Algebra

বিভিন্ন গুরুত্বপূর্ণ সুত্রাবলী:

বর্গ এর সুত্রাবলী:-	
1. $(a+b)^2 = a^2+2ab+b^2$ $= (a-b)^2 + 4ab$	**8.** $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$ অথবা, $a^2+b^2+c^2 = (a+b+c)^2-2(ab+bc+ca)$ অথবা, $2(ab+bc+ca) = (a+b+c)^2-(a^2+b^2+c^2)$
2. $(a-b)^2 = a^2-2ab+b^2$ $= (a+b)^2-4ab$	**9.** $(x+a)(x+b) = x^2+(a+b)x+ab$
3. $a^2+b^2 = (a+b)^2-2ab$ $= (a-b)^2+2ab$	ঘন এর সুত্রাবলী: **10.** $(a+b)^3 = a^3+3a^2b+3ab^2+b^3$ $= a^3+b^3+3ab(a+b)$
4. $2(a^2+b^2) = (a+b)^2 + (a-b)^2$	**11.** $(a-b)^3 = a^3-3a^2b+3ab^2-b^3$ $= a^3-b^3-3ab(a-b)$
5. $(a^2+b^2) = \frac{(a+b)^2 + (a-b)^2}{2}$	**12.** $a^3+b^3 = (a+b)^3-3ab(a+b)$ $= (a+b)(a^2-ab+b^2)$
6. $a^2-b^2 = (a+b)(a-b)$ **7.** $4ab = (a+b)^2-(a-b)^2$	**13.** $a^3-b^3 = (a-b)^3+3ab(a-b)$ $= (a-b)(a^2+ab+b^2)$
8. $ab = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$	**14.** $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

Square related:

1. $x+y = 7$ এবং $xy = 10$ হলে, x^2+y^2 এর মান কত ? *Bangladesh Bank AD-1990 (Written)]*

Solution:

Given that, $x+y=7$, $xy=10$

$\therefore x^2+y^2 = (x+y)^2-2xy \Rightarrow (7)^2-2\times10 \Rightarrow 49-20=29$ **Ans. 29**

2. If $x+\frac{2}{x} = 1$, then evaluate the value of $\frac{x^2+x+2}{x^2(1-x)}$? *[Rupali Bank- (SO)-2019 (Written)]*

Solution: $x+\frac{2}{x} = 1 \Rightarrow x^2+2 = x$ again, $x+\frac{2}{x} = 1$ then, $(1-x) = \frac{2}{x}$

[প্রশ্নে যা দেয়া আছে তা থেকে প্রশ্নে যার মান বের করতে বলা হয়েছে তার মত করে সাজিয়ে নিলে সহজে মান বসানো যাবে]

Now, $\frac{x^2+x+2}{x^2(1-x)} = \frac{(x^2+2)+x}{x^2(1-x)} = \frac{x+x}{x^2\times\frac{2}{x}}$ (মান বসিয়ে) $= \frac{2x}{2x} = 1$ **Ans: 1**

3. $\mathbf{x-\frac{1}{x}=4}$ **হলে, প্রমাণ করে যে,** $\mathbf{x^4+\frac{1}{x^4}}$ **= 322** *[Class 9-10 : 3.1]*

Solution: দেওয়া আছে, $x-\frac{1}{x}=4$

বামপক্ষ: $=x^4+\frac{1}{x^4}=(x^2)^2+\left(\frac{1}{x^2}\right)^2=\left(x^2+\frac{1}{x^2}\right)^2-2.x^2.\frac{1}{x^2}=\left\{\left(x-\frac{1}{x}\right)^2+2.x.\frac{1}{x}\right\}^2-2$

$=(4^2+2)^2-2=(18)^2-2=324-2=322=$ **(ডানপক্ষ)** $\therefore$ বামপক্ষ = ডানপক্ষ (দেখানো হলো)

4. If a+b = 19 and a – b = 11, Calculate the value of (a^2+b^2) and (ab)? *[Premier Bank (JO) cash: 2011]*

Solution:

Given that, (a+b) = 19 and (a – b) = 11

$\therefore a^2+b^2=\frac{(a+b)^2+(a-b)^2}{2}=\frac{(a+b)^2+(a-b)^2}{2}=\frac{361+121}{2}=241$

$\therefore ab=\frac{(a+b)^2-(a-b)^2}{4}=\frac{(19)^2-(11)^2}{4}=\frac{361-121}{4}=60$ Ans: 241 and 60

5. $\mathbf{a^4+a^2b^2+b^4=8}$ **এবং** $\mathbf{a^2+ab+b^2=4}$ **হলে, (i)** $\mathbf{a^2+b^2}$**, (ii) ab-এর মান নির্ণয় কর।**

Solution:

(i) দেওয়া আছে, $a^4+a^2b^2+b^4=8$

বা, $(a^2)^2+2\times a^2\times b^2+(b^2)^2-a^2b^2=8$

বা, $(a^2+b^2)^2-(ab)^2=8$ বা, $(a^2+ab+b^2)(a^2-ab+b^2)=8$ বা, $4(a^2-ab+b^2)=8$ $\therefore a^2-ab+b^2=2$

এখন, $a^2+ab+b^2=4$ এবং $a^2-ab+b^2=2$ যোগ করে পাই, $2(a^2+b^2)=4+2$ বা, $(a^2+b^2)=\frac{6}{2}$ $\therefore a^2+b^2=\mathbf{3}$

(ii) এখন, $a^2+ab+b^2=4$

এবং $a^2-ab+b^2=2$

[বিয়োগ করে] $2ab=4-2$ বা, $ab=\frac{2}{2}$ $\therefore ab=1$ **Ans: 3, 1**

6. $\mathbf{a+b=\sqrt{7}}$ **এবং** $\mathbf{a-b=\sqrt{5}}$ **হলে, প্রমাণ কর যে,** $\mathbf{8ab(a^2+b^2)=24}$ *[BKB (SO)-2015]*

Solution:

বামপক্ষ, $8ab(a^2+b^2)$

$=4ab.2(a^2+b^2)$

$=\{(a+b)^2-(a-b)^2\}\{(a+b)^2+(a-b)^2\}$

$=\{(\sqrt{7})^2-(\sqrt{5})^2\}\{(\sqrt{7})^2+(\sqrt{5})^2\}$

$=(7-5)(7+5)=2\times12=\mathbf{24}=$ **ডানপক্ষ** সুতরাং বামপক্ষ = ডানপক্ষ (**প্রমাণিত)**

❒Self Task:

7. $\mathbf{a+b=\sqrt{5}}$ **এবং** $\mathbf{a-b=\sqrt{2}}$ **হলে,** $\mathbf{24ab(a^2+b^2)}$ **এর মান কত?** **Ans: 63**

8. $a-\frac{1}{a}=3$ **হলে** , $a^2+\frac{1}{a^2}$ **এর মান কত ?** *[Agrani Bank (SO)1992(Written)]*

✍**Solution:**

$a-\frac{1}{a}=3$ হলে $\Rightarrow (a-\frac{1}{a})^2=3^2 \Rightarrow a^2-2.a.\frac{1}{a}+\frac{1}{a^2}=9 \Rightarrow a^2+\frac{1}{a^2}=9+2 \therefore a^2+\frac{1}{a^2}=11$ **Ans. 11**

❐**Self Task:**

9. $x-\frac{1}{x}=2$ **হলে** $x^2+\frac{1}{x^2}=$ **কত ?** *[Bangladesh Bank Banker's Recruting Exam-1986 (Written)]*

✍**Solution:** $x^2+\frac{1}{x^2}=(x-\frac{1}{x})^2+2\times x\times\frac{1}{x}=(2)^2+2=4+2=6$ **Ans.6**

10. $\sqrt{x}+\frac{1}{\sqrt{x}}=a$ **then find the value of** $x^2+\frac{1}{x^2}=$ *[Shajalal Islamic Bank (TSO) – 2016 (Written)]*

✍**Solution:**

শুরুর ভাবনা: প্রশ্নে প্রদত্ত রাশিতে রুট দেয়া আছে। আবার বের করতে বলা হয়েছে বর্গের মান। তাই রুট কে একবার বর্গ করলে রুট উঠে যাবে এবং পরের বার বর্গ করলে বর্গ রাশির মান বের হবে। অর্থাৎ দু'বার বর্গ করার টার্গেটে অংক করা শুরু করতে হবে।

Given , $\sqrt{x}+\frac{1}{\sqrt{x}}=a$

$\Rightarrow \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2=a^2$

$\Rightarrow (\sqrt{x})^2+2\times\sqrt{x}\times\frac{1}{\sqrt{x}}+\left(\frac{1}{\sqrt{x}}\right)^2=a^2 \Rightarrow x+2+\frac{1}{x}=a^2$

$\Rightarrow x+\frac{1}{x}=a^2-2 \Rightarrow \left(x+\frac{1}{x}\right)^2=(a^2-2)^2 \Rightarrow x^2+2\times x\times\frac{1}{x}+\left(\frac{1}{x}\right)^2=(a^2)^2-2\times a^2\times 2+2^2$

$\Rightarrow x^2+2+\frac{1}{x^2}=a^4-4a^2+4 \Rightarrow x^2+\frac{1}{x^2}=a^4-4a^2+4-2 \therefore x^2+\frac{1}{x^2}=a^4-4a^2+2$ **Ans. a^4-4a^2+2**

11. If $2x+\frac{2}{x}=3$ Then the value of $x^2+\frac{1}{x^2}$ *[sadharon Bima AM 16]*

✍**Solution**: Given that, $2x+\frac{2}{x}=3$

$\Rightarrow 2\left(x+\frac{1}{x}\right)=3 \quad \Rightarrow x^2+\frac{1}{x^2}+2.\,x\,.\,\frac{1}{x}=\frac{9}{4}$

$\Rightarrow x+\frac{1}{x}=\frac{3}{2} \quad \Rightarrow \left(x+\frac{1}{x}\right)^2=\left(\frac{3}{2}\right)^2$ [By squaring both sides]

$\Rightarrow x^2+\frac{1}{x^2}+2.\,x\,.\,\frac{1}{x}=\frac{9}{4}$

$\therefore x^2+\frac{1}{x^2}=\frac{9}{4}-2=\frac{9-8}{4}=\frac{1}{4}$ Ans: $\frac{1}{4}$

12. Given x = 3+$\sqrt{8}$, Find the value of $x^2+\frac{1}{x^2}$. *[National Bank Ltd. (PO)-2015-(Written)]*

✍**Solution:** Given, $x=3+\sqrt{8}$ ……………………….(i)

$\Rightarrow \frac{1}{x}=\frac{1}{3+\sqrt{8}}$ [বিপরীতকরণ করে।]

$\Rightarrow \frac{1}{x} = \frac{3-\sqrt{8}}{(3+\sqrt{8})(3-\sqrt{8})}$ [হর ও লবকে $3-\sqrt{8}$ কে দিয়ে গুণ করে (+ থাকলে - দিয়ে গুণ করতে হয়।)]

$\Rightarrow \frac{1}{x} = \frac{3-\sqrt{8}}{(3)^2-(\sqrt{8})^2}$

$\Rightarrow \frac{1}{x} = \frac{3-\sqrt{8}}{9-8}$

$\Rightarrow \frac{1}{x} = \frac{3-\sqrt{8}}{1}$ $\therefore \frac{1}{x} = 3-\sqrt{8}$ ……………….(ii)

> **পরামর্শ:** এরকম প্রশ্ন প্রিলি এবং রিটেন উভয় পরীক্ষায় প্রচুর পরিমানে আসে। এক্ষেত্রে x এর মান দেয়া থাকলে প্রথমে তা থেকেই $\frac{1}{x}$ এর মান বের করে হিসেব করতে হয়। সাধারণত (+) মান থাকলে তা (-) হয়ে যায়।

অতএব, (i)+(ii) হতে পাই $\Rightarrow x+\frac{1}{x} = 3+\sqrt{8}+3-\sqrt{8}$ $\therefore x+\frac{1}{x} = 6$ ……………….(iii)

এখন, $x^2+\frac{1}{x^2} = \left(x+\frac{1}{x}\right)^2 - 2x\times\frac{1}{x} = 6^2-2$ [(iii) নং হতে] $= 36-2 = 34$ **Ans.34**

13. If $x = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $y = \frac{\sqrt{5}-1}{\sqrt{5}+1}$ find the value of $\frac{(x^2+xy+y^2)}{(x^2-xy+y^2)}$ *[PKB-EO-(Written)-2019]*

✍**Solution**:

Given, $x = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $y = \frac{\sqrt{5}-1}{\sqrt{5}+1}$ or, $\frac{1}{y} = \frac{\sqrt{5}+1}{\sqrt{5}-1} = x$ So, $y = \frac{1}{x}$

Now, $\frac{x^2+xy+y^2}{x^2-xy+y^2} = \frac{x^2+2xy+y^2-xy}{x^2-2xy+y^2+xy} = \frac{(x+y)^2-xy}{(x-y)^2+xy} = \frac{(x+y)^2-x\times\frac{1}{x}}{(x-y)^2+x\times\frac{1}{x}} = \frac{(x+y)^2-1}{(x-y)^2+1}$

Here, $(x+y) = \frac{\sqrt{5}+1}{\sqrt{5}-1}+\frac{\sqrt{5}-1}{\sqrt{5}+1} = \frac{(\sqrt{5}+1)^2+(\sqrt{5}-1)^2}{(\sqrt{5}-1)(\sqrt{5}+1)} = \frac{2\{(\sqrt{5})^2+(1)^2\}}{5-1} = \frac{2(5+1)}{4} = \frac{2\times6}{4} = 3$

Again, $(x-y) = \frac{\sqrt{5}+1}{\sqrt{5}-1} - \frac{\sqrt{5}-1}{\sqrt{5}+1} = \frac{(\sqrt{5}+1)^2-(\sqrt{5}-1)^2}{(\sqrt{5}-1)(\sqrt{5}+1)} = \frac{4.\sqrt{5}.1}{(\sqrt{5})^2-1^2} = \frac{4.\sqrt{5}.1}{4} = \sqrt{5}$

So, $\frac{(x+y)^2-1}{(x-y)^2+1} = \frac{(3)^2-1}{(\sqrt{5})^2+1} = \frac{9-1}{5+1} = \frac{8}{6} = \mathbf{\frac{4}{3}}$ **Ans. $\frac{4}{3}$**

[**Note:** এই অংকে ব্যবহৃত গুরুত্বপূর্ণ দুটি সূত্র হলো: $(a+b)^2+(a-b)^2 = \mathbf{2(a^2+b^2)}$ এবং $(a+b)^2-(a-b)^2 = \mathbf{4ab}$]

❒ Self Task:

14. If $x = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ and $y = \frac{\sqrt{3}-1}{\sqrt{3}+1}$ then find out the value of (x^2+y^2). *[Al-Arafah Islami Bank Ltd.(MTO)-2013(Written)]*

✍**Solution:** $(x^2+y^2) = \left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)^2 + \left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)^2$ *[Putting the value of x and y]*

$= \frac{4}{2}+\frac{2}{4} = 2+\frac{1}{2} = \frac{4+1}{2} = \frac{5}{2}$ **Ans. $\frac{5}{2}$**

15. Given $x = 3+2\sqrt{2}$, find the value of $\sqrt{x}-\frac{1}{\sqrt{x}}$ *[Janata Bank –(AEO-RC)-2018-(Written)] & [Standard Bank –(TAO-General)-2018-(Written)]*

✍**Solution:**

Given, $x = 3+2\sqrt{2}$

$\frac{1}{x} = \frac{1}{3+2\sqrt{2}} = \frac{3-2\sqrt{2}}{(3+2\sqrt{2})(3-2\sqrt{2})} = \frac{3-2\sqrt{2}}{3^2-(2\sqrt{2})^2} = \frac{3-2\sqrt{2}}{9-8} = 3-2\sqrt{2}$

$x+\frac{1}{x} = 3+2\sqrt{2}+3-2\sqrt{2} = 6$

$\therefore \left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 = (\sqrt{x})^2 - 2.\sqrt{x}.\frac{1}{\sqrt{x}} + \left(\frac{1}{\sqrt{x}}\right)^2 = \left(x+\frac{1}{x}\right) - 2 = 6-2 = 4$

$\therefore \sqrt{x}-\frac{1}{\sqrt{x}} = \sqrt{4} = 2$ **Ans: 2**

❒**Self Task:**

16. Given $a = 3+2\sqrt{2}$, find the value of $\sqrt{a}-\frac{1}{\sqrt{a}}$ *[Janata Bank (AEO-RC) -2017 (W]* **Ans: 2**

17. If $x^2 = 13+\sqrt{168}$ then determine the value of $x^2-\frac{1}{x^2} = ?$

✍**Solution**: (যে মান দেয়া থাকবে তা থেকে যার মান বের করতে হবে তার মত করে সাজিয়ে নিতে হবে)

Given that, $x^2 = 13+\sqrt{168}$

$\Rightarrow x^2 = 7+2\sqrt{42}+6 \Rightarrow x^2 = (\sqrt{7})^2 + 2.\sqrt{7}.\sqrt{6} + (\sqrt{6})^2 \Rightarrow x^2 = (\sqrt{7}+\sqrt{6})^2$

$\Rightarrow x = \sqrt{7}+\sqrt{6} \Rightarrow \frac{1}{x} = \frac{1}{\sqrt{7}+\sqrt{6}} \Rightarrow \frac{1}{x} = \frac{\sqrt{7}-\sqrt{6}}{(\sqrt{7}+\sqrt{6})(\sqrt{7}-\sqrt{6})} \therefore \frac{1}{x} = (\sqrt{7}-\sqrt{6})$

Given expression = $x^2-\frac{1}{x^2} = \left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$

$= (\sqrt{7}+\sqrt{6}+\sqrt{7}-\sqrt{6})\ (\sqrt{7}+\sqrt{6}-\sqrt{7}+\sqrt{6}) = 2\sqrt{7}\times 2\sqrt{6} = 4\sqrt{42}$ (**Ans**.)

18. If $x+\frac{1}{x} = 2$, what is the value of $\frac{1}{x^2+x-1}$? *[Shajalal Islami Bank (TO Cash)-2016(Written]*

✍**Solution:** $x+\frac{1}{x} = 2 \Rightarrow \frac{x^2+1}{x} = 2$

$\Rightarrow x^2+1 = 2x \Rightarrow x^2-2x+1 = 0 \Rightarrow x^2-2.x.1+1^2 = 0 \Rightarrow (x-1)^2 = 0 \Rightarrow x-1 = 0 \therefore x = 1$

$\therefore \frac{1}{x^2+x-1} = \frac{1}{1^2+1-1} = \frac{1}{1} = 1$ The value of x = 1. **Ans: 1**

❒**Self Task:**

19. $x - \frac{6}{x} = 1$ হলে, $\frac{6}{x^2+x+1}$ এর মান কত?

Solution: $x - \frac{6}{x} = 1 \Rightarrow x^2 - 6 = x \Rightarrow x^2 - x - 6 = 0 \Rightarrow x^2 - 3x + 2x - 6 = 0$

$\Rightarrow x(x-3) + 2(x-3) = 0 \Rightarrow (x-3)(x+2) = 0$

অতএব, $x - 3 = 0$ অথবা $x + 2 = 0$

$x = 3$ $\Rightarrow x = -2$

$x = 3$ হলে, $\frac{6}{x^2+x+1} = \frac{6}{3^2+3+1} = \frac{6}{13}$

এবং $x = -2$ হলে, $\frac{6}{x^2+x+1} = \frac{6}{(-2)^2-2+1} = \frac{6}{3} = 2$ **Ans: 2 অথবা** $\frac{6}{13}$

20. $\mathbf{x^2+\frac{1}{x^2}=1}$**, find the value** $\mathbf{x^{102}+x^{96}+x^{90}+x^{84}+x^{78}+x^{72}+5}$**?** *[Janata Bank (AEO-Teller)-2020 Written]*

Solution: (যে রাশিটির মান বের করতে বলা হয়েছে তার x এর সবগুলো পাওয়ারের মধ্যে কমন নিলে x^6 আসবে)

Given that, $x^2+\frac{1}{x^2} = 1 \Rightarrow \frac{x^4+1}{x^2} = 1 \Rightarrow x^4+1 = x^2 \Rightarrow x^4-x^2 = -1$

$\Rightarrow (x^4-x^2)(x^4+x^2) = -(x^4+x^2)$ *[Multiply both sides by (x^4+x^2)]*

$\Rightarrow x^8-x^4 = -x^4 - x^2 \Rightarrow x^8 = -x^2 \Rightarrow \frac{x^8}{x^2} = \frac{-x^2}{x^2} \therefore x^6 = -1$

Now, $x^{102} + x^{96} + x^{90}+x^{84}+x^{78}+x^{72} + 5$

$= (x^6)^{17} + (x^6)^{16}+(x^6)^{15}+(x^6)^{14}+(x^6)^{13}+(x^6)^{12} + 5$ [x^6 এর মান বের করায় এখানে মান বসানো সহজ]

$= (-1)^{17} + (-1)^{16}+(-1)^{15}+(-1)^{14}+(-1)^{13}+(-1)^{12} + 5 = -1 + 1 - 1 + 1 - 1 + 1 + 5$ **= 5 (Ans.)**

Alternative Solution:

Given that, $x^2+\frac{1}{x^2} = 1 \Rightarrow \left(x+\frac{1}{x}\right)^2 - 2.x\frac{1}{x} = 1 \Rightarrow \left(x+\frac{1}{x}\right)^2 = 3 \Rightarrow x+\frac{1}{x} = \sqrt{3}$

$\therefore \left(x+\frac{1}{x}\right)^3 = (\sqrt{3})^3$

$\Rightarrow x^3+\frac{1}{x^3} + 3.x.\frac{1}{x}\left(x+\frac{1}{x}\right) = 3\sqrt{3} \Rightarrow x^3+\frac{1}{x^3} = 3\sqrt{3} - 3\sqrt{3} = 0 \Rightarrow \frac{x^6+1}{x^3} = 0 \Rightarrow x^6+1 = 0$

Now, $x^{102} + x^{96} + x^{90} + x^{84} + x^{78} + x^{72} + 5$

$= x^{96}(x^6+1) + x^{84}(x^6+1) + x^{72}(x^6+1) + 5 = x^{96}\times 0 + x^{84}\times 0 + x^{72}\times 0 + 5 = 5$ **Ans: 5**

❒Self Task:

21. If $\left(x+\frac{1}{x}\right)^2 = 3$, then the value of $(x^{72} + x^{66} + x^{54} + x^{24} + x^6 + 1)$ is?

Solution: $\left(x+\frac{1}{x}\right)^2 = 3$

$\Rightarrow x + \frac{1}{x} = \sqrt{3} \Rightarrow x^3 + \frac{1}{x^3} + 3\sqrt{3} = 3\sqrt{3} \Rightarrow x^3 + \frac{1}{x^3} = 0 \Rightarrow x^6 + 1 = 0 \;\therefore x^6 = -1$

Given expression $= x^{72} + x^{66} + x^{54} + x^{24} + x^6 + 1$

$= (x^6)^{12} + (x^6)^{11} + (x^6)^9 + (x^6)^4 + x^6 + 1$

$= (-1)^{12} + (-1)^{11} + (-1)^9 + (-1)^4 - 1 + 1 = 1 - 1 - 1 + 1 - 1 + 1 = \mathbf{0}$ **Ans: 0**

22. If $x = \sqrt{5} + 2$, then the value of $\frac{2x^2 - 3x - 2}{3x^2 - 4x - 3}$ is ?

✍**Solution**:

$x = \sqrt{5} + 2 \Rightarrow x - 2 = \sqrt{5} \Rightarrow (x - 2)^2 = 5$ [by squaring]

$\Rightarrow x^2 - 4x + 4 - 5 = 0 \Rightarrow x^2 - 4x - 1 = 0 \Rightarrow x - \frac{1}{x} = 4$

Now, $\frac{2x^2 - 3x - 2}{3x^2 - 4x - 3} = \frac{2x\left(x - \frac{1}{x} - \frac{3}{2}\right)}{3x\left(x - \frac{1}{x} - \frac{4}{3}\right)} = \frac{2\left(4 - \frac{3}{2}\right)}{3\left(4 - \frac{4}{3}\right)} = \frac{5}{8}$

23. If $x + \frac{1}{x} = 5$, then the value of $\frac{x^4 + 3x^3 + 5x^2 + 3x + 1}{x^4 + 1} = ?$

✍**Solution**:

$x + \frac{1}{x} = 5$ (By squaring both sides)

$x^2 + \frac{1}{x^2} + 2.x.\frac{1}{x} = (5)^2 \Rightarrow x^2 + \frac{1}{x^2} = 23$

Now, $\frac{x^4 + 3x^3 + 5x^2 + 3x + 1}{x^4 + 1}$ [Divided by x^2]

$= \frac{\frac{x^4}{x^2} + \frac{3x^3}{x^2} + \frac{5x^2}{x^2} + \frac{3x}{x^2} + \frac{1}{x^2}}{\frac{x^4}{x^2} + \frac{1}{x^2}} = \frac{x^2 + 3x + 5 + \frac{3}{x} + \frac{1}{x^2}}{x^2 + \frac{1}{x^2}} = \frac{23 + 3(5) + 5}{23} = \frac{43}{23}$

24. If $\frac{x^2 - x + 1}{x^2 + x + 1} = \frac{3}{2}$, then the value of $\left(x + \frac{1}{x}\right)^2$ is?

✍**Solution**:

Given that, $\frac{x^2 - x + 1}{x^2 + x + 1} = \frac{3}{2}$

$\Rightarrow 3x^2 + 3x + 3 = 2x^2 - 2x + 2$

$\Rightarrow x^2 + 5x + 1 = 0$

$\Rightarrow \frac{x^2 + 5x + 1}{x} = \frac{0}{x}$ [x দ্বারা ভাগ করে]

$\Rightarrow x + \frac{1}{x} + 5 = 0 \quad \Rightarrow x + \frac{1}{x} = -5 \quad \therefore \left(x + \frac{1}{x}\right)^2 = $ **25 (Ans.)**

25. If $p^2 + q^2 + r^2 - 2p + 2q + 2r = -3 = 0$ then determine the value of $p^2 + q^4 + r^6$?

✍**Solution**:

Given that, $p^2 + q^2 + r^2 - 2p + 2q + 2r + 3 = 0$

$\Rightarrow p^2 - 2p + 1 + q^2 + 2q + 1 + r^2 + 2r + 1 = 0$

$\Rightarrow (p-1)^2 + (q+1)^2 + (r+1)^2 = 0$

So,

$(p-1)^2 = 0$	$(q+1)^2 = 0$	$(r+1)^2 = 0$
$\Rightarrow p - 1 = 0$	$\Rightarrow q + 1 = 0$	$\Rightarrow r + 1 = 0$
$\Rightarrow p = 1$	$\Rightarrow q = -1$	$\Rightarrow r = -1$

$\therefore$ Given expression $= p^2 + q^4 + r^6 = (1)^2 + (-1)^4 + (-1)^6 = 1 + 1 + 1 =$ **3 (Ans.)**

26. If x = 2015, y = 2014, z = 2013, then the value of $x^2 + y^2 + z^2 - xy - yz - zx$ is?

✍**Solution:**

Given that, x = 2015, y = 2014, z = 2013

$\therefore x^2 + y^2 + z^2 - xy - yz - zx$

$= \frac{1}{2}\left[(x-y)^2 + (y-z)^2 + (z-x)^2\right]$

$= \frac{1}{2}\left[(2015-2014)^2 + (2014-2013)^2 + (2013-2015)^2\right] = \frac{1}{2}(1+1+4) =$ **3 (Ans)**

🗁 Fourth or even power related:

27. If $a - \frac{1}{a} = m$, prove that $a^4 + \frac{1}{a^4} = m^4 + 4m^2 + 2$ *[Basic Bank (AO)-2009-(Written)]*

✍**Solution:**

Given , $a - \frac{1}{a} = m$

$\Rightarrow (a - \frac{1}{a})^2 = m^2$ *[Square both side]*

$\Rightarrow a^2 - 2 \times a \times \frac{1}{a} + (\frac{1}{a})^2 = m^2$

$\Rightarrow a^2 + \frac{1}{a^2} = m^2 + 2$

$\Rightarrow (a^2 + \frac{1}{a^2})^2 = (m^2 + 2)^2$ *[Square both side again]*

$\Rightarrow (a^2)^2 + 2 \times a^2 \times \frac{1}{a^2} + (\frac{1}{a^2})^2 = m^4 + 2.m^2.2 + 2^2 \Rightarrow a^4 + \frac{1}{a^4} = m^4 + 4m^2 + 4 - 2$

$\therefore a^4 + \frac{1}{a^4} = m^4 + 4m^2 + 2$ So, L.H.S = R.H.S **(Proved)**

28. If $x = 9 + 4\sqrt{5}$ then prove that $x^4 - 322x^2 + 1 = 0$

Solution: Given that, $x = 9 + 4\sqrt{5}$

$\Rightarrow \frac{1}{x} = \frac{1}{9+4\sqrt{5}} \Rightarrow \frac{1}{x} = \frac{9-4\sqrt{5}}{(9)^2-(4\sqrt{5})^2} \quad \therefore \frac{1}{x} = 9-4\sqrt{5}$

Now, $x + \frac{1}{x} = 9 + 4\sqrt{5} + 9 - 4\sqrt{5} = 18$

$\Rightarrow \left(x+\frac{1}{x}\right)^2 = (18)^2 \Rightarrow x^2 + 2.x.\frac{1}{x} + \frac{1}{x^2} = 324 \Rightarrow x^2 + \frac{1}{x^2} = 324 - 2$

$\Rightarrow x^2 + \frac{1}{x^2} = 322 \Rightarrow \frac{x^4+1}{x^2} = 322 \Rightarrow x^4 - 322x^2 + 1 = 0$ **(Proved)**

29. Find the value of $x^4 + \frac{1}{x^4}$; If $x = \sqrt{5} - \sqrt{4}$ *[Janata Bank Ltd.(EO-FA)-2015-(Written)] & [Dhaka Bank-MTO-2017]*

Solution:

Given that, $x = \sqrt{5} - \sqrt{4}$ ----------(i)

$\Rightarrow \frac{1}{x} = \frac{1}{\sqrt{5}-\sqrt{4}}$ [বিপরীত করণ করে]

$\Rightarrow \frac{1}{x} = \frac{1\times(\sqrt{5}+\sqrt{4})}{(\sqrt{5}-\sqrt{4})(\sqrt{5}+\sqrt{4})}$

$\Rightarrow \frac{1}{x} = \frac{\sqrt{5}+\sqrt{4}}{(\sqrt{5})^2-(\sqrt{4})^2} \Rightarrow \frac{1}{x} = \frac{\sqrt{5}+\sqrt{4}}{(\sqrt{5})^2-(\sqrt{4})^2} \quad \therefore \frac{1}{x} = \sqrt{5}+\sqrt{4}$ ---------(ii)

by (i) + (ii) $x + \frac{1}{x} = \sqrt{5} - \sqrt{4} + \sqrt{5} + \sqrt{4} = 2\sqrt{5}$

Now, $x^4 + \frac{1}{x^4} = (x^2)^2 + \left(\frac{1}{x^2}\right)^2 = \left(x^2 + \frac{1}{x^2}\right)^2 - 2.x^2.\frac{1}{x^2} = \left\{\left(x+\frac{1}{x}\right)^2 - 2.x.\frac{1}{x}\right\}^2 - 2$

$= \left\{(2\sqrt{5})^2 - 2\right\}^2 - 2 = (20-2)^2 - 2 = 324 - 2 = 322$ **Ans: 322**

❒Self Task:

30. Prove that $x^4 = 527 - \frac{1}{x^4}$, When $x = 5 - \frac{1}{x}$

Solution:

Given that, $x = 5 - \frac{1}{x} \Rightarrow x + \frac{1}{x} = 5 \Rightarrow \left(x+\frac{1}{x}\right)^2 = (5)^2 \Rightarrow x^2 + \frac{1}{x^2} + 2 = 25$

$\Rightarrow \left(x^2+\frac{1}{x^2}\right)^2 = (23)^2 \Rightarrow x^4 + \frac{1}{x^4} + 2 = 529 \Rightarrow x^4 = 527 - \frac{1}{x^4}$ **(Proved)**

31. If $x = \sqrt{7}+\sqrt{6}$, Then prove that $x^4 - \frac{1}{x^4} = 104\sqrt{42}$

✍**Solution:** Given that, $x = \sqrt{7}+\sqrt{6}$

$$\Rightarrow \frac{1}{x} = \frac{1}{\sqrt{7}+\sqrt{6}} = \frac{1\times(\sqrt{7}-\sqrt{6})}{(\sqrt{7}+\sqrt{6})(\sqrt{7}-\sqrt{6})} = \frac{(\sqrt{7}-\sqrt{6})}{(\sqrt{7})^2-(\sqrt{6})^2} \therefore \frac{1}{x} = \sqrt{7}-\sqrt{6}$$

$$\text{L.H.S} = x^4 - \frac{1}{x^4} = (x^2)^2 - \left(\frac{1}{x^2}\right)^2 = \left(x^2+\frac{1}{x^2}\right)\left(x^2-\frac{1}{x^2}\right)$$

$$= \left\{\left(x+\frac{1}{x}\right)^2 - 2.x.\frac{1}{x}\right\}\left\{\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)\right\}$$

$$= \left\{\left(\sqrt{7}+\sqrt{6}+\sqrt{7}-\sqrt{6}\right)^2 - 2\right\}\times\left\{(\sqrt{7}+\sqrt{6}+\sqrt{7}-\sqrt{6})(\sqrt{7}+\sqrt{6}-\sqrt{7}+\sqrt{6})\right\}$$

$$= \{(2\sqrt{7})^2 - 2\}\times 2\sqrt{7}\times 2\sqrt{6}$$

$$= (28-2)\times 2\sqrt{7}\times 2\sqrt{6} = 26\times 2\sqrt{7}\times 2\sqrt{6} = 104\sqrt{42} = \text{R.H.S}$$ $\therefore$ **L.H.S = R.H.S (Proved)**

32. If $P = 3+\frac{1}{p}$, then prove that $p^4 = 119 - \frac{1}{p^4}$?

✍**Solution**:

Given that, $P = 3+\frac{1}{p}$

$$\Rightarrow P-\frac{1}{P} = 3 \Rightarrow \left(P-\frac{1}{P}\right)^2 = (3)^2 \Rightarrow P^2+\frac{1}{P^2} - 2 = 9 \Rightarrow P^2+\frac{1}{P^2} = 11$$

$$\Rightarrow \left(P^2+\frac{1}{P^2}\right)^2 = (11)^2 \Rightarrow P^4+\frac{1}{P^4}+2 = 121 \Rightarrow P^4+\frac{1}{P^4} = 119 \therefore P^4 = 119-\frac{1}{P^4}$$ **(Proved)**

❒**Self Task:**

33. $P^4 = 119 - \frac{1}{p^4}$ then prove that $P = 3+\frac{1}{P}$

34. If $x+\frac{1}{x} = \sqrt{3}$ then determine the value of $x^6+\frac{1}{x^6}$ = ?

✍**Solution**:

Given that, $x+\frac{1}{x} = \sqrt{3} \Rightarrow \left(x+\frac{1}{x}\right)^2 = (\sqrt{3})^2 \Rightarrow x^2+2.x.\frac{1}{x}+\frac{1}{x^2} = 3$

$$\Rightarrow x^2+\frac{1}{x^2} = 1 \quad \Rightarrow \left(x^2+\frac{1}{x^2}\right)^3 = (1)^3 \quad \Rightarrow (x^2)^3+\left(\frac{1}{x^2}\right)^3+3.x^2.\frac{1}{x^2}\left(x^2+\frac{1}{x^2}\right) = 1$$

$$\Rightarrow x^6+\frac{1}{x^6}+3(1) = 1 \quad \Rightarrow x^6+\frac{1}{x^6} = 1-3 \quad \therefore x^6+\frac{1}{x^6} = -2$$ **(Ans:)**

35. If $x - \frac{1}{x} = \sqrt{5}$ then $x^8 + \frac{1}{x^8} = ?$

✍**Solution**: Given that, $x - \frac{1}{x} = \sqrt{5} \Rightarrow \left(x - \frac{1}{x}\right)^2 = (\sqrt{5})^2 \Rightarrow x^2 + \frac{1}{x^2} - 2.x.\frac{1}{x} = 5 \Rightarrow x^2 + \frac{1}{x^2} = 7$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = (7)^2 \Rightarrow x^4 + \frac{1}{x^4} + 2.x^2.\frac{1}{x^2} = 49 \Rightarrow x^4 + \frac{1}{x^4} = 47 \Rightarrow \left(x^4 + \frac{1}{x^4}\right)^2 = (47)^2$

$\Rightarrow x^8 + \frac{1}{x^8} + 2.x^4.\frac{1}{x^4} = 2209 \quad \Rightarrow x^8 + \frac{1}{x^8} = 2209 - 2 \therefore x^8 + \frac{1}{x^8} =$ **2207 (Ans.)**

36. If $x^2 = \sqrt{5}\,x - 1$ Then prove that $47x^8 - x^{16} = 1$

✍**Solution:**

Given that, $x^2 = \sqrt{5}x - 1$

$\Rightarrow x^2 + 1 = \sqrt{5}x \Rightarrow \frac{x^2 + 1}{x} = \frac{\sqrt{5}x}{x} \Rightarrow x + \frac{1}{x} = \sqrt{5} \Rightarrow \left(x + \frac{1}{x}\right)^2 = (\sqrt{5})^2$

$\Rightarrow x^2 + \frac{1}{x^2} + 2.x.\frac{1}{x} = 5 \Rightarrow x^2 + \frac{1}{x^2} = 5 - 2 \Rightarrow x^2 + \frac{1}{x^2} = 3 \Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = (3)^2$

$\Rightarrow \left(x^4 + \frac{1}{x^4}\right) + 2 = 9 \Rightarrow x^4 + \frac{1}{x^4} = 7 \Rightarrow \left(x^4 + \frac{1}{x^4}\right)^2 = (7)^2 \Rightarrow x^8 + \frac{1}{x^8} + 2 = 49$

$\Rightarrow x^8 + \frac{1}{x^8} = 47 \Rightarrow \frac{x^{16} + 1}{x^8} = 47 \Rightarrow x^{16} + 1 = 47x^8 \Rightarrow 47x^8 - x^{16} = 1$ **(Proved)**

Cube Related:

37. If $\frac{a}{b} + \frac{b}{a} = 1$, then find the value of $a^3+b^3 = ?$

✍**Solution**:

Given that, $\frac{a}{b} + \frac{b}{a} = 1 \Rightarrow \frac{a^2 + b^2}{ab} = 1 \Rightarrow a^2+b^2 = ab \Rightarrow a^2+b^2 - ab = 0$

Given expression = $a^3+b^3 = (a+b)(a^2-ab+b^2) = (a+b) \times 0$ **= 0** **(Ans:)**

38. $2x - \frac{2}{x} = 3$ হলে, দেখাও যে, $8\left(x^3 - \frac{1}{x^3}\right) = 63$ ***[GM 9-10: 3.2]***

✍**Solution:**

দেওয়া আছে, $\left(2x - \frac{2}{x}\right) = 3 \Rightarrow 2\left(x - \frac{1}{x}\right) = 3 \therefore \left(x - \frac{1}{x}\right) = \frac{3}{2}$

প্রমাণ করতে হব যে, $8\left(x^3 - \frac{1}{x^3}\right) = 63$ **L.H.S** = $8\left(x^3 - \frac{1}{x^3}\right)$

$$=8\left\{\left(x-\frac{1}{x}\right)^3+3.x.\frac{1}{x}\left(x-\frac{1}{x}\right)\right\}=8\left\{\left(\frac{3}{2}\right)^3+3\left(\frac{3}{2}\right)\right\}=8\left\{\frac{27}{8}+\frac{9}{2}\right\}=8\left\{\frac{27+36}{8}\right\} = 63 = \textbf{R.H.S}$$

$\therefore$ **L.H.S =R.H.S (Proved)**

39. $x-3=-\frac{1}{x}$ **হলে ,** $x^3+\frac{1}{x^3}$ **এর মান কত ?** *[Karmahgangsthan Bank (AO)Cash-2011(Written)]*

✍**Solution:**

দেয়া আছে, $x-3=-\frac{1}{x}$

$\therefore x+\frac{1}{x}=3$ [পক্ষান্তর করে]

এখন , $x^3+\frac{1}{x^3}=(x+\frac{1}{x})^3-3\times x\times\frac{1}{x}(x+\frac{1}{x})$ = $3^3-3\times(3)$ [মান বসিয়ে] = 27–9 =18 **Ans. 18**

40. $x-\frac{1}{x}=\sqrt{5}$ হলে $x^3-\frac{1}{x^3}$ **এর মান নির্ণয় করুন।** *[Ministry of Food (AP)-2020 (Written)]*

✍**Solution:** দেওয়া আছে, $x-\frac{1}{x}=\sqrt{5}$

আমরা জানি, $\left(x+\frac{1}{x}\right)^2=\left(x-\frac{1}{x}\right)^2+4.x.\frac{1}{x}=\left(\sqrt{5}\right)^2+4=5+4 \Rightarrow \left(x+\frac{1}{x}\right)^2=9 \quad \therefore x+\frac{1}{x}=3$

এখন, $x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)^3-3.x.\frac{1}{x}.\left(x+\frac{1}{x}\right)$ = $(3)^3-3.3$ = 27 – 9 = **18** **Ans. 18**

❒**Self Task:**

41. If $a-\frac{1}{a}=\sqrt{5}$**, what is the value of** $a\sqrt{a}+\frac{1}{a\sqrt{a}}$?

✍**Solution**: Given that, $a-\frac{1}{a}=\sqrt{5}$

$\therefore (a+\frac{1}{a})^2=(a-\frac{1}{a})^2+4.a.\frac{1}{a}=(\sqrt{5})^2+4$ = 5+4 = 9

$\therefore a+\frac{1}{a}=3 \Rightarrow (\sqrt{a})^2+(\frac{1}{\sqrt{a}})^2=3$

$\Rightarrow (\sqrt{a}+\frac{1}{\sqrt{a}})^2-2=3 \quad \Rightarrow (\sqrt{a}+\frac{1}{\sqrt{a}})^2=5 \quad \therefore \sqrt{a}+\frac{1}{\sqrt{a}}=\sqrt{5}$

Given expression = $a\sqrt{a}+\frac{1}{a\sqrt{a}}=\sqrt{a}.\sqrt{a}.\sqrt{a}+\frac{1}{\sqrt{a}.\sqrt{a}.\sqrt{a}}=(\sqrt{a})^3+\frac{1}{(\sqrt{a})^3}$

$=(\sqrt{a}+\frac{1}{\sqrt{a}})^3-3.\sqrt{a}.\frac{1}{\sqrt{a}}(\sqrt{a}+\frac{1}{\sqrt{a}})=(\sqrt{5})^3-3\sqrt{5}=5\sqrt{5}-3\sqrt{5}=2\sqrt{5}$ **(Ans:)**

42. যদি x = $\sqrt{3}+\sqrt{2}$ **হয়, তবে প্রমাণ কর যে,** $x^3+\frac{1}{x^3}=18\sqrt{3}$ *[GM 9-10: 3.2] [Dhaka bank (TO):2017]*

সমাধান: দেয়া আছে $x=\sqrt{3}+\sqrt{2}$ বা, $\frac{1}{x}=\frac{1}{\sqrt{3}+\sqrt{2}}$ বা, $\frac{1}{x}=\frac{1(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$

বা, $\frac{1}{x}=\frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3})^2-(\sqrt{2})^2}$ $\therefore \frac{1}{x}=(\sqrt{3}-\sqrt{2})$ সুতরাং $x+\frac{1}{x}=\sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2}=2\sqrt{3}$

প্রদত্ত রাশি = $x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)^3-3.x.\frac{1}{x}\left(x+\frac{1}{x}\right)$

$=\left(2\sqrt{3}\right)^3-3\times 2\sqrt{3}=8\times 3\sqrt{3}-6\sqrt{3}=24\sqrt{3}-6\sqrt{3}=18\sqrt{3}$ **Ans.** $\mathbf{18\sqrt{3}}$

❒Self Task:

43. $x^3+\frac{1}{x^3}=18\sqrt{3}$ **হলে, প্রমাণ কর যে, x =** $\sqrt{3}+\sqrt{2}$

Solution:

দেওয়া আছে, $x^3+\frac{1}{x^3}=18\sqrt{3}\Rightarrow\frac{x^6+1}{x^3}=18\sqrt{3}\Rightarrow x^6+1-18\sqrt{3}x^3=0$

$\Rightarrow (x^3)^2-2.x^3.9\sqrt{3}+(9\sqrt{3})^2-243+1=0\Rightarrow(x^3-9\sqrt{3})^2=242\Rightarrow(x^3-9\sqrt{3})^2=(11\sqrt{2})^2$

$\Rightarrow x^3-9\sqrt{3}=11\sqrt{2}$ [বর্গমূল করে]

$\Rightarrow x^3=9\sqrt{3}+11\sqrt{2}$

$\Rightarrow x^3=3\sqrt{3}+9\sqrt{2}+6\sqrt{3}+2\sqrt{2}\Rightarrow x^3=(\sqrt{3})^3+3.(\sqrt{3})^2.\sqrt{2}+3.\sqrt{3}.(\sqrt{2})^2+(\sqrt{2})^3$

$\Rightarrow x^3=(\sqrt{3}+\sqrt{2})^3$ $\therefore x=\sqrt{3}+\sqrt{2}$ [প্রমাণিত]

Alternative solution:

Given, $x^3+\frac{1}{x^3}=18\sqrt{3}$ ....(i)

$\therefore\left(x^3+\frac{1}{x^3}\right)^2=(18\sqrt{3})^2$

$\Rightarrow\left(x^3-\frac{1}{x^3}\right)^2+4.x^3.\frac{1}{x^3}=324\times 3=972$

$\Rightarrow\left(x^3-\frac{1}{x^3}\right)^2+4.x^3.\frac{1}{x^3}=972-4$

$\Rightarrow x^3-\frac{1}{x^3}=\sqrt{968}=\sqrt{11\times 11\times 2\times 2\times 2}$

$\therefore x^3-\frac{1}{x^3}=\sqrt{968}=22\sqrt{2}$ ....(ii)

(i) no + (ii) no

$x^3 + \frac{1}{x^3} + x^3 - \frac{1}{x^3} = 18\sqrt{3} + 22\sqrt{2}$ $2x^3 = 18\sqrt{3} + 22\sqrt{2} \Rightarrow x^3 = 3\sqrt{3} + 11\ 11\sqrt{2}$

$\Rightarrow x^3 = (\sqrt{3})^3 + 3.\ (\sqrt{3})^2.\ (\sqrt{2}) + 3.\ (\sqrt{3}).\ (\sqrt{2})^2 + (\sqrt{2})^3$

$\Rightarrow x^3 = (\sqrt{3} + \sqrt{2})^3\ \therefore x = \sqrt{3} + \sqrt{2}$ **(Proved)**

44. If $x = \sqrt{5} + \sqrt{3}$ then $x^3 + \frac{1}{x^3}$ is equal to what ?

✍**Solution**: Here, $x = \sqrt{5} + \sqrt{3} \Rightarrow \frac{1}{x} = \frac{1}{\sqrt{5} + \sqrt{3}} = \frac{\sqrt{5} - \sqrt{3}}{(\sqrt{5})^2 - (\sqrt{3})^2} \Rightarrow \frac{1}{x} = \frac{\sqrt{5} - \sqrt{3}}{2}$

Now, $x + \frac{1}{x} = \sqrt{5} + \sqrt{3} + \frac{\sqrt{5} - \sqrt{3}}{2} = \frac{2\sqrt{5} + 2\sqrt{3} + \sqrt{5} - \sqrt{3}}{2} = \frac{3\sqrt{5} + \sqrt{3}}{2}$

Given expression = $x^3 + \frac{1}{x^3}$

$= (x + \frac{1}{x})^3 - 3.x.\frac{1}{x}(x + \frac{1}{x})$

$= (\frac{3\sqrt{5} + \sqrt{3}}{2})^3 - 3.(\frac{3\sqrt{5} + \sqrt{3}}{2})$

$= \frac{(3\sqrt{5} + \sqrt{3})^3}{(2)^3} - \frac{9\sqrt{5} + 3\sqrt{3}}{2})$

$= \frac{1}{8}[(3\sqrt{5})^3 + 3.(3\sqrt{5})^2.\sqrt{3} + 3.3\sqrt{5}(\sqrt{3})^2 + (\sqrt{3})^3] - \frac{9\sqrt{5} + 3\sqrt{3}}{2}$

$= \frac{1}{8}[27 \times 5\sqrt{5} + 3 \times 9 \times 5\sqrt{3} + 9 \times 3\sqrt{5} + 3\sqrt{3}] - \frac{9\sqrt{5} + 3\sqrt{3}}{2}$

$= \frac{1}{8}[162\sqrt{5} + 138\sqrt{3}] - \frac{1}{2}[9\sqrt{5} + 3\sqrt{3}]$

$= \frac{162\sqrt{5} + 138\sqrt{3} - 36\sqrt{5} - 12\sqrt{3}}{8} = \frac{126\sqrt{5} + 126\sqrt{3}}{8} = \frac{126(\sqrt{5} + \sqrt{3})}{8} = \frac{63(\sqrt{5} + \sqrt{3})}{4}$

$\therefore x^3 + \frac{1}{x^3} = \frac{63(\sqrt{5} + \sqrt{3})}{4}$ **(Ans:)**

❐**Self Task:**

45. $x = \sqrt{5} + \sqrt{3}$ হলে, $x^3 + \frac{8}{x^3}$ এর মান নির্ণয় করুন। *[GM 9-10: 3.2]*

✍**Solution:** দেওয়া আছে, $x = \sqrt{5} + \sqrt{3}$

$\Rightarrow \frac{1}{x} = \frac{1}{\sqrt{5} + \sqrt{3}} \Rightarrow \frac{1 \times 2}{x} = \frac{1 \times 2}{\sqrt{5} + \sqrt{3}}$ [উভয়পক্ষের লবকে ২ দ্বারা গুণ করে]

$\Rightarrow \frac{2}{x} = \frac{2}{\sqrt{5}+\sqrt{3}} \quad = \frac{2(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$ [ডানপক্ষের লব ও হরকে $(\sqrt{5}-\sqrt{3})$ দ্বারা গুণ করে]

$= \frac{2(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2} \quad = \frac{2(\sqrt{5}-\sqrt{3})}{5-3} \quad = \sqrt{5}-\sqrt{3}$

$\therefore x + \frac{2}{x} = \sqrt{5}+\sqrt{3}+\sqrt{5}-\sqrt{3} = 2\sqrt{5}$

প্রদত্ত রাশি, $x^3 + \frac{8}{x^3} = (x)^3 + \left(\frac{2}{x}\right)^3 = \left(x+\frac{2}{x}\right)^3 - 3.x.\frac{2}{x}\left(x+\frac{2}{x}\right) = \left(2\sqrt{5}\right)^3 - 6\left(2\sqrt{5}\right) = 28\sqrt{5}$ **(Ans.)**

46. $a = \sqrt{6} + \sqrt{5}$ হলে, $\frac{a^6-1}{a^3}$ এর মান নির্ণয় কর। ***[GM 9-10: 3.2]***

✍**Solution:** দেওয়া আছে, $a = \sqrt{6} + \sqrt{5}$

$\Rightarrow \frac{1}{a} = \frac{1}{\sqrt{6}+\sqrt{5}}$ [বিপরীতকরণ করে]

$\Rightarrow \frac{1}{a} = \frac{\sqrt{6}-\sqrt{5}}{\left(\sqrt{6}+\sqrt{5}\right)\left(\sqrt{6}-\sqrt{5}\right)} \Rightarrow \frac{1}{a} = \frac{\sqrt{6}-\sqrt{5}}{\left(\sqrt{6}\right)^2-\left(\sqrt{5}\right)^2}$ বা, $\frac{1}{a} = \frac{\sqrt{6}-\sqrt{5}}{6-5} \therefore \frac{1}{a} = \sqrt{6}-\sqrt{5}$

$\therefore a - \frac{1}{a} = \sqrt{6}+\sqrt{5}-\sqrt{6}+\sqrt{5} = 2\sqrt{5}$

প্রদত্ত রাশি $= \frac{a^6-1}{a^3}$

$= \frac{a^6}{a^3} - \frac{1}{a^3} = a^3 - \frac{1}{a^3} = \left(a-\frac{1}{a}\right)^3 + 3.a.\frac{1}{a}\left(a-\frac{1}{a}\right) = \left(2\sqrt{5}\right)^3 + 3.\left(2\sqrt{5}\right) = 8\times5\sqrt{5}+6\sqrt{5} = 46\sqrt{5}$ **(A)**

47. If a^2- $\sqrt{3}a$ + 1 = 0 what is the value of $a^3 + \frac{1}{a^3}$? *[Rupali Bank Ltd. (SO) – 2013 (Written)]*

✍**Solution:**

Given that, $a^2 - \sqrt{3}a + 1 = 0$

$\Rightarrow a^2 + 1 = \sqrt{3}a$ [Side change]

$\Rightarrow \frac{a^2+1}{a} = \frac{\sqrt{3}a}{a}$ [Dividing by a]

$\Rightarrow a + \frac{1}{a} = \sqrt{3}$ …..(i)

Now, $a^3 + \frac{1}{a^3} = \left(a+\frac{1}{a}\right)^3 - 3a\times\frac{1}{a}\left(a+\frac{1}{a}\right) = \left(a+\frac{1}{a}\right)^3 - 3\left(a+\frac{1}{a}\right) = \left(\sqrt{3}\right)^3 - 3\sqrt{3}$ [From (i)]

$= \left(\sqrt{3}\right)^2\left(\sqrt{3}\right) - 3\sqrt{3} = 3\sqrt{3} - 3\sqrt{3} = 0$ **Ans: 0**

❒**Self Task:**

48. If $x^4+\frac{1}{x^4}=119$, then prove that, $x^3-\frac{1}{x^3}=36$

Solution:

Given that, $x^4+\frac{1}{x^4}=119$

$\Rightarrow (x^2)^2+(\frac{1}{x^2})^2=119$

$\Rightarrow (x^2+\frac{1}{x^2})^2-2.x^2.\frac{1}{x^2}=119$

$\Rightarrow (x^2+\frac{1}{x^2})^2=121$

$\Rightarrow x^2+\frac{1}{x^2}=11 \quad \Rightarrow (x)^2+(\frac{1}{x})^2=11 \Rightarrow (x-\frac{1}{x})^2+2.x.\frac{1}{x}=11 \Rightarrow (x-\frac{1}{x})^2=9$

$\Rightarrow x-\frac{1}{x}=3 \Rightarrow (x-\frac{1}{x})^3=(3)^3 \Rightarrow x^3-\frac{1}{x^3}-3.x.\frac{1}{x}(x-\frac{1}{x})=27$

$\Rightarrow x^3-\frac{1}{x^3}-3.3=27 \quad \Rightarrow x^3-\frac{1}{x^3}=27+9=36$ **(Proved)**

49. If $x-\frac{1}{x}=2$ then prove that $x^5-\frac{1}{x^5}=82$

গুরুত্বপূর্ণ সূত্র:

☞ $x^5+\frac{1}{x^5}=(x^3+\frac{1}{x^3})(x^2+\frac{1}{x^2})-(x+\frac{1}{x})$

☞ $x^5-\frac{1}{x^5}=(x^3-\frac{1}{x^3})(x^2+\frac{1}{x^2})-(x-\frac{1}{x})$

Solution:

L.H.S $=x^5-\frac{1}{x^5}=(x^2+\frac{1}{x^2})(x^3-\frac{1}{x^3})-(x-\frac{1}{x})$

$=\{(x-\frac{1}{x})^2+2\times x.\frac{1}{x}\}\{(x-\frac{1}{x})^3+3.x.\frac{1}{x}(x-\frac{1}{x})\}-2$

$=(2^2+2)(2^3+3.2)-2=(6\times14)-2=84-2=82$ = R.H.S **(Proved)**

❒Self Task:

50. If $x=\sqrt{13+2\sqrt{42}}$ then determine the value of $x^5+\frac{1}{x^5}$?

Solution:

$x=\sqrt{13+2\sqrt{42}} \quad \Rightarrow x^2=13+2\sqrt{42}=(\sqrt{7})^2+2\times\sqrt{7}\times\sqrt{6}+(\sqrt{6})^2=(\sqrt{7}+\sqrt{6})^2$

$\therefore x=\sqrt{7}+\sqrt{6} \therefore \frac{1}{x}=\sqrt{7}-\sqrt{6}$

Now, $x^5+\frac{1}{x^5}=(x^3+\frac{1}{x^3})(x^2+\frac{1}{x^2})-(x+\frac{1}{x}) = 50\sqrt{7}\times26-2\sqrt{7}=1298\sqrt{7}$ **(Ans:)**

51. If $\left(x+\frac{1}{x}\right)=3$,then the value of $\left(x^6+\frac{1}{x^6}\right)$ *[Bangladesh Bank AD-2015+ sonali (off)2018]*

✍**Solution:**

$$\left(x+\frac{1}{x}\right)=3$$

Now, $\left(x^6+\frac{1}{x^6}\right)=(x^2)^3+(\frac{1}{x^2})^3=\left(x^2+\frac{1}{x^2}\right)^3-3\times x^2\times\frac{1}{x^2}\left(x^2+\frac{1}{x^2}\right)$

$=\left(x^2+\frac{1}{x^2}\right)^3-3\left(x^2+\frac{1}{x^2}\right)=\left\{\left(x+\frac{1}{x}\right)^2-2\times x\times\frac{1}{x}\right\}^3-3\left\{\left(x+\frac{1}{x}\right)^2-2\times x\times\frac{1}{x}\right\}$

$=\{(3)^2-2\}^3-3\{(3)^2-2\}$ [putting the value]

$=(9-2)^3-3(9-2)=(7)^3-3\times7=343-21=322$ **Ans. 322**

52. $\left(a+\frac{1}{a}\right)^2=3$ হলে, $\left(a^7+\frac{1}{a^7}\right)$ **এর মান কত? [$a\in N$]**

✍**Solution:**

দেওয়া আছে, $\left(a+\frac{1}{a}\right)^2=3\Rightarrow a+\frac{1}{a}=\sqrt{3}$

প্রদত্ত রাশি $=a^7+\frac{1}{a^7}$

$=\left(a^3+\frac{1}{a^3}\right)\left(a^4+\frac{1}{a^4}\right)-\left(a+\frac{1}{a}\right)$ [এই তিনটির আলাদা আলাদা মান বের করে উত্তর বের করতে হবে]

এখন, $a^3+\frac{1}{a^3}=\left\{\left(a+\frac{1}{a}\right)^3-3.a.\frac{1}{a}\left(a+\frac{1}{a}\right)\right\}$

$=(\sqrt{3})^3-3.\sqrt{3}=3\sqrt{3}-3\sqrt{3}=0$

আবার, $a^4+\frac{1}{a^4}=(a^2)^2+\left(\frac{1}{a^2}\right)^2=(a^2)^2+\left(\frac{1}{a^2}\right)^2$

$=\left\{\left(a+\frac{1}{a}\right)^2-2\right\}^2-2=\left\{(\sqrt{3})^2-2\right\}^2-2=(3-2)^2-2=-1$

$\therefore a^7+\frac{1}{a^7}=(0\times-1)-\sqrt{3}=-\sqrt{3}$ (Ans.)

53. If x = $1+\sqrt{2}+\sqrt{3}$ then determine the value of $2x^4-8x^3-5x^2+26x-28=?$

✍**Solution**:

Given that, $x=1+\sqrt{2}+\sqrt{3}$

$\Rightarrow$ x -1 = $\sqrt{2}+\sqrt{3}$

$\Rightarrow (x-1)^2 = (\sqrt{2}+\sqrt{3})^2$

$\Rightarrow x^2-2x+1 = 2+2.\sqrt{2}.\sqrt{3}+3$

$\Rightarrow x^2-2x+1-5 = 2\sqrt{6}$

$\Rightarrow x^2-2x-4 = 2\sqrt{6}$ ………… (i)

$\Rightarrow (x^2-2x-4)^2 = (2\sqrt{6})^2$

$\Rightarrow (x^2)^2+(-2x)^2+(-4)^2 + 2.x^2(-2x)+2(-2x)(-4)+2(-4)(x^2) = 24$

$\Rightarrow x^4 + 4x^2 + 13 - 4x^3 + 16x - 8x^2 - 24 = 0$

$\Rightarrow x^4-4x^2- 4x^3+16x-8 = 0$

$\Rightarrow 2x^4-8x^3-8x^2 +32x -16 = 0$ [multiplying both side by 2]

$\Rightarrow 2x^4-8x^3- 5x^2 -3x^2+26x+6x-28+12 = 0$

$\Rightarrow 2x^4-8x^3- 5x^2 +26x-28 =3x^2 -6x-12$

$\Rightarrow 2x^4-8x^3- 5x^2 +26x-28 =3(x^2 -2x-4)$

$\Rightarrow 2x^4-8x^3- 5x^2 +26x-28 =3\times2\sqrt{6}$ [From equation (i)]

$\Rightarrow 2x^4-8x^3- 5x^2 +26x-28 = 6\sqrt{6}$ $\therefore 2x^4-8x^3- 5x^2 +26x-28 = 6\sqrt{6}$ **(Ans:)**

54. If a+b+c = 0, the determine the value of $a^3+b^3+c^3$ - 3abc = ?

✍**Solution**:

Given Expression = $a^3+b^3+c^3$ - 3abc

= (a+b+c) $(a^2+b^2+c^2-ab-bc-ca)$

= 0 × $(a^2+b^2+c^2-ab-bc-ca)$ = 0 **(Ans:)**

55. If $x^a.x^b.x^c = 1$, then find the value of $a^3+b^3+c^3$ = ?

✍**Solution**:

Given that, $x^a.x^b.x^c = 1 \Rightarrow x^{a+b+c} = (x)^0$ $\therefore$ a+b+c = 0

Given Expression = $a^3+b^3+c^3$

= $a^3+b^3+c^3$ - 3abc + 3abc

= (a+b+c) $(a^2+b^2+c^2-ab-bc-ca)+3abc$

= 0 × $(a^2+b^2+c^2-ab-bc-ca)+3abc$ = 3abc **(Ans:)**

56. If x+y+z = 10, $x^3+y^3+z^3 = 75$ and xyz = 15 then find the value of $x^2+y^2+z^2-xy-yz-zx$?

✍**Solution**:

Given that , x + y + z = 10, $x^3+y^3+z^3 = 75$ and xyz = 15

We know that, (x+y+z)$(x^2+y^2+z^2-xy-yz-zx) = x^3+y^3+z^3-3xyz$

$\Rightarrow 10\ (x^2+y^2+z^2-xy-yz-zx)$ = 75-3×15

$\therefore x^2+y^2+z^2-xy-yz-zx$ = 30 ÷ 10 = 3 **(Ans:)**

Others:

57. If $x+\frac{1}{x}=2$; find the value of $x^{17}+\frac{1}{x^{19}}=?$ *[Janata Bank –(AEO)-2020 (Written)]*

Solution: (এরকম অসমান পাওয়ার থাকলে সবসময় শুধু x এর মান বের করে সমাধান করার চেষ্টা করতে হবে)

Given, $x+\frac{1}{x}=2$

$\Rightarrow \frac{x^2+1}{x}=2 \Rightarrow x^2+1=2x \Rightarrow x^2-2.x.1+1^2=0 \Rightarrow (x-1)^2=0 \Rightarrow x-1=0 \therefore x=1$

Now, $x^{17}+\frac{1}{x^{19}} = (1)^{17}+\frac{1}{(1)^{19}} = 1+\frac{1}{1} = 1+1=\mathbf{2}$

58. If $x+\frac{1}{x}=-2$ then find the value of $x^{72}+\frac{1}{x^{78}}=?$

Solution:

Given that, $x+\frac{1}{x}=-2$

$\Rightarrow x^2+1=-2x \Rightarrow x^2+2x+1=0 \Rightarrow (x+1)^2=0 \Rightarrow x+1=0 \therefore x=-1$

Now, $x^{72}+\frac{1}{x^{78}} = (-1)^{72}+\frac{1}{(-1)^{78}} = 1+\frac{1}{1} = 1+1=2$ (Ans)

59. If $a+\frac{1}{a}+1=0$ $(a\neq 0)$, then the value of (a^4-a) is?

Solution: $a+\frac{1}{a}+1=0 \Rightarrow \left(a+\frac{1}{a}\right)^2=(-1)^2$

$\Rightarrow a^2+\frac{1}{a^2}+2=1 \Rightarrow a^2+\frac{1}{a^2}=-1 \Rightarrow a^2+1=-\frac{1}{a^2}$ ------- (i)

$\Rightarrow a+\frac{1}{a}=-1$ (Given) $\therefore a^2+1=-a$ -------- (ii)

For equation (i) and (ii) we can write

$-a=\frac{-1}{a^2} \Rightarrow a^3=1$ [by $\times(-a^2)$]

$\Rightarrow a^3-1=0 \Rightarrow a(a^3-1)=0\times a \Rightarrow a^4-a=0$

60. If $x+y=a$, $x^2+y^2=b^2$ and $x^3+y^3=c^3$, then show that $a^3+2c^3=3ab^2$ *[Modhumoti Bank Ltd. (PO) – 2016 (Written)]*

Solution:

L.H.S.= a^3+2c^3

$=(x+y)^3+2(x^3+y^3)=x^3+y^3+3x^2y+3xy^2+2x^3+2y^3$

$=3x^3+3y^3+3x^2y+3xy^2=3(x^3+y^3+x^2y+xy^2)$

$= 3(x^3 + x^2y + xy^2 + y^3) = 3\{x^2(x+y) + y^2(x+y)\} = 3(x+y)(x^2+y^2) = 3ab^2$ = R.H.S.

$\therefore a^3 + 2c^3 = 3ab^2$ (**Showed**)

61. If $\frac{x^{24}+1}{x^{12}} = 7$, then the value of $\frac{x^{72}+1}{x^{36}} = ?$ [Rupali + Janata officer 2020]

✍**Solution**:

(Given that = $\frac{x^{24}+1}{x^{12}} = 7 \Rightarrow \frac{x^{24}}{x^{12}} + \frac{1}{x^{12}} = 7 \Rightarrow x^{12} + \frac{1}{x^{12}} = 7 \Rightarrow \left(x^{12} + \frac{1}{x^{12}}\right)^3 = 7^3$

$\Rightarrow x^{36} + \frac{1}{x^{36}} + 3 \times x^{12} \times \frac{1}{x^{12}}\left(x^{12} + \frac{1}{x^{12}}\right) = 343 \Rightarrow x^{36} + \frac{1}{x^{36}} + 3\ (7) = 343$

$\Rightarrow x^{36} + \frac{1}{x^{36}} = 343 - 21 \Rightarrow x^{36} + \frac{1}{x^{36}} = 322 \Rightarrow \frac{x^{72}+1}{x^{36}} = 322$

62. If $x^3 + \frac{3}{x} = 4 \times (a^3+b^3)$; $3x + \frac{1}{x^3}$ = $4(a^3-b^3)$ Value of a^2-b^2=? *[PKB-EO-(Cash)-2019-(Written)]*

✍**Solution**: (এই প্রশ্নটির সমাধানের সময় আমাদের মূল টার্গেট a^2-b^2 এর জন্য a+b এবং a-b এর মান অথবা এককভাবে a এবং b এর মান বের করার জন্য শুরুতে প্রদত্তর রাশিগুলোকে সেভাবেই সাজানো)

$x^3 + \frac{3}{x} = 4a^3 + 4b^3$ ------ (i)

$3x + \frac{1}{x^3} = 4a^3 - 4b^3$ ----- (ii)

$x^3 + \frac{3}{x} + 3x + \frac{1}{x^3} = 8a^3$ (by Adding)

$\Rightarrow x^3 + 3x + \frac{3}{x} + \frac{1}{x^3} = (2a)^3$

$\Rightarrow x^3 + 3.x^2.\frac{1}{x} + 3.x.\frac{1}{x^2} + \frac{1}{x^3} = (2a)^3$ [সূত্রের মত করে মিলিয়ে নেয়া হলো]

$\Rightarrow \left(x + \frac{1}{x}\right)^3 = (2a)^3 \quad \therefore x + \frac{1}{x} = 2a$ ------(iii)

Again, by subtracting (ii) from (i) we get,

$x^3 + \frac{3}{x} - 3x - \frac{1}{x^3} = 8b^3$

$\Rightarrow x^3 - 3x + \frac{3}{x} - \frac{1}{x^3} = 8b^3$

$\Rightarrow x^3 - 3.x^2\frac{1}{x} + 3.x.\frac{1}{x^2} - \frac{1}{x^3} = (2b)^3$[সূত্রের মত করে মিলিয়ে নেয়া হলো]

$\Rightarrow \left(x - \frac{1}{x}\right)^3 = (2b)^3 \qquad \therefore x - \frac{1}{x} = 2b$------(iv)

Now, by adding (iii) and (iv) we get,

$2a + 2b = x + \frac{1}{x} + x - \frac{1}{x} \Rightarrow 2(a+b) = 2x \quad \therefore a + b = x$

By subtracting (iv) from (iii) we get,

$2a - 2b = x + \frac{1}{x} - x + \frac{1}{x} \quad \Rightarrow 2(a-b) = 2.\frac{1}{x} \quad \therefore a - b = \frac{1}{x}$

$\therefore a^2 - b^2 = (a + b)(a - b) = x\times\frac{1}{x} = \mathbf{1}$ **Ans: 1**

63. If x is real, $x + \frac{1}{x} = 23 \neq 0$ an $x^3 + \frac{1}{x^3} = 0$, then the value of $\left(x+\frac{1}{x}\right)^4$ is?

✍**Solution**: $x^3 + \frac{1}{x^3} = 0$

$\Rightarrow \left(x+\frac{1}{x}\right)^3 - 3\times x\times\frac{1}{x}\left(x+\frac{1}{x}\right) = 0 \Rightarrow \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right) = 0 \Rightarrow \left(x+\frac{1}{x}\right)^3 = 3\left(x+\frac{1}{x}\right)$

$\Rightarrow \left(x+\frac{1}{x}\right)^2 = 3 \Rightarrow \left[\left(x+\frac{1}{x}\right)^2\right]^2 = (3)^2 \quad \therefore \left(x+\frac{1}{x}\right)^4 = 9$

64. If $4b^2 + \frac{1}{b^2} = 2$, then the value of $8b^3 + \frac{1}{b^3}$ is?

✍**Solution**: $4b^2 + \frac{1}{b^2} = 2$

$\Rightarrow (2b)^2 + \left(\frac{1}{b}\right)^2 + 4 - 4 = 2 \Rightarrow \left(2b+\frac{1}{b}\right)^2 - 4 = 2 \Rightarrow \left(2b+\frac{1}{b}\right)^2 = 6 \Rightarrow 2b + \frac{1}{b} = \sqrt{6}$

$\Rightarrow \left(2b+\frac{1}{b}\right)^3 = \left(\sqrt{6}\right)^3 \Rightarrow 8b^3 + \frac{1}{b^3} + 6\sqrt{6} = 6\sqrt{6} \Rightarrow 8b^3 + \frac{1}{b^3} = 6\sqrt{6} - 6\sqrt{6} \Rightarrow 8b^3 + \frac{1}{b^3} = 0$

65. $f(x - y) = \frac{x^2 - 2xy + y^2}{x^2 - y^2}$ then find the value of f(– x, – y) *[BEZA-(AM)-2020(Written)]*

✍**Solution**:

Given, $f(x, y) = \frac{x^2 - 2xy + y^2}{x^2 - y^2} = \frac{(x-y)^2}{(x+y)(x-y)} = \frac{(x-y)}{(x+y)}$

$\therefore f(-x, -y) = \frac{-x-(-y)}{-x+(-y)} = \frac{-x+y}{-x-y} = \frac{-(x-y)}{-(x+y)} = \frac{(x-y)}{(x+y)}$ **Ans**: $\frac{(\mathbf{x-y})}{(\mathbf{x+y})}$

66. What should be the values of a and b for which $64x^3 - 9ax^2 + 108x - b$ will be a perfect cube? *[BKB – (Cash)-2018 –(Written)]*

অর্থ: a এবং b এর মান কত হলে $64x^3 - 9ax^2 + 108x - b$ রাশিটি একটু পূর্ণ ঘন রাশি হবে?

✍**Solution:** (বাংলা আলোচনা থেকে আগে ব্যসিকটা ক্লিয়ার করতে আলোচনাগুলো পড়ুন. তারপর প্রশ্নটি সমাধান করলে খুব সহযে হয়ে যাবে।)

বীজগণিতের ক্ষেত্রে কোন রাশি নিম্নোক্ত যে কোন একটি রাশির সাথে মিলে গেলে তাকে পূর্ণ বর্গ রাশি বলা হবে।

1.$(a+b)^2$ 2. $(a-b)^2$ 3. $(a+b+c)^2$ 4. $(a-b-c)^2$

যেমন: What must be added to the expression $(4a^2+9b^2)$ so that the sum is a perfect square? এখানে$(4a^2+9b^2)$ কে , .**$(a+b)^2$** **সুত্রের সাথে মেলানোর জন্য প্রয়োজন** $(4a^2+9b^2) = (2a)^2+2.2a.3b+(3b)^2 - 12ab$ অর্থাৎ রাশিটিকে সুত্রে ফেলানোর জন্য আরো 12ab প্রয়োজন ।

তেমনিভাবে Perfect cube বানানোর জন্য একটি রাশিকে $(a+b)^3$ বা $(a-b)^3$ এর সুত্রে ফেলতে হবে। সুত্রের সাথে রাশিটিকে মিলিয়ে দেয়ার জন্য যে মান প্রয়োজন সেটিই উত্তর।

We know,
$(x-y)^3 = x^3 -3x^2y+3xy^2 -y^3$ (যেহেতু perfect cube বানাতে হবে তাই এই সুত্রের সাথে মিলিয়ে দিতে হবে।)

Given that,
$64x^3 -9ax^2+108x-b$
$= (4x)^3 - 9ax^2+ 3.4x. 3^2 -b$ (প্রথমে x^3 মিলে গেছে, সে অনুযায়ী ৩য় অংশ $3xy^2=3.4x. 3^2$ মেলাতে গিয়ে দেখা যাচ্ছে যে y = 3 পাওয়া গেছে।)

> **♦ মনে রাখবেন:**
> পরীক্ষায় শুধু ইংরেজী অংশগুলো লিখতে হবে। বাংলা লাইনগুলো শুধুমাত্র নতুন প্রশ্নটি সহজে বোঝার জন্য ব্যাখ্যা।

Comparing with 1st term,
We get, x = 4x (4x হচ্ছে প্রথম অংশ)

Comparing with 3rd term,
We get, $y^2= 3^2 \therefore y = 3$

Comparing with 4th term ,
$b= y =3^3=27$ (আমরা সুত্রানুসারে পেলাম y=3 কিন্তু রাশিটিতে b এর স্থলে y^3 বসবে যার মান হবে $3^3=27$)

Comparing with 2nd term We get, $9ax^2 = 3x^2y$ (আমাদের জানা আছে $3x^2y$ এর ৩ হচ্ছে সুত্রের, x^2 হচ্ছে প্রথম অংশের বর্গ এবং y হচ্ছে অন্য রাশির মান। তাই আমরা যে মানগুলো জানি তা বসিয়ে পাই $3.(4x)^2.3 = 9.16x^2 = 144x^2$ কিন্তু এখানে $9ax^2$ বা, $9x^2$. a ছিল, সুতরাং a এর জায়গায় 144÷9 = 16 বসালে রাশিটি পূর্ণবর্গ সুত্রের সাথে মিলে যাবে এবং একটি পূর্ণবর্গ রাশি হয়ে যাবে।)

Or, $9ax^2 = 3(4x)^2 \times 3$ Or $9ax^2 = 9\times16x^2 \therefore a = 16$ **Ans: a= 16 and b = 27**

======⇔=======⇔=======⇔=======⇔=======

Practice Part

1. $\frac{A+B}{\sqrt{AB}} = 10\sqrt{2}$ **হলে** $\frac{A-B}{\sqrt{AB}}$ **এর মান কত হবে?**

✍**Solution:**

Given that, $\frac{A+B}{\sqrt{AB}} = 10\sqrt{2}$

$\Rightarrow \left(\frac{A+B}{\sqrt{AB}}\right)^2 = \left(10\sqrt{2}\right)^2$

$\Rightarrow \frac{(A+B)^2}{AB} = 200 \Rightarrow (A+B)^2 = 200AB \quad \Rightarrow (A-B)^2 + 4AB = 200AB$

$\Rightarrow (A-B)^2 = 196AB \;\Rightarrow \frac{(A-B)^2}{AB} = 196 \Rightarrow \left(\frac{A-B}{\sqrt{AB}}\right)^2 = (14)^2 \;\Rightarrow \frac{A-B}{\sqrt{AB}} = 14$ (Ans.)

2. If $5x+\frac{1}{3x}=4$ Then $9x^2+\frac{1}{25x^2}=?$

✍**Solution:**

Given that, $5x+\frac{1}{3x}=4 \;\Rightarrow \frac{3}{5}\times\left(5x+\frac{1}{3x}\right)=\frac{3}{5}\times 4 \Rightarrow 3x+\frac{1}{5x}=\frac{12}{5}$

Given expression = $9x^2+\frac{1}{25x^2}=(3x)^2+\left(\frac{1}{5x}\right)^2=\left(3x+\frac{1}{5x}\right)^2$-2.3x. $\frac{1}{5x}=\left(\frac{12}{5}\right)^2-\frac{6}{5}=\frac{114}{25}$

3. If $x^{\frac{1}{4}}+\frac{1}{x^{\frac{1}{4}}}=\sqrt{3}$, then determine the value of $x^2+\frac{1}{x^2}$?

✍**Solution**:

Given that, $x^{\frac{1}{4}}+\frac{1}{x^{\frac{1}{4}}}=\sqrt{3}$

$\Rightarrow \left(x^{\frac{1}{4}}+\frac{1}{x^{\frac{1}{4}}}\right)^2=(\sqrt{3})^2 \;\Rightarrow \left(x^{\frac{1}{4}}\right)^2+\left(\frac{1}{x^{\frac{1}{4}}}\right)^2=3-2 \Rightarrow x^{\frac{1}{2}}+\frac{1}{x^{\frac{1}{2}}}=1 \Rightarrow \sqrt{x}+\frac{1}{\sqrt{x}}=1$

$\Rightarrow \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2=(1)^2 \;\Rightarrow x+\frac{1}{x}+2=1 \Rightarrow x+\frac{1}{x}=-1 \;\Rightarrow \left(x+\frac{1}{x}\right)^2=(-1)^2$

$\Rightarrow x^2+\frac{1}{x^2}+2=1 \qquad \Rightarrow x^2+\frac{1}{x^2}=-1$ (**Ans**.)

4. If $4.x^{\frac{9}{4}}-9.x^{\frac{9}{8}}+4=0$ then determine the value of $x^{\frac{9}{4}}+x^{\frac{-9}{4}}$?

✍**Solution**:

Let, $m = x^{\frac{9}{8}} \Rightarrow m^2 = \left(x^{\frac{9}{8}}\right)^2 \;\Rightarrow m^2 = x^{\frac{9}{4}} \;\;\therefore 4.m^2 - 9.m + 4 = 0$

$\Rightarrow \frac{4m^2 - 9m + 4}{m} = \frac{0}{m} \Rightarrow 4m + \frac{4}{m} = 9 \Rightarrow m + \frac{1}{m} = \frac{9}{4}$

Given expression = $x^{\frac{9}{4}} + x^{\frac{-9}{4}} = m^2 + m^{-2} = m^2 + \frac{1}{m^2} = \left(m + \frac{1}{m}\right)^2 - 2 = \left(\frac{9}{4}\right)^2 - 2 = \frac{49}{16}$ (**Ans.**)

5. If $a\left(2+\sqrt{3}\right) = b\left(2-\sqrt{3}\right) = 1$, then the value of $\frac{1}{a^2+1} + \frac{1}{b^2+1} = ?$

✍**Solution**:

Given that, $a\left(2+\sqrt{3}\right) = b\left(2-\sqrt{3}\right) = 1$ So, $a = \frac{1}{\left(2+\sqrt{3}\right)}$ and $b = \frac{1}{\left(2-\sqrt{3}\right)}$

$\therefore a = \frac{1}{b}$ [যেহেতু $a = \frac{1}{2+\sqrt{3}} = \frac{2-\sqrt{3}}{(2-\sqrt{3})(2+\sqrt{3})} = 2 - \sqrt{3}$]

Now, $\frac{1}{a^2+1} + \frac{1}{b^2+1} = \frac{1}{\frac{1}{b^2}+1} + \frac{1}{b^2+1} = \frac{1}{\frac{1+b^2}{b^2}} + \frac{1}{b^2+1} = \frac{b^2}{b^2+1} + \frac{1}{b^2+1} = \frac{b^2+1}{b^2+1} = 1$ (Ans)

6. If $a^2 + \frac{2}{a^2} = 16$, then find the value of $\frac{72a^2}{a^4+2+8a^2}$

✍**Solution**:

Given expression = $\frac{72a^2}{a^4+2+8a^2} = \frac{\frac{72a^2}{a^2}}{\frac{a^4+2+8a^2}{a^2}} = \frac{72}{a^2 + \frac{2}{a^2} + 8} = \frac{72}{16+8} = \frac{72}{24} = 3$ (**Ans.**)

7. If $\frac{2x}{3} - \frac{2}{3x} = 1$, the determine the value of $8 \times \left(x^2 - \frac{1}{x^2}\right)$

✍**Solution:**

Given that, $\frac{2x}{3} - \frac{2}{3x} = 1 \Rightarrow \frac{2}{3}\left(x - \frac{1}{x}\right) = 1 \Rightarrow \left(x - \frac{1}{x}\right) = \frac{3}{2}$ ....(i)

By squaring equation we get,

$\left(x - \frac{1}{x}\right)^2 = \left(\frac{3}{2}\right)^2 \Rightarrow \left(x + \frac{1}{x}\right)^2 - 4 = \frac{9}{4} \Rightarrow \left(x + \frac{1}{x}\right)^2 = \frac{25}{4} \quad \therefore x + \frac{1}{x} = \frac{5}{2}$

Given expression $= 8 \times \left(x^2 - \frac{1}{x^2}\right) = 8 \times \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right) = 8 \times \frac{3}{2} \times \frac{5}{2}$ **= 30 (Ans.)**

8. If $x^2 = 17 + 12\sqrt{2}$ then the value of $x + \frac{1}{x} = ?$

✍**Solution**:

Given that, $x^2 = 17+12\sqrt{2} \Rightarrow x^2 = 9 + 12\sqrt{2} + 8 \Rightarrow x^2 = (3)^2 + 2.3.2\sqrt{2} + (2\sqrt{2})^2$

$\Rightarrow x^2 = (3+2\sqrt{2})^2 \Rightarrow x = 3+2\sqrt{2} \Rightarrow \frac{1}{x} = \frac{1}{3+2\sqrt{2}} = \frac{3-2\sqrt{2}}{(3)^2-(2\sqrt{2})^2} \quad \therefore \frac{1}{x} = 3-2\sqrt{2}$

Given expression = $x+\frac{1}{x} = 3+2\sqrt{2}+3-2\sqrt{2} = 6$ (**Ans.**)

9. $x^2 = 5+2\sqrt{6}$ হলে $\sqrt{x}+\frac{1}{\sqrt{x}}$ এর মান কত?

✍**Solution**:

Given that, $x^2 = 5+2\sqrt{6}$

$\Rightarrow x^2 = \left(\sqrt{3}+\sqrt{2}\right)^2 \Rightarrow x = \sqrt{3}+\sqrt{2} \Rightarrow \frac{1}{x} = \frac{1}{\sqrt{3}+\sqrt{2}} \Rightarrow \frac{1}{x} = \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2} = \sqrt{3}-\sqrt{2}$

We know that

$\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 = (\sqrt{x})^2 + 2.\sqrt{x}.\frac{1}{\sqrt{x}} + \left(\frac{1}{\sqrt{x}}\right)^2$

$= x+2+\frac{1}{x} = \sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2}+2 = 2\sqrt{3}+2$

$\Rightarrow \sqrt{x}+\frac{1}{\sqrt{x}} = \sqrt{2(\sqrt{3}+1)}$ (**Ans.**)

10. $x = \sqrt{5+2\sqrt{6}}$ হলে $x^4+\frac{1}{x^4}$ এর মান কত?

✍**Solution**:

Given that, $x = \sqrt{5+2\sqrt{6}}$

$\Rightarrow x^2 = 5+2\sqrt{6} \Rightarrow x^2 = 3+2\sqrt{6}+2 \Rightarrow x^2 = (\sqrt{3})+2.\sqrt{3}.\sqrt{2}+(\sqrt{2})^2$

$\Rightarrow x^2 = (\sqrt{3}+\sqrt{2})^2 \Rightarrow x = \sqrt{3}+\sqrt{2} \Rightarrow \frac{1}{x} = \frac{1}{\sqrt{3}+\sqrt{2}} \Rightarrow \frac{1}{x} = \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})} \quad \therefore \frac{1}{x} = \sqrt{3}-\sqrt{2}$

Given expression, $= x^4+\frac{1}{x^4}$

$= (x^2)^2+\left(\frac{1}{x^2}\right)^2 = \left(x^2+\frac{1}{x^2}\right)^2 - 2.x^2.\frac{1}{x^2} = \left(x^2+\frac{1}{x^2}\right)^2 - 2 = \left\{(x)^2+\left(\frac{1}{x}\right)^2\right\}^2 - 2$

$= \left\{\left(x+\frac{1}{x}\right)^2 - 2.x.\frac{1}{x}\right\}^2 - 2 = \left\{\left(\sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2}\right)-2\right\}^2 - 2 \qquad = \left\{\left(2\sqrt{3}\right)^2-2\right\}^2 - 2$

$= \{(4\times 3) - 2\}^2 - 2 \qquad = (10)^2 - 2 = \mathbf{98}$ (**Ans.**)

11. If $x = \sqrt{5}+1$, $y = \sqrt{5}-1$ the determine the value of $\frac{x^2}{y^2}+\frac{y^2}{x^2}+4\left(\frac{x}{y}\right)+4\left(\frac{y}{x}\right)+6$

✍**Solution**:

$$\frac{x}{y}+\frac{y}{x}=\frac{x^2+y^2}{xy}=\frac{(\sqrt{5}+1)^2+(\sqrt{5}-1)^2}{(\sqrt{5}+1)(\sqrt{5}-1)}=3$$

Given expression = $\frac{x^2}{y^2}+\frac{y^2}{x^2}+4\left(\frac{x}{y}\right)+4\left(\frac{y}{x}\right)+6$

$=\left(\frac{x}{y}\right)^2+\left(\frac{y}{x}\right)^2+4\left(\frac{x}{y}+\frac{y}{x}\right)+6=\left(\frac{x}{y}\right)^2+\left(\frac{y}{x}\right)^2+2+4\left(\frac{x}{y}+\frac{y}{x}\right)+4$

$=\left(\frac{x}{y}\right)^2+\left(\frac{y}{x}\right)^2+2.\frac{x}{y}.\frac{y}{x}+4\left(\frac{x}{y}+\frac{y}{x}\right)+4=\left(\frac{x}{y}+\frac{y}{x}\right)^2+4\left(\frac{x}{y}+\frac{y}{x}\right)+4$

$= (3)^2 + 4.3 + 4$ **= 25(Ans.)**

12. If $x = a+\frac{1}{a}$ and $y = a-\frac{1}{a}$, then the value of $x^4 + y^4 – 2x^2y^2$ is? *[Examveda.com]*

✍**Solution:**

Given that , $x = a+\frac{1}{a}$ and $y = a-\frac{1}{a}$

$\therefore (x+y) = a+\frac{1}{a} + a-\frac{1}{a} = 2a$ again, $(x-y) = a+\frac{1}{a} - a+\frac{1}{a} = \frac{2}{a}$

$\therefore x^4 + y^4 – 2x^2y^2 = (x^2-y^2)^2 = [(x+y)(x-y)]^2 = (2a\times\frac{2}{a})^2 = 4^2 = 16$ **Ans: 16**

13. If $(x, y) = \left(\sqrt{a}+\frac{1}{\sqrt{a}}, \sqrt{a}-\frac{1}{\sqrt{a}}\right)$ then the value of $x^4 + y^4 – 2x^2y^2$ is?

✍**Solution**: We can write, $x=\sqrt{a}+\frac{1}{\sqrt{a}}$ and $y=\sqrt{a}-\frac{1}{\sqrt{a}}$

Given expression = $x^4 + y^4 – 2x^2y^2$

$= (x^2)^2 – 2.x^2.y^2 + (y^2)^2 = (x^2 – y^2)^2 = [(x + y)(x – y)]^2$

$=\left[(\sqrt{a}+\frac{1}{\sqrt{a}}+\sqrt{a}-\frac{1}{\sqrt{a}})(\sqrt{a}+\frac{1}{\sqrt{a}}-\sqrt{a}+\frac{1}{\sqrt{a}})\right]^2=\left(2\sqrt{a}\times2\frac{1}{\sqrt{a}}\right)^2=(4)^2=$ **16 (Ans.)**

14. If $x^2 – 2\sqrt{30} – 11 = 0$ then determine the volume of $x^4 – 2x^2y^2 + y^4$ where $y = \frac{1}{x}$

✍**Solution**:

$x^2 – 2\sqrt{30} – 11 = 0 \Rightarrow x^2 = 2\sqrt{30}+11 \Rightarrow x^2 = 6+2\sqrt{30}+5 \Rightarrow x^2 = (\sqrt{6})^2 + 2.\sqrt{6}.\sqrt{5}+(\sqrt{5})^2$

$\Rightarrow x^2 = (\sqrt{6}+\sqrt{5})^2 \quad \therefore x=\sqrt{6}+\sqrt{5}$

Now, $\frac{1}{x}=\frac{1}{\sqrt{6}+\sqrt{5}}=\frac{\sqrt{6}-\sqrt{5}}{(\sqrt{6}+\sqrt{5})(\sqrt{6}-\sqrt{5})} \quad \therefore \frac{1}{x}=\sqrt{6}-\sqrt{5}$

Given expression = $x^4 – 2x^2y^2 + y^4 = (x^2)^2 – 2.x^2.y^2 + (y^2)^2$

$= (x^2 - y^2)^2 = \{(x + y)(x - y)\}^2 = \left\{(x + \frac{1}{x})(x - \frac{1}{x})\right\}^2$

$= \left\{(\sqrt{6} + \sqrt{5} + \sqrt{6} - \sqrt{5})(\sqrt{6} + \sqrt{5} - \sqrt{6} + \sqrt{5})\right\}^2$

$= \left\{2\sqrt{6} \times 2\sqrt{5}\right\}^2 = (4\sqrt{30})^2 = 16 \times 30 = 480$ (**Ans.**)

15. If $\frac{12x^2 - 27x + 28}{4x^2 - 9x + 9} = \frac{16}{5}$ then determine the value of $\left(x^2 + \frac{1}{x^2}\right)^{\frac{1}{2}}$?

✍**Solution**:

Given that $= \frac{12x^2 - 27x + 28}{4x^2 - 9x + 9} = \frac{16}{5}$

$\Rightarrow 60x^2 - 135x + 140 = 64x^2 - 144x + 144$

$\Rightarrow 4x^2 - 9x + 4 = 0 \Rightarrow \frac{4x^2 - 9x + 4}{x} = \frac{0}{x} \Rightarrow x + \frac{1}{x} = \frac{9}{4}$

Given expression $= \left(x^2 + \frac{1}{x^2}\right)^{\frac{1}{2}}$

$= \left[\left(x + \frac{1}{x}\right)^2 - 2.x.\frac{1}{x}\right]^{\frac{1}{2}} = \left[\left(\frac{9}{4}\right)^2 - 2\right]^{\frac{1}{2}} = \left[\frac{81}{16} - 2\right]^{\frac{1}{2}} = \left[\frac{49}{16}\right]^{\frac{1}{2}} = \frac{7}{4}$ (**Ans.**)

16. If $\frac{p}{a} + \frac{q}{b} + \frac{r}{c} = 1$ and $\frac{a}{p} + \frac{b}{q} + \frac{c}{r} = 0$, where p, q, r, a, b and c are non-zero, the value of $\frac{p^2}{a^2} + \frac{q^2}{b^2} + \frac{r^2}{c^2}$ is.

✍**Solution**: Let, $x = \frac{p}{a}, y = \frac{q}{b}$ and $z = \frac{r}{c}$

The given expressions are simplified to,

$x + y + z = 1$ ....(i) and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$ ....(ii)

from (i) $x + y + z = 1 \Rightarrow (x + y + z)^2 = (1)^2 \Rightarrow x^2 + y^2 + z^2 + 2(xy + yz + zx) = 1$

$\Rightarrow x^2 + y^2 + z^2 + 2 \times 0 = 1$ $\quad [\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0 \Rightarrow \frac{xy + yz + zx}{xyz} = 0 \therefore xy + yz + zx = 0]$

$\Rightarrow x^2 + y^2 + z^2 = 1 \quad \therefore \frac{p^2}{a^2} + \frac{q^2}{b^2} + \frac{r^2}{c^2} = \mathbf{1}$ **(Ans.)**

17. If pq + qr + rp = 0 then determine the value of $\frac{p^2}{p^2 - qr} + \frac{q^2}{q^2 - rp} + \frac{r^2}{r^2 - pq} = ?$

✍**Solution**:

Given that = pq + qr + rp = 0

$\therefore$ pq + qr = - rp or, pq + rp = - qr or, qr + rp = - pq

Given expression = $\frac{p^2}{p^2 - qr} + \frac{q^2}{q^2 - rp} + \frac{r^2}{r^2 - pq}$

$= \frac{p^2}{p^2 + pq + rp} + \frac{q^2}{q^2 + pq + qr} + \frac{r^2}{r^2 + qr + rp}$

$= \frac{p^2}{p(p+q+r)} + \frac{q}{q(p+q+r)} + \frac{r^2}{r(p+q+r)}$

$= \frac{p}{p+q+r} + \frac{q}{p+q+r} + \frac{r}{p+q+r} = \frac{p+q+r}{p+q+r} =$ **1 (Ans.)**

18. If $x^2 = \frac{5x}{2} + \frac{1}{6}$ = then determine the value of a $\frac{216x^6 - 1}{27x^3}$ = ?

✍**Solution**:

Given that , $x^2 = \frac{5x}{2} + \frac{1}{6}$

$\Rightarrow x^2 = \frac{15x+1}{6} \Rightarrow 6x^2 = 15x+1 \Rightarrow 6x^2 - 1 = 15x \Rightarrow \frac{6x^2 - 1}{3x} = \frac{15x}{3x} \Rightarrow 2x - \frac{1}{3x} = 5$

Given expression, $\frac{216x^6 - 1}{27x^3} = \frac{216x^6}{27x^3} - \frac{1}{27x^3}$

$= 8x^3 - \frac{1}{27x^3} = (2x)^3 - (\frac{1}{3x})^3$

$= (2x - \frac{1}{3x})^3 + 3.2x.\frac{1}{3x}(2x - \frac{1}{3x}) = (5)^3 + 2\times 5 = 125 + 10 =$ **135** **(Ans:)**

19. If $p + \frac{1}{p}$ = 2, the value of $p^{\frac{3}{2}} + p^{\frac{-3}{2}}$ is ?

✍**Solution**:

Given that, $p + \frac{1}{p} = 2 \Rightarrow (\sqrt{p})^2 + (\frac{1}{\sqrt{p}})^2 = 2$

$\Rightarrow (\sqrt{p} + \frac{1}{\sqrt{p}})^2 - 2\sqrt{p}.\frac{1}{\sqrt{p}} = 2 \Rightarrow (\sqrt{p} + \frac{1}{\sqrt{p}})^2 = 4 \Rightarrow \sqrt{p} + \frac{1}{\sqrt{p}} = 2$

Given expression = $p^{\frac{3}{2}} + p^{\frac{-3}{2}} = p^{\frac{3}{2}} + \frac{1}{p^{\frac{3}{2}}} = (p^{\frac{1}{2}})^3 + \frac{1}{(p^{\frac{1}{2}})^3} = (\sqrt{p})^3 + \frac{1}{(\sqrt{p})^3}$

$= (\sqrt{p})^3 + (\frac{1}{\sqrt{p}})^3 = (\sqrt{p} + \frac{1}{\sqrt{p}})^3 - 3.\sqrt{p}.\frac{1}{\sqrt{p}}(\sqrt{p} + \frac{1}{\sqrt{p}}) = (2)^3 - 3.2 =$ **2 (Ans:)**

20. If p = 124, then the value of $\sqrt[3]{p(p^2+3p+3)+1}$, is

✍**Solution:**

Given that, p = 124

Now, $\sqrt[3]{p(p^2+3p+3)+1} = \sqrt[3]{p^3+3p^2+3p+1} = \sqrt[3]{(p+1)^3}$ = p + 1= 124+1 = **125**

21. If (a-b) = 3, (b-c) = 5 and (c-a) = 1 then, $\frac{a^3+b^3+c^3-3abc}{a+b+c} = ?$

✍**Solution**:

Given that, a-b = 3, b-c = 5 & c-a = 1

Given expression = $\frac{a^3+b^3+c^3-3abc}{a+b+c} = \frac{(a+b+c)(a^2+b^2+c^2-ab-bc-ca)}{(a+b+c)}$

$= \frac{1}{2}(2a^2+2b^2+2c^2-2ab-2bc-2ca)$

$= \frac{1}{2}\{(a-b)^2+(b-c)^2+(c-a)^2\} = \frac{1}{2}\{(3)^2+(5)^2+(1)^2\} = \frac{35}{2}$ **(Ans:)**

🗀Factorization:

1. **Factorise $4t^2+35t-9$** *[Dhaka Bank Ltd.-(MTO) -2018 (Written)]*

✍**Solution:**

$4t^2+35t-9 \Rightarrow 4t^2+36t-t-9 \Rightarrow 4t(t+9)-1(t+9) \Rightarrow$ **(t+9)(4t-1)** (**Ans**)

❐**Self Task:**

2. **$a^2-30a+216$**

✍**Solution**:

$a^2-30a+216 = a^2-18a-12a+216 = a(a-18)-12(a-18)$ **= (a-18)(a-12)**

3. **$3(a^2+2a)^2-22(a^2+2a)+40$**

✍**Solution**: (x ধরে সমাধান)

$3(a^2+2a)^2-22(a^2+2a)+40$

$= 3x^2-22x+40$ [$a^2+2a = x$ ধরে]

$= 3x^2-12x-10x+40$

$= 3x(x-4)-10(x-4)$

$= (x-4)(3x-10) = (a^2+2a-4)\{3(a^2+2a)-10)$ (x এর মান বসিয়ে) = **$(a^2+2a-4)\{3a^2+6a-10)$ (Ans)**

✍**বিকল্প সমাধান:** (মিডিল টার্ম নিয়মে সমাধান)

$3(a^2+2a)^2-22(a^2+2a)+40$

$= 3(a^2+2a)^2-12(a^2+2a)-10(a^2+2a)+40$

$= 3(a^2+2a)(a^2+2a-4)-10(a^2+2a-4) = (a^2+2a-4)(3a^2+6a-10)$ **(Ans)**

❐**Self Task:**

4. **$(a-1)x^2+a^2xy+(a+1)y^2$**

✍**Solution**:

ধরি, (a-1) = p .... (i) এবং (a+1) = q .... (ii)

(i) ও (ii) নং গুণ করে পাই, $a^2-1 = pq \quad \therefore a^2 = pq+1$

তাহলে প্রদত্ত রাশিটি দাঁড়ায়

$px^2 + (pq + 1)xy + qy^2$

$= px^2 + pqxy + xy + qy^2$

$= px(x+qy) + y(x+qy)$

$= (x+qy)(px+y) = \{x+(a+1)y\}\{(a-1)x+y\}$ [মান বসিয়ে] $=$ **(x+ay+y) (ax-x+y) (Ans.)**

5. **উৎপাদকে বিশ্লেষণ করুন:** $x^2 - \left(\frac{2}{a} - 3a\right)x - 6$ *[Ministry of Food (AP)-2020 (Written)]*

✍**সমাধান:** $x^2 - \left(\frac{2}{a} - 3a\right)x - 6 = x^2 - \frac{2x}{a} + 3ax - 6 = x\left(x - \frac{2}{a}\right) + 3a\left(x - \frac{2}{a}\right) = \left(x - \frac{2}{a}\right)(x + 3a)$ **(Ans)**

6. **Resolve into factor :** $\mathbf{a^2 + \frac{1}{a^2} + 2 - 2a - \frac{2}{a}}$ *[BKB – (Cash)-2018 –(Written)]*

✍**Solution:**

$a^2 + \frac{1}{a^2} + 2 - 2a - \frac{2}{a} = \left(a + \frac{1}{a}\right)^2 - 2.a.\frac{1}{a} + 2 - 2(a + \frac{1}{a}) = (a+\frac{1}{a})^2 - 2(a+\frac{1}{a})$

$= (a+\frac{1}{a})(a+\frac{1}{a}-2)$ [$(a+\frac{1}{a})^2$ থেকে ১টি $(a+\frac{1}{a})$ নিলে আরেকটি থাকে)] **Ans.** $(a+\frac{1}{a})(a+\frac{1}{a}-2)$

❒**Self Task:**

7. $\mathbf{4a^2 + \frac{1}{4a^2} - 2 + 4a - \frac{1}{a}}$

✍**Solution**: $4a^2 + \frac{1}{4a^2} - 2 + 4a - \frac{1}{a} = (2a)^2 + \left(\frac{1}{2a}\right)^2 - 2 + 4a - \frac{1}{a}$

$= \left(2a - \frac{1}{2a}\right)^2 + 2.2a.\frac{1}{2a} - 2 + 4a - \frac{1}{a} = \left(2a - \frac{1}{2a}\right)^2 + 2 - 2 + 4a - \frac{1}{a}$

$= \left(2a - \frac{1}{2a}\right)\left(2a - \frac{1}{2a}\right) + 2\left(2a - \frac{1}{2a}\right) = \left(\mathbf{2a - \frac{1}{2a}}\right)\left(\mathbf{2a - \frac{1}{2a} + 2}\right)$ **(Ans)**

8. **উৎপাদকে বিশ্লেষণ করুন: x(x + 1) (x + 2) (x + 3) – 15** *[Ministry of Food (AP)-2020 (Written)]*

✍**সমাধান:**

$x(x + 1)(x + 2)(x + 3) - 15$

$= x(x + 3)(x + 1)(x + 2) - 15$

$= (x^2 + 3x)(x^2 + 3x + 2) - 15$

ধরি, $(x^2 + 3x) = a$

প্রদত্ত রাশি,

$a(a + 2) - 15 = a^2 + 2a - 15 = a^2 + 5a - 3a - 15 = a(a + 5) - 3(a + 5)$

$= (a + 5)(a - 3) = (x^2 + 3x + 5)(x^2 + 3x - 3)$ [a এর মান বসিয়ে পাই] **(Ans)**

❒**Self Task:**

9. **(x+2)(x+3)(x+4)(x+5)-48**

✍**Solution:**

$(x+2)(x+3)(x+4)(x+5) - 48$
$= (x+2)(x+5)(x+3)(x+4) - 48$
$= (x^2+5x+2x+10)(x^2+4x+3x+12) - 48$
$= (x^2+7x+10)(x^2+7x+12) - 48$
ধরি, $x^2+7x + 10 = a$
$= a(a + 2) – 48$
$= a^2 + 2a – 48$
$= a^2 + 8a – 6a - 48$
$= a(a + 8) – 6(a + 8)$
$= (a + 8) (a – 6)$
$= (x^2 + 7x + 10 + 8) (x^2 + 7x + 10 – 6)$ [$a = x^2+7x + 10$ বসিয়ে]
$= (x^2 + 7x + 18) (x^2 + 7x + 4)$ (**Ans.**)

10. উৎপাদকে বিশ্লেষণ করুনঃ $x^2+ax–(3a–2)(4a–2)$ *[Karmahgangsthan Bank (Officer)-2011-(Written)]*

✍**Solution:**

$x^2+ax–(3a–2)(4a–2)$
$= x^2+ax–(3a–2)(3a–2+a)$
$= x^2+ax–p(p+a)$ [3a-2= p ধরে]
$= x^2–p^2+ax–ap$
$= (x+p)(x–p)+a(x–p)$
$= (x-p)(x+p+a) = \{x-(3a–2)\}\{x+(3a–2)+a\}= (x–3a+2)(x+3a–2+a) =$ **$(x–3a+2)(x+4a–2)$ (Ans)**

❒**Self Task:**

11. উৎপাদকে বিশ্লেষণ করুন: $p^2+ mp –(3m- 2) (4m- 2)$ *[Karmashangsthan Bank Ltd.(SO)-2013(Written)]*

Ans. $(p – 3m + 2) (p + 4m – 2)$

12. $x^2 – x – (a + 1) (a + 2)$

✍**Solution:**

$x^2 – x – (a + 1) (a + 2)$
$= x^2 – x – (a + 1) (a + 1 + 1)$
$= x^2 – x – p(p + 1)$ [যখন $a + 1 = p$]
$= x^2 – x – p^2 – p$
$= x^2 – p^2 – x – p$
$= (x + p) (x – p) – 1(x + p)$
$= (x + p) (x – p – 1) = (x + a + 1) (x – a – 1– 1)$ [p এর মান বসিয়ে] $= (x + a + 1) (x – a – 2)$ (**Ans**)

❒**Self Task:**

13. উৎপাদকে বিশ্লেষেণ করঃ $ax^2+(a^2+1)x+a$ *[Karmahgangsthan Bank (AO)Cash-2011]*

✍**Solution:**

$ax^2+(a^2+1)x+a =ax^2+a^2x+x+a = ax(x+a)+1(x+a) =(x+a)(ax+1)$ **Ans. $(x+a)(ax+1)$**

14. $(a – b)^3 + (b – c)^3 + (c – a)^3$ কে উৎপাদকে বিশ্লেষণ করুন।

✍**Solution**:

মনে করি,
$x = a – b, y = b – c, z = c – a$
ফলে, $x + y + z = a – b + b – c + c – a = 0$
সুতরাং প্রদত্ত রাশি $= (a – b)^3 + (b – c)^3 + (c – a)^3$

$= x^3 + y^3 + z^3$
$= x^3 + y^3 + z^3 - 3xyz + 3xyz$
$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) + 3xyz$
$= 0 \times (x^2 + y^2 + z^2 - xy - yz - zx) + 3xyz = 3xyz$ (**Ans**.)

❒Self Task:

15. $(x + y + 2z)^3 + (y + z - 2x)^3 + (z + x - 2y)^3$

✍**Solution**:

মনে করি,
$x + y + 2z = a$
$y + z - 2x = b$
$z + x - 2y = c$
$0 = a + b + c$ [যোগ করে]

∴ প্রদত্ত রাশি $= a^3 + b^3 + c^3 - 3abc + 3abc$
$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$
$= 0 \times (a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$
$= 3(x + y - 2z)(y + z - 2x)(z + x - 2y)$ (**Ans**)

16. Factorize $x^3 - 21x + 20$ *[Rupali Bank (Off)—2019-(Written)]*

✍**Solution:**

Let, $F(x) = x^3 - 21x + 20$
So, $F(1) = 1^3 - 21\times1 + 20 = 21-21 = 0$
∴ $(x-1)$ is a factor of $x^3 - 21x + 20$
Now, $x^3 - 21x + 20$
$= x^3 - x^2 + x^2 - x - 20x + 20$
$= x^2(x - 1) + x(x - 1) - 20(x - 1)$ [প্রথম লাইনের পর আগে ৩য় লাইন লিখে সেখান থেকে ২য় লাইনটি মেলাতে হয়।]
$= (x - 1)(x^2 + x - 20)$
$= (x - 1)(x^2 + 5x - 4x - 20) = (x - 1)\{x(x+5) - 4(x+5)\}$ = **$(x - 1)(x - 4)(x + 5)$** (**Ans**)

❒Self Task:

17. $x^3 - x - 6$ কে উৎপাদকে বিশ্লেষণ কর।

✍**সমাধান:**

এখানে, $f(2) = x^3 - x - 6 = 2^3 - 2 - 6 = 8 - 8 = 0$
সুতরাং, $x - 2$, $f(x)$
∴ $f(x) = x^3 - x - 6$
$= x^3 - 2x^2 + 2x^2 - 4x + 3x - 6 = x^2(x-2) + 2x(x-2) + 3(x-2) =$ **$(x-2)(x^2 + 2x + 3)$**

18. $3a^3+2a+5$

✍**Solution:**

$3a^3 + 2a + 5$
ধরি, $f(-1) = 3(-1)^3 + 2(-1) + 5 = -3 - 2 + 5 = 5 - 5 = 0$
∴ $(a + 1)$, $f(a)$ এর একটি উৎপাদক।
এখন, $3a^3 + 2a + 5$
$= 3a^3 + 3a^2 - 3a^2 - 3a + 5a + 5 = 3a^2(a + 1) - 3a(a + 1) + 5(a + 1) =$ **$(a + 1)(3a^2 - 3a + 5)$**

19. $x^3 - 7xy^2 - 6y^3$ *[৯ম-১০ম শ্রেণী অনু:৩.৪]*

✍**Solution:**

$x^3 - 7xy^2 - 6y^3$

ধরি, $f(-y) = (-y)^3 - 7(-y).y^2 - 6y^3$

$= -y^3 + 7y^3 - 6y^3 = 7y^3 - 7y^3 = 0$

$\therefore (x + y)$, $f(x)$ এর একটি উৎপাদক।

এখন, $x^3 - 7xy^2 - 6y^3$

$= x^3 + x^2y - x^2y - xy^2 - 6xy^2 - 6y^3$

$= x^2(x + y) - xy(x + y) - 6y^2(x + y)$

$= (x + y)(x^2 - xy - 6y^2)$

$= (x + y)(x^2 - 3xy + 2xy - 6y^2) = (x + y)\{x(x - 3y) + 2y(x - 3y)\} = \mathbf{(x + y)(x + 2y)(x - 3y)}$

20. $x^6 - x^5 + x^4 - x^3 + x^2 - x$

✍Solution:

প্রদত্ত রাশি $= x^6 - x^5 + x^4 - x^3 + x^2 - x = x(x^5 - x^4 + x^3 - x^2 + x - 1)$

এখন, মনে করি, $f(x) = x^5 - x^4 + x^3 - x^2 + x - 1$

$\therefore$ $x = 1$ বসালে পাই, $f(1) = (1)^5 - (1)^4 + (1)^3 - (1)^2 + (1) - 1 \quad = 1 - 1 + 1 - 1 + 1 - 1 = 0$

$\therefore$ ভাগশেষ উপপাদ্য অনুসারে $(x - 1)$, $f(x)$ এর একটি উৎপাদক।

এখন, $f(x) = x^5 - x^4 + x^3 - x^2 + x - 1$

$= x^4(x - 1) + x^2(x - 1) + 1(x - 1)$

$= (x - 1)(x^4 + x^2 + 1)$

$= (x - 1)\{(x^2)^2 + 2.x^2.1 + (1)^2 - x^2\}$

$= (x - 1)\{(x^2 + 1)^2 - (x)^2\}$

$= (x - 1)\{(x^2 + 1) + x)\}\{(x^2 - 1) - x)\}$

$= (x - 1)(x^2 + 1 + x)(x^2 + 1 - x) \quad \mathbf{= (x - 1)(x^2 + x + 1)(x^2 - x + 1)}$

$\therefore x^6 - x^5 + x^4 - x^3 + x^2 - x = x(x^5 - x^4 + x^3 - x^2 + x + 1)$

$= x(x - 1)(x^2 + x + 1)(x^2 - x + 1)$

21. k এর মান কত হলে $(3x - 2)$; $15x^2 - kx - 14$ এর একটি উৎপাদক হবে?

✍Solution:

ধরি, $f(x) = 15x^2 - kx - 14$ $(3x - 2)$; $f(x)$ এর একটি উৎপাদক হলে $f\left(\frac{2}{3}\right) = 0$

$\therefore f\left(\frac{2}{3}\right) = 15\left(\frac{2}{3}\right)^2 - k.\frac{2}{3} - 14 \Rightarrow 0 = 15\frac{4}{9} - \frac{2k}{3} - 14 \Rightarrow -\frac{2k}{3} = \frac{22}{3} \quad \therefore k = -11$

22. Find the HCF (গ.সা.গু) of $x^3 - 16x$, $2x^3 + 9x^2 + 4x$, $2x^3 + x^2 - 28x$. *[Agrani Bank (SO-Auditor)-2018-(Written)]*

✍Solution:

At first case, $\mathbf{x^3 - 16x} = x(x^2 - 16) \Rightarrow x(x^2 - 4^2) \Rightarrow x(x+4)(x-4)$

At second case, $\mathbf{2x^3 + 9x^2 + 4x}$

$= x(2x^2 + 9x + 4) = x(2x^2 + 8x + x + 4) = x\{2x(x+4) + 1(x+4)\} = x(x+4)(2x+1)$

At third case, $\mathbf{2x^3 + x^2 - 28x}$

$= x(2x^2 + x - 28) = x(2x^2 + 8x - 7x - 28) = x\{2x(x+4) - 7(x+4)\} = x(x+4)(2x-7)$

So H.C.F (গ.সা.গু) = x(x+4) (গ.সা.গু বের করার সময় শুধু কমন রাশিগুলো নিতে হয়।) **Ans. x(x+4)**

Simplification:

1. Simplify : $\dfrac{3\frac{1}{2}-2\frac{1}{6}}{\frac{1}{4}\text{of}(\frac{1}{5}+\frac{1}{7})}\div 15\frac{5}{9}$ *[AB Bank Ltd.(PO)-Recruiment Test-1990(Written)]*

Solution: $\dfrac{3\frac{1}{2}-2\frac{1}{6}}{\frac{1}{4}\text{of}(\frac{1}{5}+\frac{1}{7})}\div 15\frac{5}{9} = \dfrac{\frac{7}{2}-\frac{13}{6}}{\frac{1}{4}\text{of}(\frac{7+5}{35})}\div\dfrac{140}{9} = \dfrac{\frac{21-13}{6}}{\frac{1}{4}\times\frac{12}{35}}\div\dfrac{140}{9} = \dfrac{\frac{8}{6}}{\frac{3}{35}}\div\dfrac{140}{9}$

$= \dfrac{8}{6}\times\dfrac{35}{3}\times\dfrac{9}{140} = 1$ **Ans. 1**

2. Simplify: $\dfrac{a^{\frac{1}{2}}+a^{-\frac{1}{2}}}{1-a}+\dfrac{1-a^{-\frac{1}{2}}}{1+\sqrt{a}}$ *[Standard Bank Ltd. (AO)Cash-2011]*

Solution:

$\dfrac{a^{\frac{1}{2}}+a^{-\frac{1}{2}}}{1-a}+\dfrac{1-a^{-\frac{1}{2}}}{1+\sqrt{a}} = \dfrac{\sqrt{a}+\frac{1}{\sqrt{a}}}{1-a}+\dfrac{1-\frac{1}{\sqrt{a}}}{1+\sqrt{a}} = \dfrac{\frac{a+1}{\sqrt{a}}}{1-a}+\dfrac{\frac{\sqrt{a}-1}{\sqrt{a}}}{1+\sqrt{a}} = \dfrac{a+1}{\sqrt{a}(1-a)}+\dfrac{\sqrt{a}-1}{\sqrt{a}(1+\sqrt{a})}$

$= \dfrac{a+1}{\sqrt{a}(1-a)}+\dfrac{(\sqrt{a}-1)^2}{\sqrt{a}(1+\sqrt{a})(\sqrt{a}-1)}$ (লব ও হরের সাথে $\sqrt{a}-1$ গুণ করে)

$= \dfrac{a+1}{\sqrt{a}(1-a)}+\dfrac{a-2\sqrt{a}+1}{\sqrt{a}(a-1)}$ (উপরে উপরে বর্গের সূত্র এবং নিচে $(x+y)(x-y) = x^2 - y^2$ সূত্র প্রয়োগ করে)

$= \dfrac{a+1-a+2\sqrt{a}-1}{\sqrt{a}(1-a)} = \dfrac{2\sqrt{a}}{\sqrt{a}(1-a)} = \dfrac{2}{1-a}$ **Ans.** $\mathbf{\dfrac{2}{1-a}}$

3. Simplify: $\sqrt[3]{a^{-2}b}\times\sqrt[3]{b^{-2}c}\times\sqrt[3]{c^{-2}a}$

Solution: $\sqrt[3]{a^{-2}b}\times\sqrt[3]{b^{-2}c}\times\sqrt[3]{c^{-2}a}$

$= (a^{-2})^{\frac{1}{3}}b\times(b^{-2})^{\frac{1}{3}}c\times(c^{-2})^{\frac{1}{3}}a$

$= a^{-\frac{2}{3}}.a\times b^{-\frac{2}{3}}.b\times c^{-\frac{2}{3}}.c$

$= a^{-\frac{2}{3}+1}\times b^{-\frac{2}{3}+1}\times c^{-\frac{2}{3}+1} = a^{\frac{1}{3}}.b^{\frac{1}{3}}.c^{\frac{1}{3}} = abc^{\frac{1}{3}} = \sqrt[3]{abc}$ **(Ans)**

4. Simplify: $\left[\dfrac{4^{m+\frac{1}{4}}\times\sqrt{2.2^m}}{2\sqrt{2^{-m}}}\right]^{\frac{1}{m}}$ **= ?**

✍**Solution:** $\left[\dfrac{4^{m+\frac{1}{4}}\times\sqrt{2.2^{m}}}{2\sqrt{2^{-m}}}\right]^{\frac{1}{m}}$

$$=\left[\frac{2^{2m+\frac{1}{2}}\times 2^{\frac{1}{2}}.2^{\frac{m}{2}}}{2.2^{-\frac{m}{2}}}\right]^{\frac{1}{m}}=\left[\frac{2^{2m+\frac{1}{2}+\frac{1}{2}+\frac{m}{2}}}{2^{1-\frac{m}{2}}}\right]^{\frac{1}{m}}=\left[2^{2m+1+\frac{m}{2}-1+\frac{m}{2}}\right]^{\frac{1}{m}}=\left[2^{3m}\right]^{\frac{1}{m}}=2^{3}=\mathbf{8\ (Ans.)}$$

Exponent:

5. **If $2^x = \sqrt{1024}$, what is the value of x?** (যদি $2^x = \sqrt{1024}$ হয়, তাহলে x এর মান কত?) *[Microcredit Regulator Authority (AD)-2021(Written)]*

✍**Solution:** $2^x = \sqrt{1024} \Rightarrow 2^x = 32 \Rightarrow 2^x = 2^5$ $\therefore$ **x = 5** **Ans: 5**

6. **If 10000 = 10^{p+r} and 100 = 10^{p-r} then find the value of p and r** ? ***[****Bangladesh Krishi Bank (SO) Examination –2011- (Written)]*

✍**Solution:**

$10000 = 10^{p+r} \Rightarrow 10^4 = 10^{p+r} \Rightarrow 4 = p+r$ $\therefore$ p+r = 4…… (i)

Again, $100 = 10^{p-r}$ $10^2 = 10^{p-r}$ $\therefore$ p- r = 2…….(ii)

Adding (i) & (ii) we get 2p = 6 $\therefore$ p = 3

From (i) we have, p+ r = 4 ⇒3+r = 4 $\therefore$ r = 1

Required value p = 3, r = 1 **Ans: 3 & 1**

7. **If 2^n=128 and y = 5, then** $\dfrac{(2^{n-1})(5^{n-2})}{\sqrt{y}-1}$ **=?** *[Uttara bank Ltd.(PO)-2009(Written)]*

✍**Solution:**

Given, 2^n=128 or, $2^n = 2^7$ $\therefore$ n=7 Now n = 7 & y = 5

Now, $\dfrac{(2^{n-1})(5^{n-2})}{\sqrt{y}-1}$

$$=\frac{(2^{7-1})(5^{7-2})}{\sqrt{5}-1}=\frac{2^6\times5^5}{\sqrt{4}}=\frac{2^6\times5^5}{2}=2^5\times5^5=(2\times5)^5=10^5=100000$$ **Ans. 100000**

8. **Simplify the following algebraic expression** $\dfrac{9(4^x)^2}{16^{x+2}-2^{x+1}(8^x)}$ *[One Bank Ltd. (PO) – 2008 – (Written)]*

✍**Solution:**

$$\frac{9(4^x)^2}{16^{x+2}-2^{x+1}(8^x)}$$

$= \frac{9\times 2^{2\times x\times 2}}{16^x\times 16^2 - 2^x\times 2^1\times 2^{3x}} = \frac{9\times 2^{4x}}{2^{4x}\times 256 - 2\times 2^{x+3x}} = \frac{9\times 2^{4x}}{2^{4x}\times(256-2)} = \frac{9}{254}$ **Ans:** $\mathbf{\frac{9}{254}}$

9. Find the value of x $\sqrt[3]{8x^2\sqrt{32x\sqrt{4x^2}}} = 4$ **?**

✍Solution:

$\sqrt[3]{8x^2\sqrt{32x.2x}} = 4$

$\Rightarrow \sqrt[3]{8x^2\sqrt{64x^2}} = 4 \Rightarrow \sqrt[3]{8x^2.8x} = 4 \Rightarrow \sqrt[3]{64x^3} = 4 \Rightarrow \{(4x)^3\}^{\frac{1}{3}} = 4 \Rightarrow 4x = 4$ ∴ **x = 1 (Ans)**

10. Simplify: $\frac{2^{x+4} - 4.2^{x+1}}{2^{x+2}\div 2}$ *[৯ম-১০ম শ্রেনী-(সাধারণ গণিত অনু:৪.১)]*

✍Solution:

$\frac{2^{x+4} - 4.2^{x+1}}{2^{x+2}\div 2}$ *(উপরে মাঝখানে (-) আছে তাই (-) এর দু পাশে আলাদা ভাবে কাজ করতে হবে)*

$= \frac{2^{x+4} - 2^2.2^{x+1}}{2^x.2^2\div 2} = \frac{2^x.2^4 - 2^{2+x+1}}{2^x.2^{2-1}} = \frac{2^x.2^4 - 2^{3+x}}{2^x.2} = \frac{2^x(2^4 - 2^3)}{2^x.2} = \frac{16-8}{2} = \frac{8}{2} = \mathbf{4}$ **(Ans)**

11. If a = xy^{p-1} , b = xy^{q-1} , c = xy^{r-1} and p+q+r = 3 Then prove that **$a^{q-r}.b^{r-p}.c^{p-q} = 1$**

[Agrani Bank Officer (Cash)-2018-(Written)] & [Rupali Bank – (Cash)-2018-(Written)]

✍Solution:

পরামর্শ: $a^{q-r}.b^{r-p}.c^{p-q} = 1$ এখানে দেখতে হবে যে, প্রশ্নে প্রদত্ত ক্লু এর সাথে এই রাশিটির মিল কোথায় মিল না থাকলে মিলিয়ে নেয়ার জন্য যা করা উচিৎ তাই করতে হবে। মনে রাখবেন শুধু এই প্রশ্ন সমাধান করলে আবার এই প্রশ্নটিই কমন আসবে অন্য পরীক্ষায় এমন খুব কম সময় ই হয়। তাই সমাধানটি দেখে বোঝার চেষ্টা করুন। যাতে এরকম অন্য যে কোন প্রশ্ন সমাধান করতে পারেন।

Hence,

$a = xy^{p-1}$

$a^{q-r} = \{xy^{(p-1)}\}^{(q-r)}$ (a^{q-r} এটা প্রশ্নে আছে, তাই গুণ)

$a^{q-r} = x^{q-r}\times y^{(p-1)(q-r)}$

And $b = xy^{q-1}$

$b^{r-p} = x^{r-p}\times y^{(q-1)(r-p)}$ (উপরের নিয়মেই)

Again, $c = xy^{r-1}$

$c^{p-q} = x^{(p-q)}\times y^{(r-1)(p-q)}$

(এখন এই মানগুলো প্রশ্নে প্রদত্ত রাশির সাথে মিলে যাওয়ায় সেখানে বসিয়ে হিসেব করলেই উত্তর মিলে যাবে।)

L.H.S. $= a^{q-r}.\ b^{r-p}.c^{(p-q)}$

$= x^{q-r}\times y^{(p-1)(q-r)}\times x^{r-p}\times y^{(q-1)(r-p)}\times x^{(p-q)}\times y^{(r-1)(p-q)}$ (ঠান্ডা মাথায় একটার পর একটা মান বসিয়ে হিসেব)

$= \{x^{q-r}.\ x^{r-p}.x^{p-q}\}\times\{y^{(p-1)(q-r)}\times y^{(q-1)(r-p)}\times y^{(r-1)(p-q)}\}$ (x গুলোকে বামে y গুলোকে ডানে রাখা হলো)

$= (x^{q-r+r-p+p-q})\times(y^{pq-pr-q+r+qr-pq-r+p+pr-qr-p+q})$ (গুণ অবস্থায় থাকার জন্য উপরের পাওয়ারগুলো যোগ)

$= x^0.y^0$ (সবগুলো কেটে দিয়ে শেষে শুধু ০ ই থাকে)

$= 1\times 1$ (প্রশ্নে প্রদত্ত p+q+r = 3 মানটি ব্যবহার করার প্রয়োজন হচ্ছে না।)

$=1$ = R.H.S . ∴ **L.H.S. = R.H.S. (proved)**

12. দেখাও যে, যদি x= $a^{q+r}b^p$, y = $a^{r+p}b^q$, z = $a^{p+q}b^r$, হয়, তবে $x^{q-r}.y^{r-p}.z^{p-q} = 1$.

✍Solution:

দেওয়া আছে, $x = a^{q+r}b^p$, $y = a^{r+p}b^q$ এবং $z = a^{p+q}b^r$

বামপক্ষ $= x^{q-r}.y^{r-p}.z^{p-q}$

$= (a^{q+r}b^p)^{q-r}.(a^{r+p}b^q)^{r-p}.(a^{p+q}b^r)^{p-q}$ [x, y, z এর মান বসিয়ে]

$= a^{(q+r)(q-r)}.b^{pq-pr}.a^{(r+p)(r-p)}.b^{qr-qp}.a^{(p+q)(p-q)}.b^{pr-qr}$

$= a^{q^2-r^2}.a^{r^2-p^2}.a^{p^2-q^2}.b^{pq-pr}.b^{qr-qp}.b^{pr-qr}$

$= a^{q^2-r^2+r^2-p^2+p^2-q^2}.b^{pq-pr+qr-qp+pr-qr}$

$= a^0.b^0 = 1.1 = 1 =$ ডানপক্ষ $\therefore x^{q-r}.y^{r-p}.z^{p-q} = 1$ [দেখানো হলো]

13. $p = xy^{a-1}$, $q = xy^{b-1}$, $r = xy^{c-1}$ হলে, $\left(\frac{p}{q}\right)^c \times \left(\frac{q}{r}\right)^a \times \left(\frac{r}{p}\right)^b =$ কত?

Solution:

দেওয়া আছে, $p = xy^{a-1}$, $q = xy^{b-1}$,$r = xy^{c-1}$

প্রদত্ত রাশি, $\left(\frac{p}{q}\right)^c \times \left(\frac{q}{r}\right)^a \times \left(\frac{r}{p}\right)^b$

$= \left(\frac{xy^{a-1}}{xy^{b-1}}\right)^c \times \left(\frac{xy^{b-1}}{xy^{c-1}}\right)^a \times \left(\frac{xy^{c-1}}{xy^{a-1}}\right)^b$

$= (y^{a-1-b+1})^c \times (y^{b-1-c+1})^a \times (y^{c-1-a+1})^b$

$= (y^{a-b})^c \times (y^{b-c})^a \times (y^{c-a})^b$

$= y^{ac-bc} \times y^{ab-ca} \times y^{bc-ab} = y^{ac-bc+ab-ca+bc-ab} = y^0 = 1$ $\therefore$ নির্ণেয় মান = 1

14. Prove that, $\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} . \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} . \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} = 1$

Solution:

$L.H.S = \left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} . \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} . \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}}$

$= x^{(a-b)\frac{1}{ab}} . x^{(b-c)\frac{1}{bc}} . x^{(c-a)\frac{1}{ca}} = x^{\frac{a-b}{ab}} . x^{\frac{b-c}{cb}} . x^{\frac{c-a}{ca}} = x^{\frac{a-b}{ab}+\frac{b-c}{cb}+\frac{c-a}{ca}}$

$= x^{\frac{c(a-b)+a(b-c)+b(c-a)}{abc}} = x^{\frac{ac-bc+ab-ac+bc-ab}{abc}} = x^{\frac{0}{abc}} = x^0 = 1 =$ L.H.S $\therefore$ **L.H.S = R. H.S (Proved)**

15. Prove that, $\left(\frac{x^a}{x^b}\right)^{a+b} . \left(\frac{x^b}{x^c}\right)^{b+c} . \left(\frac{x^c}{x^a}\right)^{c+a} = 1$

Solution:

$L.H.S = \left(\frac{x^a}{x^b}\right)^{a+b} . \left(\frac{x^b}{x^c}\right)^{b+c} . \left(\frac{x^c}{x^a}\right)^{c+a}$

$= x^{(a-b)(a+b)} . x^{(b-c)(b+c)} . x^{(c-a)(c+a)} = x^{a^2-b^2} . x^{b^2-c^2} . x^{c^2-a^2}$

$= x^{a^2-b^2+b^2-c^2+c^2-a^2} = x^0 = 1 =$ R.H.S $\therefore$ **L.H.S = R. H.S (Proved)**

16. Prove that, $\left(\frac{x^p}{x^q}\right)^{p+q-r} . \left(\frac{x^q}{x^r}\right)^{q+r-p} . \left(\frac{x^r}{x^p}\right)^{r+p-q} = 1$

✍**Solution**:

$$\text{L.H.S} = \left(\frac{x^p}{x^q}\right)^{p+q-r} . \left(\frac{x^q}{x^r}\right)^{q+r-p} . \left(\frac{x^r}{x^p}\right)^{r+p-q}$$

$$= x^{(p-q)(p+q-r)} . x^{(q-r)(q+r-p)} . x^{(r-p)(r+p-q)}$$

$$= x^{p^2+pq-pr-pq-q^2+qr} . x^{q^2+qr-pq-qr-r^2+pr} . x^{r^2+pr-qr-pr-p^2+pq}$$

$$= x^{p^2-pr-q^2+qr} . x^{q^2-pq-r^2+pr} . x^{r^2-qr-p^2+pq}$$

$$= x^{p^2-pr-q^2+qr+q^2-pq-r^2+pr+r^2-qr-p^2+pq} = x^0 = 1 = \text{R.H.S} \quad \therefore \textbf{L.H.S = R. H.S (Proved)}$$

17. Simplify: $\sqrt[bc]{\dfrac{x^{\frac{b}{c}}}{x^{\frac{c}{b}}}} \times \sqrt[ca]{\dfrac{x^{\frac{c}{a}}}{x^{\frac{a}{c}}}} \times \sqrt[ab]{\dfrac{x^{\frac{a}{b}}}{x^{\frac{b}{a}}}}$

✍**Solution:**

$$\sqrt[bc]{\frac{x^{\frac{b}{c}}}{x^{\frac{c}{b}}}} \times \sqrt[ca]{\frac{x^{\frac{c}{a}}}{x^{\frac{a}{c}}}} \times \sqrt[ab]{\frac{x^{\frac{a}{b}}}{x^{\frac{b}{a}}}}$$

$$= \sqrt[bc]{x^{\frac{b}{c}-\frac{c}{b}}} \times \sqrt[ca]{x^{\frac{c}{a}-\frac{a}{c}}} \times \sqrt[ab]{x^{\frac{a}{b}-\frac{b}{a}}}$$

$$= \sqrt[bc]{x^{\frac{b^2-c^2}{bc}}} \times \sqrt[ca]{x^{\frac{c^2-a^2}{ac}}} \times \sqrt[ab]{x^{\frac{a^2-b^2}{ab}}}$$

$$= \left(x^{\frac{b^2-c^2}{bc}}\right)^{\frac{1}{bc}} \times \left(x^{\frac{c^2-a^2}{ac}}\right)^{\frac{1}{ca}} \times \left(x^{\frac{a^2-b^2}{ab}}\right)^{\frac{1}{ab}}$$

$$= x^{\frac{b^2-c^2}{b^2c^2}} \times x^{\frac{c^2-a^2}{a^2c^2}} \times x^{\frac{a^2-b^2}{a^2b^2}}$$

$$= x^{\frac{b^2-c^2}{b^2c^2}+\frac{c^2-a^2}{a^2c^2}+\frac{a^2-b^2}{a^2b^2}}$$

$$= x^{\frac{a^2(b^2-c^2)+b^2(c^2-a^2)+c^2(a^2-b^2)}{a^2b^2c^2}} = x^{\frac{a^2b^2-a^2c^2+b^2c^2-a^2b^2+a^2c^2-b^2c^2}{a^2b^2c^2}} = x^{\frac{0}{a^2b^2c^2}} = \mathbf{x^0 = 1}$$

18. $\sqrt[3]{8x^2\sqrt{32x\sqrt{4x^2}}} = 4$ **then the value of x.** *[Premier bank ltd.(MTO)-2012(Written)]*

✍**Solution:**

$$\sqrt[3]{8x^2\sqrt{32x\sqrt{4x^2}}} = 4$$

$$\Rightarrow \sqrt[3]{8x^2\sqrt{32x \times 2x}} = 4 \Rightarrow \sqrt[3]{8x^2\sqrt{64x^2}} = 4 \Rightarrow \sqrt[3]{8x^2 \times 8x} = 4$$

$$\Rightarrow \sqrt[3]{64x^3} = 4 \quad \Rightarrow \sqrt[3]{(4x)^3} = 4 \quad \Rightarrow (4x)^{3\times\frac{1}{3}} = 4 \quad \Rightarrow 4x = 4 \quad \therefore x=1$$

Ans: 1

19. If $\sqrt[3]{x^2\sqrt{x\sqrt{\frac{1}{x}}}} = \cfrac{1}{\cfrac{1}{\cfrac{1}{\cfrac{1}{x}}}}$ **Find the value of x** . *[SocialIslami Bank Ltd. (PO)-2014(Written)]*

✍**Solution:**

L.H.S= $\sqrt[3]{x^2\sqrt{x\sqrt{\frac{1}{x}}}} = \sqrt[3]{x^2\sqrt{x\sqrt{x^{-1}}}} = \sqrt[3]{x^2\sqrt{x\times x^{-\frac{1}{2}}}} = \sqrt[3]{x^2\sqrt{x^{1-\frac{1}{2}}}}$

$= \sqrt[3]{x^2\sqrt{x^{\frac{1}{2}}}} \quad = \sqrt[3]{x^2\times x^{\frac{1}{2}\times\frac{1}{2}}} \quad = \sqrt[3]{x^2\times x^{\frac{1}{4}}} \quad = \sqrt[3]{x^{2+\frac{1}{4}}} \quad = \left(x^{\frac{9}{4}}\right)^{\frac{1}{3}} = x^{\frac{3}{4}}$

R.H.S= $\cfrac{1}{\cfrac{1}{\cfrac{1}{\cfrac{1}{x}}}} = \cfrac{1}{\cfrac{1}{x}}$ [As $\cfrac{1}{\cfrac{1}{x}} = x$] $= x$ [Since $\cfrac{1}{\cfrac{1}{x}}$ =x]

Now, $\Rightarrow x^{\frac{3}{4}} = x$ (উপরে প্রাপ্ত দুটি মান) $\Rightarrow x^{\frac{3}{4}\times 4} = x^4 \Rightarrow x^3=x^4$ (power 4 both side)

$\Rightarrow x^4 - x^3 = 0 \Rightarrow x^3(x-1) = 0 \Rightarrow x-1=0 \quad \therefore x=1$ **Ans. 1**

20. Prove that, $\frac{2^{2p+1}.3^{2p+q}.5^{p+q}.6^p}{3^{p-2}.6^{2p+2}.10^p.15^q} = \frac{1}{2}$

✍**Solution**:

L.H.S= $\frac{2^{2p+1}.3^{2p+q}.5^{p+q}.6^p}{3^{p-2}.6^{2p+2}.10^p.15^q}$

$= \frac{2^{2p+1}.3^{2p+q}.5^{p+q}.(3\times 2)^p}{3^{p-2}.(3\times 2)^{2p+2}.(5\times 2)^p.(5\times 3)^q}$

$= \frac{2^{2p+1}.3^{2p+q}.5^{p+q}.3^p.2^p}{3^{p-2}.3^{2p+2}.2^{2p+2}.5^p.2^p.5^q.3^q}$

$= \frac{2^{2p+1+p}.3^{2p+q+p}.5^{p+q}.}{2^{2p+2+p}3^{p-2+2p+2+q}.5^{p+q}}$

$= 2^{(2p+1+p)-(2p+2+p)} \times 3^{(2p+q+p)-(p-2+2p+2+q)} \times 5^{(p+q)-(p+q)}$

$=2^{(2p+1+p-2p-2-p)} \times 3^{(2p+q+p-p+2-2p-2-q)} \times 5^{(p+q-p-q)}$

$= 2^{-1} \times 3^0 \times 5^0 = \frac{1}{2}$ = **R.H.S** $\therefore$ **L.H.S = R.H.S (Proved)**

21. **যদি a =** $2^{\frac{1}{3}} + 2^{\frac{-1}{3}}$ **হয়, তবে দেখাও যে,** $2a^3 - 6a = 5$

✍**Solution:** দেওয়া আছে, $a = 2^{\frac{1}{3}} + 2^{\frac{-1}{3}}$

$\Rightarrow a^3 = \left(2^{\frac{1}{3}} + 2^{-\frac{1}{3}}\right)^3$ [ঘন করে]

$\Rightarrow a^3 = \left(2^{\frac{1}{3}}\right)^3 + \left(2^{\frac{-1}{3}}\right)^3 + 3.2^{\frac{1}{3}}.\ 2^{\frac{-1}{3}}\left(2^{\frac{1}{3}} + 2^{\frac{-1}{3}}\right)$

$\Rightarrow a^3 = 2 + 2^{-1} + 3.\ 2^{\frac{1}{3}-\frac{1}{3}}.a \quad [\because a = 2^{\frac{1}{3}} + 2^{\frac{-1}{3}}]$

$\Rightarrow a^3 = 2 + \frac{1}{2} + 3.2^0.a \Rightarrow a^3 = 2 + \frac{1}{2} + 3.1.a$

$\Rightarrow a^3 = 2 + \frac{1}{2} + 3a \Rightarrow a^3 = \frac{4+1+6a}{2} \Rightarrow 2a^3 = 5 + 6a \Rightarrow 2a^3 - 6a = 5$ [দেখানো হলো]

22. সমাধান কর: $3.27^x = 9^{x+4}$

✍**Solution:**

$3.27^x = 9^{x+4}$

$\Rightarrow 3.(3^3)^x = (3^2)^{x+4} \Rightarrow 3.3^{3x} = 3^{2(x+4)} \Rightarrow 3^{3x+1} = 3^{2x+8} \Rightarrow 3x + 1 = 2x + 8 \quad \therefore$ **x = 7 (Ans)**

23. Solve: $4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$

✍**Solution:**

$4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$

$\Rightarrow 4^x + 2^{2x-1} = 3^{x+\frac{1}{2}} + 3^{x-\frac{1}{2}}$

$\Rightarrow (2^2)^x + 2^{2x}.2^{-1} = 3^x.3^{\frac{1}{2}} + 3^x.3^{\frac{-1}{2}}$

$\Rightarrow 2^{2x} + 2^{2x}.\frac{1}{2} = 3^x\left(3^{\frac{1}{2}} + \frac{1}{3^{\frac{1}{2}}}\right)$

$\Rightarrow 2^{2x}\left(1 + \frac{1}{2}\right) = 3^x\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)$

$\Rightarrow (2^2)^x\left(\frac{2+1}{2}\right) = 3^x\left(\frac{3+1}{\sqrt{3}}\right)$

$\Rightarrow 4^x.\frac{3}{2} = 3^x.\frac{4}{\sqrt{3}}$

$\Rightarrow \frac{4^x}{3^x} = \frac{4}{\sqrt{3}} \times \frac{2}{3}$

$$\Rightarrow \left(\frac{4}{3}\right)^x = \frac{8}{3\sqrt{3}} \Rightarrow \left\{\left(\frac{2}{\sqrt{3}}\right)^2\right\}^x = \left(\frac{2}{\sqrt{3}}\right)^3 \Rightarrow \left(\frac{2}{\sqrt{3}}\right)^{2x} = \left(\frac{2}{\sqrt{3}}\right)^3 \Rightarrow 2x = 3 \therefore x = \frac{3}{2} \textbf{ (Ans)}$$

24. সমাধান করুন : $4^x - 3(2^{x+2}) + 2^5 = 0$ *[৩৮ তম বিসিএস (লিখিত)]*

✍**Solution:**

. দেওয়া আছে,

$4^x - 3(2^{x+2}) + 2^5 = 0$

$\Rightarrow (2^x)^2 - 12.2^x + 32 = 0$

$\Rightarrow (2^x.2^x) - 8.2^x - 4.2^x + 32 = 0$

$\Rightarrow 2^x(2^x - 8) - 4(2^x - 8) = 0$

$\Rightarrow (2^x - 8)(2^x - 4) = 0$

$\Rightarrow 2^x - 8 = 0$ অথবা, $2^x - 4 = 0$

$\Rightarrow 2^x = 2^3$ বা, $2^x = 2^2$

$\therefore x = 3$ $\therefore x = 2$ $\therefore$ **নির্ণেয় সমাধান $x=2$ অথবা $x = 3$**

25. $3(9^x - 4.3^{x-1}) + 1 = 0$

✍**Solution:**

$3(9^x - 4.3^{x-1}) + 1 = 0$

$\Rightarrow 3.9^x - 4.3.3^{x-1} + 1 = 0$

$\Rightarrow 3.(3^2)^x - 4.3^{x-1+1} + 1 = 0$

$\Rightarrow 3.3^{2x} - 4.3^x + 1 = 0$

$\Rightarrow 3.(3^x)^2 - 4.3^x + 1 = 0$

$\Rightarrow 3a^2 - 4a + 1 = 0$ [$3^x = a$ ধরে]

$\Rightarrow 3a^2 - 3a - a + 1 = 0$

$\Rightarrow 3a(a-1) - 1(a-1) = 0$

$\Rightarrow (a-1)(3a-1) = 0$

$\therefore a - 1 = 0$ অথবা $3a - 1 = 0$

$\Rightarrow a = 1$ $\Rightarrow 3a = 1$

$\Rightarrow 3^x = 1$ [a এর মান বসিয়ে] $\Rightarrow 3.3^x = 1$

$\Rightarrow 3^x = 3^0$ $\Rightarrow 3^{x+1} = 3^0$

$\therefore x = 0$ $\Rightarrow x + 1 = 0$

$\therefore x = -1$ $\therefore$ নির্ণেয় সমাধানঃ $x = 0, -1$

🗁Logarithm:

26. If $\log_{\sqrt{2}} x = a$ then determine the value of $\log_{2\sqrt{2}} x$?

✍**Solution:**

Given that $\log_{\sqrt{2}} x = a \Rightarrow x = (\sqrt{2})^a$

Now, $\log_{2\sqrt{2}} x = \log_{2\sqrt{2}} (\sqrt{2})^a = \log_{2^{\frac{3}{2}}} (\sqrt{2})^a = \frac{2}{3} \times \log_2 2^{\frac{a}{2}} = \frac{2}{3} \times \frac{a}{2} \log_2 2 = \frac{a}{3}$ **(Ans)**

27. Determine the value: $8^{\log_2 \sqrt[3]{121}+\frac{1}{3}}$

✍**Solution:**

$= 8^{\log_2 (121)^{\frac{1}{3}}}.8^{\frac{1}{3}} = 2^{3\log_2 (121)^{\frac{1}{3}}}.2^{3\frac{1}{3}} = 2^{\log_2 (121)^{3.\frac{1}{3}}}.2 = 2^{\log_2 (121)}.2 = 121 \times 2 =$ **242 (Ans)**

28. $\log_9 3 \log_2 (1+\log_3(1+2\log_2 x) = \frac{1}{2}$ **হলে, x = ?**

✍**Solution:**

$\log_9 3 \log_2 (1+\log_3(1+2\log_2 x) = \frac{1}{2}$

$\Rightarrow 3 \log_2(1+\log_3(1+2\log_2 x)) = 9^{\frac{1}{2}} = 3$

$\Rightarrow \log_2(1+\log_3(1+2\log_2 x)) = 1$

$\Rightarrow 1 + \log_3 (1+2\log_2 x) = 2^1$

$\Rightarrow \log_3(1+2\log_2 x) = 1$

$\Rightarrow 1 + 2 \log_2 x = 1 \quad \Rightarrow 1+2 \log_2 x = 3 \Rightarrow 2\log_2 x = 2 \Rightarrow \log_2 x = 1 \quad \Rightarrow x = 2^1 =$ **2**

29. যদি, $xy^2 = 4$ **এবং** $\log_3(\log_2 x) + \log_{\frac{1}{3}}\left(\log_{\frac{1}{3}} y\right) = 1$ **হয় তবে x এর মান নির্ণয় কর।**

✍**Solution:**

$\log_3(\log_2 x) + \log_{\frac{1}{3}}\left(\log_{\frac{1}{3}} y\right) = 1$

$\Rightarrow \log_3\left(\log_2 \frac{4}{y^2}\right) - \log_3\left(\log_{\frac{1}{2}} y\right) = 1$

$\Rightarrow \frac{\log_2 \frac{4}{y^2}}{\log_{\frac{1}{2}} y} = 3^1 \Rightarrow \log_2 \frac{4}{y^2} = 3\log_{\frac{1}{2}} y \Rightarrow \log_2 \frac{4}{y^2} = 3\log_2 y^{-3} \Rightarrow \frac{4}{y^2} = y^{-3} \quad \Rightarrow y = \frac{1}{4}$

$\therefore y = \frac{1}{4}$ এর মান বসিয়ে, $xy^2 = 4$ এ বসিয়ে $x.\frac{1}{16} = 4 \quad \therefore$ **x = 64 (Ans)**

30. Prove that, $\log_{\sqrt{a}} b \times \log_{\sqrt{b}} c \times \log_{\sqrt{c}} a = 8$

✍**Solution:**

বামপক্ষ $= \log_{\sqrt{a}} b \times \log_{\sqrt{b}} c \times \log_{\sqrt{c}} a$

$= \log_{\sqrt{a}}(\sqrt{b})^2 \times \log_{\sqrt{b}}(\sqrt{c})^2 \times \log_{\sqrt{c}}(\sqrt{a})^2$

$= 2\log_{\sqrt{a}} \sqrt{b} \times 2\log_{\sqrt{b}} \sqrt{c} \times 2\log_{\sqrt{c}} \sqrt{a}$

$= 8\log_{\sqrt{a}} \sqrt{b} \times (\log_{\sqrt{b}} \sqrt{c} \times \log_{\sqrt{c}} \sqrt{a})$

$= 8\log_{\sqrt{a}} \sqrt{b} \times \log_{\sqrt{b}} \sqrt{a}$

$= 8\log_{\sqrt{a}}\sqrt{a} = 8.1$ [$\because \log_a a = 1$] $= 8 =$ ডানপক্ষ $\therefore \log_{\sqrt{a}} b \times \log_{\sqrt{b}} c \times \log_{\sqrt{c}} a$ **= 8** [দেখানো হলো]

31. If $5^{\log_{10}x} + x^{\log_{10}5} = 50$ then value of x = ?

✍**Solution:**

$5^{\log_{10}x} + x^{\log_{10}5} = 50 \quad \Rightarrow 5^{\log_{10}x} + 5^{\log_{10}x} = 50 \quad \Rightarrow 2 \times 5^{\log_{10}x} = 50$

$\Rightarrow 5^{\log_{10}x} = 25 \quad \Rightarrow 5^{\log_{10}x} = 5^2 \quad \Rightarrow \text{Log}_{10}x = 2 \quad \therefore x = 10^2 = 100$ (Ans)

32. Solve the equation: $2 + \log_3 27 = \log_3 (4x+7)$

✍**Solution:**

$2 + \log_3 27 = \log_3 (4x+7)$

$\Rightarrow 2 = \log_3(4x+7) - \log_3 27$

$\Rightarrow 2 = \log_3 \dfrac{4x+7}{27} \quad \Rightarrow 3^2 = \dfrac{4x+7}{27} \quad \Rightarrow 4x + 7 = 9 \times 27 \quad \Rightarrow x = 59$

33. Solve the equation: $\log_3(x+1) + \log_3(x+3) = 1$

✍**Solution:**

$\Rightarrow \log_3 [(x+1)(x+3)] = 1$

$\Rightarrow \log_3(x^2+4x+3) = 1$

$\Rightarrow x^2 + 4x + 3 = 3^1$

$\Rightarrow x^2 + 4x + 3 = 0$

$\Rightarrow x(x+4) = 0 \quad \therefore x = 0 , \; x = -4$

but x = - 4 is not acceptable as (x+1) and (x+1) will be negative] $\therefore$ **x = 0 (Solved)**

34. Simplify: $\dfrac{\log_{10}\sqrt{27} + \log_{10} 8 - \log_{10}\sqrt{1000}}{\log_{10} 1.2}$ *[ICB capital management (AP): 2019]*

✍**Solution**:

$$\frac{\log_{10}\sqrt{27} + \log_{10}8 - \log_{10}\sqrt{1000}}{\log_{10}1.2}$$

$$= \frac{\log_{10}(3^3)^{\frac{1}{2}} + \log_{10}8 - \log_{10}(10^3)^{\frac{1}{2}}}{\log_{10}\frac{12}{10}}$$

$$= \frac{\log_{10}3^{\frac{3}{2}} + \log_{10}2^3 - \log_{10}(10)^{\frac{3}{2}}}{\log_{10}12 - \log_{10}10}$$

$$= \frac{\frac{3}{2}\log_{10}3 + 3\log_{10}2 - \frac{3}{2}\log_{10}10}{\log_{10}(3\times 2^2) - \log_{10}10} = \frac{\frac{3}{2}(\log_{10}3 + 2\log_{10}2 - 1)}{(\log_{10}3 + 2\log_{10}2 - 1)} \; [\because \log_{10}10 = 1] = \frac{3}{2} \quad \textbf{Ans. } \frac{3}{2}$$

35. যদি $x = \log_{2a}a$, $y = \log_{3a}2a$ এবং $z = \log_{4a}3a$ হয় তাহলে প্রমাণ করুণ যে, $xyz + 1 = 2yz$

✍**Solution:**

L.H.S = xyz + 1

$= \text{Log}_{2a}a \; \text{Log}_{3a}2a \,.\, \text{Log}_{4a}3a + 1$

$= \text{Log}_{3a}a \,.\, \text{Log}_{4a}3a + 1$

$= \text{Log}_{4a}a + 1$

$= \text{Log}_{4a}a + \text{Log}_{4a}4a = \text{Log}_{4a}(a.4a)$

$= \text{Log}_{4a}4a^2$

$= \text{Log}_{4a}(2a)^2 = 2\text{Log}_{4a}2a = 2 \times \text{Log}_{3a}2a \times 2 \times \text{Log}_{4a}3a = 2yz =$ **R.H.S**

36. If $\frac{a}{(q-r)} = \frac{b}{(r-p)} = \frac{c}{(p-q)}$ then show that, a+b+c = pa+qb+rc *[BDBL-(SO)-2018-(Written)]*

✍Solution:

Suppose, $\frac{a}{(q-r)} = \frac{b}{(r-p)} = \frac{c}{(p-q)} = x$

or, $\frac{a}{(q-r)} = x, \frac{b}{(r-p)} = x, \frac{c}{(p-q)} = x$

So, a = qx-rx, b = rx-px, c = px-qx

Now L.H.S.,

a+b+c = qx-rx+rx-px+px-qx = 0 (a, b এবং c এর মান বসিয়ে যোগ বিয়োগ করে উত্তর:০)

and, R.H.S.,

pa + qb + rc

= p(qx-rx) + q(rx-px) + r(px-qx) (a, b এবং c এর মান আলাদা ভাবে বসানো হলো।)

= pqx-prx + qrx-pqx + prx-qrx

= 0 (উভয় পাশের উত্তর ০) **So, L.H.S. = R.H.S. (SHOWED)**

যোজন বিয়োজনের প্রশ্ন:

37. If $x=\frac{4}{5}$, then $\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}$ =? *[RAKUB-(Officer)-2014(Written)]*

✍Solution:

দেওয়া আছে , $x = \frac{4}{5}$

$\Rightarrow \frac{1}{x} = \frac{5}{4}$ [বিপরীত করণ করে]

$\Rightarrow \frac{1+x}{1-x} = \frac{5+4}{5-4}$ [যোজন বিয়োজন করে]

$\Rightarrow \frac{1+x}{1-x} = \frac{9}{1}$

$\Rightarrow \sqrt{\frac{1+x}{1-x}} = \sqrt{9}$ [বর্গমূল করে]

$\Rightarrow \frac{\sqrt{1+x}}{\sqrt{1-x}} = 3$

$\Rightarrow \dfrac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}} = \dfrac{3+1}{3-1}$ [পুণরায় যোজন-বিয়োজন করে]

$\Rightarrow \dfrac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}} = \dfrac{4}{2}$ $\Rightarrow \dfrac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}} = 2$ **Ans. 2**

38. $\dfrac{\sqrt{2+x}+\sqrt{2-x}}{\sqrt{2+x}-\sqrt{2-x}} = 2$. **Find the value of x.** *[Bangladesh Bank Ltd. (Officer)-2015(Written)]*

✍**Solution:**

$\dfrac{\sqrt{2+x}+\sqrt{2-x}}{\sqrt{2+x}-\sqrt{2-x}} = 2$

$\Rightarrow \dfrac{\sqrt{2+x}+\sqrt{2-x}+\sqrt{2+x}-\sqrt{2-x}}{\sqrt{2+x}+\sqrt{2-x}-\sqrt{2+x}+\sqrt{2-x}} = \dfrac{2+1}{2-1}$ [Doing invertend and dividend]

$\Rightarrow \dfrac{2\sqrt{2+x}}{2\sqrt{2-x}} = \dfrac{3}{1}$

$\Rightarrow \left(\sqrt{\dfrac{2+x}{2-x}}\right)^2 = (3)^2$ [Squaring Both Sides]

$\Rightarrow \dfrac{2+x}{2-x} = 9$ $\Rightarrow$ 2+x=18–9x $\Rightarrow$ 10x=16 $\Rightarrow x = \dfrac{16}{10}$ $\therefore x = \dfrac{8}{5}$ **Ans.** $\dfrac{8}{5}$

🗀Single & Double equation:

1. Find the value of 'x' if $(2x^2 - 1) = (3x^2 - 2x)$ *[B.B. AD- 2004; Written]*

✍**Solution:**

Given that, $(2x^2 - 1) = (3x^2 - 2x)$

$\Rightarrow 2x - 1 = 3x^2 - 2x^2$

$\Rightarrow 2x - 1 = x^2 \Rightarrow x^2 - 2x + 1 = 0 \Rightarrow (x-1)^2 = 0 \Rightarrow x - 1 = 0 \Rightarrow x = 1$ **Ans: 1**

2. x এর মান বের করুনঃ $\dfrac{1}{2-\dfrac{x}{1-x}} = \dfrac{1}{2}$ *[BB Banker's Recruting Examination-1986 (Written)]*

✍**Solution:**

$\dfrac{1}{2-\dfrac{x}{1-x}} = \dfrac{1}{2} \Rightarrow \dfrac{1}{\dfrac{2-2x-x}{1-x}} = \dfrac{1}{2} \Rightarrow \dfrac{1}{\dfrac{2-3x}{1-x}} = \dfrac{1}{2} \Rightarrow \dfrac{1-x}{2-3x} = \dfrac{1}{2}$

$\Rightarrow$ 2-2x= 2-3x $\Rightarrow$ -2x+3x= 2-2 $\therefore$ x= 0 **Ans. নির্ণেয় মান 0.**

3. If 2x = 4y = 8z and $\dfrac{1}{2x}+\dfrac{1}{4y}+\dfrac{1}{4z} = 4$**, then find the value of x.** *[UCBL Bank Ltd. (Officer)-2010(Written)]*

✍**Solution:**

Given , 2x = 4y = 8z $\Rightarrow$ x = 2y = 4z

Now, $\frac{1}{2x}+\frac{1}{4y}+\frac{1}{4z}=4 \Rightarrow \frac{1}{2x}+\frac{1}{2x}+\frac{1}{x}=4$ [Since 4z = x]

$\Rightarrow \frac{1}{x}(\frac{1}{2}+\frac{1}{2}+1)=4 \Rightarrow \frac{2}{x}=4 \Rightarrow 4x=2 \therefore x=\frac{2}{4}=\frac{1}{2}$ **Ans.** $\mathbf{\frac{1}{2}}$

4. If $\frac{2x - y}{x+2y}=\frac{1}{2}$**, then the value of** $\frac{3x - y}{3x+y}$ **is?***[Examveda.com]*

✍**Solution:**

$\frac{2x - y}{x+2y}=\frac{1}{2} \Rightarrow 4x\text{-}2y = x+2y \Rightarrow 3x = 4y \Rightarrow x:y = 4:3$

Now, $\frac{3x - y}{3x+y}=\frac{3\times4 - 3}{3\times4+3}=\frac{12 - 3}{12+3}=\frac{9}{15}=\frac{3}{5}$ **(Ans:)** $\frac{3}{5}$

5. Solve the following equation. $\mathbf{\frac{2}{x-2}+\frac{3}{x+3}=1}$ *[One Bank Ltd. (PO) – 2008 –(Written)]*

✍**Solution:**

$\frac{2}{x-2}+\frac{3}{x+3}=1$

$\Rightarrow \frac{2x+6+3x-6}{(x-2)(x+3)}=1$

$\Rightarrow \frac{5x}{x^2+x-6}=1$

$\Rightarrow x^2 + x \text{ -}6 = 5x$

$\Rightarrow x^2 - 4x \text{ -}6 = 0$

$\Rightarrow x=\frac{-(-4)\pm\sqrt{-(4)^2-4\times1\times(-6)}}{2.1}$ [সুত্র : $x=\frac{-b\pm\sqrt{-b^2-4ac}}{2a}$]

$\Rightarrow x=\frac{4\pm\sqrt{40}}{2} \Rightarrow x=\frac{4\pm2\sqrt{10}}{2} \quad \therefore x=2\pm\sqrt{10}$ **Ans:** $2\pm\sqrt{10}$

6. If $\mathbf{\frac{x+2y}{a+3b}=\frac{y+3x}{a+4b}}$**, then prove that the** $\mathbf{\frac{x}{y}=\frac{a+5b}{2a+5b}}$ *[Bank Asia Ltd.(MTO)-2005(Written)]*

✍**Solution:**

$\frac{x+2y}{a+3b}=\frac{y+3x}{a+4b}$,

$\Rightarrow$ (x+2y)(a+4b)=(y+3x)(a+3b)

$\Rightarrow$ ax+4bx+2ay+8by=ay+3by+3ax+9bx

$\Rightarrow$ ax+4bx–3ax–9bx=ay+3by–2ay–8by

$\Rightarrow$ –2ax–5bx= –ay–5by

$\Rightarrow$ 2ax+5bx=ay+5by [-1 দ্বারা গুণ করে।]

$\Rightarrow$ x(2a+5b)=y(a+5b) $\Rightarrow \frac{x}{y} = \frac{a+5b}{2a+5b}$ **(Proved)**

7. x - এর মান নির্ণয় করুন- $\mathbf{\frac{x-a}{b+c} + \frac{x-b}{c+a} + \frac{x-c}{a+b} = 3}$ *[BKB (Cashier)-2012(Written)]*

✍Solution:

$\frac{x-a}{b+c} + \frac{x-b}{c+a} + \frac{x-c}{a+b} = 3$

$\Rightarrow \frac{x-a}{b+c} - 1 + \frac{x-b}{c+a} - 1 + \frac{x-c}{a+b} - 1 = 0$

$\Rightarrow \frac{x-a-b-c}{b+c} + \frac{x-b-c-a}{c+a} + \frac{x-c-a-b}{a+b} = 0$

$\Rightarrow \frac{x-(a+b+c)}{b+c} + \frac{x-(a+b+c)}{c+a} + \frac{x-(a+b+c)}{a+b} = 0$

$\Rightarrow x-(a+b+c)(\frac{1}{b+c} + \frac{1}{c+a} + \frac{1}{a+b}) = 0$

$\Rightarrow x-(a+b+c) = 0$ [(Since) $(\frac{1}{b+c} + \frac{1}{c+a} + \frac{1}{a+b}) \neq 0$]

$\therefore x = a+b+c$ **Ans. a+b+c**

8. **Suppose 81p+62q = 138 and 62p+81q = 5. Find out the value of p and q.** *[one Bank Ltd. (Sco)-2012(Written)]*

✍Solution:

81p+62q=138…………………(i)
62p+81q = 5…………………..(ii)

Multiply (i) by 62 and (ii) by 81 (৬২ এবং ৮১ এর মধ্যে কোন সংখ্যার মিল না থাকায় ৬২ ও ৮১ দিয়েই গুণ)
Then we get
5022p -5022p + 3844q-6561q = 8556– 405 $\Rightarrow$ –2717q = 8151 $\therefore$ q= –3
Putting the value of q in equation (i)

81p+62(–3) = 138 or, 81p = 138+186 or, p = $\frac{324}{81}$ $\therefore$ p = 4 **Ans. p = 4, q = –3**

🗁Single equation:

9. Find the value of 'a' if (a-3) = $\frac{10}{a}$ *[BB (AD)-2004(Written)]*

✍Solution:

$(a - 3) = \frac{10}{a}$

$\Rightarrow a^2 - 3a = 10$

$\Rightarrow a^2 - 3a - 10 = 0$

$\Rightarrow a^2 - 5a + 2a - 10 = 0$

$\Rightarrow a(a - 5) + 2(a - 5) = 0 \Rightarrow (a - 5)(a + 2) = 0$ So, a = 5 or, a = - 2 **Ans: 5 or -2**

10. If $\frac{a}{b}=\frac{1}{3}$ then $\frac{3a+2b}{3a-2b}$**?** *[Premier bank(TJO): 2013, Shajalal Islami bank(TSO): 2011]*

✍Solution:

Given that, $\frac{a}{b} = \frac{1}{3}$ $\therefore 3a = b$ Now, $\frac{3a+2b}{3a-2b} = \frac{b+2b}{b-2b} = \frac{3b}{-b} = -3$ **Ans: -3**

11. A system of equation is given below:
x + l = 6, x – m = 5; x + p = 4; x – q = 3, What is the value of l + m + p + q? *[United commercial bank(MTO): 2013]*

✍Solution:

যেহেতু যেটার মান বের করতে বলা হয়েছে সেখানে কোনো x নেই তাই এমনভাবে যোগ-বিয়োগ করতে হবে যেন x বাদ যায়। সেক্ষেত্রে প্রথমটি থেকে দ্বিতীয়টি বাদ এবং তৃতীয়টি থেকে চতুর্থটি বাদ দিয়ে যোগ করলে উত্তর পাওয়া যাবে।

Given that, **x + l = 6 .... (i)**
x – m = 5............. (ii)
x + p = 4............. (iii)
x – q = 3............. (iv)
Now, (i) – (ii) + (iii) – (iv) ⇒
$(x + 1) – (x – m) + (x + p) – (x – q) = 6 – 5 + 4 – 3$ $\therefore 1 + m + p + q = 2$ **(Ans.)**

23. Consider the equation y=kx+3, where k is a constant. If y=17 when x=2 , what is the value of y when x = 4. *[Southeast Bank Ltd.(PO)-2012(Written)]*

✍Solution:

দেওয়া আছে , y = kx+3
y=17 ও x=2 হলে
$17=k.2+3 \Rightarrow 2k=17-3 \Rightarrow 2k=14 \therefore k=7$
আবার , x = 4 এবং k = 7 হলে
$Y=7.4+3=28+3=31$ ∴ যখন, x = 4 then y =31 **Ans. 31**

12. $\frac{4}{2x+1}+\frac{9}{3x+2}=\frac{25}{5x+4}$ *[Al-Arafah Islami Bank (MTO)-2017(Written)]& [Modhumoti Bank – (PO)-2018-(Written)]*

✍Solution :

> **ডানের ২৫ কে ভাঙ্গানোর নিয়ম:** বাম পাশের উপরের ৪ দিয়ে ডান পাশের নিচের (x) এর সহগ ৫ কে গুণ করলে ২০ হয় এখন বাম পাশের নিচের (x) এর সহগ ২ দিয়ে ভাগ করলে১০ হয়। তাই ২৫ = ১৫+১০

$$\frac{4}{2x+1}+\frac{9}{3x+2}=\frac{25}{5x+4}$$

$$\Rightarrow \frac{4}{2x+1}+\frac{9}{3x+2}=\frac{15}{5x+4}+\frac{10}{5x+4}$$

$$\Rightarrow \frac{4}{2x+1}-\frac{10}{5x+4}=\frac{15}{5x+4}-\frac{9}{3x+2}$$ **(উপরের জোড়া সংখ্যাগুলোকে একপাশে বেজোড় গুলোকে এক পাশে:)**

$$\Rightarrow \frac{20x+16-20x-10}{(2x+1)(5x+4)}=\frac{45x+30-45x-36}{(5x+4)(3x+2)}$$

$$\Rightarrow \frac{6}{(2x+1)(5x+4)} = \frac{-6}{(5x+4)(3x+2)}$$

$$\Rightarrow \frac{1}{2x+1} = \frac{-1}{3x+2} \Rightarrow 3x + 2 = -2x-1 \Rightarrow 3x + 2x = -1-2 \Rightarrow 5x = -3 \quad \therefore x = -\frac{3}{5} \quad \textbf{Ans: } -\frac{3}{5}$$

13. সরল করঃ $\frac{3x}{4} + \frac{5x-2}{6} = \frac{4x+5}{8}$ *[Karmahgangsthan Bank Ltd.(SO)-2013(Written)]*

✍Solution:

$$\frac{3x}{4} + \frac{5x-2}{6} = \frac{4x+5}{8}$$

$$\Rightarrow \frac{9x+10x-4}{12} = \frac{4x+5}{8}$$

$$\Rightarrow \frac{19x-4}{12} = \frac{4x+5}{8} \Rightarrow \frac{19x-4}{3} = \frac{4x+5}{2} \Rightarrow 38x-8=12x+15 \Rightarrow 26x=23 \quad \therefore x=\frac{23}{26} \textbf{ Ans.}$$

14. Solve the equation : $2\left(\frac{x+3}{x-3}\right)^2 - 7\left(\frac{x+3}{x-3}\right)+6 = 0$ *[BKB- (officer)-2017- (Written)]*

✍Solution: Suppose, $\left(\frac{x+3}{x-3}\right) = a$

So, $2\left(\frac{x+3}{x-3}\right)^2 - 7\left(\frac{x+3}{x-3}\right) + 6 = 0$

$\Rightarrow 2a^2 - 7a + 6 = 0 \Rightarrow 2a^2 - 4a - 3a + 6 = 0 \Rightarrow 2a(a-2) - 3(a-2) = 0 \Rightarrow (a-2)(2a-3) = 0$

Now, either

$a-2 = 0 \Rightarrow a = 2 \Rightarrow \frac{x+3}{x-3} = 2 \Rightarrow 2x - 6 = x + 3 \quad \therefore x = 9.$

or, $2a-3 = 0 \Rightarrow 2a = 3 \quad \therefore a=\frac{3}{2}$

Putting the value of $a=\left(\frac{x+3}{x-3}\right) \Rightarrow \frac{x+3}{x-3}=\frac{3}{2} \Rightarrow 3x - 9 = 2x + 6 \quad \therefore x = 15$ **Ans: 9 or 15**

15. Solve the question : $\frac{1}{x+2} + \frac{1}{x+5} = \frac{1}{x+4} + \frac{1}{x+3}$ *[Social Islami Bank - (PO)-2017 (Written)]*

✍Solution:

$$\frac{1}{x+2} + \frac{1}{x+5} = \frac{1}{x+4} + \frac{1}{x+3}$$

$$\Rightarrow \frac{1}{x+2} - \frac{1}{x+4} = \frac{1}{x+3} - \frac{1}{x+5}$$ [জোড় সংখ্যা গুলোকে একসাথে নেয়া হয়েছে, তবে বেজোড়গুলোকে নিলেও হবে]

$$\Rightarrow \frac{x+4-(x+2)}{(x+2)(x+4)} = \frac{(x+5)-(x+3)}{(x+3)(x+5)}$$

$$\Rightarrow \frac{x+4-x-2}{x^2+4x+2x+8} = \frac{x+5-x-3}{x^2+5x+3x+15}$$

$\Rightarrow \frac{2}{x^2+6x+8}=\frac{2}{x^2+8x+15}$

$\Rightarrow \frac{1}{x^2+6x+8}=\frac{1}{x^2+8x+15}$

$\Rightarrow x^2+8x+15=x^2+6x+8 \Rightarrow x^2+8x-x^2+6x = 8-15 \Rightarrow 2x = -7 \quad \therefore x = -\frac{7}{2}$ **(Ans)**

16. Solve the question : $\frac{1}{x+2}+\frac{1}{x+5}=\frac{1}{x+4}+\frac{1}{x+3}$ *[Social Islami Bank - (PO)-2017 (Written)] & [One Bank (SCO)- 2018-(Written)]*

✍Solution:

$\frac{1}{x+2}+\frac{1}{x+5}=\frac{1}{x+4}+\frac{1}{x+3}$

$\Rightarrow \frac{1}{x+2}-\frac{1}{x+4}=\frac{1}{x+3}-\frac{1}{x+5}$ [জোড় সংখ্যা গুলোকে একসাথে নেয়া হয়েছে, তবে বেজোড়গুলোকে নিলেও হবে]

$\Rightarrow \frac{x+4-(x+2)}{(x+2)(x+4)}=\frac{(x+5)-(x+3)}{(x+3)(x+5)}$

$\Rightarrow \frac{x+4-x-2}{x^2+4x+2x+8}=\frac{x+5-x-3}{x^2+5x+3x+15}$

$\Rightarrow \frac{2}{x^2+6x+8}=\frac{2}{x^2+8x+15}$

$\Rightarrow \frac{1}{x^2+6x+8}=\frac{1}{x^2+8x+15}$

$\Rightarrow x^2+8x+15=x^2+6x+8 \Rightarrow x^2+8x-x^2+6x=8-15 \Rightarrow 2x=-7 \quad \therefore x = -\frac{7}{2}$ **Ans : x=** $-\frac{7}{2}$

17. Solved the equation $\frac{4}{2x+3}+\frac{15}{5x+4}=\frac{35}{7x+6}$ *[One Bank (SCO)-2017-(Written)]*

✍Solution:

$\frac{4}{2x+3}+\frac{15}{5x+4}=\frac{35}{7x+6}$

$\Rightarrow \frac{4}{2x+3}+\frac{15}{5x+4}=\frac{21}{7x+6}+\frac{14}{7x+6}$ *(বাম পাশের ৪ দিয়ে ডানের ৭ কে গুণ করে ২ দিয়ে ভাগ = ১৪)*

$\Rightarrow \frac{4}{2x+3}-\frac{14}{7x+6}=\frac{21}{7x+6}-\frac{15}{5x+4}$

$\Rightarrow \frac{28x+24-28x-42}{(7x+6)(2x+3)}=\frac{105x+84-105x-90}{(5x+4)(7x+6)}$

$\Rightarrow \frac{-18}{2x+3}=\frac{-6}{5x+4}$

$\Rightarrow \frac{3}{2x+3}=\frac{1}{5x+4} \Rightarrow 15x+12 = 2x+3 \Rightarrow 13x = -9 \therefore x= -\frac{9}{13}$ **Ans.** $-\frac{9}{13}$

18. Solve the equation: $\frac{1}{2x-5}+\frac{1}{2x-11}=\frac{1}{2x-7}+\frac{1}{2x-9}$ *[Modhumoti Bank Ltd. (PO)-2017]*

Solution:

$$\frac{1}{2x-5}+\frac{1}{2x-11}=\frac{1}{2x-7}+\frac{1}{2x-9}$$
$$\Rightarrow \frac{1}{2x-5}-\frac{1}{2x-9}=\frac{1}{2x-7}-\frac{1}{2x-11}$$
$$\Rightarrow \frac{2x-9-(2x-5)}{(2x-5)(2x-9)}=\frac{2x-11-(2x-7)}{(2x-7)(2x-11)}$$
$$\Rightarrow \frac{2x-9-2x+5}{4x^2-18x-10x+45}=\frac{2x-11-2x+7}{4x^2-22x-14x+77}$$
$$\Rightarrow \frac{-4}{4x^2-28x+45}=\frac{-4}{4x^2-36x+77}$$
$$\Rightarrow \frac{1}{4x^2-28x+45}=\frac{1}{4x^2-36x+77}$$ [উভয় পক্ষকে - 4 দিয়ে ভাগ করে]
$$\Rightarrow 4x^2-28x+45 = 4x^2-36x+77$$
$$\Rightarrow 4x^2-28x-4x^2+36x=77-45 \Rightarrow 8x=32 \quad \therefore x=4$$ **Ans: 4**

19. Solve the equation: $\frac{x-4}{x-1}+\frac{x-7}{x-3}+\frac{x-2}{x-9}=3$ *[Bank Asia Ltd.(MT)-2017]*

Solution:

$$\frac{x-4}{x-1}+\frac{x-7}{x-3}+\frac{x-2}{x-9}=3$$
$$\Rightarrow \frac{x-4}{x-1}+\frac{x-7}{x-3}=3-\frac{x-2}{x-9}$$
$$\Rightarrow \frac{(x-4)(x-3)+(x-7)(x-1)}{(x-1)(x-3)}=\frac{3(x-9)-(x-2)}{x-9}$$
$$\Rightarrow \frac{x^2-3x-4x+12+x^2-x-7x+7}{x^2-3x-x+3}=\frac{3x-27-x+2}{x-9}$$
$$\Rightarrow \frac{2x^2-15x+19}{x^2-4x+3}=\frac{2x-25}{x-9}$$
$$\Rightarrow (2x^2-15x+19)(x-9)=(x^2-4x+3)(2x-25)$$
$$\Rightarrow 2x^3-15x^2+19x-18x^2+135x-171=2x^3-8x^2+6x-25x^2+100x-75$$
$$\Rightarrow -33x^2+154x-171 = -33x^2+106x-75$$
$$\Rightarrow -33x^2+33x^2+154x-106x = -75+171 \Rightarrow 48x=96 \Rightarrow x=\frac{96}{48} \quad \therefore x=2$$ **Ans. 2**

20. Solve the equation: $\frac{3}{x+1}+\frac{6}{2x+1}=\frac{18}{3x+1}$. *[Janata Bank Ltd.(EO-FA)-2015- (Written)]*

Solution:

$$\frac{3}{x+1}+\frac{6}{2x+1}=\frac{18}{3x+1}$$

> **ভাঙ্গানোর নিয়ম:** বাম পাশের উপরের ৩ দিয়ে ডান পাশের নিচের (x) এর সহগ ৩ কে গুণ করলে ৯ হয় এখন বাম পাশের নিচের (x) এর সহগ ১ দিয়ে ভাগ করলে ৯ হয়। তাই ১৮ = ৯+৯

$\Rightarrow \frac{3}{x+1}+\frac{6}{2x+1}=\frac{9}{3x+1}+\frac{9}{3x+1}$

$\Rightarrow \frac{3}{x+1}-\frac{9}{3x+1}=\frac{9}{3x+1}-\frac{6}{2x+1}$

$\Rightarrow \frac{3(3x+1)-9(x+1)}{(x+1)(3x+1)}=\frac{9(2x+1)-6(3x+1)}{(3x+1)(2x+1)}$

$\Rightarrow \frac{9x+3-9x-9}{(x+1)(3x+1)}=\frac{18x+9-18x-6}{(3x+1)(2x+1)}$

$\Rightarrow \frac{-6}{(x+1)(3x+1)}=\frac{3}{(3x+1)(2x+1)}$

$\Rightarrow \frac{-2}{x+1}=\frac{1}{2x+1}$ *[Dividing both side by* $\frac{3}{3x+1}$*]*

$\Rightarrow x+1=-4x-2 \Rightarrow 5x=3 \therefore x=-\frac{3}{5}$ **Ans.** $-\frac{3}{5}$

21. Solve the equation: $\frac{8}{2x-1}+\frac{9}{3x-1}=\frac{7}{x+1}$ *[Bank Asia-2016-(Writte)n] & [National Bank -(P0)- 2017- (Written)]* ***[Board book – Class-08]***

✍Solution:

$\frac{8}{2x-1}+\frac{9}{3x-1}=\frac{7}{x+1}$

> ◆ভাঙ্গানোর **নিয়ম:** বাম পাশের উপরের ৮ দিয়ে ডান পাশের নিচের (x) এর সহগ ১ কে গুণ করলে ৮ হয় এখন বাম পাশের নিচের (x) এর সহগ ২ দিয়ে ভাগ করলে ৪ হয়। তাই উপরের ৭ = ৪+৩

Or, $\frac{8}{2x-1}+\frac{9}{3x-1}=\frac{4}{x+1}+\frac{3}{x+1}$

Or, $\frac{8}{(2x-1)}-\frac{4}{(x+1)}=\frac{3}{(x+1)}-\frac{9}{(3x-1)}$ (উপরের জোড় সংখ্যাগুলোকে একপাশে বিজোড়গুলোকে আরেকপাশে)

Or, $\frac{8x+8-8x+4}{(2x-1)(x+1)}=\frac{9x-3-9x-9}{(x+1)(3x-1)}$

Or, $\frac{12}{(2x-1)(x+1)}=\frac{-12}{(x+1)(3x-1)}$

Or, $\frac{1}{(2x-1)}=\frac{-1}{(3x-1)}$ *[Dividing both side by* $\frac{12}{x+1}$*]*

Or, 3x-1= -2x+1 Or, 5x = 2 $\therefore x=\frac{2}{5}$ **Ans:** $\frac{2}{5}$

22. Simplify: $\frac{5x+2}{x^2-x-20}+\frac{2x-1}{x^2-4x-5}$ *[Agrani Bank (SO-Auditor)-2018-(Written)]*

✍Solotion:

$\frac{5x+2}{x^2-x-20}+\frac{2x-1}{x^2-4x-5}$

$\Rightarrow \frac{5x+2}{(x^2-5x+4x-20)}+\frac{2x-1}{x^2-5x+x-5}$

$\Rightarrow \frac{5x+2}{x(x-5)+4(x-5)}+\frac{2x-1}{x(x-5)+1(x-5)}$

$\Rightarrow \frac{5x+2}{(x-5)(x+4)}+\frac{2x-1}{(x-5)(x+1)}$

$\Rightarrow \frac{(5x+2)(x+1)+(2x-1)(x+4)}{(x-5)(x+4)(x+1)}$

$\Rightarrow \frac{5x^2+5x+2x+2+2x^2+8x-x-4}{(x-5)(x+4)(x+1)}$ $\Rightarrow \mathbf{\frac{7x^2+14x-2}{(x-5)(x+4)(x+1)}}$ **(Ans)**

23. সমাধান কর : $\mathbf{\frac{2}{2x-1}+\frac{3}{3x-1}=\frac{8}{4x+1}}$ ***[Board book – Class-08]***

✍সমাধান :

$\frac{2}{2x-1}+\frac{3}{3x-1}=\frac{8}{4x+1}$

$\Rightarrow \frac{2}{2x-1}+\frac{3}{3x-1}=\frac{4}{4x+1}+\frac{4}{4x+1}$

$\Rightarrow \frac{2}{2x-1}-\frac{4}{4x+1}=\frac{4}{4x+1}-\frac{3}{3x-1}$

$\Rightarrow \frac{8x+2-8x+4}{(2x-1)(4x+1)}=\frac{12x-4-12x-3}{(4x+1)(3x-1)}$

$\Rightarrow \frac{6}{(2x-1)(4x+1)}=\frac{-7}{(4x+1)(3x-1)}$

$\Rightarrow \frac{6}{(2x-1)}=\frac{-7}{(3x-1)}$ [$(4x+1)$ দ্বারা উভয় পক্ষকে গুণ করে]

$\Rightarrow$18x-6 = -14x+7 [আড়গুণ করে]

$\Rightarrow$18x+14x = 7+6 [পক্ষান্তর করে] $\Rightarrow$32x= 13 $\therefore x = \frac{13}{32}$ $\therefore$ নির্ণেয় সমাধান : $x = \frac{13}{32}$ **উত্তর :** $\mathbf{\frac{13}{32}}$

> ◆ভাঙ্গানোর **নিয়ম:** বাম পাশের উপরের ২ দিয়ে ডান পাশের নিচের (x) এর সহগ ৪ কে গুণ করলে ৮ হয় এখন বাম পাশের নিচের (x) এর সহগ ২ দিয়ে ভাগ করলে ৪ হয়। তাই উপরের ৮ = ৪+৪

🕮Practice from Board Book:

24. Solve: $\frac{3x+7}{4}+\frac{5x-4}{7}=x+3\frac{1}{2}$ *[Board book – Class-07]*

$\Rightarrow \frac{3x+7}{4}+\frac{5x-4}{7}-x=\frac{7}{2}$ [by transposing]

$\Rightarrow \frac{21x+49+20x-16-28x}{28}=\frac{7}{2}$ [by distributive law]

$\Rightarrow \frac{13x+33}{28}=\frac{7}{2} \Rightarrow 13x + 33 = 98 \Rightarrow 13x = 98 - 33 \Rightarrow 13x = 65 \therefore x = 5$ **Ans: 5**

25. সমাধান কর : $\frac{5}{x+2}+\frac{7}{x-3}=\frac{12}{x-1}$ *[Board book – Class-08]*

সমাধান :

$\frac{5}{x+2}+\frac{7}{x-3}=\frac{12}{x-1}$

$\Rightarrow \frac{5}{x+2}+\frac{7}{x-3}=\frac{5}{x-1}+\frac{7}{x-1}$

$\Rightarrow \frac{5}{x+2}-\frac{5}{x-1}=\frac{7}{x-1}-\frac{7}{x-3}$ [পক্ষান্তর করে]

$\Rightarrow \frac{5x-5-5x-10}{(x+2)(x-1)}=\frac{7x-21-7x+7}{(x-1)(x-3)}$

$\Rightarrow \frac{-15}{(x+2)(x-1)}=\frac{-14}{(x-1)(x-3)}$

$\Rightarrow \frac{15}{(x+2)}=\frac{14}{(x-3)}$ [উভয় পক্ষকে $\frac{-1}{x-1}$ দ্বারা ভাগ করে]

$\Rightarrow 15x - 45 = 14x+28$ [আড়গুণ করে]

$\Rightarrow 15x-14x = 28+45 \therefore x = 73 \therefore$ নির্ণেয় সমাধান : $x = 73$ **উত্তর : 73**

26. সমাধান কর : $\frac{6}{x+1}+\frac{5}{x+5}=\frac{11}{x+3}$ *[Board book – Class-08]*

সমাধান : $\frac{6}{x+1}+\frac{5}{x+5}=\frac{11}{x+3}$

$\Rightarrow \frac{6}{x+1}+\frac{5}{x+5}=\frac{6}{x+3}+\frac{5}{x+3}$

$\Rightarrow \frac{6}{x+1}-\frac{6}{x+3}=\frac{5}{x+3}-\frac{5}{x+5}$ [পক্ষান্তর করে]

$\Rightarrow \frac{6x+18-6x-6}{(x+1)(x+3)}=\frac{5x+25-5x-15}{(x+3)(x+5)}$

$\Rightarrow \frac{12}{(x+1)(x+3)}=\frac{10}{(x+3)(x+5)}$

$\Rightarrow \frac{12}{(x+1)}=\frac{10}{(x+5)}$ [উভয় পক্ষকে $(x+3)$ গুণ করে]

$\Rightarrow 12x+60 = 10x+10 \Rightarrow 2x = -50 \therefore x = -25$ নির্ণেয় সমাধান : $x = -25$ **উত্তর : x = -25**

27. সমাধান কর : $\frac{10}{2x-5}+\frac{1}{x+5}=\frac{18}{3x-5}$ *[Board book – Class-08]+ [Premier Bank (TJO)-2018-(Written)]*

সমাধান :

$\frac{10}{2x-5}+\frac{1}{x+5}=\frac{18}{3x-5}$ (বাম পাশের উপরের ১০ দিয়ে ডানের ৩ কে গুণ করে বাম পাশে ২দিয়ে ভাগ করে ১৫+৩)

$\Rightarrow \frac{10}{2x-5}+\frac{1}{x+5}=\frac{15}{3x-5}+\frac{3}{3x-5}$

$\Rightarrow \frac{10}{2x-5}-\frac{15}{3x-5}=\frac{3}{3x-5}-\frac{1}{x+5}$ [পক্ষান্তর করে]

$\Rightarrow \frac{30x-50-30x+75}{(2x-5)(3x-5)}=\frac{3x+15-3x+5}{(3x-5)(x+5)}$

$\Rightarrow \frac{25}{(2x-5)(3x-5)}=\frac{20}{(3x-5)(x+5)}$

$\Rightarrow \frac{5}{(2x-5)}=\frac{4}{(x+5)}$ [উভয় পক্ষকে $\frac{5}{3x+5}$ দ্বারা ভাগ করে]

$\Rightarrow 8x-20 = 5x+25 \Rightarrow 8x-5x = 25+20 \Rightarrow 3x = 45 \therefore x = \frac{45}{3} = 15$ নির্ণেয় সমাধান : x= 15 (**উত্তর)**

Double Equation:

28. সমাধান করুন: $x+\frac{1}{y}=\frac{3}{2}$ এবং $x+\frac{1}{x}=3$ *[Ministry of Food (AP)-2020 (Written)]*

সমাধান: দেওয়া আছে, $x+\frac{1}{y}=\frac{3}{2}$ ------------- (i) এবং $x+\frac{1}{x}=3$----------- (ii)

From equation (ii) $y = 3 - \frac{1}{x} \Rightarrow y = \frac{3x-1}{x}$ ----------- (iii) $\therefore \frac{1}{y}=\frac{x}{3x-1}$

এখন $\frac{1}{y}$ এর মান (i) নং সমীকরণে বসাই

$\Rightarrow x+\frac{x}{3x-1}=\frac{3}{2} \Rightarrow \frac{3x^2-x+x}{3x-1}=\frac{3}{2} \Rightarrow \frac{3x^2}{3x-1}=\frac{3}{2} \Rightarrow \frac{x^2}{3x-1}=\frac{1}{2}$

$\Rightarrow 2x^2 = 3x-1$

$\Rightarrow 2x^2-3x+1=0 \Rightarrow 2x^2-2x-x+1=0 \Rightarrow 2x(x-1)-1(x-1)=0 \Rightarrow (x-1)(2x-1)=0$

হয়, $x-1=0 \Rightarrow x=1$ অথবা, $2x-1=0 \Rightarrow x=\frac{1}{2}$

x = 1 হলে (iii) নং সমীকরণ থেকে পাই, $y=\frac{3.1-1}{1}=2$ আবার, $x=\frac{1}{2}$ হলে,

$\therefore y = \dfrac{3.\frac{1}{2} - 1}{\frac{1}{2}} = \dfrac{\frac{3-2}{2}}{\frac{1}{2}} = 1$ ∴ নির্ণয় সমাধান (x, y) = (1, 2) $(\frac{1}{2}, 1)$

29. Solve: $\mathbf{\frac{x}{2} + \frac{6}{y} = 9}$, $\mathbf{\frac{x}{3} + \frac{2}{y} = 4}$ *[BDBL-(SO)-2018- (Written)]*

✍**Solution:** $\frac{x}{2} + \frac{6}{y} = 9 \Rightarrow \frac{xy+12}{2y} = 9$ ∴ xy+12 = 18y …..(i)

And $\frac{x}{3} + \frac{2}{y} = 4 \Rightarrow \frac{xy+6}{3y} = 4$ ∴ xy+6 = 12y …..(ii)

Now (i) - (ii) we have

xy+12 = 18y

<u>xy+6 = 12y</u>

6 = 6y ∴ y = 1

Putting the value of 'y' in (ii), (x×1) + 6= 12× 1 ⇒ x = 12-6 ∴ x = 6 **∴(x,y) = (6, 1)**

30. If $\mathbf{\frac{x}{2}} + \mathbf{\frac{y}{3}}$ = 1 and $\mathbf{\frac{x}{3}} + \mathbf{\frac{y}{2}}$ = 1 **Find the value of x and y** *[Agrani Bank (Off-Cash)-2018-(Written)]*

✍**Solution:**

Here, $\mathbf{\frac{x}{2}} + \mathbf{\frac{y}{3}}$ = 1 or, $\frac{3x+2y}{6} = 1$ ∴ 3x+2y = 6……….(i)

Again, $\mathbf{\frac{x}{3}} + \mathbf{\frac{y}{2}}$ = 1 or, $\frac{2x+3y}{6} = 1$ ∴ 2x+3y = 6……….(ii)

By (i) ×3 and (ii) ×2 then subtracting (ii) from (i) We get,

9x+6y = 18

<u>4x+6y = 12</u>

5x = 6 ∴ **x=** $\frac{6}{5}$ Putting the Value of x in (i)

$3\times\frac{6}{5} + 2y = 6 \Rightarrow 2y = 6 - \frac{18}{5} \Rightarrow 2y = \frac{12}{5}$ ∴ **y =** $\frac{6}{5}$ So, (x,y) = $(\frac{6}{5}, \frac{6}{5})$ **Ans.** $\mathbf{(\frac{6}{5}, \frac{6}{5})}$

31. $\mathbf{\frac{x}{3} + \frac{y}{2} = 6}$ এবং $\mathbf{\frac{x}{2} + \frac{y}{4} = 5}$ ***[Board book – Class-08]***

✍**সমাধান :** $\mathbf{\frac{x}{3} + \frac{y}{2} = 6}$ ----------------- (i) এবং $\mathbf{\frac{x}{2} + \frac{y}{4}}$ = **5** --------------- (ii)

প্রথম সমীকরণকে 3 দিয়ে এবং দ্বিতীয় সমীকরণকে 2 দ্বারা গুণ করে, বিয়োগ করে পাই

$\frac{3y}{2} - \frac{y}{2} = 8 \Rightarrow \frac{3y-y}{2} = 8 \Rightarrow \frac{2y}{2} = 8 \Rightarrow 2y = 16$ ∴ y = 8

প্রথম সমীকরণে y = 8 বসিয়ে পাই $\frac{x}{3}+\frac{8}{2}=6$ $\Rightarrow \frac{x}{3}+4=6 \Rightarrow \frac{x+12}{3}=6$ $\Rightarrow$ x+12 = 18 $\therefore$ **x = 6**

নির্ণেয় সমাধান : x = 6, y = 8 **উত্তর : (x,y) = (6,8)**

32. $\mathbf{\frac{x}{2}+\frac{y}{3}=3}$ এবং $\mathbf{x+\frac{y}{6}=3}$ *[Board book – Class-08]*

✍সমাধান : $\mathbf{\frac{x}{2}+\frac{y}{3}=3}$ ---------------- (i) এবং $\mathbf{x+\frac{y}{6}=3}$ ----------------- (ii)

প্রথম সমীকরণকে 2 দ্বারা গুণ করার পর বিয়োগ করে পাই

$\frac{2y}{3}-\frac{y}{6}=3$ $\Rightarrow \frac{4y-y}{6}=3$ $\Rightarrow \frac{3y}{6}=3$ $\Rightarrow$3y= 18 $\therefore$ **y = 6**

প্রথম সমীকরণকে y = 6 বসিয়ে পাই $\frac{x}{2}+\frac{6}{3}=3 \Rightarrow \frac{x}{2}+2=3 \Rightarrow \frac{x}{2}=1$ $\therefore$ **x = 2** **উত্তর : (x.y) = (2,6)**

33. $\mathbf{\frac{x}{3}-\frac{2}{y}=1}$ এবং $\mathbf{\frac{x}{4}+\frac{3}{y}=3}$ *[Board book – Class-08]*

✍**সমাধান :** $\mathbf{\frac{x}{3}-\frac{2}{y}=1}$ ---------------- (i) এবং $\mathbf{\frac{x}{4}+\frac{3}{y}=3}$ ---------------- (ii)

প্রথম সমীকরণকে 3 দিয়ে এবং দ্বিতীয় সমীকরণকে 2 দ্বারা গুন যোগ করে পাই

$x+\frac{x}{2}=9 \Rightarrow \frac{2x+x}{2}=9 \Rightarrow \frac{3x}{2}=9$ $\Rightarrow$3x = 18 $\therefore$ x = 6

প্রথম সমীকরণে x = 6 বসিয়ে পাই $\frac{6}{3}-\frac{2}{y}=1 \Rightarrow 2-\frac{2}{y}=1$ $\Rightarrow 2-1=\frac{2}{y}$ $\Rightarrow 1=\frac{2}{y}$ $\therefore$ **y =2**

নির্ণেয় সমাধান : x = 6, y = 2 **উত্তর : (x.y) = (6,2)**

34. Solve: $\mathbf{x^2 - yx = 7, y^2 + xy = 30}$ *[Sonali Bank (SO) – 2018-(Written)]*

✍**Solution:** Given, $x^2 - yx = 7$, Or, $x(x-y) = 7$ Or, $x-y = \frac{7}{x}$

Or, $y = x-\frac{7}{x}$ ....(i) Again, $y^2+xy = 30$……..(ii)

Now,

$x^2-xy=7$

$\underline{y^2+xy=30}$

$x^2+y^2=37$ [by adding]

Or, $x^2+\left(x-\frac{7}{x}\right)^2 = 37$ Or, $x^2+x^2-2.x.\frac{7}{x}+\left(\frac{7}{x}\right)^2 = 37$

Or, $2x^2-14+\frac{49}{x^2} = 37$ Or, $\frac{2x^4+49}{x^2}=51$ Or, $2x^4+49 = 51x^2$

Or, $2x^4-51x^2+49 = 0$ Or, $2x^4-49x^2-2x^2+49 = 0$

Or, $x^2(2x^2-49)-1(2x^2-49) = 0$ Or, $(2x^2-49)(x^2-1) = 0$

$\therefore 2x^2 - 49 = 0$

Or, $x^2 = \frac{49}{2}$ $\therefore x = \frac{7}{\sqrt{2}}$

When $x = \frac{7}{\sqrt{2}}$,

$Y = x - \frac{7}{x}$

$= \frac{7}{\sqrt{2}} - \frac{7}{\frac{7}{\sqrt{2}}}$

$= \frac{7}{\sqrt{2}} - \sqrt{2} = \frac{7-2}{\sqrt{2}} = \frac{5}{\sqrt{2}}$

$\therefore x^2 - 1 = 0$

Or, $x^2 = 1$ $\therefore x = 1$

When x = 1,

$Y = x - \frac{7}{x} = 1 - \frac{7}{1} = \frac{1-7}{1} = -6$

$\therefore (x,y) = \left(\frac{7}{\sqrt{2}}, \frac{5}{\sqrt{2}}\right)$ Or (1, - 6)

Ans: $(x,y) = \left(\frac{7}{\sqrt{2}}, \frac{5}{\sqrt{2}}\right)$ Or (1, - 6)

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