CAPSTONE

Mathematics: Arithmetic

Lecture 07

Overview	Next Lecture
◆ Probability ◆ Permutation ◆ Combination	◆ Speed ◆ Train ◆ Boat

Name: **Batch:**

IBA Regular Batch

Reach Us

Panthapath : 01972-277 866
Mouchak : 01999-017 011
Mirpur : 01970-985 421
Chattogram : 01970-985 420

www.capstonebd.com Capstone Education

Simple Probability

In general, the probability of an event is the number of favorable outcomes divided by the total number of possible outcomes.

Probability = (# of favorable outcomes) / (# of possible outcomes)

Example 1: What is the probability that a card drawn at random from a deck of cards will be an ace?

Solution: In this case there are four favorable outcomes:

(1) The ace of spades (2) The ace of hearts (3) The ace of diamonds (4) The ace of clubs

Since each of the 52 cards in the deck represents a possible outcome, there are 52 possible outcomes. Therefore, the probability is $\frac{4}{52}$ or $\frac{1}{13}$.

Example 2: What is the probability that when a pair of six-sided dice are thrown, the sum of the number equals 12?

Solution: We already know the total number of possible outcomes is 36, and since there is only one outcome that sums to 12, (6, 6-- you need to roll double sixes), the probability is simply $\frac{1}{36}$.

Probability of Multiple Events

For questions involving single events, the formula for simple probability is sufficient. **For questions involving multiple events, the answer combines the probabilities for each event in ways that may seem counter-intuitive.** The following strategy is excellent for acquiring a better feel for probability questions involving multiple events or for **making a quick guess** if time is short. We will focus on questions involving two events.

- If two events have to occur together, generally an **"and"** is used. Take a look at **Statement 1:** "I will only be happy today if I get email and win the lottery." The "and" means that **both events are expected to happen together**.

- If both events do **not necessarily have to occur together**, an **"or"** may be used as in **Statement 2,** "I will be happy today if I win the lottery or have email."

1. A **and** B: the probability < than the **individual** probabilities of **either** A **or** B.

2. A **or** B must occur: the probability > the **individual** probabilities of either A or B. This is an excellent strategy for eliminating certain answer choices. These two types of probability are formulated as follows:

Probability of A and B:

$P(A \text{ and } B) = P(A) \times P(B)$.

In other words, the probability of A and B both occurring is the product of the probability of A and the probability of B.

Probability of A or B:

$P(A \text{ or } B) = P(A) + P(B)$.

In other words, the probability of A or B occurring is the sum of the probability of A and the probability of B.

Example 3: If a coin is tossed twice, what is the probability that on the first toss the coin lands heads and on the second toss the coin lands tails? A. $\frac{1}{6}$ B. $\frac{1}{3}$ C. $\frac{1}{4}$ D. $\frac{1}{2}$ E. 1

Solution: First note the "and" in between event A (heads) and event B (tails). That means we expect both events to occur together, and that means fewer options, a less likely occurrence, and a lower probability. Expect the answer to be less than the individual probabilities of either event A or event B, so less than $\frac{1}{2}$. Therefore, eliminate D and E. Next we follow the rule P(A and B) = P(A) × P(B). If event A and event B have to happen together, we multiply individual probabilities. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. Answer C is correct.

Example 4: If a coin is tossed twice what is the probability that it will land either heads both times or tails both times? A. $\frac{1}{8}$ B. $\frac{1}{6}$ C. $\frac{1}{4}$ D. $\frac{1}{2}$ E. 1

Solution: To figure out the probability for event A or B, consider all the possible outcomes of tossing a coin twice: heads, heads; tails, tails; heads, tails; tails, heads. We can see that the probability for event **A is $\frac{1}{4}$ and that the probability for event B is $\frac{1}{4}$.** Now we use the rule to get the exact answer. P(A or B) = P(A) + P(B). If either event 1 or event 2 can occur, the individual probabilities are added: $\frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$. Answer D is correct.

The following chart summarizes the "and's" and "or's" of probability:

Probability	Formula	Expectation
P(A and B)	P(A) × P(B)	Lower than P(A) or P(B)
P(A or B)	P(A) + P(B)	Higher than P(A) or P(B)

Independent and Dependent Events

The types of events that we have discussed so far are all independent events. By independent we mean that the first event does not affect the probability of the second event. Coin tosses are independent. They cannot affect each other's probabilities; the probability of each toss is independent of a previous toss and will always be $\frac{1}{2}$. Separate drawings from a deck of cards are independent events if you put the cards back. **An example of a dependent event, one in which the probability of the second event is affected by the first, is drawing a card from a deck but not returning it. By not returning the card, you've decreased the number of cards in the deck by 1, and you've decreased the number of whatever kind of card you drew.** If you draw an ace of spades, there are 1 fewer aces and 1 fewer spades.

If A and B are not independent, then the probability of A and B is

P(A and B) = P(A) × P(B|A)

Where P(B|A) is the conditional probability of B given A.

Example 5: If someone draws a card at random from a deck and then, without replacing the first card, draws a second card, what is the probability that both cards will be aces?

Solution: Event A is that the first card is an ace. Since 4 of the 52 cards are aces, $P(A) = \frac{4}{52} = \frac{1}{13}$. Given that the first card is an ace, what is the probability that the second card will be an ace as well? Of the 51 remaining cards, 3 are aces. Therefore, $P(B|A) = \frac{3}{51} = \frac{1}{17}$, and the probability of A and B is $\frac{1}{13} \times \frac{1}{17} = \frac{1}{221}$.

Mutually Exclusive Events

Another type of probability deals with mutually exclusive events. What do we mean by mutually exclusive events? And what does it mean for two events not to be mutually exclusive? Consider the following example of drawing cards:

Example 6: What is the probability that a card from a deck will be either an ace or a king?

A. $\frac{1}{169}$ B. $\frac{1}{26}$ C. $\frac{2}{13}$ D. $\frac{4}{13}$ E. $\frac{8}{13}$

Solution: The question asks for either an ace or a king. Since there are four kings and four aces in a deck, the probabilities for event A and event B are the same, $\frac{4}{52} = \frac{1}{13}$. Our answer must be more than this, so eliminate A and B. Do kings and aces have anything to do with each other? Is there such a thing as an ace of kings or a king of aces? No, so we don't have to worry about having over-counted; the events are mutually exclusive. The probability is straightforward: $P(A \text{ or } B) = P(A) + P(B) = \frac{1}{13} + \frac{1}{13} = \frac{2}{13}$. C is correct.

Again we could have used simple probability. Count the total number of kings and aces (4+4) and divide by the total number of cards in a deck: $\frac{8}{52} = \frac{2}{13}$.

Conditional Probabilities

A conditional probability is the probability of an event given that another event has occurred.

Example 7: What is the probability that the total of two dice will be greater than 8 **given that** the first die is a 6?

Solution: This can be computed by considering only outcomes for which the first die is a 6. Then, determine the proportion of these outcomes that total more than 8. All the possible outcomes for two dice are shown in the section on simple probability. There are 6 outcomes for which the first die is a 6: (6,1), (6,2), (6,3), (6,4), (6,5), (6,6), and of these, there are four that total more than 8. The probability of a total greater than 8 given that the first die is 6 is therefore $\frac{4}{6} = \frac{2}{3}$

Combinations

Suppose that a job has two different parts. There are m different ways of doing the first part, and there are n different ways of doing the second part. The problem is to find the number of ways of doing the entire job. For each way of doing the first part of the job, there are **n ways** of doing the **second part**. Since there are **m ways** of doing the **first part**, the total number of ways of doing the entire job is m x n. The formula that can be used is : **Number of ways = m x n** For any problem that involves **two actions or**

two objects, each with a number of CHOICES and asks for **the number of combinations,** the above formula can be used.

Combinations are where small groups of size r are made from a larger group of size n. Order is not important in combinations. **Formula:** $\frac{n!}{r!(n-r)!}$

Example 8: This time we have 7 different books and we want to give three to a friend for Christmas. How many different gifts of three books could we give? We have a large group of seven and want to see how many smaller groups of three we could make. You can see that order is not important, since the gift is the same no matter which of the 3 books is on the top or the bottom. The answer is $\frac{7!}{3!(7-3)!}$ or $5040 \div 6 \times 24 = 35$.

Example 9: William wants a sandwich and a drink for lunch. If a restaurant has 4 choices of sandwiches and 3 choices of drinks, how many different ways can he order his lunch?

Solution: Since there are 4 choices of sandwiches and 3 choices of drinks, using the formula: Number of ways = 4(3) = 12 Can order lunch 12 different ways.

Permutations

Permutations of Unlike Objects of Number n taken r at a time. **Formula P(n,r) = n!/(n-r)!**

In permutations there is no choosing, you are just arranging

The order of the objects is important,

When arranging numerals, 123 is different from 321 and 213. The rule here is that the number of ways to "arrange" n DIFFERENT objects is n! UNLESS you want to arrange them in smaller groups, then you follow the rule of $\frac{n!}{(n-r)!}$

There is no choosing.

The order of the objects is important,

You have some objects which are the same.

Example 10: You want to arrange 10 books on a shelf but you have 2 copies of one book and 3 copies of another. The number of ways you can arrange them is $\frac{10!}{2!\times 3!}$ which equals $3{,}628{,}800 \div 2 \times 6 = 302{,}400$.

Notes:

- Circular Arrangement: In general, n objectives can be arranged in a circle in $(n-1)!$ ways.
- Special Combinations:

i. ${}^{n}C_{n} = 1$; Example: ${}^{7}C_{7} = 1$

ii. ${}^{n}C_{0} = 1$; Example: ${}^{7}C_{0} = 1$

iii. ${}^{n}C_{1} = n$; Example: ${}^{7}C_{1} = 7$

iv. ${}^{n}C_{(n-1)} = n$; Example: ${}^{7}C_{6} = 7$

Practice Test

1. There are three different road from Shahbag to Farmgate and 4 different roads from Farmgate to Banani. How many different route are there from Shahbag to Banani which go through Farmgate?

A. 4 B. 3 C. 7 D. 12 E. None of these

2. How many different six digit numbers can be formed using all of the following digits 3, 3, 4, 4, 4, 5?

A. 25 B. 40 C. 50 D. 60 E. None of these

3. In how many ways can Sadib, Tazul, Abir, Hamid, Banna, Omar be seated if Sadib and Tazul cannot be seated next to each other?

A. 240 B. 360 C. 480 D. 600 E. 650

4. How many 3 digit numbers can be formed from the digits 2, 3, 5, 6, 7 and 9, which are divisible by 5 and none of the digits is repeated?

A. 5 B. 10 C. 15 D. 20 E. 25

5. At a party, everyone shook hands with everybody else. If there were 66 handshakes, how many people were at the party?

A. 9 B. 15 C. 10 D. 12 E. 13

6. A school committee consists of 2 teachers and 4 students. The number of different committee that can be formed from 5 teachers and 10 students is -

A. 220 B. 5100 C. 2100 D. 3200 E. 3500

7. The number of different signals which can be given from 6 flags of different colors taken one or more at a time is -

A. 1954 B. 1956 C. 1958 D. 1962 E. 1966

8. Maliha has 16 marbles in her pocket. She has 8 red ones, 4 green ones and 4 blue ones. What is the minimum number of marbles she must take out of her pocket to ensure that she has one of each color?

A. 4 B. 8 C. 12 D. 13 E. 11

9. Two integers will be randomly drawn from the sets $A = [2, 3, 4, 5]$ and $B = [4, 5, 6, 7, 8]$, one integer from set A and one integer from set B. What is the probability that the sum of the two integers will be 9?

A. 0.05 B. 0.15 C. 0.20 D. 0.25 E. 0.30

10. In a class there are 15 students from roll number 1 to 15. If you randomly pick 2 students, what is the probability that roll number of both students will be odd? [MBA Dec'15]

A. $\frac{8}{15}$ B. $\frac{3}{5}$ C. $\frac{4}{15}$ D. $\frac{2}{5}$ E. None of these

11. A jar contains 4 red toothpicks, 10 blue toothpicks and 6 yellow toothpicks. If three toothpicks are removed from the bag at random and no toothpick is returned to the bag after removed, what is the probability that all three toothpicks will be blue?

A. $\frac{1}{2}$ B. $\frac{1}{8}$ C. $\frac{3}{20}$ D. $\frac{2}{19}$ E. $\frac{1}{10}$

12. In a workshop there are 4 kinds of beds, 3 kinds of closets, 2 kinds of shelve and 7 kinds of chairs. In how many ways can a person decorate his room if he wants to buy in the workshop one shelf, one bed and one of the following: a chair or a closet?

A. 168 B. 16 C. 80 D. 48 E. 56

13. Three people are to be seated on a bench. How many different sitting arrangements are possible if Arif must sit next to Jony?

A. 2 B. 4 C. 6 D. 8 E. 10

14. In a flower shop, there are five different types of flowers. Two of the flower are blue, two are red and one is yellow. In how many different combinations of different colors can a 3 flower garland be made?

A. 4 B. 20 C. 3 D. 5 E. 6

15. There are 10 women and 3 men in room A. One person is picked at random from room A and moved to room B. Where there are already 3 women and 5 men. If a single person is then to be picked up from room B, what is the probability that a woman will be picked? [MBA June 2016]

A. $\frac{13}{21}$ B. $\frac{49}{117}$ C. $\frac{40}{117}$ D. $\frac{15}{32}$ E. None of these

16. There are six different models who are to appear in a fashion show. Two are from Europe, two are from South America and two are from North America. If all the models from the same continent are to stand next to each other, how many ways can the fashion show organizer arrange the models? [BBA 13-14]

A. 48 B. 64 C. 24 D. 8 E. 72

17. Event E is defined to be rolling an even number on a 6 sided die and event F is defined to be rolling a 1, 2 or 3. Calculate the probability of rolling a die such that events E and F occur simultaneously on a single roll of the die? [BBA 13-14]

A. $\frac{1}{2}$ B. $\frac{1}{6}$ C. 0 D. $\frac{5}{6}$ E. 1

18. The probability of rolling any number on a weighted 6 sided die, with faces numbered 1 through 6 is directly proportional to the number rolled. What is the probability of getting 5, if the die is rolled only once? [BBA 15-16]

A. $\frac{1}{6}$ B. $\frac{5}{6}$ C. $\frac{5}{16}$ D. $\frac{5}{21}$ E. None of these

19. A committee of 6 is chosen from 8 men and 5 women, so as to contain at least 2 men and 3 women. How many different committee could be formed if two of the men refuse to serve together?

A. 3510 B. 2620 C. 1404 D. 700 E. 635

20. Virat is one among the 15 members of a certain club. What is the ratio of the 4 member committees that can be formed from the members of the club including Virat to the number of 4 member committees that can be formed from the members of the club excluding Virat?

A. 3 to 2 B. 4 to 11 C. 1 to 10 D. 4 to 5 E. 1 to 15

21. In how many ways 5 different chocolates be distributed to 4 children such that any child can get any number of chocolate?

A. 20 B. 24 C. 120 D. 625 E. 1024

22. A term of 8 students goes on an excursion, in two cars, of which one can seat 5 and the other only 4. In how many ways can they travel?

A. 120 B. 126 C. 146 D. 156 E. 166

23. A certain bag contains 6 marbles of which 4 are red and 2 are white in color. If Babu is to pick out 2 marbles from the bag simultaneously and at random, What is the probability that one is red and the other is white? [BBA 16-17]

A. $\frac{1}{5}$ B. $\frac{1}{3}$ C. $\frac{1}{15}$ D. $\frac{8}{15}$ E. None of these

24. A box contain 2 white balls, 3 black balls and 4 red balls. In how many ways can 3 balls be drawn from the box, if at least one black ball is to be included in the draw?

A. 32 B. 48 C. 64 D. 96 E. None of these

25. There are 4 women and 4 men sitting in a waiting room for a job interview. If two of the interviewers are selected at random, what is the probability that both will be women? [MBA Dec'16]

A. $\frac{1}{2}$ B. $\frac{3}{7}$ C. $\frac{3}{4}$ D. $\frac{3}{14}$ E. None of these

Home Task

1. From a group of 7 men and 6 women five persons are to be selected to form a committee so that at least 3 men are there on the committee, in how many ways can it be done?

A. 756 B. 735 C. 645 D. 564 E. 654

2. What is the probability that the product of two integers (not necessarily different integers) randomly selected from the numbers 1 to 20, inclusive, is odd?

A. $\frac{1}{3}$ B. $\frac{1}{4}$ C. $\frac{1}{2}$ D. $\frac{1}{5}$ E. $\frac{2}{5}$

3. A box contains 10 electric bulbs from which 2 bulbs are defective. Two bulbs are chosen at random. What is the probability the one of them is defective?

A. $\frac{3}{10}$ B. $\frac{16}{45}$ C. $\frac{25}{68}$ D. $\frac{8}{33}$ E. $\frac{29}{68}$

4. 8 directors, the vice chairman and the chairman are to be seated around a circular table. If the chairman should sit between a director and the vice chairman, in how many ways can they be seated?

A. 9! B. 7!*2 C. 9!*2 D. 8!*2 E. 8!

5. The jones family is going on vacation. The members of the family will sit on one side of the aisle and three seats on the other. The two oldest children will sit together on one side that has only two seats. The parents and the youngest child will sit together on the other side. How many different seating arrangements are possible?

A. 5 B. 10 C. 12 D. 60 E. 120

6. Samia cannot completely remember her four-digit ATM pin number. She does remember the first two digit and she knows that each of the last two digits is greater than 5. The ATM will allow her three tries before it blocks further access. If she randomly guesses the last two digits, what is the probability that she will get access to her account?

A. $\frac{1}{2}$ B. $\frac{1}{4}$ C. $\frac{3}{16}$ D. $\frac{3}{18}$ E. $\frac{1}{32}$

7. The retirement plan for a company allows employees to invest in 10 different mutual funds. Six of the 10 funds grew by at least 10% over the last year. If Shahadat randomly selected 4 of the 10 funds, what is the probability that 3 of Shahadat's 4 funds grew by least 10% over last year?

A. ${}^{6}C_3 \div {}^{10}C_4$ B. $({}^{6}C_3 \times {}^{4}C_1) \div {}^{10}C_4$ C. $({}^{6}C_3 \times {}^{4}C_1) \div {}^{10}P_4$ D. $({}^{6}P_3 \times {}^{4}P_1) \div {}^{10}C_4$ E. $({}^{6}P_3 \times {}^{4}P_1) \div {}^{10}P_4$

8. How many 3 digit numbers satisfy the following condition: The first digit is different from zero and the other digits are all different from each other?

A. 648 B. 504 C. 576 D. 810 E. 672

9. Gracy has 8 shirts and 9 pants. How many clothing combinations does Gracy have, if she doesn't wear 2 specific shirts with 3 specific pants?

A. 41 B. 66 C. 36 D. 70 E. 56

10. Out of a box that contain, 4 black and 6 white mice, three are randomly chosen. What is the probability that all three will be black?

A. $\frac{8}{125}$ B. $\frac{1}{30}$ C. $\frac{2}{5}$ D. $\frac{1}{720}$ E. $\frac{3}{10}$

11. Dolly, Moly, Jolly flipped a coin 5 times and each time the coin landed on 'head'. Dolly bet that on the sixth time. The coin will land on 'tails', what is the probability that she's right?

A. 1 B. $\frac{1}{2}$ C. $\frac{3}{4}$ D. $\frac{1}{4}$ E. $\frac{1}{3}$

12. There are 18 balls in a jar. You take out 3 blue balls without putting them back inside and now the probability of putting out a blue ball is $\frac{1}{5}$. How many blue balls were there in the beginning?

A. 9 B. 8 C. 7 D. 12 E. 6

13. In a box there are A green balls, 3A + 6 red balls and 2 yellow one. If there are no other colors, what is the probability of taking out a green or a yellow ball?

A. $\frac{1}{5}$ B. $\frac{1}{2}$ C. $\frac{1}{3}$ D. $\frac{1}{4}$ E. $\frac{2}{3}$

14. The probability of Anik passing the exam is $\frac{1}{4}$. The probability of Anik passing the exam and Mimi passing the driving test is $\frac{1}{6}$, What is the probability Mimi passing her driving test?

A. $\frac{1}{24}$ B. $\frac{1}{2}$ C. $\frac{1}{3}$ D. $\frac{2}{3}$ E. $\frac{2}{5}$

15. Out of a classroom of 6 boys and 4 girls the teacher pick a president for the student board, a vice president and a secretary. What is the probability that only girls will be elected?

A. $\frac{8}{125}$ B. $\frac{2}{5}$ C. $\frac{1}{30}$ D. $\frac{1}{720}$ E. $\frac{13}{48}$

16. Two dice are rolled. What is the probability that the sum will be greater than 10?

A. $\frac{1}{9}$ B. $\frac{1}{12}$ C. $\frac{5}{36}$ D. $\frac{1}{6}$ E. $\frac{1}{5}$

17. The probability of having a girl is identical to the probability of having a boy. In a family with three children, what is the probability that all the children are of the same gender?

A. $\frac{1}{8}$ B. $\frac{1}{6}$ C. $\frac{1}{3}$ D. $\frac{1}{5}$ E. $\frac{1}{4}$

18. On one side of a coin there is the number 0 and on the other side the number is 1. What is the probability that the sum of three coin tosses will be 2?

A. $\frac{1}{8}$ B. $\frac{1}{2}$ C. $\frac{1}{5}$ D. $\frac{3}{8}$ E. $\frac{1}{3}$

19. A bag of 10 marbles contain 3 red marble and 7 blue marble. If two marbles are selected at random, what is the probability that at least one marble is blue?

A. $\frac{21}{50}$ B. $\frac{3}{13}$ C. $\frac{47}{50}$ D. $\frac{14}{15}$ E. $\frac{1}{5}$

20. How many different ways can 2 students be seated in a row of 4 desks, so that there is always at least one empty desk between the students?

A. 2 B. 3 C. 4 D. 6 E. 12

21. A committee of 3 people is to be chosen from the president and vice president of four different companies. What is the number of different committees that can be chosen if two people who work for the same company cannot both serve on the committee?

A. 16 B. 24 C. 28 D. 32 E. 40

22. A die is rolled and a coin is tossed, find the probability that the die shows an odd number and the coin shows a head.

A. 1 B. $\frac{1}{2}$ C. $\frac{1}{4}$ D. $\frac{1}{8}$ E. $\frac{1}{16}$

23. Three unbiased coins are tossed. What is the probability of getting at most two heads?

A. $\frac{3}{4}$ B. $\frac{1}{4}$ C. $\frac{3}{8}$ D. $\frac{7}{8}$ E. None of these

24. Abir has 4 painting in the basement. He is going to bring up 2 of them and hang 1 in his den and 1 in his bedroom. In how many ways can he choose which paintings go in each room?

A. 4 B. 6 C. 12 D. 16 E. 24

25. There are 50% chance that Rafiq will go to a fast food restaurant for lunch and 50% chance that he will skip lunch. If he goes to a fast food restaurant there is 60% chance that he will take only a sandwich and nothing else. If he decides not to take a sandwich, he will either take a burger or a chicken pie. What is the probability that he will take a burger? [BBA 16-17]

A. 5% B. 10% C. 20% D. 25% E. None of these