CAPSTONE

Mathematics: Geometry

Lecture 11

Overview

- Advance Geometry:
  1. Circle
  2. Solid Geometry

Name: **Batch:**

IBA Regular Batch

Reach Us

Panthapath : 01972-277 866
Mouchak : 01999-017 011
Mirpur : 01970-985 421
Chattogram : 01970-985 420

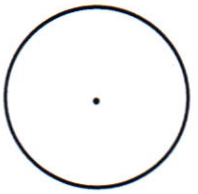
www.capstonebd.com Capstone Education

Math Lecture Sheet: 11

Circle Basics

A circle is named as per the center's name. This circle is circle A:

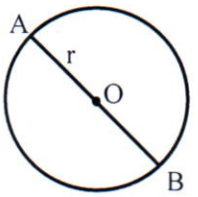
To define a circle you need only 2 things:



- ✓ Where is its center?
- ✓ What is its radius?

Here, in the figure below, AB = diameter = d, Diameter = 2 × Radius

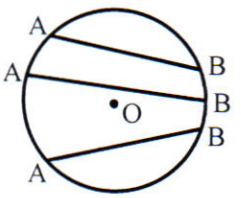
AO = radius = r, O being center of the circle.



Circumference = $2\pi r$

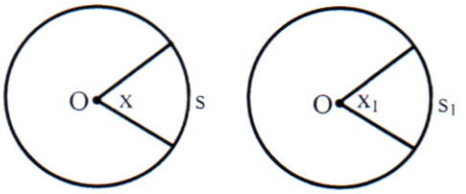
Area = πr^2

Chord: Chord is the straight line connecting two points on the circumference of the circle. The line AB is chord, in every instance below:



The diameter is the longest chord in any circle.

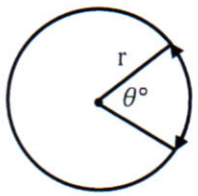
Arc: Arc is part of circumference. The length of arc and the degree measure of the central angle are proportional.



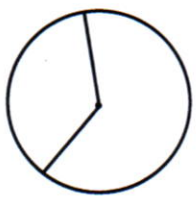
Central angle: Central angle is formed when two points on the circumference make an angle at the center.

Inscribed Angle: When that two points create an angle at the circumference, it is called Inscribed Angle.

A central angle creates **an arc** and **an area of sector.**



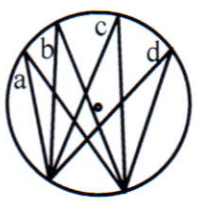
$$\text{Length of arc} = \frac{\theta}{360} \times 2\pi r$$



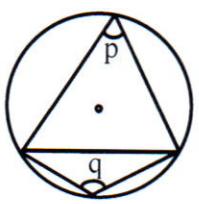
$$\text{Area of sector} = \frac{\theta}{360} \times \pi r^2$$

Facts:

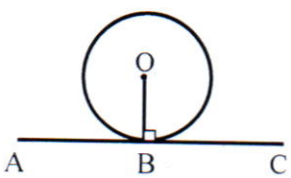
- All the inscribed angles standing on a same chord are equal. Here a=b=c=d.



- The inscribed angles should be formed on the **same side** of the chord. So, In the figure below $p \neq q$ here, but in such case, the relationship between p and q is: $p + q = 180^\circ$.



Tangent is an external line, that touches a circle only at one point.



The line AC is a tangent for the circle O. AC touches the circle at point B.

B is called the point of tangency. OB will be perpendicular on AC. That is the line connecting the center and the tangency point are perpendicular to each other.

Clock-Hand problems

The most common type of problem is asking degree value of the angle formed by two hand at a certain time.

E.g what is the angle formed by the hour hand and the minute hand at the time 03:07?

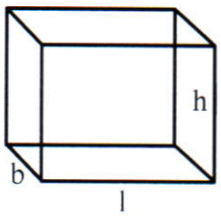
If the value of hour is h. value of minute is m and the angle formed by two hands is θ then.

$\theta = |30h - 5.5m|$

If the value of θ is above 180, subtract it from 360 to get perfect answer.

Volume & Surface Area:

1. Cuboid



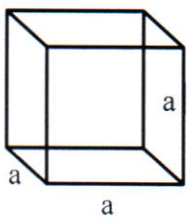
Let, length = l, breadth = b and height = h units, then:

i. Volume = (l × b × h) cubic units

ii. Surface area = 2(lb + bh + lh) sq. units

iii. Diagonal = $\sqrt{l^2 + b^2 + h^2}$ units

2. Cube



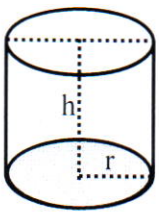
Let, each edge of a cube be of length a, then:

i. Volume = a^3 cubic units

ii. Surface area = $6a^2$ sq. units

iii. Diagonal = $\sqrt{3}a$ units

3. Cylinder



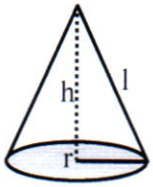
Let, radius of base = r and height (or length) = h, then:

i. Volume ($\pi r^2 h$) cubic units

ii. Curved surface area = ($2\pi rh$) sq. units

iii. Total surface area = $2\pi r(h + r)$ sq. units

4. Cone



Let, radius of base = r and height = h, then:

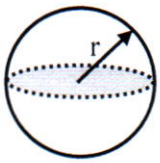
i. Slant height (হেলানো তলের উচ্চতা), $l = \sqrt{h^2 + r^2}$ units

ii. Volume = $\frac{1}{3}\pi r^2 h$ cubic units

iii. Curved surface area = πrl sq. units

iv. Total surface area = $(\pi rl + \pi r^2)$ sq. units

5. Sphere

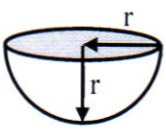


Let, the radius of the sphere be r, then:

i. Volume = $\frac{4}{3}\pi r^3$ cubic units

ii. Surface area = $4\pi r^2$ sq. units

6. Hemisphere



Let, the radius of a hemisphere be r, then:

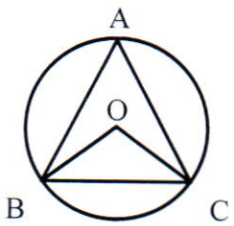
i. Volume = $\left(\frac{2}{3}\pi r^3\right)$ cubic units

ii. Curved surface area = $(2\pi r^2)$ sq. units

iii. Total surface area = $(3\pi r^2)$ sq. units

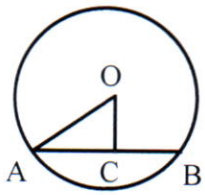
Practice Test

1. In the figure, O is the center of circle and $\angle OCB = 35°$. What is the value of $\angle BAC = ?$



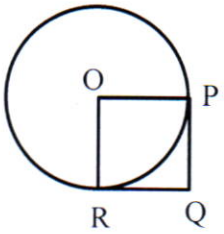
A. 65° B. 60° C. 55° D. 50° E. None of these

2. In the figure, O is the center of the circle. OC is perpendicular to AB and is 2 cm less than AC. If the diameter of the circle is 20 cm, what is the length of AB in cm?



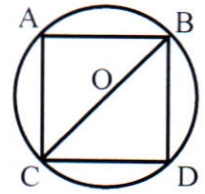
A. 85 B. 10 C. 15 D. 35 E. None of these

3. In the figure below, if the area of the square OPQR is 2, find the area of the circle with center O.



A. 2 B. 4 C. π D. 2π E. None of these

4. If a square is inscribed in a circle of radius r as shown below, then the area of the square region is -



A. $\frac{r^2}{2\pi}$ B. $\frac{\pi r^2}{2\pi}$ C. πr^2 D. $2r^2$ E. None of these

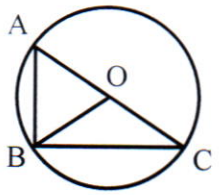
5. What is the angle formed by the hour hand and the minute hand at the time 2:45 pm?

A. 187.5° B. 175° C. 172.5° D. 178.5° E. 183.5°

6. A bicycle wheel has a diameter of 60 cm. approximately how many times does the wheel rotate at 2.5 km long trip?

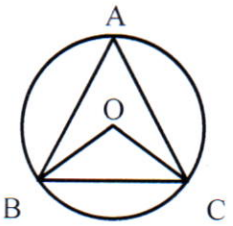
A. 1326 B. 133 C. 763 D. 1600 E. None of these

7. In the figure below, O is the center of the circle. If OC = BC, what is the value of angle BAC?



A. 22.5° B. 30° C. 45° D. 60° E. None of these

8. O is the center of the circle. If ∠BAC is 55°, what is the value of ∠OCB?

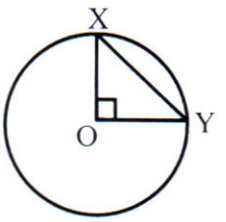


A. 55° B. 45° C. 35° D. 25° E. None of these

9. A cow is tied to the corner of a square of side 15 m with a rope of length 14. Find the area the cow can graze and the area which it cannot.

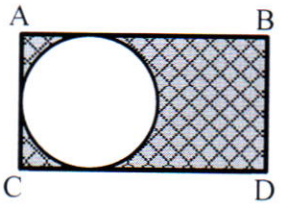
A. 144 m^2, 77 m^2 B. 135 m^2, 73 m^2 C. 164 m^2, 77 m^2 D. 154 m^2, 71 m^2 E. None of these

10. O is the center of the circle at the right. XO is perpendicular to YO and the area of triangle XOY is 32. What is the area of circle O?



A. 16π B. 32π C. 64π D. 128π E. 256π

11. In the rectangle ABDC, 2AB = 3BD. If the radius of the circle is $\sqrt{7}$ find the area of the shaded region.



A. 42 – 7π B. 32 – 5π C. 13 – 7π D. 56 – 7π E. None of these

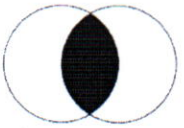
12. A certain cake recipe states that the cake should be baked in a pan 8 inches in diameter. If Salma wants to use the recipe to make a cake of the same depth but 12 inches in diameter, by what factor should she multiply the recipe ingredients?

A. 2.5 B. 2.25 C. 1.5 D. 1.33 E. None of these

13. An equilateral triangle is inscribed in a circle, as shown below. If the radius of the circle is 2, what is the area of the triangle?

A. $\frac{\sqrt{2}}{2}$ B. $\sqrt{2}$ C. $3\sqrt{3}$ D. $10\sqrt{3}$ E. None of these

14. In the figure, each of the circle has radius 4 and the area enclosed by both circles is 28π. What is the area of the shaded region?



A. 0 B. 2π C. 4π D. $4\pi^2$ E. None of these

15. A certain recipe makes enough batter (a semi liquid mixture of one or more grains used to prepare various foods) for exactly 8 circular pancakes that are each 10 inches in diameter. How many circular pancakes, each 5 inches in diameter and of the same thickness as the 10 inches pancakes, should the recipe make?

[BBA 14-15]

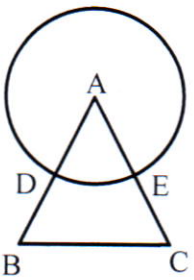
A. 4 B. 15 C. 24 D. 32 E. 40

16. A motorcycle stunt man belonging to a fair rides over a circular well at an average speed of 54 km/h for 5 minutes. If the radius of the well is 5 meters then the distance travelled is -

A. 2.5 km B. 3.5 km C. 5 km D. 4.5 km E. 5.5 km

17. In the following figure, ABC is an equilateral triangle where points D and E are midpoints of sides AB and AC respectively. If point A is the center of the circle and BC = 42 cm, what is the area of the arc (sector) ADE?

[June 2018]



A. 462 cm^2 B. 346.5 cm^2 C. 231 cm^2 D. 154 cm^2 E. None of these

18. The length width and depth of a rectangular box are 8 feet, 7 feet and 9 feet respectively. A hose supplies water at a rate of 12 cubic feet per minute. How much time in minute it takes to fill a conical box whose volume is three times the volume of the rectangular box? [June 2018]

A. 42 B. 126 C. 205 D. 235 E. None of these

19. A circular well with 18 m inside diameter is dug out 14 m deep. The soil taken out of it has been spread out over a square shaped field with a diagonal of $18\sqrt{2}$ m. The level of the field will rise by how much? [June 2018]

A. 18 m B. 16 m C. 15 m D. 12 m E. 11 m

20. Water is poured into an empty cylindrical tank at a constant rate. In 15 minutes, the height of the water increased by 9 feet. The radius of the tank is 10 feet. What is the rate at which the water is poured? [June 2018]

A. 11π cft/min B. 54π cft/min C. $60\ \pi$ cft/min D. $90\ \pi$ cft/min E. None of these

21. If the length of an edge of cube X is thrice the length of an edge of cube Y, what is the ratio of the volume of cube Y to the volume of cube X?

A. $\frac{1}{2}$ B. $\frac{1}{4}$ C. $\frac{1}{6}$ D. $\frac{1}{27}$ E. None of these

22. A metallic sheet is of rectangular shape with diameter 48m × 36m. From each of its corners, a square is cut off so as to make an open box. If the length of the square is 8m, the volume of the box (in m^3) is - [BBA 13-14]

A. 5120 B. 6420 C. 8960 D. 4830 E. None of these

23. A cylindrical rod of iron, whose height is equal to its radius, is melted and cast into spherical balls whose radius is half the radius of the rod. Find the number of balls?

A. 3 B. 4 C. 5 D. 6 E. None of these

24. If a rectangular block that is 4 inches by 4 inches by 10 inches is placed inside a right circular cylinder of radius 3 inches and height of 10 inches, the volume of the unoccupied portion of the cylinder is how many cubic inches? [BBA 13-14]

A. $6\pi - 16$ B. $9\pi - 16$ C. $160\pi - 30\pi$ D. $6\pi - 160$ E. $90\pi - 160$

25. The length of one edge of a cube equals 4. What is the distance between the center of the cube and one of its vertices?

A. 2 B. $2\sqrt{2}$ C. $2\sqrt{3}$ D. $4\sqrt{2}$ E. None of these

Home Task

1. When the diameter of a circle a is tripled, the arc of the circle will be increased by -

A. 3 time B. 6 time C. 9 time D. 12 time E. None of these

2. The area of circle A is 6.25π sq. inch. If the radius of the circle is doubled, what is the new area of circle A in sq. inch?

A. 5π B. 12.5π C. 25π D. 39.0625π E. None of these

3. The radius of a circle is increased by 10% then the area is increased by what percent?

A. 10% B. 20% C. 21% D. 100% E. None of these

4. The side length of a square inscribed in a circle is 2. What is the area of the circle?

A. π B. $\sqrt{2}\pi$ C. 2π D. $2\sqrt{2}\pi$ E. None of these

5. A wheel that has 6 cogs is meshed with a wheel of 14 cogs. When the smaller wheel has made 21 revolutions, the number of revolution made by the larger wheel is -

A. 4 B. 9 C. 12 D. 49 E. 54

6. A circular garden with diameter of 20 meters is surrounded by a walk way of width 1 meter. What is the area of the walk way?

A. $41\pi m^2$ B. $41m^2$ C. $21\pi m^2$ D. $21m^2$ E. None of these

7. A goat is tied to one corner of a square plot of side 12 m by a rope 7 long. Find the area it can graze?

A. 19.25 sq. m B.155 sq. m C. 144 sq. m D. 38.48 sq. m E. None of these

8. The length of a rope, to which a cow is tied, is increased from 19 m to 30 m. How much additional ground will it be able to graze? Assume that the cow is able to move on all sides with equal case?

A. 696 sqm B. 1694 sqm C. 1594 sqm D. 1756 sqm E. None of these

9. You own a Rubik's cube with a volume of 343 cm^3. What is the edge length of the cube?

A. 14 cm B. 7 cm C. 49 cm D. 43 cm E. 9 cm

10. Which of the following has the largest area?

i. A circle of radius $\sqrt{2}$ ii. An equilateral triangle of side 4. iii. A triangle whose sides are 3, 4 and 5

A. i B. ii C. iii D. i and ii E. ii and iii

11. If the volume of a cube is 27 cubic meters, find the surface area of the cube?

A. 9 sq. m B. 54 sq. m C. 18 sq. m D. 3 sq. m E. None of these

12. A box is made in the form of a cube. If a second cubical box with dimension three time those of the first box, how many times as much does the second box contain?

A. 6 B. 9 C. 12 D. 27 E. None of these

13. A room of size 5m × 3m and height 3m requires walls and ceiling painting. What is the area to painted?

A. 63 B. 70 C. 75 D. 64 E. 90

14. The number of square units in the surface area of a cube is twice as large as the number of cubic units in its volume. What is the cube's volume, in cubic units?

A. 108 B. 216 C. 36 D. 9 E. 27

15. A piece of wood measuring 3cm × 3cm × 2cm is divided into 18 square shaped cubes of identical shape. What is the total surface area of all the identical cubes (in cm^2)? [BBA 14-15]

A. 42 B. 48 C. 96 D. 108 E. None of these