

Number Theory

Negative + Natural + whole

Integer

$[-3, -2, -1, 0, 1, 2, 3]$
 $[+\alpha, 0, -\alpha]$

Natural

$^{-3, -2, -1, 0, 1, 2, 3}$
 $^{+ve} 1, 2, 3, 4, 5 \dots +\alpha$

Whole

$0, 1, 2, 3, 4, 5, 6 \dots +\alpha$
 $0, +ve + \text{Natural}$

$$\begin{array}{r} 3 \overline{) 385} \\ \underline{300} \\ 85 \\ \underline{84} \\ 1 \end{array}$$

Integer

$$9 = \textcircled{3} < 200$$

Prime or not?

only even

$$\begin{array}{r} 5 \overline{) 385} \\ \underline{350} \\ 35 \\ \underline{35} \\ 0 \end{array}$$

$$\sqrt{400} = 20$$

$$\begin{array}{r} \sqrt{\text{non prime}} \\ 385 \\ \boxed{400} \end{array}$$

- | |
|-----------------------------|
| $\textcircled{2}, 3, 5, 7,$ |
| $11, 13, 17, 19$ |

$$200$$

$$\begin{array}{r} 200 \\ \textcircled{19} \textcircled{196} \end{array}$$

$$\begin{array}{r} 3 \overline{) 10} \\ \underline{9} \\ 1 \end{array}$$

Divisibility

$$\boxed{1 \ 2 \ 3 \ 4}$$

$$1 + 2 + 3 + 4$$

$$\boxed{= 10}$$

$$\sqrt{5} \rightarrow [0/5]$$

$$\begin{array}{r} 48 \textcircled{5} \\ \hline 100 \\ \hline \end{array}$$

$$\frac{9}{\boxed{2 \ 3 \ 4 \ 5 \ 4}} \quad \checkmark$$

$$2 + 3 + 4 + 5 + 4 = 18$$

$$\begin{array}{r|l} 9 & 18 \\ \hline & 18 \\ \hline & 0 \end{array} \quad | \quad 2$$

Even Odd Even/Odd

$\textcircled{2/4}$ $\textcircled{1/3}$ $\textcircled{3}$ ↓
Even + Odd = Odd

Even + Even = Even

Odd + Odd = Even ✓

Odd + Even = Odd

1 + 2 = 3

Even Odd = product

Even \times Odd = Even

Even \times Even = Even

Odd \times Even = Even
 $1 \times 2 = 2$

Odd \times Odd = Odd

HCF

LCM

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$$\begin{array}{r}
 2 \overline{) 18} \\
 \underline{12} \\
 6 \\
 3 \overline{) 6} \\
 \underline{6} \\
 0
 \end{array}$$

12, 18

$$\begin{array}{r}
 2 \overline{) 12} \\
 \underline{6} \\
 6 \\
 2 \overline{) 6} \\
 \underline{6} \\
 0
 \end{array}
 = \frac{2^2 \times 3^1}{2^1 \times 3^1}, \frac{3^2 \times 2^1}{3^1 \times 2^1}$$

$$\frac{2^1 \times 3^1}{2^1 \times 3^1}$$

$$= \textcircled{6} \text{ Least}$$



Maximum

Product of two numbers

$$= \underline{\text{H.C.F}} \times \text{L.C.M.},$$

$$\begin{array}{ccc} 12 \downarrow & & 18 \\ \text{HCF } 6 & \begin{array}{c} 2 \\ 2 \times 3 \end{array} & \begin{array}{c} 3 \\ 3 \times 2 \end{array} \\ & & \underline{\text{LCM}} \end{array}$$

$$\frac{12 \times 18}{216} = \frac{6 \times 36}{216}$$

$$\begin{aligned} &= 2^2 \times 3 \\ &= 4 \times 9 \\ &= 36 \end{aligned}$$

01) If x is an odd integer, for which of the following equations must y be an even integer?

A) $xy=5$

B) $x + y=8$

~~C) $x+2y=7$~~

~~D) $2x+y=6$~~

E) None of these

~~$x = 1$
 $8 - 3 = 5$~~

~~$x = 3$
 $3 + 2y = 7$
 $2y = 4$
 $y = 2$~~

~~$2y = 6$
 $y = 3$
 $2y = 4$
 $y = 2$~~

~~y - even
 x - odd~~

$2x + y = 6$
 $2x + 3 + y = 6$
 $\Rightarrow y = 0$

$2x \Rightarrow$

$2x + 3 = 6$ 6 - even
 $\Rightarrow 2x = 3$

02) If x , y , and z are consecutive non-zero integers, and if $x < y < z$, which of the following must be a positive odd integer?

~~A) $x - z$~~

~~B) $x + y + 1$~~

C) $x + y + z$

D) $xz - y$

E) None of these

$\sqrt{3 \times 5 - 4 = 11} \rightarrow \begin{array}{ccc} -1 & -2 & -3 \\ x & y & z \\ 3 & 4 & 5 \end{array}$

$\sqrt{2 \times 4 - 3 = 5} \rightarrow \begin{array}{ccc} x & y & z \\ 2 & 3 & 4 \end{array}$

$[1, 3, 5, 7]$

03) If x and y are positive integers, each greater than 1, and if $\underline{13(x-1)} = \underline{17(y-1)}$, what is the least possible value of $(x+y)$.

$$\underline{18+14=32}$$

~~A) 32~~

B) 30

C) 26

D) 25

E) None of these

$$\begin{aligned} \checkmark \quad \frac{13 \times (x-1)}{17} &= \frac{17 \times (y-1)}{13} \\ 13 &= 17-1 & 17 &= x-1 \\ y &= 14 & x &= 18 \end{aligned}$$

04) If the product of three consecutive integers is 120, then the sum of the integers is:

A) 12

B) 14

~~C) 15~~

D) 18

E) None of these

05) The sum of squares of 3 consecutive integers is less than 97. What is the greatest possible value of the smallest one?

~~A) 4~~

B) 5

C) 6

D) 7

E) None of these

$$\begin{array}{l} 1^2 + 2^2 + 3^2 = 14 < 97 \\ 2^2 + 3^2 + 4^2 = 29 < 97 \\ 3^2 + 4^2 + 5^2 = 50 < 97 \\ \textcircled{4}^2 + 5^2 + 6^2 = 77 < 97 \\ \hline 5^2 + 6^2 + 7^2 = 110 > 97 \end{array}$$

06) When the positive integer n is divided by 8, the remainder is 3. What is the remainder if $3n$ is divided by 8?

A) 0

~~B) 1~~

C) 2

D) 3

E) None of these

$$\begin{array}{r} 8 \overline{) 1111} \\ \underline{8} \\ 3 \end{array}$$

$$\begin{array}{r} 8 \overline{) 8110} \\ \underline{80} \\ 10 \end{array}$$

$$\begin{array}{r} 8 \overline{) 3314} \\ \underline{32} \\ 14 \end{array}$$

$$3n = 3 \times 11 = 33$$

$$3n = 3 \times 27 = 81$$

07) $2\overline{pq}5$ is a four digit number divisible by 25. If the number formed from two digits \overline{pq} is a multiple of 13. Then $\overline{pq}=?$

~~A) 10~~

~~B) 25~~

C) 52

D) 65 ✓

E) None of these

$$\begin{array}{r}
 2525 \\
 \hline
 \overline{pq} \\
 \hline
 25 \overline{) 2525} \\
 \underline{25} \\
 25 \\
 \underline{25} \\
 0
 \end{array}$$

$$\begin{array}{l}
 13 = 13, 26, 39, \underline{52} \\
 = 65, 78, \dots
 \end{array}$$

$$\begin{array}{r}
 25 \overline{) 27658} \\
 \underline{25} \\
 158 \\
 \underline{150} \\
 8
 \end{array}$$

08) If both ~~112~~¹¹² and ~~33~~³³ are factors of the number $a \cdot 4^3 \cdot 6^2 \cdot 13^{11}$, then what is the smallest possible value of 'a'?

A) 121

B) 3267

~~C) 363~~

D) 33

E) None of these

$a \cdot 4^3 \cdot 6^2 \cdot 13^{11}$

$2^2 \cdot 2^2 \cdot 2^2 \cdot 2^2 \cdot 2^2$

~~$3^3 \cdot 2$~~

~~$3 \cdot 11 \cdot 2$~~

~~$3 \cdot 12 \cdot 2$~~

$a \cdot 4^3 \cdot 6^2 \cdot 13^{11}$

$(3 \cdot 3) \cdot a \cdot (2^2)^3 \cdot (3 \cdot 2)^{11}$

$a \cdot 2^8 \cdot 3 \cdot 13^{11}$

gcf lcf

09) The greatest common factor of two positive integers is A. the least common multiple of the two integers is B. if one of the integers is C, what is the other number?

~~A) $\frac{AB}{C}$~~

B) $\frac{BC}{A}$

C) $\frac{C}{A} + B$

D) $A + \frac{B}{C}$

E) None of these

$C \times x = \frac{A \times B}{AB}$

$\frac{A}{HCF} \times LCM = C \times x$
 $\Rightarrow x = \frac{A \times B}{C}$

$HCF / GCF = A$
 $LCM = B$

10) What is the minimum number of chocolates that must be added to the existing stock of 260 chocolates so that the total stock can equally be divided among 1, 4, or 6 persons?

~~A) 4~~

B) 8

C) 12

D) 16

E) None these

$$\begin{array}{r} 12 \overline{) 260} \\ \underline{24} \\ 20 \\ \underline{12} \\ 8 \end{array}$$

$$\boxed{260}$$

$$260 - 8 = 252$$

$$\begin{array}{l} ? \overline{) 1, 4, 6} \\ \underline{1, 2, 3} \\ \hline 2 \times 3 = 12 \\ 12 - 8 = 4 \\ 8 \\ 264 \end{array}$$

11) The light of Shahbag signal flashes every 120 seconds and that of Green road and Bata signal flash every 60 and 90 seconds. If they all flash at 8:30 am, when will they flash again simultaneously?

+6

~~A) 8:36 am~~

B) 8:35 am

C) 8:33 am

D) 8:40 am

E) None of these

60 -

$$\begin{array}{r} 15 \overline{) 120, 60, 90} \\ \underline{120} \\ 0 \\ 2 \overline{) 8, 4, 6} \\ \underline{8} \\ 0 \\ 2 \overline{) 4, 2, 3} \\ \underline{4} \\ 0 \\ 2 \end{array}$$

360

12) A number when divided by 195 leaves a remainder 47. If the same number is divided by 15, the remainder will be?

- A) 1 ~~B) 2~~ C) 3 D) 4 E) None of these

Divisor + Remainder

$$195 + 47 = 242$$
$$15 \overline{) 242} \begin{array}{r} 16 \\ \underline{15} \\ 92 \\ \underline{90} \\ 2 \end{array}$$
$$195 \overline{) 242} \begin{array}{r} 1 \\ \underline{195} \\ 47 \end{array}$$
$$195 \text{ O } + 47$$
$$15 \overline{) 1997} \begin{array}{r} 133 \\ \underline{15} \\ 2 \end{array}$$

13) The number of terms between 11 and 200 which are divisible by 7 but not by 3 are

A) 18

B) 19

C) 27

D) 28

E) None of these

$$a + (n-1)d$$

$$21 + (n-1) \cdot 7 = 189$$

$$\Rightarrow 21 + 7n - 7 = 189$$

$$\Rightarrow 7n = 175$$

$$\Rightarrow n = 25$$

$$11 \quad 200$$

$$\begin{array}{r} 14 \cdot 27 \\ \hline 28 \end{array}$$

$$14 + (n-1) \cdot 7 = 196$$

$$14 + 7n - 7 = 196$$

$$7n = 189$$

$$\Rightarrow n = 27$$

$$6 = 2 \cdot 3$$

$$21 = 3 \cdot 7$$

- 21, 42, 63, 84, 105, 126, 147, 168, 189

$$\frac{21}{189}$$

$$196$$

(7, 3)

L.C.M = $7 \times 3 = 21$

only 3

14) A number when divided by 3 leaves a remainder 1. When the quotient is divided by 2, it leaves a remainder 1. What will be the remainder when the number is divided by 6?

- A) 2 B) 3 ~~C) 4~~ D) 5 E) None of these

10

$$\begin{array}{r} 6 \overline{) 10} \\ \underline{6} \\ 4 \end{array}$$

$$\begin{array}{r} 3 \overline{) 10} \\ \underline{9} \\ 1 \end{array}$$

$$\begin{array}{r} 2 \overline{) 3} \\ \underline{2} \\ 1 \end{array}$$

$$\begin{array}{r} 1 \overline{) 6} \\ \underline{0} \\ 1 \end{array} \begin{array}{l} \text{Quotient} \\ \text{Remainder} \end{array}$$