Triangles

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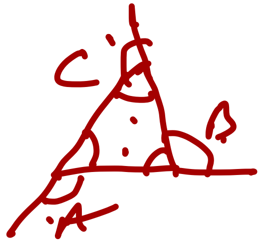
Basic Insight

Properties:

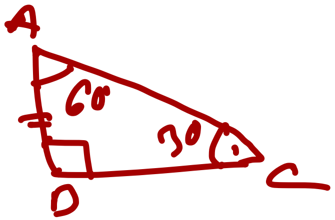


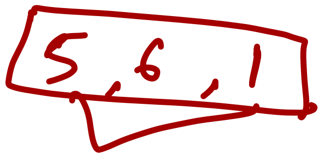
A+B+C= 360°

1. A triangle has three sides and three angles.
2. The sum of the angles of a triangle is always **180 degrees**.
3. The exterior angles of a triangle always add up to **360 degrees**.
4. The sum of consecutive interior and exterior angle is supplementary.
5. The sum of the lengths of any two sides of a triangle is greater than the length of the third side. Similarly, the difference between the lengths of any two sides of a triangle is less than the length of the third side.
6. The shortest side is always opposite the smallest interior angle. Similarly, the longest side is always opposite the largest interior angle.

∠ACB

∠ACD



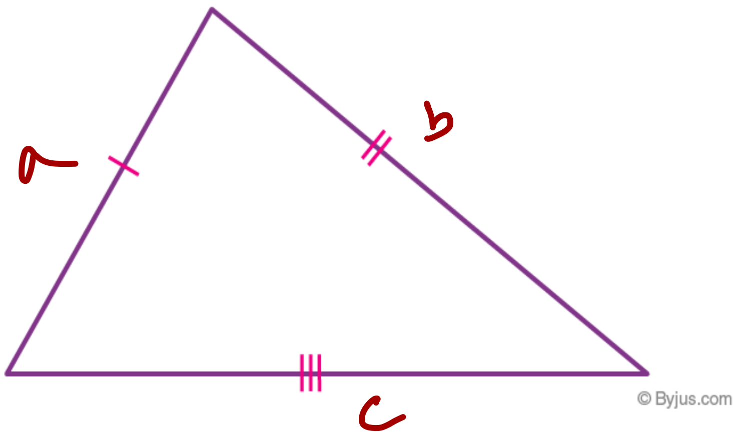


6 = 6

Types:

Scalene Triangle:

A scalene triangle is a type of triangle, in which all the three sides have different side measures. Due to this, the three angles are also different from each other.



$$s = \frac{a+b+c}{2}$$

$$s(s-a)(s-b)(s-c)$$

Semi-Perimeter = (a+b+c)/2

Area = s(s-a)(s-b)(s-c)

Isosceles Triangle:

In an isosceles triangle, the lengths of two of the three sides are equal. So, the angles opposite the equal sides are equal to each other. In other words, an isosceles triangle has two equal sides and two equal angles. The figure given below illustrates an isosceles triangle.

$45^\circ, 45^\circ, 90^\circ$

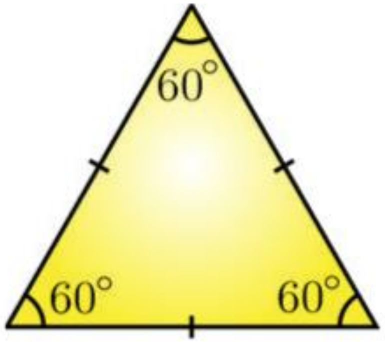
$x \quad x \quad x\sqrt{2}$

$30^\circ, 60^\circ, 90^\circ$

$x \quad x\sqrt{3} \quad 2x$

Equilateral Triangle:

In an equilateral triangle, all the lengths of the sides are equal. In such a case, each of the interior angles will have a measure of 60 degrees. Since the angles of an equilateral triangle are same, it is also known as an **equiangular triangle**. The figure given below illustrates an equilateral triangle.



$$s = \frac{a+b+c}{2} = \frac{3a}{2}$$

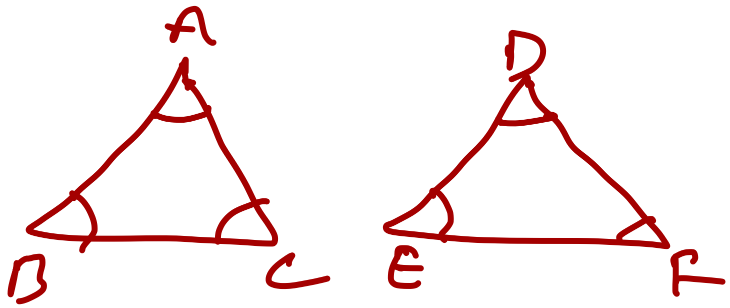
$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{s(s-a)^3}$$

$$=$$

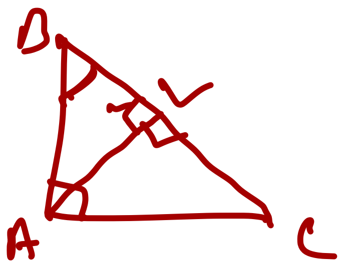
Area $= \frac{\sqrt{3}}{4}a^2$

Preliminary



1. In a triangle ABC, $\angle A = 90°$, AL is drawn perpendicular to BC, Then $\angle BAL$ is equal to:

A. $\angle ALC$
B. $\angle ACB$
C. $\angle BAC$
D. $\angle B - \angle BAL$



$\angle BAL$

$\angle BAL + \angle ABL + 90° = 180$

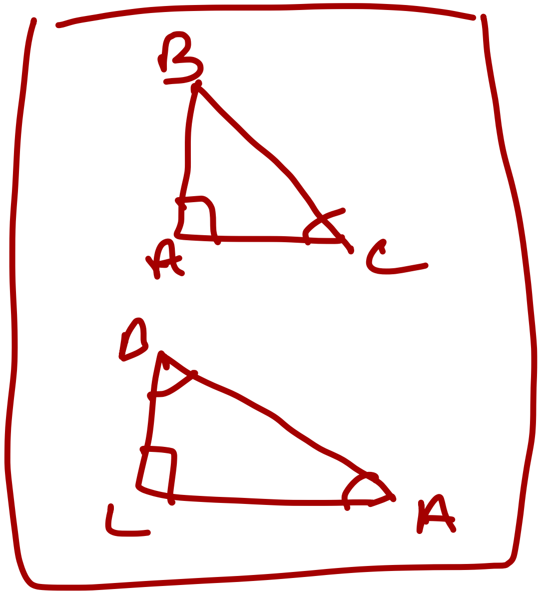
$\Rightarrow \angle BAL = 90° - \angle ABL$

$\angle ACB + \angle B + 90 = 180$

$\Rightarrow \angle ACB = 90° - \angle B$

$\Rightarrow \angle ACB = 90° - \angle ABL$

$= \angle BAL$



$\angle L = \angle A$

$\angle B = \angle B$

$\angle A = \angle C$

$\triangle ABC \sim \triangle BAL$

2. The sides of a triangle are in the ratio 3 : 4 : 6. The triangle is:

A. Acute-angled

B. Right-angled

C. Obtuse-angled

D. Either acute-angled or right-angled

$a^2 + b^2 > c$ acute.

$a^2 + b^2 = c^2$ Right

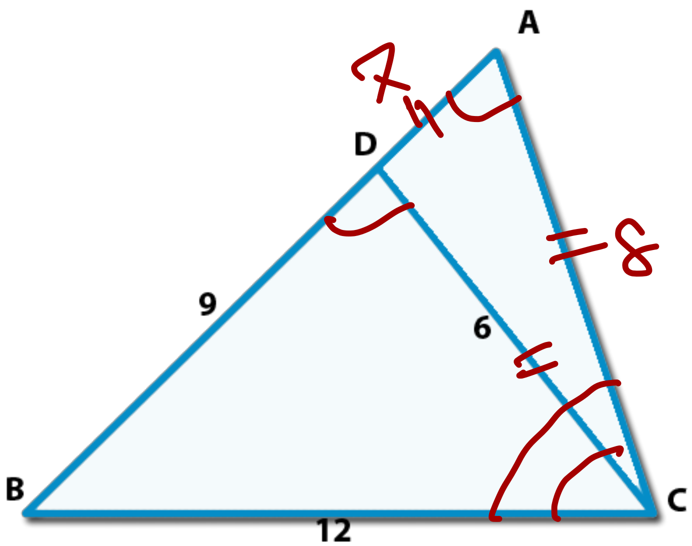
$a^2 + b^2 < c^2$ obtuse

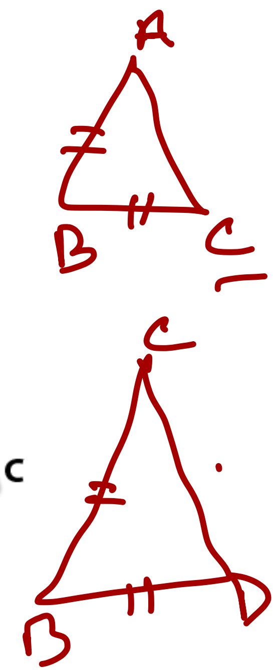
$3^2 + 4^2$ 6^2

$9 + 16$ 36

$25 < 36$

3. Consider the triangle shown in the figure where BC = 12 cm, DB = 9 cm, CD = 6 cm and ∠BCD = ∠BAC. What is the ratio of the perimeter of the triangle ADC to that of the triangle BDC ?





$\triangle ABC \quad \triangle BCD$

$\angle A = \angle C$

$\angle B = \angle B$

$\angle C = \angle D$

$$\boxed{\frac{AB}{BC} = \frac{BC}{BD} = \frac{AC}{CD}}$$

$$\frac{AB}{12} = \frac{12}{9} \qquad AB = 16$$

$$\frac{AC}{CD} = \frac{BC}{BD} \Rightarrow AC = \frac{12 \times 6}{9} = 8$$

A. 7 : 9
B. 8 : 9
C. 6 :9
D. 5 : 9
E. None of these

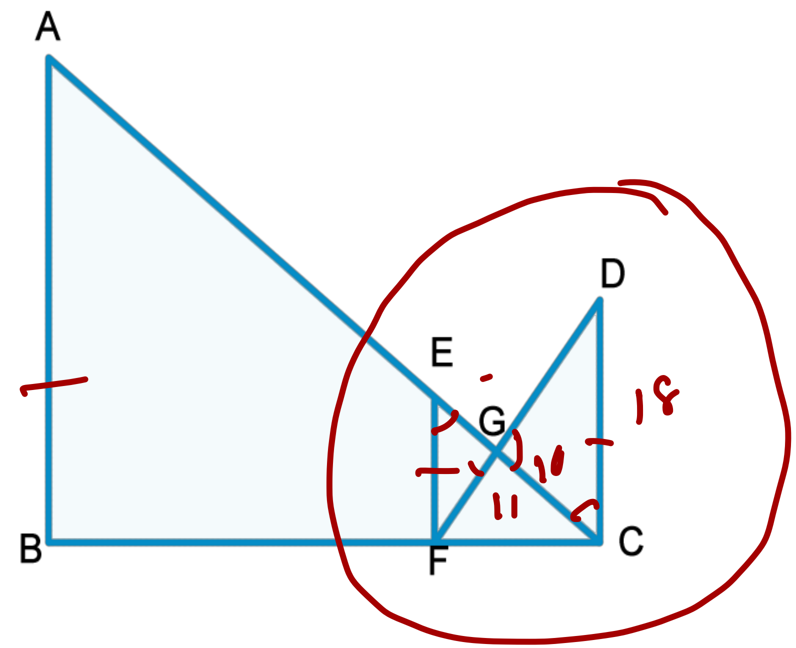
perimeter of $\triangle DC = 9+6+12 = 27$

" " $ADC = 7+8+6$

$= 21$

$$\frac{21}{27} = \cancel{7}\ 13$$

4. In the adjoining figure AB, EF and CD are parallel lines. Given that GE = 5 cm, GC = 10 cm and DC = 18 cm, then EF is equal to:



A. 11 cm
B. 5 cm
C. 6 cm
~~**D.**~~ 9 cm

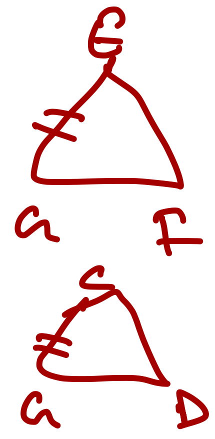
$\triangle GEF \quad \triangle GCD$

$EF \parallel CD \quad EC$

$\angle E = \angle C$

$\angle G = \angle G$

$\angle F = \angle D$

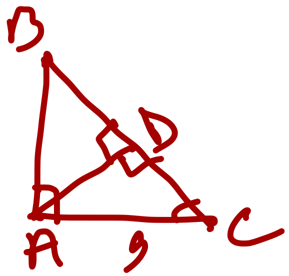


$$\frac{GE}{CG} = \frac{EF}{CD}$$

$$EF = \frac{GE \times CD}{CG} = \frac{5 \times 18}{10} = 9$$

5. **ΔABC be a right-angled triangle where ∠A = 90° and AD ⊥ BC. If area(ΔABC) = 40 cm², area (ΔACD) = 10 cm² and AC = 9 cm, then the length of BC is**

A. 12 cm
B. 18 cm
C. 4 cm
D. 6 cm



$\Delta ABC = 40\ cm^2$

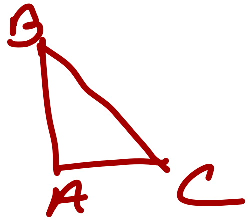
$\Delta ACD = 10\ cm^2$

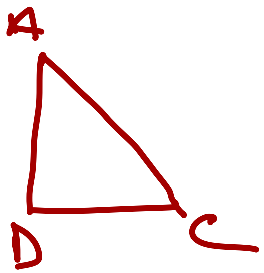
$\Delta ABC \sim \Delta ACD$

$\angle A = \angle D$

$\angle C = \angle C$

$\angle B = \angle A$





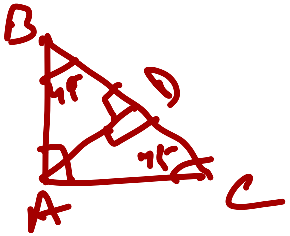
$$\frac{40}{10} = \boxed{\frac{AB^2}{AD^2}} = \frac{BC^2}{AC^2}$$

$$4 = \frac{BC^2}{81}$$

$$BC = 2 \times 9 = 18$$

6. In a triangle ABC, ∠BAC = 90° and AD is perpendicular to BC. If AD = 6 cm and BD = 4 cm then the length of BC is:

A. 8 cm
B. 10 cm
C. 9 cm
D. 13 cm

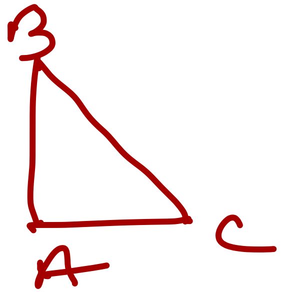


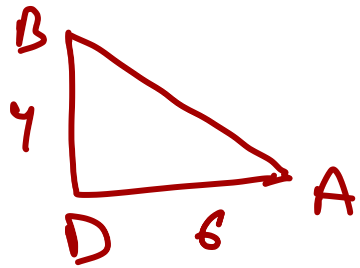
$\triangle ADC \sim \triangle ABC$

$\angle A = \angle D$

$\angle D = \angle B$

$\angle C = \angle A$





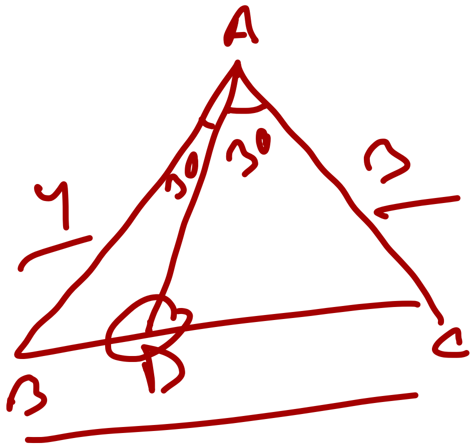
$$\frac{BC}{AB} = \frac{AB}{BD}$$

$$\Rightarrow BC = \frac{AB^2}{BD} = \frac{52}{4} = 13$$

$$AB = \sqrt{4^2 + 6^2} = \sqrt{52}$$

7. In a triangle ABC, the internal bisector of the angle A meets BC at D. If AB = 4, AC = 3 and ∠A = 60°, then length of AD is

A. $2\sqrt{3}$
B. $12\sqrt{3}/7$
C. $15\sqrt{3}/8$
D. $6\sqrt{3}/7$



$$AD = \frac{2ab\cos(A/2)}{a+b}$$

$$= \frac{2\times 4\times 3\times \cos 30}{4+3}$$

$$= \frac{\overset{12}{\cancel{24}}\times \sqrt{3}/\cancel{2}}{7}$$

$$= \frac{12\sqrt{3}}{7}$$

	0	30°	45°	60°	90°
sin	$\sqrt{0/4}$	$\sqrt{1/4}$	$\sqrt{2/4}$	$\sqrt{3/4}$	$\sqrt{4/4}$
	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
	$\sqrt{4/4}$	$\sqrt{3/4}$	$\sqrt{2/4}$	$\sqrt{1/4}$	$\sqrt{0/4}$

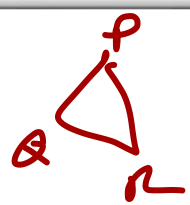
8. **In triangle PQR length of the side QR is less than twice the length of the side PQ by 2 cm. Length of the side PR exceeds the length of the side PQ by 10 cm. The perimeter is 40 cm. The length of the smallest side of the triangle PQR is :**

A. 6 cm
B. 8 cm
C. 7 cm
D. 10 cm

$PQ = x$

$QR = 2x - 2$

$PR = x + 10$



$PQ + PR + QR = 40$

$\Rightarrow x + x + 10 + 2x - 2 = 40$

$\Rightarrow 4x = 32$

$x = 8$

9. If the circumradius of an equilateral triangle be 10 cm, then the measure of its in-radius is-

A. 5 cm
B. 10 cm
C. 20 cm
D. 15 cm

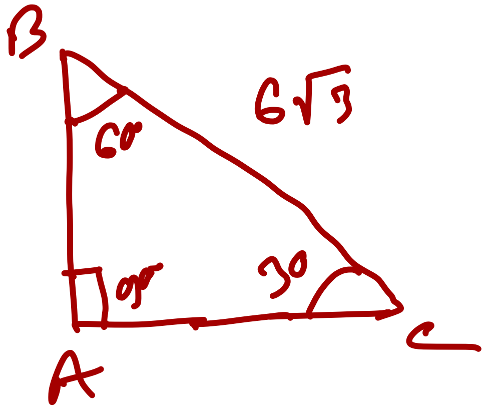
$$\boxed{R = 2r}$$

$$\Rightarrow 10 = 2r$$

$$\therefore r = 5$$

10. **For a triangle base is $6\sqrt{3}$ cm and two base angles are 30° and 60°. Then height of the triangle is**

A. $3\sqrt{3}$ cm
B. 4.5 cm
C. $4\sqrt{3}$ cm
D. $2\sqrt{3}$ cm



AC

30°	60°	90°
x	$x\sqrt{3}$	$2x$

$$\sin 30° = \frac{AB}{6\sqrt{3}}$$

$$\Rightarrow 6\sqrt{3} \times \tfrac{1}{2} = AB$$

$$\therefore AB = \underline{3\sqrt{3}}$$

$$2x = 6\sqrt{3}$$

$$x = \underline{3\sqrt{3}}$$

Written

1. A triangle has sides of lengths 5 cm, 6 cm, and 7 cm. What is its area?

(1) Let,

$a = 5$ cm

$b = 6$ cm

$c = 7$ cm

$\therefore$ Semi-perimeter $S = \frac{a+b+c}{2} = \frac{5+6+7}{2} = 9$ cm

We know,

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{9(9-5)(9-6)(9-7)} = \sqrt{9 \times 4 \times 3 \times 2}$

$= \sqrt{216}$

$= 6\sqrt{6}$ cm^2

Ans.

2. In a triangle ABC, angle A=60^0, side b=8 cm, and side c=5 cm. What is the area of the triangle?

we know,

$$\text{Area} = \frac{1}{2} \times b \times c \times \sin 60^\circ$$

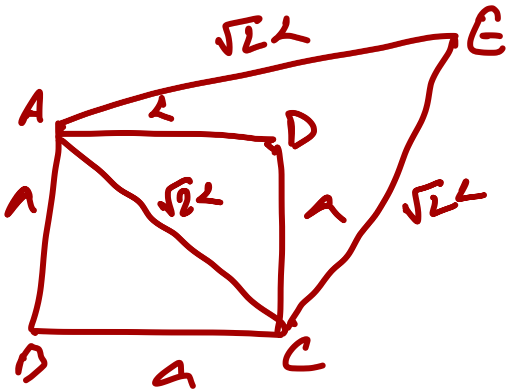
$$= \frac{1}{2} \times 8 \times 5 \times \frac{\sqrt{3}}{2}$$

$$= 10\sqrt{3} \text{ cm}^2$$

Ans.

3. An equilateral triangle is described on the diagonal of a square. What is the ratio of the area of the triangle to that of the square ?

Let,
a cm be the each sides of the square



$\therefore$ Diagonal $AC = \sqrt{2}\,a$

$\therefore$ Area of the equilateral triangle will be,

$= \frac{\sqrt{3}}{4}(\sqrt{2}a)^2$

$= \sqrt{3}/4 \times 2a^2$

$= \dfrac{\sqrt{3}a^2}{2}$

$\therefore$ area of the square is a^2

$\therefore$ Required Ratio $= \sqrt{3}a^2/2 : a^2$

$= \sqrt{3} : 2$

Ans

4. The radius of a circle is 20% more than the height of a right-angled triangle. The base of the triangle is 36 cm. If the area of triangle and circle be equal, what will be area of circle ?

Let, the height of the triangle be x cm

$\therefore$ radius of the circle will be $= \frac{6x}{5}$ cm

ATQ,

$\frac{1}{2} \times 36 \times x = \frac{22}{7} \times \left(\frac{6x}{5}\right)^2$

$\Rightarrow 18x = \frac{22}{7} \times \frac{36x^2}{25}$

$\Rightarrow 25 \times 7 = 44x$

$\Rightarrow x = \frac{25 \times 7}{44}$

$\therefore$ Radius $= \dfrac{\overset{3}{\cancel{6}}}{\cancel{5}} \times \dfrac{\overset{5}{\cancel{25}} \times 7}{\underset{22}{\cancel{49}}}$

$$= \frac{105}{22}$$

$\therefore$ Area of the circle will be,

$$= \frac{\cancel{22}}{\cancel{7}} \times \frac{\overset{15}{\cancel{105}}}{\cancel{22}} \times \frac{105}{22}$$

$$= 71.60\ cm^2$$

Ans.

5. The area of two similar triangles are 12 cm^2 and 48 cm^2. If the height of the smaller one is 2.1 cm, then the corresponding height of the bigger one is :

Let,

x cm be the height of the bigger triangle

ATQ,

$$\frac{12}{48} = \frac{(2.1)^2}{x^2}$$

$$\Rightarrow x^2 = 4.41 \times 4$$

$$\Rightarrow x = 2.1 \times 2$$

$$= 4.2 \text{ cm}$$

$\therefore$ The height of the bigger triangle will be 4.2 cm Ans

Thank You