Mensuration(Written)

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1. The length of a rectangle is halved, while its breadth is tripled. What is the percentage change in area?

Let,

x be the original length

y " " " breadth

$\therefore$ original area will be $= xy$

$\therefore$ new length $= x/2$

$\therefore$ " breadth $= 3y$

$\therefore$ new area will be $= \left(x/2 \times 3y\right)$

$\therefore$ % change $= \dfrac{3xy/2 - xy}{xy} \times 100 = \dfrac{3xy - 2xy}{2xy} \times 100$

$$= \frac{\cancel{xy}}{\cancel{2xy}} \times \cancel{100}\, 50$$

$$= 50\%$$

$\therefore$ The area will be increased by 50%.

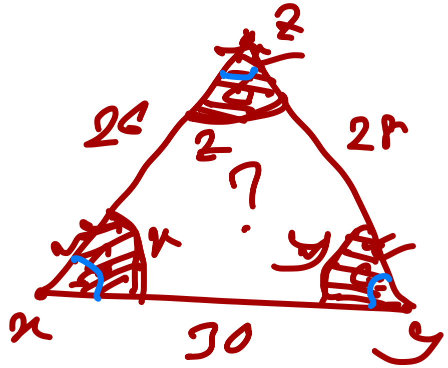
2. At each corner of a triangular field of sides 26 m, 28 m and 30 m, a cow is tethered by a rope of length $\boxed{7\text{ m}}$, the area (in m^2) ungrazed by the cows is

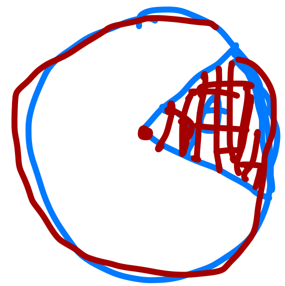
Semi-perimeter of the triangle is,

$$= \frac{26+28+30}{2}\ m$$

$$= \frac{84}{2}\ m$$

$$= 42\ m.$$





$$\frac{\theta}{360} \times \pi r^2$$

$\therefore$ Area of the triangle will be,

$$= \sqrt{42(42-26)(42-28)(42-30)}$$

$$= \sqrt{42 \times 16 \times 14 \times 12}$$

$$= \sqrt{3 \times 2 \times 7 \times 2^4 \times 7 \times 2 \times 2^2 \times 3}$$

$$= 2^4 \times 3 \times 7 = 336\ m^2$$

Let,
the angle created by rope in each corner of the triangle be x, y and z.

$\therefore$ Area of the grazed region will be,

$$= \frac{x}{360} \times \pi (r)^2 + \frac{y}{360} \times \pi (r)^2 + \frac{z}{360} \times \pi (r)^2$$

$$= \frac{49\pi}{360}(x+y+z)$$

$$= \frac{49\pi}{\cancel{360}_{2}} \times \cancel{180}$$

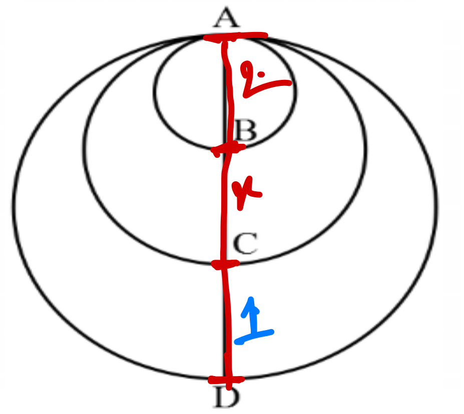
$$= \frac{\cancel{49} \times \cancel{22}}{\cancel{2} \times \cancel{7}} = 77 \text{ m}^2$$

$\therefore$ Area of the ungrazed region $= (336 - 77) \text{ m}^2$

$= 259 \text{ m}^2$

Ans

3. ABCD passes through the centres of the three circles as shown in the figure. AB = 2 cm and CD = 1 cm. If the area of middle circle is the average of the areas of the other two circles, then what is the length (in cm) of BC?



Let,

$BC = x$

ATQ

$$\pi\left(\frac{2+x}{2}\right)^2 = \frac{\pi\left(\frac{2}{2}\right)^2 + \pi\left(\frac{3+x}{2}\right)^2}{2}$$

$$\Rightarrow \pi\frac{(2+x)^2}{4} = \frac{\pi\times\frac{4}{4} + \pi\frac{(3+x)^2}{4}}{2}$$

$$\Rightarrow \frac{\pi}{4}(2+x)^2 = \frac{\frac{\pi}{4}\{4+(3+x)^2\}}{2}$$

$$\Rightarrow 2(2+x)^2 = \frac{\cancel{\frac{\pi}{4}}\{4+(3+x)^2\}}{\cancel{\frac{\pi}{4}}}$$

$$\Rightarrow 2(4+4x+x^2) = 4+9+6x+x^2$$

$\Rightarrow 8 + 8x + 2x^2 = 13 + 6x + x^2$

$\Rightarrow x^2 + 2x - 5 = 0$

$ax^2 + bx + c = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1$

$b = 2$

$c = -5$

$\therefore x = \dfrac{-2 \pm \sqrt{4 + 20}}{2}$

$= \dfrac{-2 \pm \sqrt{24}}{2}$

$= \dfrac{-2 \pm 2\sqrt{6}}{2}$

$= -1 \pm \sqrt{6}$

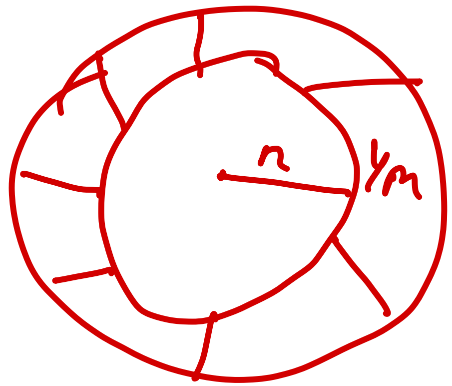
$= -1 + \sqrt{6} = \sqrt{6} - 1$

$\therefore$ value of BC will be $(\sqrt{6} - 1)$ Ans

4. A circular swimming pool is surrounded by a concrete wall 4 m wide. If the area of the concrete wall surrounding the pool is 11/25 that of the pool, then the radius (in m) of the pool:

Let,
the radius of the pool be r m

$\therefore$ the radius of the pool with the wall will be $= (r+4)$ m



ATQ,

$$\pi(r+4)^2 - \pi r^2 = \frac{11}{25}\pi r^2$$

$$\Rightarrow \cancel{\pi}\{(r+4)^2 - r^2\} = \frac{11}{25}\cancel{\pi} r^2$$

$$\Rightarrow \cancel{r^2} + 8r + 16 - \cancel{r^2} = \frac{11}{25} r^2$$

$$\Rightarrow 200r + 400 = 11r^2$$

$\Rightarrow 11n^2 - 200n - 400 = 0$ 4400

$\Rightarrow 11n^2 - 220n + 20n - 400 = 0$

$\Rightarrow 11n(n-20) + 20(n-20) = 0$

$\Rightarrow (n-20)(11n+20) = 0$

$n - 20 = 0$

$\therefore n = 20$

$11n + 20 = 0$

$\Rightarrow 11n = -20$

$\therefore n = -\frac{20}{11}$ [radius can not be negative)

$\therefore$ Radius of the pool will be 20m

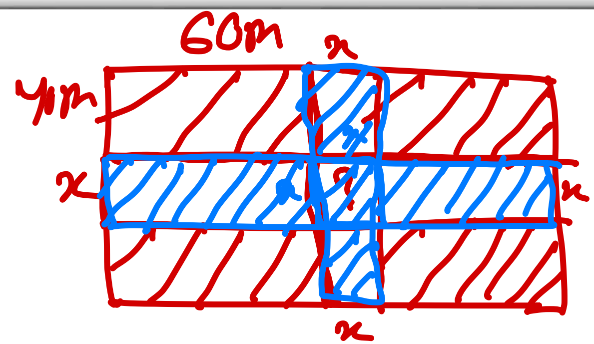
Ans

5. A rectangular park 60 m long and 40 m wide has two concrete crossroads running in the middle of the park and rest of the park had been used as a lawn. If the area of the lawn is 2109 m^2 then the width of the road is

given that,

Length of the park is 60 m

width of " " " 40 m.



$\therefore$ Area of the park is $= (60 \times 40) m^2$

$= 2400 m^2$

Let, the width of the crossroad be x m.

ATQ

$60 \times x + 40 \times x - x^2 = 2400 - 2109$

$\Rightarrow 100x - x^2 = 291$

$\Rightarrow x^2 - 100x + 291 = 0$

$\Rightarrow x^2 - 97x - 3x + 291 = 0$

$\Rightarrow x(x - 97) - 3(x - 97) = 0$

$\Rightarrow (x - 97)(x - 3) = 0$

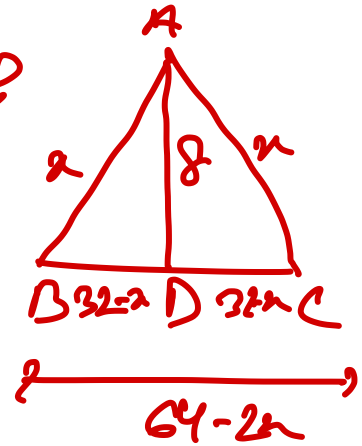
$x = 97$ [The width of the park can not be longer than the length of the park]

$\therefore x = 3$

$\therefore$ the width of the park will be 3m Ans

6. The altitude drawn to the base of an isosceles triangle is 8 cm and its perimeter is 64 cm. The area (in cm^2) of the triangle is

Let,
$\triangle ABC$ be the triangle where x m is the equal sides.



$$\therefore \text{base} = (64 - x - x)\,m$$
$$= (64 - 2x)\,m$$

In $\triangle ABD$,

$$x^2 = 8^2 + (32 - x)^2$$
$$\Rightarrow \cancel{x^2} = 64 + 1024 - 64x + \cancel{x^2}$$
$$\Rightarrow 64x = 1088$$
$$\therefore x = 17$$

$\therefore$ the base of the triangle will be,

$= (64 - 2\times17)\,cm$

$= 64 - 34\ cm$

$= 30\,cm.$

$\therefore$ Area $= \cancel{\tfrac{1}{2}} \times 30 \times \cancel{8}^{4}$

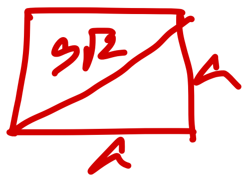
$= 120\,cm^2$

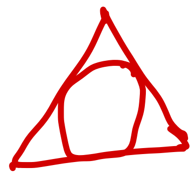
Ans

7. The length of diagonal of a square is 9√2 cm, the square is reshaped to form a triangle. What is the area (in cm^2) of largest incircle that can be formed in that triangle?

Let,

x be the sides of the square





$\therefore x^2 + x^2 = (9\sqrt{2})^2$

$\Rightarrow 2x^2 = 9^2 \times 2$

$\Rightarrow x^2 = 9^2$

$\therefore x = 9$

$\therefore$ perimeter of the square will be $= 4 \times x = 4 \times 9 = 36$ cm

Since the circle of maximum radius is only exist in equilateral triangle, the triangle will be equilateral triangle.

$\therefore$ Sides of the triangle will be $= \frac{36}{3}$ cm

$= 12$ cm

We know,

$$R = \frac{\text{Sides of the triangle}}{2\sqrt{3}}$$

$$R = \frac{a}{2\sqrt{3}}$$

$$\Rightarrow r = \frac{\cancel{12}\,6}{\cancel{2}\sqrt{3}} = \frac{2\times 3}{\sqrt{3}} = \frac{2\times\cancel{\sqrt{3}}\times\sqrt{3}}{\cancel{\sqrt{3}}} = 2\sqrt{3}$$

$\therefore$ Area of the circle will be,

$$= \pi(2\sqrt{3})^2$$

$$= 12\pi \text{ cm}^2$$

Ans

$$\boxed{R = \frac{A}{S}}$$

$$\boxed{h = \frac{\sqrt{3}}{2}a}$$

$$r = \frac{a}{2\sqrt{3}}$$

8. In an isosceles triangle, the measure of each of equal sides is 10 cm and the angle between them is 45°, then area of the triangle is

given that,
length of the equal sides of the triangle is 10 cm
angle between the equal sides = 45°

we knows

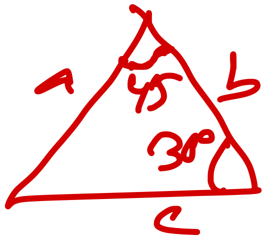
$$\text{Area} = \frac{1}{2} \times 10 \times 10 \times \sin 45^\circ$$

$$= 50 \times \frac{1}{\sqrt{2}}$$

$$= 25 \times \cancel{\sqrt{2}} \times \cancel{\sqrt{2}} \times \frac{1}{\cancel{\sqrt{2}}}$$

$$= 25\sqrt{2}\ cm^2$$

Ans



$$\frac{1}{2} \times a \times b \times \sin 45^\circ$$

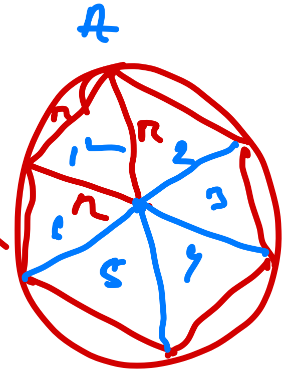
$$\frac{1}{2} \times b \times c \times \sin 30^\circ$$

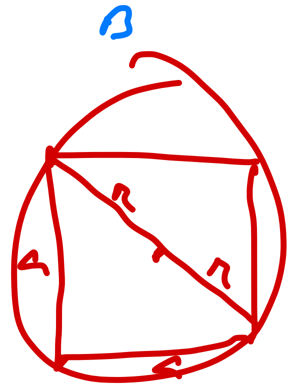
9. A square and a regular hexagon are drawn such that all the vertices of the square and the hexagon are on a circle of radius r cm. The ratio of area of the square and the hexagon is

given that,

radius of the circle is r cm.

$\therefore$ diagonal of the square will be $2r$ cm





let,

the sides of the square be a cm.

$$\therefore a^2 + a^2 = (2r)^2$$

$$\Rightarrow 2a^2 = 4r^2$$

$$\therefore a^2 = 2r^2$$

$\therefore$ Area of the square will be $2r^2$ cm^2

∴ area of the Hexagon will be,

$$= \cancel{6}^{3} \times \frac{\sqrt{3}}{\cancel{4}_{2}} r^2$$

$$= \frac{3\sqrt{3}\, r^2}{2}\ cm^2$$

∴ Required ratio $= 2\cancel{r^2} : \frac{3\sqrt{3}\,\cancel{r^2}}{2}$

$= 4 : 3\sqrt{3}$

Ans

10. The length of one side of a rhombus is 6.5 cm and its altitude is 10 cm. If the length of its diagonal be 26 cm, the length of the other diagonal will be:

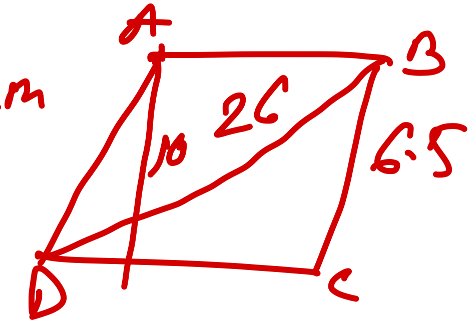
given that,

Length of one side of the rhombus is 6.5 cm

altitude/height = 10 cm.

one diagonal $d_1 = 26$

Let the other diagonal be d_2 cm.



we know,

Area = base × height

$= (6.5 \times 10)\ cm^2$

$= 65\ cm^2$

ATQ,

$$\frac{1}{\cancel{2}} \times \overset{13}{\cancel{26}} \times d_2 = 65$$

$$\Rightarrow \quad d_2 = \frac{65}{13}$$

$$\therefore d_2 = 5 \text{ cm.}$$

$\therefore$ the other diagonal will be 5cm.

Ans

Thank You