CAPSTONE

Mathematics: Arithmetic

Lecture 03

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◆ Average	◆ Ratio Equivalence

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Math Lecture Sheet: 03

INEQUALITIES

An inequality is a statement that uses one of the following symbols:

$\neq$ not equal to

$>$ greater than

$\geq$ greater than or equal to

$<$ less than

$\leq$ less than or equal to

Some examples of inequalities are $5x - 3 < 9, 6x \geq y, \frac{1}{2} < \frac{3}{4}$.

Solving a linear inequality with one unknown is similar to solving an equation; the unknown is isolated on one side of the inequality.

Notes: Multiplying or dividing an inequality by a negative number reverses the order of the inequality.

for example, $6 > 2; but\ (-1)6 < (-1)(2)$

Order of the inequality reverses if we reciprocate the number from both sides.

for example, $6 > 2$

but $\frac{1}{6} < \frac{1}{2}$

so, solve the inequality $3x - 2 > 5$ for x, isolate x by using the following steps:

$3x - 2 > 5$

$3x > 7$ (adding 2 to both sides)

$x > \frac{7}{3}$ (diving both side by 3)

To solve the inequality $\frac{5x-1}{-2} < 3$ for x, isolate x by using the following steps:

$$\frac{5x - 1}{-2} < 3$$

$5x - 1 > -6$ (multiplying both side by -2)

$5x > -5$ (adding 1 to both sides)

$x > -1$ (dividing both side by 5)

Key point:

When solving an inequality:

➔ You can add the same quantity to each side.

➔ You can subtract same quantity from each side.

➔ You can multiplying or divide each side by the same **positive** quantity.

➔ If you multiply or divide each side by negative quantity, the inequality symbol must be reversed.

Inequalities used with a modulus symbol

Inequalities often appear in conjunction with the modulus or absolute value symbol. |11|, for example, in a statement such as $|x| < 2$

Recall that the modulus of a number is simply its magnitude or absolute value. Regardless of its sign. So

$|2| = 2$ and $|-2| = 2$

So, if the absolute value of x is less than 2, then this means that x must lie between 2 and -2. We can write this as $-2 < x < 2$.

Example: solve $|5x - 8| \leq 12$

This means $-12 \leq 5x - 8 \leq 12$

Now, $-12 \leq 5x - 8$

$\Rightarrow -4 \leq 5x$ [adding 8 to both sides]

$\Rightarrow -\frac{4}{5} \leq x$ [diving by 5]

and $5x - 8 \leq 12$

$\Rightarrow 5x \leq 20$ [adding 8 to bot side]

$\Rightarrow x \leq 4$ [dividing by 5]

putting these results together gives the solution

$$-\frac{4}{5} \leq x \leq 4$$

Example: Let x and y are integers. If $x + 1 > 0$ and $xy > 0$, which of the following must be true? [MBA 2016]

A. $y > 0$ B. $y < 0$ C. $y > x$ D. $y < x$ E. None of these

Solution: Given, $x + 1 > 0$

$\therefore x > -1$

here, x is an integer and $xy > 0$; so x can be zero and x must be positive.

so, y is also positive. That is $y > 0$

Answer: A

AVERAGE

The average value or average of a number of similar quantities is their sum divided by their number. Hence,

$$Average = \frac{Sum\ of\ the\ quantities}{Number\ of\ quantities}$$

Some common types of questions

Change in average by inclusion or deletion in whole

Example: The average weight of students in a class of 35 is 50 kg. If however, the weight of the teacher be included, the average will be increased by 0.5 kg. Find the weight of the teacher.

Solution: Average of 35 boys = 50 kg

Total weight of 35 boys = $35 \times 50 = 1750$ kg

New average of class = 50.5 kg

Total weight of 35 boys and teacher = $50.5 \times 36 = 1818$ kg

$\therefore$ weight of teacher = $(1818 - 1750)$ kg = 68 kg

Shortcut: Weight of the teacher = $50 + (36 \times 0.5) = 68$ kg

Calculating the middle number:

Example - 2: The average of 11 numbers is 109. If average of the first six numbers is 105 and that of the last six numbers is 114, what is the middle number?

Solution: Total of 11 numbers = $109 \times 11 = 1199$

Total of first six numbers = $105 \times 6 = 630$

Total of last six numbers = $114 \times 6 = 684$

Total of 12 numbers = $684 + 630 = 1314$

Hence, middle number = $1314 - 1199 = 115$

Finding the number

Example: The average age of a board of eight directors of a company remain the same as if was three years ago when one of them is replaced by a new member. Find how much younger is he than the director in whose place has been elected?

Solution: Number of directors = 8

Total increase in years = $8 \times 3 = 24$ years

But the average remains the same.

Hence, the new member must be 24 years younger.

Example: A batsman makes a score of 87 runs in the 17^{th} innings and thus increases his average by 3. What is his average after 17^{th} inning?

A. 23 B. 34 C. 39 D. 44 E. None of these

Solution: Let score up to 16^{th} innings be x.

$\therefore$ Score after 17^{th} innings will be $x + 3$

$\therefore 16x + 87 = 17(x + 3)$

$\Rightarrow 16x + 87 = 17x + 51$

$\Rightarrow x = 36$

$\therefore$ Average after 17^{th} innings $= 36 + 3 = 39$

Answer: C. 39

Practice Test

1. Karim's school is 10 kms from his home. From there he travels 4 kms to go to his friends house and then travels 3 kms to get to the cricket ground. If he is than x miles from home, what is the range of possible values of x?

A. $3 \leq x < 12$ B. $3 \leq x \leq 17$ C. $3 \leq x \leq 10$ D. $3 \leq x \leq 14$ E. None of these

2. Moyna has n chocolate, where n is an integer such that $20 < n < 50$. If Moyna divides the chocolate equally among 5 childrens, she will have 2 chocolate remaining. If she divides the chocolate among 6 children, she will have 1 chocolate remaining. How many chocolate will remain if she divides the chocolate among 7 children?

A. 0 B. 1 C. 2 D. 3 E. 4

3. If $2 < x < 3$ and $7 < y < 8$, which of the following expressions will give the largest value?

A. x^2y B. xy^2 C. 5xy D. $\frac{4x^2y}{3}$ E. $\frac{3x^2y}{4}$

4. If $|x - 12| = 4x$, then $x =$? [MBA 15]

A. -8 B. -4 C. 1 D. 4 E. None of these

5. If $|x - y| + x = y$, which of the following must be true? [MBA 15-16]

A. $x = 0$ B. $x = -y$ C. $x = y$ D. $x \leq y$ E. None of these

6. If x and y non-zero integers such that $3x = 2y$, then which of the following must be true? [MBA 16-17]

i. $\frac{x}{y} > 1$ ii. xy is positive iii. $x - y$ is positive

A. i B. ii C. both i & ii D. both ii and iii E. i, ii and iii

7. If x and y are both positive integers and $10 < x < 20$ and $7y - 2x = 0$, what is the value of $x - y$? [MBA 16-17]

A. 7 B. 8 C. 9 D. 10 E. None of these

8. In a class of 25 students, 10 have less than 6 marbles, 10 have more than 7 marbles and 4 have more than 8 marbles. How many students have more than 5 marbles but less than 9 marbles? [MBA 17]

A. 10 B. 11 C. 12 D. 13 E. None of these

9. The weight of a box is estimated by three persons. According to A, the weight lies between 50 and 60 kg. According to B, the weight is more than 45 kg but less than 58 kg. C estimate that the weight cannot be greater than 56 kg. If all of them are correct in their estimation, what is the average of different probable weight of the box? [MBA 17-18]

A. 52 B. 53 C. 54 D. 55 E. None of these

10. Given $y = (x-6)(x-5)(x-4)(x-3)$ and x is a positive integer. If $y > 0$, than which of the following must be true? [MBA 18]

A. $x < 3$ B. $x < 6$ C. $3 < x < 0$ D. $x > 3$ E. None of these

11. Three students P, Q, R are standing in a line. There are 5 students standing in the line between P and Q, and 8 students standing in the line between Q and R. If there are 3 students standing in the line before R, and 21 students standing in the line behind P, then what is the minimum number of the students standing in the line? [MBA 18]

A. 41 B. 40 C. 28 D. 27 E. None of these

12. c individuals pledged to pay equal contribution so that a charity's goal of $x could be reached. If d of the contributors failed to pay their share, which of the following represents the additional number of dollars that each of the remaining individuals would have to pay in order to allow the charity to reach its goal?

A. $\frac{cx}{d(c-x)}$ B. $\frac{x}{c-d}$ C. $\frac{d}{c-dx}$ D. $\frac{x}{c(c-d)}$ E. $\frac{dx}{c(c-d)}$

13. A circular jogging track forms the edge of a circular lake that has a diameter of 2 miles. Johanna walked once around the track at the average speed of 3 miles per hour. If t represents the number of hours it took Johanna to walk completely around the lake, which of the following is correct statement?

A. $0.5 < t < 0.75$ B. $1.75 < t < 2.0$ C. $2.0 < t < 2.5$ D. $2.5 < t < 3.0$ E. $3 < t < 3.5$

14. If $|x - 1| > 2$, which of the following must be true? [BBA 14-15]

I. $|x| > 3$ II. $x^2 > 9$ III. $x > 3$

A. I only B. II only C. I and II only D. III only E. None of these

15. Jack and Jill have certain number of apples with them. The total number of apples with both of them is less than 80. If Jill gives a certain number of apples to Jack, than Jack will have 4 times the no of apples Jill has. If Jack gives the same number of apple to Jill, then Jack will have thrice the number of apples that Jill has. What can be the number of apple that Jack initially has?

A. 31 B. 36 C. 45 D. 62 E. 63

16. The average of M number is A and the average of N number is B. What is the average of all the number?

A. $\frac{MA+NB}{M+N}$ B. $\frac{MB+NA}{A+B}$ C. $\frac{AB+MN}{A+B}$ D. $\frac{AB+MN}{M+N}$ E. None of these

17. The average of the two-digit numbers, which remain the same when the digits interchange their positions, is:

A. 33 B. 44 C. 55 D. 66 E. 77

18. If the average of seven consecutive integers is $k + 2$, then the product of the greatest and least integers is:

A. $k^2 + 4k - 5$ B. $k^2 - 9$ C. $k^2 + 6k - 9$ D. $k^2 - 2k + 1$ E. $k^2 - 4k + 5$

19. The captain of a cricket team of 11 members is 26 years old and the wicket keeper is 4 year older. If the ages of them two are excluded, the average age of the remaining players is one year less than the average age of the whole team. What is the average age of the whole team?

A. 22 B. 23 C. 24 D. 25 E. None of these

20. 3 years ago, the average age of a family of 5 members was 17 years. A baby having been born, the average age of the family is the same today. The present age of the baby is:

A. 6 month B. 1 year C. 1 year 6 month D. 2 year E. 3 years

21. Nine persons went to a hotel for taking their meals. 8 of them spent tk. 12 and each on their meals and the ninth spent tk. 8 more than the average expenditure of all the nine. What was the total money spent by them?

A. 114 B. 125 C. 119 D. 117 E. 205

22. While travelling on a train Mr. Saif noticed three different numbers were written on the roof of the train. He calculated that the average of the three numbers was V. If one numbers was Z and another was Y what was the remaining number? [BBA 09-10]

A. $ZY - V$ B. $\frac{Z}{V} - 3 - Y$ C. $\frac{Z}{3} - V - Y$ D. $3V - Z - Y$ E. $V - Z - Y$

23. The average of the five numbers is 7. If one of the numbers is multiplied by 3, the average of the numbers increases to 9.4. Which of the five numbers is multiplied by 3? [MBA 15]

A. 4 B. 5 C. 5.6 D. 4 E. None of these

24. The average age of 8 students is 20. The average age of first two students is 19 and that of the next three students is 21. If the age of the sixth student is less than that of the seventh and eighth student by 2 and 3 respectively, then find the age of the eight student. [MBA 17-18]

A. 18 B. 21 C. 24 D. 27 E. None of these

25. In a particular course Arif appeared in 10 quizzes. The average of his best 9 quizzes is 10% more than the average of all the quizzes he attended. The total marks obtained in best 9 quizzes is what percent of the total marks obtained in 10 quizzes? [BBA 14-15]

A. 80% B. 88% C. 90% D. 99% E. None of these

Home Work

1. The average age of 8 people increase by 2 years when two women are included in place of two men aged 20 and 24. Find the average age of the two women. [MBA 15]

A. 30 B. 32 C. 35 D. 40 E. None of these

2. A water filter can be filled with 8 jugs of capacity 1.3 liters each. How many jugs are required to fill the same filter, if capacity of the jug is 0.8 liters? [MBA 16-17]

A. 15 B. 13 C. 12 D. 8 E. None of these

3. The average of ten numbers is x and the average of five numbers is y. If the average of remaining five is z, then- [MBA 17-18]

A. $x = y + z$ B. $z = x + 2y$ C. $x = 2y + 2x$ D. $2x = y + z$ E. None of these

4. The average age of a group of 10 students is 15 years. When 5 more students joined the group, the average age rose by 1 year. What is the average age of the newly joined students?

A. 15 B. 16 C. 17 D. 18 E. 19

5. If $b = 9d - c$ and $d = \frac{a}{6}$, what is the average of a, b, c and d?

A. 2d B. 3d C. 4d D. 5d E. None of these

6. The average of 6 numbers is 8.5. When one number discarded, the average of the remaining numbers become 7·2. What is the discarded number?

A. 7.5 B. 7.8 C. 6.5 D. 14 E. 15

7. The average of 20 numbers is zero. Of them, at the most how many may be greater than zero?

A. 0 B. 1 C. 10 D. 19 E. 20

8. The average monthly income of P and Q is tk. 5050. The average monthly income of Q and R is tk. 6250 and the average monthly income of P and R is tk. 5200. The monthly income of P is:

A. 3500 B. 4000 C. 4050 D. 5000 E. 4500

9. Set x contain 10 consecutive integers. If the sum of the 5 smallest members of set X is 265, what is the average of the 5 largest members of set X?

A. 56 B. 58 C. 57 D. 59 E. 61

10. If $xy > 0$ and $yz < 0$, which of the following must be negative?

A. xyz B. xyz^2 C. xy^2z D. xy^2z^2 E. None of these

11. If $xy < 0$, which of the following must be true?

i. $x + y = 0$ ii. $2y - 2x < 0$ iii. $x^2 + y^2 > 0$

A. i only B. ii only C. iii only D. both ii and iii E. both i & iii

12. If $x < 10$, then it must be true that [MBA 15]

A. $-x < -10$ B. $-x - 2 < 12$ C. $-x + 2 < -8$ D. $x - 2 < 9$ E. None of these

13. Given that $xy > 0$ and $x > y$, which of the following must be true? [MBA 15]

i. $x + y > 0$ ii. $x^2 + y^2 > xy$ iii. $x^2 - y^2 > 0$

A. i B. ii C. iii D. ii and iii E. None of these

14. If $z = \frac{x+y}{2xy}$ and $0 < y < x < 1$, which of the following must be true? [MBA 15-16]

A. $z < 0.5$ B. $z > 0.5$ C. $z < 1$ D. $z > 1$ E. None of these

15. If $x \geq -1$ and $x \geq x^5$, which of the following must be true? [MBA 17]

A. $x > 1$ B. $0 < x < 1$ C. $-1 \leq x \leq 0$ D. $x > 0$ E. None of these

16. If x is an integers such that $5 < x < 11, 7 < x < 13$ and $x + 2 < 12$, then how many integers will satisfy x? [MBA 17]

A. 1 B. 2 C. 3 D. 4 E. None of these

17. If $ab < 0$, then all the following must be true, EXCEPT [MBA 17]

A. $\frac{a}{b} < 0$ B. $a^2 + b^2 > 0$ C. $a^3 + b^3 < 0$ D. $\frac{b}{a} < 0$ E. None of these

18. If x is an integer and $y = -4x + 17$, what is the least value of x for which y is less than 1? [MBA 17]

A. 2 B. 3 C. 4 D. 5 E. None of these

19. If $-8 \leq x \leq 2$ and $-4 \leq y \leq 10$ which of the following represents the range of all possible values of xy? [MBA 17]

A. $-8 \leq xy < 20$ B. $-80 \leq xy \leq 32$ C. $-32 \leq xy \leq 20$ D. $-8 \leq xy \leq 32$ E. $-80 \leq xy \leq 80$

20. If $-1 < x < 1$ and $x \neq 0$, which of the following must be true? [MBA 17-18]

i. $x^3 < x^2$ ii. $x^5 < 1 - x$ iii. $x^4 < x^2$

A. i only B. i & ii only C. ii & iii only D. i & iii only E. i, ii & iii

21. If $-1 < a < b < 0$, which of the following has the highest value? [MBA 18]

A. $\frac{a}{b}$ B. $\frac{b}{a}$ C. $\frac{b}{a^2}$ D. $\frac{a^2}{b}$ E. None of these

22. 3 people are splitting a tk. 150 bill. If A pays tk. 5 less than B, which c pays more than tk. 60, what is the most A can pay, given all of them pay integer amounts? [MBA 18]

A. 29 B. 42 C. 47 D. 61 E. None of these

23. Find the range of value of x for which $(2x + 3)(x - 1) < 0$. [MBA 18]

A. $x < -\frac{3}{2}$ B. $-\frac{3}{2} < x < 1$ C. $x > 1$ D. $x > -\frac{3}{2}$ E. None of these

24. The smallest of three consecutive even integer is 40 less than three times the largest. What is the largest of these integers?

A. 17 B. 10 C. 15 D. 14 E. None of these

25. If $a + b > 0$ and $\sqrt{a + b} > b$, which of the following must be true? [BBA 08-09]

A. $a > 0$ B. $b > 0$ C. $a < 0$ D. $b < 0$ E. None of these