

Progressions

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①

~~50 calls~~

②

1, 2, 4, 8, 16

107 57 20 8 2 2

~~107 calls~~

$$\boxed{1 + 3 + 5 + 7 + \dots + 15}$$

$$\frac{4}{2} = 2 + 1$$

$$\begin{aligned} \underline{\underline{n}} &= \frac{15 - \underline{1}}{2} + 1 = \frac{z - a}{d} \rightarrow \text{th term} \\ &= \textcircled{8} \end{aligned}$$

$$\begin{aligned} \text{Sum of AP} &= \frac{\underline{15} + 1}{2} \times \underline{\underline{n}} = \frac{z + a}{2} \times \underline{n} \\ &= \frac{16}{2} \times 8 = \textcircled{\underline{\underline{64}}} \end{aligned}$$

$$\checkmark \text{Sum (AP)} = \frac{z+a}{2} \times \underline{\underline{n}} \quad [n = \text{total number}]$$

$$\underline{\underline{n}} = \frac{z-a}{d} + 1 = \frac{z-a+d}{d}$$

$$\text{Sum} = \frac{z+a}{2} \times \frac{z-a+d}{d}$$
$$\checkmark = \frac{(z+a)(z-a+d)}{2d} \quad \checkmark \checkmark$$

$$\text{Sum} = \frac{(z+a)(z-a+d)}{2d}$$

$$\left[\begin{array}{l} z = \text{w} \\ a = \text{1st} \\ d = \text{c. r} \end{array} \right]$$

$$= \frac{(15+1)(15-1+2)}{2 \cdot 2} = \frac{16 \times 16}{4}$$

$$= 64$$

$$Sum = \overset{w}{\frac{n}{2}} \left\{ 2a + (n-1)d \right\}$$

$$\frac{1+3+5+\dots+15=?}{\downarrow}$$

$$1 + \textcircled{3} + 5 + \textcircled{7} + \dots$$

$$\textcircled{2} \quad \textcircled{2} \quad \textcircled{2}$$

1st

$$\rightarrow 1 + \underline{0 \cdot 2}$$

$$\underline{n^{\text{th}}} \rightarrow \textcircled{1} + \textcircled{(n-1)} \cdot \textcircled{2}$$

2nd

$$\rightarrow 1 + \underline{1 \cdot 2}$$

$$= a + (n-1)d$$

3rd

$$\rightarrow 1 + \textcircled{2 \cdot 2}$$

4th

$$\rightarrow 1 + \underline{\underline{3 \cdot 2}}$$

$$\text{nth term} = a + (n-1)d$$

$$a + (a+d), (a+2d), (a+3d) \dots a+(n-1)d$$

$$\text{Sum} = \frac{a + (a + (n-1)d)}{2} \times n$$

$$= \frac{n}{2} \{ 2a + (n-1)d \}$$

$$u_n = \frac{z - a}{d} + 1 \quad \text{nth term} = a + (n-1)d$$

$$Sum = \frac{z + a}{2} \times n$$

$$\checkmark Sum = \frac{(z+a)(z-a+d)}{2d}$$

1+3+5+...
22

$$\checkmark Sum = \left[\frac{n}{2} \{ 2a + (n-1)d \} \right]$$

$$4 + 8 + 16 + 32 + \dots$$

n th term = $a r^{n-1}$ (G.P) $r \Rightarrow$ common ratio

$$\text{Sum (G.P)} = \frac{a(r^n - 1)}{r - 1} \quad [r > 1]$$

$$\text{ii} = \frac{a(1 - r^n)}{1 - r} \quad [r < 1]$$

$$3^v = 9$$
$$\boxed{1+2} + 3 + 5 + \dots + n = \frac{n(n+1)}{2}$$

$$1^v + 2^v + 3^v + 5^v + \dots + n^v = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left\{ \frac{n(n+1)}{2} \right\}^2$$

$$\begin{aligned} \underline{S} &= \textcircled{1} + 2 + 3 + \textcircled{4} \\ \underline{S} &= \textcircled{4} + 3 + 2 + \textcircled{1} \end{aligned}$$

$$1+2+3+4 = \frac{4(4+1)}{2}$$

$$\textcircled{4S} = \underline{5} + \underline{5} + \underline{5} + \underline{5}$$

$$2S = 4 + 5$$

$$S = \frac{4 + \textcircled{5}}{2} = \frac{4 + (4+1)}{2}$$

Q1: What is the sum of the numbers from 1 to 50? [Bank Asia (Teller)-2023]

$$1 + 2 + 3 + \dots + 50 = \frac{25 \cancel{50} (50+1)}{2} = 25 \times 51 = \underline{1275}$$

Q2: What is the sum of the numbers from 1 to 100?

$$\frac{50 \times (100 + 1)}{2}$$

$$= 50 \times 101$$

$$= 5050$$

Q3: Set A contains all the even numbers between 2 and 50 inclusive. Set B contains all the even numbers between 102 and 150 inclusive. What is the difference between the sum of elements of set B and that of set A? [IBBL (TAO)-21]

Set A = { 2, 4, 6, ... 50 } ✓

Set B = { 102, 104, 106, ... 150 } ✓

(*) Sum =
$$\frac{(2+a)(2-a+d)}{2d} = \frac{52 \times 50}{2+2} = 650$$

102, 104, 106 - - - 150

$$2 \frac{(150 + 102)(150 - 102 + 2)}{2 + 2}$$

$$= \frac{6^3 \cdot 252 \times 50}{4} = \underline{\underline{3150}}$$

$$\underline{\underline{3150 - 650 = 2500}}$$

Q4: Which term of the series $72 + 63 + 54 + \dots$ is zero? [Jamuna Bank (MTO)-23]

-9
 -9

$4 + 8 + 16 + \dots$
Common difference

$$n^{\text{th}} \text{ term} = 0$$

$$a + (n-1)d = 0$$

$$= \frac{63 - 72}{-9}$$

$$72 + (n-1)(-9) = 0$$

$$9(n-1) = 72$$

$$n-1 = 8 \Rightarrow \underline{n = 9}$$

Common ratio

$$r = \frac{8}{4} = 2$$

Q5: The sum of first 17 terms of the series 5, 9, 13, 17.... [Sonal Bank (SO)-18]

$$\frac{n}{2} \{ 2a + (n-1)d \}$$
$$= \frac{17}{2} \{ 2 \cdot 5 + (17-1)4 \}$$
$$= \frac{17}{2} \times (10 + 64) = \frac{17}{2} \times 74$$
$$= \underline{629}$$

~~$$\frac{(7+a)(7-a+d)}{2d}$$~~

Q6: How many terms of Arithmetic Progression (A.P) 21, 18, 15, 12,... must be taken to give the sum zero? [Sonal Bank (SO)-18]

$$\frac{n}{2} \{ 2a + (n-1)d \} = 0$$

$$\frac{n}{2} \{ 2 \times 21 + (n-1)(-3) \} = 0$$

$$\frac{n}{2} (42 - 3n + 3) = 0$$

$$\textcircled{2} \quad n(45 - 3n) = 0$$

$$45 - 3n = 0$$

$$3n = 45$$

$$\textcircled{n = 15}$$

Q7: If $a + 1$, $2a + 1$, $4a - 1$ are in Arithmetic Progression, then the value of 'a' is- [Sonal Bank (Cash)-18]

$$\underline{a+1}, \underline{2a+1},$$

$$\textcircled{4a-1}$$

$$\underline{3, 5, 7}$$

$$(2a+1) - (a+1) = (4a-1) - (2a+1)$$

$$2a+1 - a - 1 = 4a - 1 - 2a - 1$$

$$\Rightarrow a = 2a - 2$$

$$\Rightarrow 2a - a = 2 \quad \Rightarrow a = 2$$

$$\underline{3, 5, 7, 9, 11}$$

$$7 - 5 = 2$$

$$9 - 7 = 2$$

$$\textcircled{7-5} = \textcircled{9-7}$$

Q8: 4th term, 10th term and nth term of an A.P. are 7, 25 and 58 respectively. Find 1st term, common difference and the value of n.

Let, 1st term = a , common difference = d

$$a + 3d = 7 \quad \dots \textcircled{1}$$

$$a + 9d = 25 \quad \dots \textcircled{2}$$

$$\textcircled{2} - \textcircled{1} \text{ we get, } 6d = 18 \Rightarrow \underline{d = 3}$$

putting the value of d in equation (1)

$$a + 3 \cdot 3 = 7 \Rightarrow a = 7 - 9 = \underline{-2}$$

Here,

$$n\text{th term} = 58$$

$$a + (n-1)d = 58$$

$$-2 + (n-1)3 = 58$$

$$n-1 = \frac{60}{3} = 20$$

$$n = 20 + 1 = \underline{\underline{21}}$$

Q9: The sum of 1st three numbers in an Arithmetic Progression is 30.

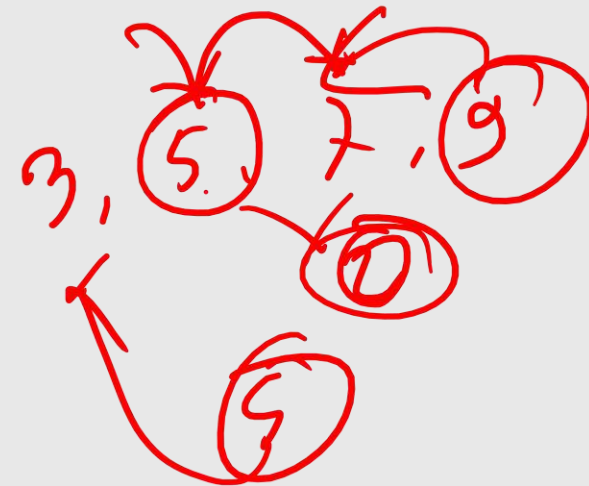
The sum of their squares is 318. Find the numbers. [BKB (Cash) 18]

$a+d =$

Let, the 2nd term be a
common difference d

\therefore 1st term = $a-d$

2nd term = $a+d$



$5+2$

Plus, $a - d + r + a + d = 30 \Rightarrow 3a = 30 \Rightarrow \underline{a = 10}$

$$(10 - d)^{\checkmark} + 10^{\checkmark} + (10 + d)^{\checkmark} = 318$$

$$\underline{100} - \cancel{2 \cdot 10 \cdot d} + \underline{d^{\checkmark}} + \underline{100} + \underline{100} + \cancel{2 \cdot 10 \cdot d} + \underline{d^{\checkmark}} = 318$$

$$(a+b)^{\checkmark} + (a-b)^{\checkmark}$$

$$= 2(a+b)^{\checkmark}$$

$$\underline{300} + 2d^{\checkmark} = \underline{318}$$

$$2d^{\checkmark} = 18$$

$$d^{\checkmark} = 9$$

$$d = \pm 3$$

$$10 - 3 = 7$$

$$10 + 3 = 13$$

$$\underline{7, 10, 13}$$

$$\sqrt{9} = \underline{+3}$$

$$9^{\infty} \text{ square } = \pm 3$$

$$\underline{\sqrt{9} = 3}$$

13, 10, 7

Q10: Three numbers x , y and z are in an A.P. (Arithmetic Progression) and their sum is 30. Also the sum of their squares is 308. Find the numbers. [BB AD 18]

H-w

Ans

8, 10, 12

22, 10, 8

Q11: Prove that the sum of the odd numbers from 1 to 125 inclusive is equal to the sum of the odd numbers from 169 to 209 inclusive. [Agrani (Cash) 18]

Ans -

$$1 + 3 + 5 + 7 + \dots + 125 = 169 + 171 + 173 + \dots + 209$$

$$\frac{(125 + 1)(125 - 1 + 2)}{2 \times 2}$$

$$\frac{126 \times 126}{2 \times 2} = 21 \times 189$$

$$\frac{(209 + 169)(209 - 169 + 2)}{2 \times 2}$$

$$\frac{378 \times 42}{2 \times 2} = 21 \times 189$$

$$1+3+5+\dots+125 = \boxed{}$$

$$169+171+173+\dots+209 = \boxed{}$$

Here, 1st series = $1+3+5+7+\dots+125$

2nd series = $169+171+\dots+209$

Q12: The first term of a geometric progression is 7 and the fifth term is 567.

All the terms are positive. Find the common ratio and the sum of the first 6 terms of the progression.

Let, 1st term, $a = 7$

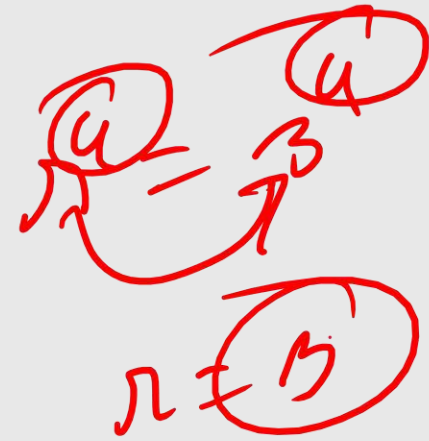
Common ratio = r

5th term = $a r^{5-1}$

$$567 = a r^4$$

$$\Rightarrow \frac{567}{7} = r^4 = 81 = \underline{\underline{3^4}}$$

$$r = 3$$



$$\text{Sum of } n^{\text{th}} \text{ term} = \frac{a(r^n - 1)}{r - 1}$$

$$\therefore \text{6th term} = \frac{7(3^6 - 1)}{3 - 1} = \frac{7}{2}(3^6 - 1) \dots$$

$$81 \times 3 \Rightarrow 243 \times 3 \\ = 729$$

$$= \frac{7}{2} \times 728$$

$$= 7 \times 364$$

$$= \underline{\underline{2548}}$$

Q13: A man agrees to refund the loan of Tk. 2500 in some parts. Each part is Tk. 2 more than the previous part. If the first part is Tk. 1, in how many parts will the man be able to refund that amount?

Given that, 1st installment = Tk. 1
2nd " " = Tk. 3
3rd " " = Tk. 5

$$\boxed{1 + 3 + 5 + \dots} = 2500$$

It's an arithmetic series.

1st term = 1 common difference = 2

We know.

$$\text{Sum of } n\text{th term} = \frac{n}{2} \{2a + (n-1)d\} = 2500$$

$$\Rightarrow \frac{n}{2} \{2 + (n-1)2\} = 2500$$

$$\Rightarrow n(2 + 2n - 2) = 5000$$

$$\Rightarrow n + 2n = 5000$$

$$\Rightarrow n\sqrt{3} = 2500$$

$$\Rightarrow n = \sqrt{2500}$$

$$= 50$$

$$V_{\text{out}} = \frac{V_{\text{in}} - V_{\text{ref}}}{R_{\text{feedback}}} + 1$$

Thank You

$$V_{\text{out}} = \frac{V_{\text{in}} + V_{\text{ref}}}{2} + V_{\text{in}} \cdot 20V$$