.** [Janata Bank Ltd. Assistant (EO)-2015]

a) 99 b) 89 c) 79 d) 69

**06. The greatest number that exactly divides 105, 1001 and 2436 is:** [BB. AD. (G.S)-2014 + PKB (EO) Cash-k2014]

a) 3 b) 7 c) 11 d) 21

**07. Find the least number exactly divisible by 12, 15, 20 and 27.** [Rupali Bank Ltd. (SO)-2013]

a) 540 b) 430 c) 320 d) 300

**08. Which of the following numbers is the least common multiple of the number 2, 3, 4 and 5?** [Social Islami Bank Ltd. Officer-2014]

a) 24 b) 30 c) 40 d) 60

**09. What will be the laest number which when doubled will be exactly divisible by 12, 18, 21 and 30?** [Exim Bank Offi-2013]

a) 196 b) 630 c) 1260 d) 2520

**10. What is the smallest number of apple that can be distributed equally [Without cutting any apple) among 6, 10, 14 or, 18 boys?** [EMBA DU-2011]

a) 1260 b) 315 c) 360 d) 630

**11. Six bells start ringing together and ring at intervals of 4, 8, 10, 12, 15 and 20 seconds respectively. How many times will they bring together in 60 minutes?** [Mercantile Bank. (MTO)-2015]

a) 15 b) 16 c) 30 d) 33

**12. The least number which is a perfect square and divisible by each of the numbers 16, 20 and 24, is:** [Meghna Bank (MTO)-2014]

a) 1600 b) 3600 c) 6400 d) 14400

**13. Find the largest number of apples not exceeding 1000 which can be divided amont 6, 15, 20 or 24 boys?** [Exim Bank (TAO)-2018]

a) 960 b) 930 c) 900 d) 870

**14. Which of the following is the lowest positive integer divisible by 2, 3, 4, 5, 6, 7, 8 and 9?**

a) 15,120 b) 3,024 c) 2,520 d) 1,890

# Lecture-10: Algebra (MCQ)

## Importan Formulas

**বর্গ এর সূত্রাবলী**

| | |
|---|---|
| 1. $(a+b)^2$ | $= a^2 + 2ab + b^2$ |
| | $= (a-b)^2 + 4ab$ |
| 2. $(a-b)^2$ | $= a^2 - 2ab + b^2$ |
| | $= (a+b)^2 - 4ab$ |
| 3. $a^2+b^2$ | $= (a+b)^2 - 2ab$ |
| | $= (a-b)^2 + 2ab$ |
| 4. $2(a^2+b^2)$ | $= (a+b)^2 + (a-b)^2$ |
| 5. $a^2-b^2$ | $= (a+b)(a-b)$ |
| 6. $4ab$ | $= (a+b)^2 - (a-b)^2$ |
| 7. $ab$ | $= \left(\frac{a+b}{2}\right)^2 \left(\frac{a-b}{2}\right)^2$ |
| 8. $(a+b+c)^2$ | $= a^2+b^2+c^2+2(ab+bc+ca)$ |
| 9. $(x+a)(x+b)$ | $= x^2+(a+b)x+ab$ |

**ঘন এর সূত্রবলী**

| | |
|---|---|
| 10. $(a+b)^3$ | $= a^3+3a^2b+3ab^2+b^3$ |
| | $= a^3+b^3+3ab(a+b)$ |
| 11. $(a-b)^3$ | $= a^3-3a^2b+3ab^2-b^3$ |
| | $= a^3-b^3-3ab)(a-b)$ |
| 12. $a^3+b^3$ | $= (a+b)^3-3ab(a+b)$ |
| | $= (a+b)(a^2-ab+b^2)$ |
| 13. $a^3-b^3$ | $= (a-b)^3 + 3ab(a-b)$ |
| | $= (a-b)(a^2+ab+b^2)$ |

## Teacher's Work | Formula of Square

**01.** If $a^2 + b^2 = 45$ and $ab = 18$ find $\frac{1}{a} + \frac{1}{b}$ [BB- Officer Cash: 2023]

a) 1/3 b) 1/2 c) 2/3 d) 1/4

**02.** If $x/y = 1/3$, then the value of $(x^2 + y^2) / (x^2-y^2)$ is- [BB-AD: 2022]

a) −10/9 b) 5/4 c) −5/4 d) −5/3

**03.** If $(3x + 2y) = 8$ and $(2x-2y) = 2$, then find the value of $(4-3x)$ [BB- Officer General: 2022]

a) 1.5 b) 3 c) -2 d) -2.5

**04.** If $\frac{x}{y} + \frac{y}{z} = 6$, the value of $\frac{x^3}{y^3} + \frac{y^3}{x^3}$ is- [21 Based Combined Officer General: 2024]

a) 198 b) 176 c) 156 d) 144

**05.** If $x - \frac{1}{x} = \sqrt{3}$ then $x + \frac{1}{x} = ?$ [BD House Building FC (SO)-2017] & [BDBL-(SO)-2017]

a) $3\sqrt{3}$ b) $\sqrt{7}$ c) $2\sqrt{3}$ d) 7

**06.** If $\frac{x}{y} + \frac{y}{x} = 4$, then what is the value of $\frac{x^2}{y^2} + \frac{y^2}{x^2}$? [AL Arafah IBL-(MTO)-2017]

a) 16 b) 17 c) 13 d) 14

**07.** If $x = a + \frac{1}{a}$ and $y = a - \frac{1}{a}$ then $x^4 + y^4 - 2x^2y^2 = ?$

a) 4 b) 8 c) 16 d) None

**08.** If $\frac{x}{(2x+y+z)} = \frac{y}{(x+2y+z)} = \frac{z}{(x+y+2z)} = a$, then find the value of a if $(x + y + z) \neq 0$ [BB-AD: 20-10-23]

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{4}$ d) $\frac{1}{8}$

## Formula of Cubes

**09.** If $x + y = a$ and $x - y = b$, then $2xy = ?$

a) $\frac{a^2 - b^2}{2}$ b) $\frac{b^2 - a^2}{2}$ c) $\frac{a - b}{2}$ d) $\frac{ab}{2}$

## Single Equation & Double Equation

**10.** If $x + y = 6$, $y + z = 4$ and $z + x = 2$, then $2y - z - x$?

a) 0 b) 2 c) 8 d) 6

**11.** At a stationary shop, it costs Tk. 185 for 4 gel-pens, 8 ball-point pens and 1 marker pen and tk. 315 for 7 gel-pens. 15 ball-point pens and 1 marker pen. What would be the cost of 1 gel-pen, 1 ball-point pen and 1 marker pen? [BB-AD: 28-10-22]

a) Tk. 45 b) Tk 55 c) Tk. 60 d) Tk. 70

**12.** For which value of $p$ will the square root of $4x^2 - px + 9$ be an integer? [BD Gas Fields - AM Exam - 2021]

a) 20 b) 9 c) 12 d) 16

**13.** Babu's income is Tk. 1,000 more than that of Selim. Their total salary is Tk. $x$. What is Selim's salary? [BPEX - AM Exam - 2017]

a) $\frac{x}{2} - 500$ b) $x - 500$

c) $\frac{(2x-500)}{2}$. d) $2x - 1{,}000$

**14.** For some value of $x$, $5(x + 2) = y$. After the value of x is increased by 3, $5(x + 2) = Z$. What is the value of $(z - y)$?

a) 18 b) 12 c) 15 d) 10

15. One third the sum of 13 and a certain number is the same as one more than twice the number. Find out the number.
a) 6 b) 2 c) 5 d) 3

16. If the sum of two numbers is 33 and their difference is 15, the smaller number is:
a) 9 b) 12 c) 15 d) 18

17. A positive number $x$ is multiplied by 2, and this product is then divided by 3. If the positive square root of the result of these two operations equals $x$, what is the value of $x$?
a) $\frac{9}{4}$ b) $\frac{3}{2}$ c) $\frac{4}{3}$ d) $\frac{2}{3}$

18. The sum of the two numbers is 12 and their product is 35. What is the sum of the reciprocals of these numbers?
a) $\frac{12}{35}$ b) $\frac{1}{35}$ c) $\frac{35}{8}$ d) $\frac{7}{32}$

19. Some chocolates were distributed among three children X, Y and Z. If X gave 8 chocolates to Y & Y gave 5 chocolates to Z, then all of them would have equal number of chocolates. It the total number of chocolates were 45, how many chocolates did Y get?
a) 10 b) 12 c) 14 d) 15

20. In a Cox's Bazar hotel, the daily rate for an economy room, which can accommodate a maximum of three persons' is Taka 800 for one person and Taka Y for each additional person. If 3 friends take the room got one day and each pays Take 450 for the room, what is Taka Y?
a) 200 b) 275 c) 350 d) 425

21. There are 200 questions in a 3-hour examination. Among questions are 50 mathematics problems. It is suggested that twice as much time be allowed for each mathematics problem as for each of the other questions. How many minutes should be spent on the mathematics problems?
a) 36 b) 60 c) 72 d) 100

22. To make an instant coffee drink in a coffee machine, W liters of water is needed for every liter of concentrated liquid coffee mix. In a certain weekday in IBA canteen, C liters of concentrated coffee mix are required and coffee is sold for S taka per liter. If coffee mix is bought for B taka per liter and water us free, what will be the gross profit per day?
a) C (S + W – B) b) S (C + W – B)
c) S (C + CW – B) d) C (S + SW – B)

23. 74 is divided into two parts so that 5 times one part and 11 times the other part are together equal to 454. The parts are?
a) 14,60 b) 60,14 c) 30,44 d) 44,30

24. One junior student is asked to divide half a number by 6 and the other half by 4 and then add the quantities. Instead of doing so, the student divides the given number by 5. If the answer is 4 short of the correct answer, then the number is?
a) 320 b) 360 c) 400 d) 480

25. In a group of buffaloes and ducks, the number of legs is 24 more than twice the number of heads. What is the number of buffaloes in the group?
a) 6 b) 8 c) 10 d) 12

26. One third of Aman's marks in Mathematics exceeds a half of his marks in English by 30. If he got 240 marks in the two subjects together, how many marks did he get in English? [BB-AD: 20-10-23]
a) 180 b) 120 c) 90 d) 60

27. A worker was hired for 7 days. Each day, he was paid Tk. 10 more than what he was paid for the previous day of work. The total amount he was paid in the first 4 days of work equaled the total amount he was paid in the last 3 days. What was his starting pay? [20 Based Combined SO: 20-01-2023]
a) Tk.90 b) Tk. 138 c) Tk. 150 d) Tk. 160

## Illustrative Question

**01.** If $a = \sqrt{3} + 2$, What is the value of $a + \frac{1}{a}$?

**Sol.** $a = \sqrt{3} + 2$

$\Rightarrow \frac{1}{a} = \frac{1}{\sqrt{3}+2} = \frac{\sqrt{3}-2}{(\sqrt{3}+2)(\sqrt{3}-2)}$

$= \frac{\sqrt{3}+2}{(\sqrt{3})^2-(2)^2}$ $\quad \therefore [(a+b)(a-b) = a^2-b^2]$

$= \frac{\sqrt{3}-2}{3-4} = \frac{\sqrt{3}-2}{-1} = 2-\sqrt{3}$

Now $= a + \frac{1}{a} = (\sqrt{3}+2+2-\sqrt{3}) = 4$ **(Ans:)**

**02.** If $x = 7 + 4\sqrt{3}$ then $\sqrt{x}$ =?

**Sol.** $x = 7 + 4\sqrt{3}$

$\Rightarrow x = 4 + 3 + 4\sqrt{3}$

$\Rightarrow x = (2)^2 + (\sqrt{3})^2 + 2.2.\sqrt{3} \Rightarrow x = (2)^2 + 2.2.\sqrt{3} + (\sqrt{3})^2$

$\Rightarrow x = (2+\sqrt{3})^2$ $\quad [a^2 + 2ab + b^2 = (a+b)^2]$

$\Rightarrow \sqrt{x} = 2 + \sqrt{3}$ **(Ans:)**

**03.** If $x^2 - \sqrt{3}x) + 1 = 0$ then $\left(x + \frac{1}{x}\right)^2$ =?

**Sol.** $x^2 - \sqrt{3}x + 1 = 0$

$\Rightarrow x^2 + 1 = \sqrt{3}x \Rightarrow \frac{x^2+1}{x} = \sqrt{3}$

$\Rightarrow x + \frac{1}{x} = \sqrt{3} \Rightarrow \left(x + \frac{1}{x}\right)^2 = (\sqrt{3})^2$

$\therefore \left(x + \frac{1}{x}\right)^2 = 3$ **(Ans:)**

**04.** If $\frac{m}{n}+\frac{n}{m}=\sqrt{2}$, the value of $\frac{m^2}{n^2}+\frac{n^2}{m^2}$=?

**Sol.** $\frac{m^2}{n^2}+\frac{n^2}{m^2}$

$=\left(\frac{m}{n}\right)^2+\left(\frac{n}{m}\right)^2$

$=\left(\frac{m}{n}+\frac{n}{m}\right)^2-2.\frac{m}{n}.\frac{n}{m}$ $[\because a^2+b^2=(a+b)^2-2ab]$

$=(\sqrt{2})^2-2\times1=2-2=0$ **(Ans:)**

**05.** If $x=\sqrt{5-2\sqrt{6}}$, what is the value of $\left(x^3+\frac{1}{x^3}\right)$ =?

**Sol.** $x=\sqrt{5-2\sqrt{6}}$

$\Rightarrow x=\sqrt{3+2-2\sqrt{6}}$

$\Rightarrow x=\sqrt{(\sqrt{3})^2+(\sqrt{2})^2-.\sqrt{3}.\sqrt{2}}$

$\Rightarrow x=\sqrt{(\sqrt{3}-\sqrt{2})^2}$ $[\because a^2+b^2-2ab=(a-b)^2]$

$\therefore x=\sqrt{3}-\sqrt{2}$

$\frac{1}{x}=\frac{1}{\sqrt{3}-\sqrt{2}}$

$=\frac{\sqrt{3}+\sqrt{2}}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$ [Multiply the numerator & denominator by $(\sqrt{3}+\sqrt{2})$]

$=\frac{\sqrt{3}+\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2}$ $[\because (a-b)(a+b)=a^2-b^2]$

$=\frac{\sqrt{3}+\sqrt{2}}{3-2}=\sqrt{3}+\sqrt{2}$

$\therefore x+\frac{1}{x}=(\sqrt{3}-\sqrt{2})+(\sqrt{3}+\sqrt{2})=2\sqrt{3}$

Now $=x^3+\frac{1}{x^3}$

$=\left(x+\frac{1}{x}\right)^3-3.x.\frac{1}{x}\left(x+\frac{1}{x}\right)$ $[a^3+b^3=(a+b)^3-3ab(a+b)]$

$=(2\sqrt{3})^3-3\times2\sqrt{3}$

$=8\times3\sqrt{3}-6\sqrt{3}=24\sqrt{3}-6\sqrt{3}=18\sqrt{3}$ **(Ans:)**

**06. Factorize of $m^8+m^4-2$?**

**Sol.** $m^8+m^4-2$

$=m^8+2m^4-m^4-2$

$=m^4(m^4+2)-1(m^4+2)$

$=(m^4+2)(m^4-1)$

$=(m^4+2)\{(m^2)^2-1^2\}$

$=(m^4+2)(m^2+1)(m^2-1)$

$=(m^4+2)(m^2+1)(m^2-1^2)$

$=(m^4+2)(m^2+1)(m+1)(m-1)$

$=(m+1)(m-1)(m^2+1)(m^4+2)$ **(Ans:)**

**07. If $f(x)=x^3+27$ then $f(-3)+f(3)$ = what?**

**Sol.** $f(x)=x^3+27$

$f(-3)=(-3)^3+27$

$=-27+27=0$

$f(3)=3^3+27$

$=27+27=54$

$\therefore f(-3)+f(3)=0+54$

$=54$ **(Ans:)**

**08. If $f(x)=54x^4+27x^3a-16x-8a$ and $f\left(-\frac{1}{2}a\right)=0$, the factor of f(x) is?**

**Sol.** $\left(-\frac{1}{2}a\right)=0$

root of $x=-\frac{1}{2}a$

$\Rightarrow 2x=-a$

$\Rightarrow 2x+a=0$

$\therefore (2x+a)$ is a factor of $(x)$

**09. $f(x)=x^3+x^2+x+1$ is a polynomial of the variable of x. What would be the remainder if f(x) is divided by (x + 1)?**

**Sol.** $f(x)=x^3+x^2+x+1$

$=x^2(x+1)+1(x+1)$

$=(x+1)(x^2+1)$

$\therefore (x+1)$, f(x) is a factor of f(x)

So remainder = 0

Alternative solution: $x+1=0$ Or, $x=-1$

Remainder $=f(-1)=(-1)^3+(-1)^2+(-1)+1$

$-1+1-1+1=0$ **(Ans:)**

**10. If $a+b=\sqrt{7}$ and $a-b=\sqrt{5}$, prove that, $8ab(a^2+b^2)=24$.**

**Sol.** $4ab=(a+b)^2-(a-b)^2$

$=(\sqrt{7})^2-(\sqrt{5})^2=7-5=2$

$a^2+b^2=\frac{1}{2}\{(a+b)^2+(a-b)^2\}$

$=\frac{1}{2}\{(\sqrt{7})^2+(\sqrt{5})^2\}=\frac{1}{2}\times(7+5)=\frac{1}{2}\times12=6$

Now $=8ab(a^2+b^2)$

$=2\times4ab\times(a^2+b^2)=2\times2\times6=24$ **(Ans:)**

**11. $a+b+c=6$ and $a^2+b^2+c^2=14$, than the value of $(a-b)^2+(b-c)^2+(c-a)^2$ =?**

**Sol.** Given that, $a+b+c=6$

and $a^2+b^2+c^2=14$

We know,

$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

or, $(6)^2=14+2(ab+bc+ca)$

or, $36-14=+2(ab+bc+ca)$

or, $2(ab+bc+ca)=22$

or, $(ab+bc+ca)=11$

Now,
$= (a-b)^2 + (b-c)^2 + (c-a)^2$
$= a^2 - 2ab + b^2 + b^2 - 2bc + c^2 + c^2 - 2ca + a^2$
$= 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca$
$= 2(a^2 + b^2 + c^2) - 2(ab + bc + ca)$
$= 2 \times 14 - 2 \times 11$
$= 28 - 22 = 6$ **(Ans:)**

**12. If $m = 7-4\sqrt{3}$, Then $\sqrt{m} + \frac{1}{\sqrt{m}} = ?$**

**Sol.** Given that, $m = 7 - 4\sqrt{3}$

$$\frac{1}{m} = \frac{1}{7-4\sqrt{3}}$$
$$= \frac{(7+4\sqrt{3})}{(7-4\sqrt{3})(7+4\sqrt{3})}$$
$$= \frac{7+4\sqrt{3}}{49-48} = 7 + 4\sqrt{3}$$
$$\left(\sqrt{m} + \frac{1}{\sqrt{m}}\right)^2 = m + \frac{1}{m} + 2$$
$$= 7-4\sqrt{3} + 7 + 4\sqrt{3} + 2 = 16$$
$$\therefore \sqrt{m} + \frac{1}{\sqrt{m}} = 4$$ **(Ans:)**

**13. If $x^2 - 3x + 1 = 0$, what is the value of $x^2 - \frac{1}{x^2}$?**

**Sol.** $x^2 - 3x + 1 = 0$

$$\Rightarrow x^2 + 1 = 3x \Rightarrow \frac{x^2+1}{x} = 3 \Rightarrow x + \frac{1}{x} = 3$$

Now $= x^2 - \frac{1}{x^2}$

Now, $\left(x - \frac{1}{x}\right)^2 = \left(x + \frac{1}{x}\right)^2 - 4 \times x \times \frac{1}{x} = 3^2 - 4$

$$\therefore \left(x - \frac{1}{x}\right)^2 = 5$$

$$\therefore \left(x - \frac{1}{x}\right) = \sqrt{5}$$

$$\therefore x^2 - \frac{1}{x^2} = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right) = 3\sqrt{5}$$ **(Ans:)**

**14.** $\frac{x}{a} + \frac{a}{x} = \frac{x}{b} + \frac{b}{x}$

**Sol:** $\frac{x}{a} + \frac{a}{x} = \frac{x}{b} + \frac{b}{x}$

or, $\frac{x}{a} - \frac{x}{b} = \frac{b}{x} - \frac{b}{x}$

or, $\frac{bx - ax}{ab} = \frac{b-a}{x}$

or, $\frac{x(b-a)}{ab} = \frac{b-a}{x}$

or, $\frac{x}{ab} = \frac{1}{x}$ [divided both side by $(b-a)$]

or, $x^2 = ab \therefore x = \pm\sqrt{ab}$

Solution, $x = -\sqrt{ab}$ or, $\sqrt{ab}$ **(Ans:)**

**15. Solve it, $x^2 + y^2 = 25$, $xy = 12$**

Now,

$x^2 + y^2 = 25$ ..... (i)

$xy = 12$ ..... (ii)

multiply (ii) by 2, at adding to (i) we get

$x^2 + y^2 + 2xy = 25 + 24$

$(x + y)^2 = 49$

$\therefore x + y = \pm 7$ ..... (iii)

again multiply (ii) by 2, subtract from (i) and get

$x^2 + y^2 - 2xy = 25 - (2 \times 12)$

or, $x^2 - 2xy + y^2 = 25 - 24$

or, $(x - y)^2 = 1$

$\therefore x - y = \pm 1$ ..... (iv)

from (iii) and (iv),

$x + y = 7$, $x - y = 1$ ..... (v)

$x + y = 7$, $x - y = -1$ ..... (vi)

$x + y = -7$, $x - y = 1$ ..... (vii)

$x + y = -7$, $x - y = -1$ ..... (viii)

solve the problem from (v) $x = 4, y = 3$

solve the problem from (vi) $x = 3, y = 4$

solve the problem from (vii) $x = -3, y = -4$

solve the problem from (viii) $x = -4, y = -3$

$\therefore$ Deterministic solution : $(x, y) = (4, 3), (3, 4), (-3, -4), (-4, -3)$. **(Ans:)**

## Home Practice : Algebra (MCQ)

**01. If 9 is $\frac{3}{4}$ of n, what number is $\frac{5}{6}$ of n?** [Janata Officer (RC) – 28-06-2024]

a) 12 b) 15 c) 10 d) 12.5

**02. The daily rate for a hotel room that sleeps 4 people is Tk. 390 for one person and X taka for each additional person. It 3 people take the room for one day and each pays Tk. 210 for the room then what is the value of X?** [21 Based Combined Officer (Cash) 2024]

a) 60 b) 120 c) 80 d) 240

**03. If $x = a + \frac{1}{a}$ and $y = a - \frac{1}{a}$ then $x^4 + y^4 - 2x^2y^2 = ?$** [21 Based Combined Officer (Cash) 2024]

a) 4 b) 8

c) 16 d) None of these

**04. The difference between two numbers is 5 and the difference between their square is 65. What is the smaller number?** [21 Based Combined Officer (Cash) 2024]

a) 4 b) 5 c) 11 d) 13

05. In a T-20 cricket match, the number of boundaries scored was twice the number of over boundaries by a team. The team took 22 single runs. no two or three runs and could not score from 38 deliveries. How many runs did the team score? [21 Based Combined Officer (Cash) 2024]
a) 124 b) 144 c) 150 d) 302

06. A boy read $3/8^{th}$ of a book on one day and $4/5^{th}$ of the remainder on another day. If there were 30 pages unread, how many pages did the book contain? [21 Based Combined Officer General: 2024]
a) 240 b) 300 c) 600 d) 800

07. If the average of 'm' numbers is $n^2$ and that of 'n' numbers is $m^2$, then the average of (m+n) numbers is- [BB-AD: 20-10-23]
a) m +n b) m – n c) m/n d) mn

08. If x/y = 1/3, then the value of $(x^2 + y^2) / (x^2 - y^2)$ is- [BB-AD: 28-10-22, 20 Based Combined SO: 20-01-2023]
a) −10/9 b) 5/4 c) −5/4 d) −5/3

09. If a + b + c – 0, the value of $\frac{a^2}{bc} + \frac{a^2}{ca} + \frac{a^2}{ab}$ is- [21 Based Combined SO: 10-11-23]
a) 3abc b) $\frac{1}{3}$ c) 1 d) 3

10. Salam earns Tk. 8.50 per hour on days other than Friday and twice than the rate of Friday. last week he worked a total of 50 hours including 8 hours on Friday. What is his earning for the week? [20 Based Combined SO: 20-01-2023]
a) Tk. 340 b) Tk. 398 c) Tk. 408 d) Tk. 493

11. Mina has 3 Taka more than Babu has, but 5 Taka less than Shelly has. If Mina has $x$ Taka, how much money Shelly and Babu have altogether? [বাখরবাদ গ্যাস এ.এম (জেনারেল) পরীক্ষা - ২০১৭]
a) $2x - 8$ b) $2x - 5$ c) $2x - 2$ d) $2x + 2$

12. Half of the people on a bus get off at each stop after the first and no one gets on after the first stop. If only 4 persons get off at stop number 4, how many people got on at the first stop? [বাখারবাদ গ্যাস এ.এম (জেনারেল) পরীক্ষা - ২০১৭]
a) 16 b) 32
c) 44 d) None of these

13. If Kamal gives 20 marbles to Shuvo, then both of them will have equal numbers of marbles in their possessions. If Shuvo gives 40 marbles to Kamal then Kamal will have twice the number of marbles that Shuvo will retain. What is the number of marbles that Kamal has? [বাখারবাদ গ্যাস এ.এম (জেনারেল) পরীক্ষা - ২০১৭]
a) 100 b) 160 c) 200 d) 260

14. Courier charges for packages to a certain destination are Tk. 65 for the first 250 grams and Tk. 10 for each additional 100 grams or part thereof. What could be the weight in grams of a package for which the charge is Tk. 155? [BPEX - AM Exam - 2017]
a) 1155 gms b) 1145 gms
c) 1040 gms d) None of these

15. A ferry can carry 24 buses or 36 cars at a time. If there are 18 buses on the ferry, how many cars can be loaded onto it? [BPEX - AM Exam - 2017]
a) 6 b) 8 c) 9 d) 12

16. If $x$ and $y$ are positive integers, each greater than 1, and if $13(x - 1) = 17(y - 1)$, what is the least possible value of $(x + y)$? [BPEX - AM Exam - 2017]
a) 32 b) 30 c) 26 d) 25

17. In a class, if 4 students sit in each bench, there are 3 empty benches, but 6 students have to stand if 3 students sit each bench. How many students are there in that class? [BD Gas Fields - AM Exam - 2021]
a) 50 b) 60 c) 26 d) 25

18. In an essay competition, a winner gets a prize of Tk. 100 and a participant who does not win gets a prize of Tk. 25. The total prize money distributed is Tk. 3,000. Find the number of winners, if the total participant is 63? [BPEX - AM Exam - 2023]
a) 15 b) 17 c) 19 d) 21

19. A television with a price of Tk. 300 is to be purchased with an initial payment of Tk. 60 and weekly payments of Tk. 30. Which of the following equations can be used to find the number of weekly payments, required to complete the puchase, assuming there are no taxes of free? [BD Petro Institute AM Exam - 2020]
a) $300 = 30w - 60$ b) $300 = 60w - 30$
c) $300 = 30w + 60$ d) $300 = 30w$

20. X, Y and Z share Tk. 1,800 is such a way that X has 2.5 times as much as Y, and Y has 4 times as much as Z. How much money (in taka) does Z receive? [Titas Gas AM (General) Exam - 2021]
a) 120 b) 132 c) 145 d) 200

21. If $m = 7-4\sqrt{3}$, then $\sqrt{m} + \frac{1}{\sqrt{m}} = ?$ [Rupali Bank-(S.O)-2019]
a) 3 b) 8 c) 4 d) 6

22. If $x^2-3x+1=0$ what is the value of $x^2-\frac{1}{x^2}$? [IFIC Bank-(TSO)-2018]
a) $4\sqrt{3}$ b) $3\sqrt{5}$ c) $4\sqrt{5}$ d) $2\sqrt{3}$

23. What is the value of $(m+n)^2-(m-n)^2$? (Agrani Bank Off. (Cash)-2013) & Agrani Bank Ltd. Seni Offi-2013)
a) $m^2$ b) $n^2$ c) $m^2-n^2$ d) 4mn

24. Evaluate: $\frac{(2.39)^2-(1.61)^2}{2.39-1.61}$ [Trust Bank Ltd. (MTO)-2016]
a) 2 b) 4 c) 6 d) 8

25. $a+b=\sqrt{3}$ এবং $a-b=\sqrt{2}$ হলে $8ab(a^2+b^2)$=? (BKB Sen. Off-2015)
a) 12 b) 10 c) 8 d) 5

26. If $a^2+b^2=45$ and $ab=18$, find $\frac{1}{a}+\frac{1}{b}$=? [IBBL-2016) & Premier Bank Ltd. TJO 2013)
a) $\frac{1}{3}$ b) $\frac{2}{3}$ c) $\frac{1}{2}$ d) None

27. If $xy=2$ and $xy^2=8$, what is the value of x? [Pubalie Bank SO 2013 + Bank Asia MTO 2011)
a) ½ b) 2 c) 4 d) 8

28. Which one of the following is the minimum value of the sum of two integers whose product is 36? [Exim bank T. off.-2015)
a) 37 b) 20 c) 15 d) 12

29. The product of two numbers is 120 and the sum of their square is 289. The sum of the number is- (Exim Bank Ltd. Off 2013)
a) 20 b) 21 c) 22 d) 23

30. The factors of $x^2-5x-6$ are: [Sonali Bank-(SO)-2018]
a) (x–6) (x+1) b) (x + 6) x-1)
c) (x–3) (x+2) d) (x–3)(x–2)

31. If (t-8) is a factor of $t^2-kt-48$, then k=? [SIBL (PO)-2017] [MTB-(MT)-2017]
a) -6 b) -2 c) 2 d) 14

32. The roots of the equation $9x^2-bx+81=C$ will be equal, if the value of b is- [Rupali Bank Off-(Cash)-2018]
a) ±9 b) ±18 c) ±27 d) ±54

33. In a test, a candidate secured 336 marks out of maximum marks x. If the maximum marks x had been converted into 400 marks, he would have secured 192 marks. What was the maximum marks of the test?

34. In a herd of cows, the numbers of legs are 14 more than twice the number of heads. The number of cows in the herd is-
a) 5 b) 7 c) 10 d) 12

35. There are some cows, bulls and 45 hens in a group. One caretaker looks after 15 animals; the number of bulls is twice the number of cows. If the number of heads is less than the total number of feet by 186 (including the caretakers), how many caretakers are there?
a) 5 b) 6 c) 8 d) 9

36. M men agree to purchase a gift for Tk D. If three drop out how much more will each have to contribute towards the purchase of the gift?
a) $\frac{D}{M-3}$ b) $\frac{MD}{3}$ c) $\frac{M}{D-3}$ d) $\frac{3D}{M^2-3M}$

37. In an examination there are 30 questions. 1 mark is given for each correct answer and 0.25 is deducted for every incorrect answer. Ankur attempted all the questions and scored 13.75. How many incorrect answers did he have?
a) 10 b) 11 c) 12 d) 13

38. If $x+y=\sqrt{5}$ and $x-y=\sqrt{3}$, what is the value of $x^2-y^2$?
a) 15 b) $\sqrt{15}$ c) $\frac{3}{5}$ d) $\sqrt{\frac{3}{5}}$

39. If $(x^2-y^2)=48$ and $(x-y)=4$, what is the average of x and y? (Dhaka bank Ltd. MTO 2011)
a) 12 b) 10 c) 8 d) 6

40. If $x+y=4$ then $x^3+y^3+12xy$ = ? Ans: 64

41. If $x-$ $=\sqrt{5}$ Then $x^3-\frac{1}{x^3}$ = ? Ans: $8\sqrt{5}$

42. If $x+\frac{2}{x}=3$ Then $x^3+\frac{8}{x^3}$ = ? Ans: 9

43. A candidate was asked to find $\frac{7}{8}$ of a positive number. He found $\frac{7}{18}$ of the same by mistake. If his answer was 770 less than the expected answer. What was the expected answer?
a) 64 b) 126 c) 136 d) None

44. $m-\frac{1}{m}=2$ হলে, $m^4+\frac{1}{m^4}$ = ? [Agrani Bank Ltd. Seni Offi-2015]
a) 34 b) 32 c) 31 d) 30

45. If $\sqrt{3}-\frac{1}{x}=x$ Then $x^3+x+\frac{1}{x}+\frac{1}{x^3}$ = ?

# Lecture 11 : Algebra (Written)

## Teacher's work | Formula of Square

**01.** If $x^2 - \sqrt{3}x + 1 = 0$, $\frac{x^6+1}{x^3} = ?$

**02.** $(p+q)^2 = \sqrt[3]{27}$ and $p^2 = \sqrt{6} + q^2$, $p^3q + pq^3 = ?$

**03.** If $y = \sqrt{2} + \sqrt{3}$, $\left(y^2 + \frac{1}{y^2}\right)\left(y^3 - \frac{1}{y^3}\right) = ?$

**04.** If $(p^2+1)^2 = 6p^2$ Show, $p3 + \frac{1}{p^3} = 3\sqrt{6}$.

**05.** Given $\frac{2}{x} + \frac{2}{xy} = \frac{1}{6}$ and $\frac{2}{x} + \frac{2}{y} = 0$, Find 'a' for which $y = ax - 4$. [21 Based Combined Senior Officer (General): 2023]

**06.** If $a^2 - \sqrt{5}a + 1 = 0$ then find...

(i) $a^5 + \frac{1}{a^5}$ (ii) $a^6 + \frac{1}{a^6}$

**07.** If $a^3 + \frac{1}{a^3} = 2$, then find $a + \frac{1}{a}$

**08.** Given $x = 3+2\sqrt{2}$, find the value of $\sqrt{x} - \frac{1}{\sqrt{x}}$ [Janata Bank–(AEO-RC)-2018 & Standard Bank- (TAO-General)-2018]

**09.** $x^2+\frac{1}{x^2}=1$, find the value $x^{102}+x^{96}+x^{90}+x^{84}+x^{78}+x^{72}+5$? [Janata Bank (AEO-Teller)-2020]

**10.** $\sqrt{x} + \frac{1}{\sqrt{x}} = a$, then find the value of $x^2 + \frac{1}{x^2}$.

**11.** If $x - \frac{1}{x} = \sqrt{5}$ Then, $x^3 - \frac{1}{x^3}$ [Ministry of Food (AP)-2020]

**12.** If $a = \sqrt{6} + \sqrt{5}$ then Find the value of $\frac{a^6-1}{a^3}$

**13.** Solve the equation $\frac{4}{2x+1} + \frac{9}{3x+2} = \frac{25}{5x+4}$

**14.** If $x = \frac{4}{5}$, then $= \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}$ ?

**15.** Factorize $x^2 - \left(\frac{2}{a} - 3a\right)x - 6$ [Ministry of Food (AP)-2020]

**16.** A bonus of Tk. 1000 is to be divided among three people so that Tamim receives twice as much as Sakib, who receives one-fifth as much as Mahmudullah,. How much money should Mahmudullah receive?

**17.** In a classroom, there are certain numbers of benches. If 6 students are made to sit on a bench, then to accommodate all of them, one more bench is needed. However, if 7 students are made to sit on a bench, then after accommodating all of them, space for 5 students is left. What is the total number of students in the class?

**18.** A man engaged a worker on the condition that he would pay him Tk. 30,000 and one uniform after one year of service. The worker served only 9 months and got Tk. 22,000 and a uniform. What is the price of the uniform? [20 Based Combined Officer (General): 2023]

## Illustrative Questions

**01. If, $a = \sqrt{13} + 2\sqrt{3}$, the value of $\frac{13a}{a^2-\sqrt{13}a+1}$ ?**

**Sol.** Given that, $a = \sqrt{13}+2\sqrt{3}$

$$\therefore \frac{1}{a} = \frac{1}{\sqrt{13}+2\sqrt{3}}$$

$$= \frac{\sqrt{13}+2\sqrt{3}}{(\sqrt{13}+2\sqrt{3})(\sqrt{13}-2\sqrt{3})}$$

[multiply the numerator and denominator by $[\sqrt{13}-2\sqrt{3}]$

$$= \frac{\sqrt{13}-2\sqrt{3}}{(\sqrt{(13)^2}-2\sqrt{3})^2}$$

$$= \frac{\sqrt{13}-2\sqrt{3}}{13-4.3} = \frac{\sqrt{13}-2\sqrt{3}}{13-12}$$

$$= \sqrt{13}-2\sqrt{3}$$

Now, $\frac{13a}{a^2-\sqrt{13}a+1}$

$$= \frac{\frac{13a}{a}}{\frac{a^2}{a} - \frac{\sqrt{13}a}{a} + \frac{1}{a}}$$

[Divide the newmerator and denomenator by a]

$$= \frac{13}{a-\sqrt{13}+\frac{1}{a}} = \frac{13}{a+\frac{1}{a}-\sqrt{13}}$$

$$= \frac{13}{\sqrt{13}+2\sqrt{3}+\sqrt{13}-2\sqrt{3}-\sqrt{13}}$$

$[a = \sqrt{13}+2\sqrt{3}$ এবং $\frac{1}{a} = \sqrt{13}-2\sqrt{3}]$

$$= \frac{13}{\sqrt{13}} = \frac{\sqrt{13}.\sqrt{13}}{\sqrt{13}} = \sqrt{13}$$

$\therefore$ Value is $\sqrt{13}$ **(Ans.)**

**02. If, $x^2-13-2\sqrt{42}=0$, the value of $x^2+\frac{1}{x^2}$ ।**

**Sol.** Given that, $x^2-13-2\sqrt{42}=0$

Or, $x^2=13+2\sqrt{42}$

Or, $x^2=7+2.\sqrt{7}.\sqrt{6}+6$

Or, $x^2=(\sqrt{7})^2+2.\sqrt{7}.\sqrt{6}+(\sqrt{6})^2$

Or, $x^2=(\sqrt{7}+\sqrt{6})^2$

$\therefore x=\sqrt{7}+\sqrt{6}$ [ square both side ]

$\therefore \frac{1}{x}=\frac{1}{\sqrt{7}+\sqrt{6}}$

$=\frac{\sqrt{7}-\sqrt{6}}{(\sqrt{7}+\sqrt{6})(\sqrt{7}-\sqrt{6})}=\frac{\sqrt{7}-\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2}$

$=\frac{\sqrt{7}-\sqrt{6}}{7-6}=\sqrt{7}-\sqrt{6}$

Now, $x^2+\frac{1}{x^2}$

$=\left(x+\frac{1}{x}\right)^2-2.x.\frac{1}{x}$

$=(\sqrt{7}+\sqrt{6}+\sqrt{7}-\sqrt{6})^2-2$

$=(2\sqrt{7})^2-2=4.7-2=28-2=26$

$\therefore x^2+\frac{1}{x^2}=26$

$\therefore$ Value is 26 **(Ans.)**

$x=\sqrt{7}+\sqrt{6}$ and $\frac{1}{x}=\sqrt{7}-\sqrt{6}$

**03. If $x=3+2\sqrt{2}$ Then showed $x^4-34x^2+1=0$**

**Sol.** Given that, $x=3+2\sqrt{2}$

$\therefore \frac{1}{x}=\frac{1}{3+2\sqrt{2}}$

$=\frac{(3-2\sqrt{2})}{(3+2\sqrt{2})(3-2\sqrt{2})}$

$=\frac{3-2\sqrt{2}}{3^2-(2\sqrt{2})^2}$

$=\frac{3-2\sqrt{2}}{9-8}=3-2\sqrt{2}$

Now, $x+\frac{1}{x}=(3+2\sqrt{2})+(3-2\sqrt{2})$

Or, $x+\frac{1}{x}=3+2\sqrt{2}+3-2\sqrt{2}$

Or, $x+\frac{1}{x}=6$

Or, $\left(x+\frac{1}{x}\right)^2=6^2$ [square both side]

Or, $x^2+2.X.\frac{1}{x}+\frac{1}{x^2}=36$

Or, $x^2+2+\frac{1}{x^2}=36$

Or, $x^2+\frac{1}{x^2}=36-2$ Or, $\frac{x^4+1}{x^2}=34$

Or, $x^4+1=34x^2$

$\therefore x^4-34x^2+1=0$ **Ans.**

**04. If $x^2+\frac{1}{x^2}=10$, Showed that $x=\sqrt{3}+\sqrt{2}$**

**Sol.** Given that, $x^2+\frac{1}{x^2}=10$

Or, $\frac{x^4+1}{x^2}=10$ Or, $x^4+1=10x^2$

Or, $x^4-10x^2+1=0$

Or, $(x^2)^2-2.x^2.5+5^2-5^2+1=0$

Or, $(x^2-5)^2-25+1=0$

Or, $(x^2-5)^2-24=0$

Or, $(x^2-5)^2=24$

Or, $x^2-5=\sqrt{6.4}$ [ square both side]

Or, $x^2-5=\sqrt{3.2.4}$ Or, $x^2-5=2\sqrt{3.2}$

Or, $x^2=2.\sqrt{3}.\sqrt{2}+5$

Or, $x^2=2.\sqrt{3}.\sqrt{2}+(\sqrt{2})^2+(\sqrt{3})^2$

Or, $x^2=(\sqrt{3}+\sqrt{2})^2$

Or, $x=\sqrt{3}+\sqrt{2}$ [square both side

$\therefore x=\sqrt{3}+\sqrt{2}$ **(Ans.)**

**05. If, $x-\frac{6}{x}=1$, the value of, $\frac{6}{x^2+x+1}$ ?**

**Sol.** $x-\frac{6}{x}=1$

Or, $\frac{x^2-6}{x}=1$ Or, $x^2-6=x$

Or, $x^2-x-6=0$ Or, $x^2-3x+2x-6=0$

Or, $x(x-3)+2(x-3)=0$ Or, $(x-3)(x+2)=0$

$\therefore x-3=0$ Or, $x+2=0$

$\therefore x=3$ Or, $x=-2$

If, $x=3$ Now, $\frac{6}{x^2+x+1}=\frac{6}{3^2+3+1}=\frac{6}{9+3+1}=\frac{6}{13}$

If, $x=-2$ Now, $\frac{6}{x^2+x+1}=\frac{6}{(-2)^2-2+1}$

$=\frac{6}{4-2+1}=\frac{6}{3}=2$

$\frac{6}{13}$ or, 2. **Ans:**

**06. If, $2a=\sqrt{7}+\sqrt{5}$ and $2b=\sqrt{7}-\sqrt{5}$, Showed that, $ab(a^4-b^4)=3\sqrt{35}$**

**Sol.** Given that, $2a=\sqrt{7}+\sqrt{5}$

$2b=\sqrt{7}-\sqrt{5}$

$\therefore 2a+2b=\sqrt{7}+\sqrt{5}+\sqrt{7}-\sqrt{5}$

Or, $2(a+b)=2\sqrt{7}$

Or, $a+b=\sqrt{7}$ [ Divide both sides by 2]

Again, $2a-2b=\sqrt{7}+\sqrt{5}-\sqrt{7}+\sqrt{5}$

Or, $2(a-b)=2\sqrt{5}$

Or, $a-b=\sqrt{5}$ [ Divide both side by 2]

L.H.S = $ab(a^4-b^4)$

$= ab\{(a^2)^2-(b^2)^2\}$

$= ab(a^2+b^2)(a^2-b^2)$

$= ab(a^2+b^2)(a+b)(a-b)$

$= \left\{\left(\frac{a+b}{2}\right)^2-\left(\frac{a-b}{2}\right)^2\right\}\left\{\frac{(a+b)^2+(a-b)^2}{2}\right\}(a+b)(a-b)$

$= \left\{\left(\frac{\sqrt{7}}{2}\right)^2-\left(\frac{\sqrt{5}}{2}\right)^2\right\}\left\{\frac{(\sqrt{7})^2+(\sqrt{5})^2}{2}\right\}.\sqrt{7}\ .\sqrt{5}$

$= \left\{\frac{7}{4}-\frac{5}{4}\right\}\left\{\frac{7+5}{2}\right\}\sqrt{35}$

$= \left(\frac{7-5}{4}\right)\left(\frac{12}{2}\right)\sqrt{35}$

$= \frac{2}{4}.6.\sqrt{35} = 3\sqrt{35}$ = R.H.S

$\therefore ab(a^4-b^4) = 3\sqrt{35}$ **(Showed)**

**07. If $a^3+b^3+c^3 = 3abc$, Proved that, $a+b+c = 0$ square or, $a = b = c$**

**Sol.** $a^3+b^3+c^3 = 3abc$

Or, $a^3+b^3+c^3-3abc = 0$

Or, $\frac{1}{2}(a+b+c)\{(a-b)^2+(b-c)^2+(c-a)^2\}=0$

Or, $(a+b+c)\{(a-b)^2+(b-c)^2+(c-a)^2\} = 0$

$\therefore a+b+c = 0$ or, $(a-b)^2+(b-c)^2+(c-a)^2=0$

Now the square root is only positive or zero. So if the sum of severel squares is zero then each of them will have value of zero.

$\therefore (a-b)^2+(b-c)^2+(c-a)^2 = 0$

$a-b=0$, $b-c=0$ & $c-a=0$

that is, $a = b$, $b=c$ & $c = a$

that is, $a = b = c$

So, $a+b+c = 0$ Or, $a = b = c$ **(Proved)**

**08. If, $x^2-2\sqrt{30}-11 = 0$, $x > 0$ Showed that, $x^3+\frac{1}{x^3} = 42\sqrt{6}$**

**Sol.** Given that, $x^2-2\sqrt{30}-11 = 0$

$\Rightarrow x^2 = 11+2\sqrt{30}$

$\Rightarrow x^2 = 6+5+2\sqrt{5\times6}$

$\Rightarrow x^2 = (\sqrt{6})^2+(\sqrt{5})^2+2\sqrt{6}.\sqrt{5}$

$\Rightarrow x^2 = (\sqrt{6})^2+2(\sqrt{6})(\sqrt{5})+(\sqrt{5})^2$

$\Rightarrow x^2 = (\sqrt{6}+\sqrt{5})^2$

$\therefore x = \sqrt{6}+\sqrt{5}$ [square both side]

$\therefore \frac{1}{x} = \frac{1}{\sqrt{6}+\sqrt{5}}$

$= \frac{\sqrt{6}-\sqrt{5}}{(\sqrt{6}+\sqrt{5})(\sqrt{6}-\sqrt{5})}$

$= \frac{(\sqrt{6}-\sqrt{5})}{(\sqrt{6})^2-(\sqrt{5})^2}$

$= \frac{\sqrt{6}-\sqrt{5}}{6-5} = \sqrt{6}-\sqrt{5}$

$\therefore x+\frac{1}{x} = \sqrt{6}+\sqrt{5}+\sqrt{6}-\sqrt{5} = 2\sqrt{6}$

Now, $x^3+\frac{1}{x^3}$

$= \left(x+\frac{1}{x}\right)^3-3.x.\frac{1}{x}\left(x+\frac{1}{x}\right)$

$= (2\sqrt{6})^3-3(2\sqrt{6})$

$= 2^3\times(\sqrt{6})^3-6\sqrt{6}$

$= 8\times6\sqrt{6}-6\sqrt{6}$

$= 48\sqrt{6}-6\sqrt{6} = 42\sqrt{6}$

$\therefore x^3+\frac{1}{x^3} = 42\sqrt{6}$ **(shown)**।

**09. If, $x = \sqrt{5+2\sqrt{6}}$ the value of, $x^3+\frac{1}{x^3}$**

**Sol.** Given that, $x = \sqrt{5+2\sqrt{6}}$

$\Rightarrow x^2 = 5+2\sqrt{6}$ [Squre both sides]

$\Rightarrow x^2 = 3+2+2\sqrt{6}$

$\Rightarrow x^2 = (\sqrt{3})^2+2.\sqrt{3}.\sqrt{2}+(\sqrt{2})^2$

$\Rightarrow x^2 = (\sqrt{3}+\sqrt{2})^2$

$\therefore x = \sqrt{3}+\sqrt{2}$

$\therefore \frac{1}{x} = \frac{1}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$

$= \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2} = \sqrt{3}-\sqrt{2}$

$\therefore x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3-3.X.\frac{1}{x}\left(x+\frac{1}{x}\right)$

$= \left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right)$

$= (\sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2})^3-3(\sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2})$

$= (2\sqrt{3})^3-3.2\sqrt{3}$

$= (2\sqrt{3})^3-6\sqrt{3}$

$= 8.3\sqrt{3}-6\sqrt{3}$

$= 24\sqrt{3}-6\sqrt{3}$

$= 18\sqrt{3}$

$\therefore$ value is $18\sqrt{3}$ **(Ans:)**

**10. IF, $x = 3+2\sqrt{2}$, the value of, $\frac{x^6+x^4+x^2+1}{x^3}$?**

**Sol.** Given that, $x = 3+2\sqrt{2}$

Now, $\frac{1}{x} = \frac{1}{3+2\sqrt{2}}$

$\Rightarrow \frac{1}{x} = \frac{3-2\sqrt{2}}{(3+2\sqrt{2})(3-2\sqrt{2})}$

$= \frac{3-2\sqrt{2}}{3^2-(2\sqrt{2})^2}$

$= \frac{3-2\sqrt{2}}{9-8} = \frac{3-2\sqrt{2}}{1}$

$\therefore \frac{1}{x} = 3-2\sqrt{2}$

Now, $\frac{x^6+x^4+x^2+1}{x^3}$

$=\frac{x^6}{x^3}+\frac{x^4}{x^3}+\frac{x^2}{x^3}+\frac{1}{x^3}$

$=x^3+x+\frac{1}{x}+\frac{1}{x^3}$

$=\left(x^3+\frac{1}{x^3}\right)+\left(x+\frac{1}{x}\right)$

$=\left(x+\frac{1}{x}\right)^3-3.x.\frac{1}{x}\left(x+\frac{1}{x}\right)+\left(x+\frac{1}{x}\right)$

$=\left(x+\frac{1}{x}\right)^3-3.\left(x+\frac{1}{x}\right)+\left(x+\frac{1}{x}\right)$

$=\left(x+\frac{1}{x}\right)^3-2\left(x+\frac{1}{x}\right)$

$=(3+2\sqrt{2}+3-2\sqrt{2})^3-2(3+2\sqrt{2}+3-2\sqrt{2})$

$(6)^3-2\times6$

$=216-12=204$

$\therefore$ deterministic value 240 **(Ans:)**

**11. If, $(x^2+1)^2=2x^2$, the value is $x^8+\frac{1}{x^8}$**

**Sol.** Given that, $(x^2+1)^2=2x^2$

$\Rightarrow(x^2)^2+2.X^2.1+(1)^2=2x^2$

$\Rightarrow x^4+2x^2+1=2x^2$

$\Rightarrow x^4+2x^2+1-2x^2=0$

$\Rightarrow x^4+1=0$

$\therefore x^4=-1$

Now, $x^8+\frac{1}{x^8}=(x^4)^2+\frac{1}{(x^4)^2}$

$=(-1)^2+\frac{1}{(-1)^2}$

$=1+1=2$

$\therefore$ value is 2 (Ans.)

**12. If $(a^2+1)^2=6a^2$ Showed that, $a^3-\frac{1}{a^3}=5\sqrt{2}$**

**Sol.** Given that, $(a^2+1)^2=6a^2$

$\Rightarrow(a^2)^2+2.a^2.1+(1)^2=6a^2$

$\Rightarrow a^4+2a^2+1-6a^2=0$

$\Rightarrow a^4-4a^2+1=0$

$\Rightarrow\frac{a^4-4a^2+1}{a^2}=0$ [divide both side by $a^2$]

$\Rightarrow\frac{a^4}{a^2}-\frac{4a^2}{a^2}+\frac{1}{a^2}=0$

$\Rightarrow a^2-4+\frac{1}{a^2}=0$

$\Rightarrow a^2+\frac{1}{a^2}=4$

$\Rightarrow a^2+2.a.\frac{1}{a}+\frac{1}{a^2}-2=4$

$\Rightarrow\left(a+\frac{1}{a}\right)^2=4+2$

$\Rightarrow\left(a+\frac{1}{a}\right)^2=6$

$\Rightarrow a+\frac{1}{a}=\sqrt{6}$ [takes positive value]

Again, $\left(a-\frac{1}{a}\right)^2=\left(a+\frac{1}{a}\right)^2-4.a.\frac{1}{a}=(\sqrt{6})^2-4=6-4=2$

$\therefore a-\frac{1}{a}=\sqrt{2}$ [Considering the rich value]

$\Rightarrow\left(a-\frac{1}{a}\right)^3=(\sqrt{2})^3$ [thickened on both side]

$\Rightarrow a^3-\frac{1}{a^3}-3.a.\frac{1}{a}\left(a-\frac{1}{a}\right)=2\sqrt{2}$

$\Rightarrow a^3-\frac{1}{a^3}-3\sqrt{2}=2\sqrt{2}$

$\Rightarrow a^3-\frac{1}{a^3}=2\sqrt{2}+3\sqrt{2}$

$\therefore a^3-\frac{1}{a^3}=5\sqrt{2}$ **(Shown)**

**13. $(a+b)x^2-2ax+(a-b)$**

**Sol.** $(a+b)x^2-2ax+(a-b)$

Suppose, $a+b=p$

$a-b=q$

$(+)\ 2a=p+q$

$\therefore$ Now, $px^2-(p+q)x+q$

$=px^2-px-qx+q$

$=px(x-1)-q(x-1)$

$=(x-1)(px-q)$

$=(x-1)\{(a+b)x-(a-b)\}$[put the value]

$=(x-1)(ax+bx-a+b)$ **(Ans:)**

**14. Factorize, $a^4-4a+3$**

**Sol.** Suppos, $f(a)=a^4-4a+3$

$\therefore f(1)=(1)^4-4.1+3$

$=1-4+3=0$

$\therefore(a-1)$, a factor, f(a)

Now, $a^4-4a+3$

$=a^4-a^3+a^3-a^2+a^2-a-3a+3$

$=a^3(a-1)+a^2(a-1)+a(a-1)-3(a-1)$

$=(a-1)(a^3+a^2+a-3)$

$=(a-1)\{a^3-a^2+2a^2-2a+3a-3\}$

$=(a-1)\{a^2(a-1)+2a(a-1)+3(a-1)\}$

$=(a-1)(a-1)(a^2+2a+3)$

$=(a-1)^2(a^2+2a+3)$ **(Ans.)**

**15. Factorize, $x^3+3x+36$**

**Sol.** here, $x^3+3x+36$

Suppose, $f(x)=x^3+3x+36$

$\therefore f(-3)=(-3)^3+3.(-3)+36$

$=-27-9+36=0$

$\therefore(x+3)$, a factorize of f(x)

Now, $x^3+3x+36$

$=x^3+3x^2-3x^2-9x+12x+36$

$=x^2(x+3)-3x(x+3)+12(x+3)$

$=(x+3)(x^2-3x+12)$ **(Ans.)**

**16.** $\frac{2x-1}{x-1}+\frac{3x-4}{x-2}=\frac{5x-12}{x-3}$

**Sol.** $\frac{2x-1}{x-1}+\frac{3x-4}{x-2}=\frac{5x-12}{x-3}$

Or, $\frac{2x-1}{x-1}-2+\frac{3x-4}{x-2}-3=\frac{5x-12}{x-3}-5$

Or, $\frac{2x-1-2x+2}{x-1}+\frac{3x-4-3x+6}{x-2}=\frac{5x-12-5x+15}{x-3}$

Or, $\frac{1}{x-1}+\frac{2}{x-2}=\frac{3}{x-3}$

Or, $\frac{1}{x-1}+\frac{2}{x-2}=\frac{1}{x-3}+\frac{2}{x-3}$

Or, $\frac{1}{x-1}-\frac{1}{x-3}=\frac{2}{x-3}-\frac{2}{x-2}$

Or, $\frac{x-3-x+1}{(x-1)(x-3)}=\frac{2x-4-2x+6}{(x-3)(x-2)}$

Or, $\frac{-2}{x-1}=\frac{2}{x-2}$

Or, $2x-2=-2x+4$

Or, $2x+2x=4+2$

Or, $4x=6$

$\therefore x=\frac{3}{2}$

$\therefore$ Value is : $x=\frac{3}{2}$ .**(Ans.)**

**17.** $\frac{x-a}{b}+\frac{x-b}{a}+\frac{x-3a-3b}{a+b}=0$

**Sol.** $\frac{x-a}{b}+\frac{x-b}{a}+\frac{x-3a-3b}{a+b}=0$

Or, $\frac{x-a}{b}+\frac{x-b}{a}+\frac{x-3(a+b)}{a+b}=0$

Or, $\frac{x-a}{b}+\frac{x-b}{a}+\frac{x}{a+b}-\frac{3(a+b)}{a+b}=0$

Or, $\frac{x-a}{b}+\frac{x-b}{a}+\frac{x}{a+b}-3=0$

Or, $\left(\frac{x-a}{b}-1\right)+\left(\frac{x-b}{a}-1\right)+\left(\frac{x}{a+b}-1\right)=0$

Or, $\frac{x-a-b}{b}+\frac{x-b-a}{a}+\frac{x-(a+b)}{a+b}=0$

Or, $\frac{x-a-b}{b}+\frac{x-a-b}{a}+\frac{x-a-b}{a+b}=0$

Or, $(x-a-b)\left(\frac{1}{b}+\frac{1}{a}+\frac{1}{a+b}\right)=0$

Is, $x-a-b=0$ or, $\frac{1}{b}+\frac{1}{a}+\frac{1}{a+b}=0$

But, $\frac{1}{b}+\frac{1}{a}+\frac{1}{a+b}\neq 0$

Result, $x-a-b=0$

$\therefore x=a+b$

$\therefore$ value is, $x=(a+b)$. **(Ans.)**

**18.** $\frac{x-a}{b+c+2a}+\frac{x-b}{c+a+2b}+\frac{x-c}{a+b+2b}+3=0$

**Sol.** $\frac{x-a}{b+c+2a}+\frac{x-b}{c+a+2b}+\frac{x-c}{a+b+2b}+3=0$

Or, $\frac{x-a}{b+c+2a}+1+\frac{x-b}{c+a+2b}+1+\frac{x-c}{a+b+2c}+1=0$

Or, $\frac{x-a+b+c+2a}{b+c+2a}+\frac{x-b+c+a+2b}{c+a+2b}+\frac{x-c+a+b+2c}{a+b+2c}=0$

Or, $\frac{x+a+b+c}{b+c+2a}+\frac{x+a+b+c}{c+a+2b}+\frac{x+a+b+c}{a+b+2c}=0$

Or, $(x+a+b+c)\left(\frac{1}{b+c+2a}+\frac{1}{c+a+2b}+\frac{1}{a+b+2c}\right)=0$

But, $\frac{1}{b+c+2a}+\frac{1}{c+a+2a}+\frac{1}{a+b+2c}\neq 0$

Result : , $x+a+b+c=0$ $\therefore x=-(a+b+c)$

$\therefore$ Value is: $x=-(a+b+c)$. **(Ans.)**

**19. Prove:**

**$a(x+y)=2ab$**

**$b(x-y)=2ab$**

**Sol.** Now: $a(x+y)=2ab$.....(i)

$b(x-y)=2ab$.....(ii)

From Equation (i) and (ii), we get,

$a(x+y)=2ab$

Or, $x+y=\frac{2ab}{a}$

Or, $x+y=2b$

$\therefore x=2b-y$.....(iii)

Now, $b(x-y)=2ab$

Or, $x-y=\frac{2ab}{b}$

Or, $(2b-y)-y=2a$ [put in value of x]

Or, $2b-y-y=2a$

Or, $2b-2y=2a$

Or, $-2y=2a-2b$

Or, $-2y=-2(b-a)$

Or, $y=\frac{-2(b-a)}{-2}$

$\therefore y=b-a$

Value of Y put in equation no- (iii) get,

$X=2b-y$

$=2b-(b-a)$

$=2b-b+a$

$=b+a$

$=a+b$

**$\therefore$ Value is: $(x, y)=(a+b, b-a)$. (Ans:)**

**20.** $\frac{x}{a}+\frac{y}{b}=2$

$ax+by=a^2+b^2$

**Sol.** Given two equation :

$\frac{x}{a}+\frac{y}{b}=2$.....(i)

$ax+by=a^2+b^2$.....(ii)

from equation (i) we get,

$\frac{y}{b}=2-\frac{x}{a}$

Or, $\frac{y}{b}=\frac{2a-x}{a}$

Or, $ay=b(2a-x)$

Or, $y=\frac{2ab-bx}{a}$ .....(iii)

substituting the value of y obtained from equation (iii) into equation (ii) we get,

$ax+b\left(\frac{2ab-bx}{a}\right)=a^2+b^2$

Or, $\frac{a^2x+2ab^2-b^2x}{a}=a^2+b^2$

Or, $a^2x+2ab^2-b^2x=a^3+ab^2$

Or, $a^2x-b^2x=a^3+ab^2-2ab^2$

Or, $x(a^2-b^2)=a^3-ab^2$

Or, $x(a^2-b^2)=a(a^2-b^2)$

Or, $x=\frac{a(a^2-b^2)}{a^2-b^2}$

$\therefore x=a$

Substituting the value of (x) in equation (iii)

Or, $y=\frac{2ab-b.a}{a}$ Or, $y=\frac{2ab-ab}{a}$

Or, $y=\frac{ab}{a}$

$\therefore y=b$

$\therefore$ **Value is:** (x, y) = (a, b). **(Ans:)**

## Home Practice

**01.** If $a+b=\sqrt{7}$ & $a-b=\sqrt{5}$ proved that $8ab(a^2+b^2)=24$ [BKB (SO)-2015]

**02.** If $a-\frac{1}{a}=3$, $a^2+\frac{1}{a^2}=?$ [Agrani Bank (SO) 1992] Ans: 11

**03.** Given $x=3+2\sqrt{2}$, find the value of $\sqrt{x}-\frac{1}{\sqrt{x}}$ [Janata Bank– (AEO-RC)-2018 & Standard Bank–(TAO)-2018]

**Ans:** $2\sqrt{2}$

**04.** If $\left(x+\frac{1}{x}\right)^2=3$, then the value of $(x^{72}+x^{66}+x^{54}+x^{24}+x^6+1)$ is? **Ans: 1**

**05.** If $x=\sqrt{5}+2$, then the value of $\frac{2x^2-3x-2}{3x^2-4x-3}$ is? **Ans: 5/8**

**06.** If $x+\frac{1}{x}=5$, then the value of $\frac{x^4+3x^3+5x^2+3x+1}{x^4+1}$ =? **Ans:** $\frac{43}{23}$

**07.** If $x=\sqrt{3}+\sqrt{2}$ proved that $x^3+\frac{1}{x^3}=18\sqrt{3}$ [GM 9-10:3.2] [Dhaka bank (TO): 2017] **Ans:** $18\sqrt{3}$

**08.** If $x^a.X^b.x^c=1$, then find the value of $a^3+b^3+c^3=?$ **Ans: 3abc**

**09.** If $x+\frac{1}{x}=2$; find the value of $x^{17}+\frac{1}{x^{19}}=?$ [Janata Bank–(AEO)-2020] **Ans: 2**

**10.** If $x+\frac{1}{x}=2$ then find the value of $x^{72}+\frac{1}{x^{78}}$ **Ans: 2**

**11.** If $a+\frac{1}{a}=2$ then find.

(i) $a^5+\frac{1}{a^5}$ (ii) $a^6+\frac{1}{a^6}$ **Ans: i) 2, ii) 2**

**12.** If $x+\frac{1}{x}=2$, $\frac{x}{x^2+x-1}=?$ **Ans:1**

**13.** If $a+b=\sqrt{3}$ and $a-b=\sqrt{2}$ Show, $8ab(a^2+b^2)=5$

**14.** If $a+\frac{1}{a}=2$ Show, $a^2+\frac{1}{a^2}=a^4+\frac{1}{a^4}$

**15.** If $a+\frac{1}{a}=2$, $\left(a^2+\frac{1}{a^2}\right)\left(a^3+\frac{1}{a^3}\right)=?$ **Ans:4**

**16.** If $2x-\frac{2}{x}=3$ Showed that $8\left(x^3-\frac{1}{x^3}\right)=63$

**17.** If $\frac{a}{b}=\frac{1}{3}$ $\therefore 3a=b$ Now, $\frac{3a-2b}{3a+2b}$? [Premier bank (TJO): 2013, Shajalal Islami bank (TSO): 2011] **Ans:** $\frac{-1}{3}$

**18.** Solved the equation $\frac{4}{2x+3}+\frac{15}{5x+4}=\frac{35}{7x+6}$ [One Bank (SCO)-2017] **Ans:** $-\frac{9}{13}$

**19.** Find the volume of x, when $\frac{x-a}{b+c}+\frac{x-b}{c+a}+\frac{x-c}{a+b}=3$ [BKB (Cashier)-2012] **Ans: x = (a + b + c)**