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MATH LECTURE - 01

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PART I: BASIC CONCEPTS

The prerequisite of doing well in Math section is to have a strong basic concept. Following are few basic concepts that will help you understand this section better. So before starting the lecture, have a look at those!

- In the number line, if you move to the right from any certain point, the value will increase gradually and vice versa.
- Real numbers are the values those include both positive and negative numbers such as −3, −2, −1, 0, 1, 2, 3, 3.5, 4.67….. etc.
- Integers are those numbers which have no fractions or decimals. (e.g. −1, −2, 0, 1 etc.)
- Natural numbers are the whole numbers used for counting (e.g. 1, 2, 3…..)
- A mixed number consists of a whole number and a fraction; e.g. $5\frac{1}{2}$ is a mixed number which means $5 + \frac{1}{2}$
- Any number divided by zero (0) becomes undefined. So a fraction cannot have 0 as a denominator.
- Consecutive integers are the integers that follow each other in order and have a certain difference between every two of them.
- Square root of a negative number cannot be a real number.
- The greater the absolute value of a negative number, the smaller it actually is. (e.g. $-10 < -9 < -8 < -7$)
- Squaring of any fraction between 0 and 1 results in a smaller number.
- Any number squared or raised to an even power is always positive.
- A prime number is that integer which has exactly two unique positive factors (1 and itself).
- 1 is not considered as a prime number.
- There is no even number other than 2 which can be a prime.
- Any integer greater than 1 is a prime or can be written as a product of primes; e.g. 30 = 2 x 3 x 5, 45= 3 x 3 x 5
- There are 25 prime numbers between 1 and 100.
- The product of even number of negative numbers is positive; e.g. $(-1)\times(-2)\times(-3)\times(-4) = 24$
- The product of odd number of negative numbers is negative; e.g. $(-1)\times(-2)\times(-3) = -6$
- If an expression has more than one set of parentheses, the inner parenthesis must be removed first and then the rest others have to be worked out.
- If two quantities are directly proportional in any equation, then when one increases, the other also increases.
- If two quantities are inversely proportional in any equation, then when one increases, the other decreases.

PART II: REVIEW LESSON

Properties of Integers:

Even-Odd: An even number is divisible by 2 and an odd number is not divisible by 2. All even numbers end with the digits 0, 2, 4, 6, or 8 while odd numbers end in the digits 1, 3, 5, 7, or 9. For example the numbers 358, 90, 18, 98, 74, and 46 are even numbers. The numbers 67, 871, 475, and 89 are odd numbers. It is important to remember the following facts:

The sum of two even numbers is even, and the sum of two odd numbers is even, but the sum of an odd and an even number is odd. **Example:** 4 + 8=12, 5 + 3 = 8 and 7 + 2 = 9.

The product of two odd numbers is odd, but the product of an even number and any other number is an even number. **Example:** $3 \times 5 = 15$ (odd); $4 \times 5 = 20$ (even); $4 \times 6 = 24$ (even).

Even numbers are expressed in the form 2k where k may be any integer. Odd numbers are expressed in the form of 2k + 1 or 2k ─ 1 where k may be any integer. For example, if k = 17, then 2k = 34 and 2k + 1 =35. If k = 6, then we have 2k = 12 and 2k + 1= 13.

Divisibility: There are various tests to see whether an integer is divisible by any certain number. These tests are listed below:

1. Any integer is divisible by 2 if the last digit of the number is 0, 2, 4, 6 or 8.
2. Any integer is divisible by 3 if the sum of its digits is divisible by 3.
3. Any integer is divisible by 4 if the number formed by the last two digits of the number is divisible by 4.
4. An integer is divisible by 5 if the last digit is either 0 or 5.
5. Any integer is divisible by 6 if it passes the divisibility tests for both 2 and 3.
6. Any integer is divisible by 8 if the number formed by the last three digits is divisible by 8. (Since 1000 is divisible by 8, you can ignore all multiples of 1000.)
7. Any integer is divisible by 9 if the sum of its digits is divisible by 9.
8. Any integer is divisible by 10 if the last digit is a 0.
9. Any integer is divisible by 11 if the sum of the odd-placed digits from the right minus the sum of the even-placed digits is either 0 or divisible by 11.

Factor: If a number P is divisible by a number Q, then Q is a factor of P. Again in this case, P will also be divisible by all the factors of Q. For example, 60 is divisible by 12, so 12 is a factor 60. Again 60 is also divisible by 2, 3, 4, and 6 which all are factors of 12.

Prime Numbers: Although the traditional definition states prime number as an integer that is divisible only by 1 and itself, the correct definition states a prime number is that integer which has exactly two unique factors. The first few prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37..... Note that the number 1 is not considered as a prime number.

To determine if a number is a prime, follow these steps:

STEP 1: Determine a rough approximate square root of the number.

STEP 2: Divide the number by all of the primes which are less than the approximate square root. If the number is not divisible by any of these primes, then it is a prime. If it is divisible by any of the primes, then it is not a prime.

Example: Is the number 97 prime?

Solution: An approximate square root of 97 is 10. All of the prime numbers less than 10 are 2, 3, 5, and 7. Divide 97 by 2, 3, 5, and 7. No integer results, so 97 is a prime.

Example: Is the number 161 prime?

Solution: An approximate square root of 161 is 13. The prime numbers less than 13 are 2, 3, 5, 7 and 11. Divide 161 by 2, 3, 5, 7 and 11. 161 is divisible by 7 (161 ÷ 7 = 23), so 161 is not a prime.

Average: The formula for calculating averages is $\dfrac{\text{Sum of the numbers}}{\text{No. of the numbers}}$

Example: What is the average of 10, 15, 24 and 32?

Solution: Sum of the numbers = 10 + 15 + 24 + 32 = 81. No. of the numbers = 4.

Average = 81/4 = 20.25

Evaluation of Expressions:

To evaluate an expression means to substitute a value in place of a letter.

Example: Evaluate $3a^2 - c^3$; if $a = -2$, $c = -3$.

Solution: $3a^2 - c^3 = 3(-2)^2 - (-3)^3 = 3(4) - (-27) = 12 + 27 = 39$

Example: Given that, $a \nabla b = ab + b^2$. Find the value of $-2 \nabla 3$.

Solution: Using the definition given, we get,

$-2 \nabla 3 = (-2)\times(3) + (3)^2 = -6 + 9 = 3$. Therefore, $-2 \nabla 3 = 3$

PART III: CLASS PRACTICE

GROUP 1: PROPERTIES OF INTEGER

1. If x is any integer, which of the following represents an odd number?
   a. 2x b. 2x + 3 c. 3x d. 2x + 2 e. x + 1

2. If u is an odd number, and v and w are different integers, which of the following must be even?
   a. uv + uw b. u + vw c. uvw d. u+v+w e. None of these

3. If n is an odd number, then which of the following best describes the number represented by $n^2 + 2n + 1$?
   a. It can be odd or even. b. It must be odd. c. It must be divisible by four.
   d. It must be divisible by six. e. Cannot be determined

4. If P is an even number, and Q and R are both odd, which of the following must be true?
   a. P.Q is an odd number. b. Q – R is an even number. c. PQ – PR is an odd number
   d. Q + R cannot equal P e. None of these

5. Which of the following represents the smallest possible value of $(M – 1/2)^2$ if M is an integer?
   a. 0.00 b. 0.25 c. 0.50 d. 0.75 e. 1.00

GROUP 2: DIVISIBILITY

6. If a number is divisible by 47, then it is also divisible by which of the following?
   a. 3 b. 7 c. 9 d. 11 e. None of these

7. Which of the following numbers is divisible by 11?
   a. 30,217 b. 44,221 c. 59,403 d. 60,411 e. None of these

8. Which of the following numbers is divisible by 36?
   a. 35,924 b. 64,530 c. 74,098 d. 152,640 e. 192,042

GROUP 3: PRIME NUMBERS

9. Which of the following numbers is a prime?
   a. 147 b. 149 c. 153 d. 155 e. 161

10. How many prime numbers are there between 35 and 70?
    a. 5 b. 6 c. 7 d. 8 e. 9

11. The sum of 5 consecutive integers is 35. How many of the five integers are prime numbers?
    a. 0 b. 1 c. 2 d. 3 e. 4

GROUP 4: AVERAGE PROBLEMS

12. If π is the first of five consecutive odd numbers, what is their average?
    a. π b. π + 1 *c. π + 2* d. π + 3 e. π + 4

13. What is the average of the following numbers: 35.5, 32.5, 34.0, 35.5, and 34.5?
    a. 33.0 b. 33.3 *c.* 34.0 d. 34.4 e. 34.5

14. What is the average of all multiples of ten from 10 to 190 inclusive?
    a. 90 b. 95 c. 100 d. 105 e. 110

15. If the average of five whole numbers is an even number, which of the following statements is not true?
   a. The sum of the five numbers must be divisible by 2.
   b. The sum of the five numbers must be divisible by 5.
   c. The sum of the five numbers must be divisible by 10.
   d. All of the five numbers must be odd.
   e. At least one of the five numbers must be even.

GROUP 5: APPROXIMATION PROBLEMS

16. Which of the following is the best approximation of the product 1.005 × 20.0025 × 0.0101?
   a. 0.02 b. 0.20 c. 2.00 d. 20.0 e. 200

17. If you multiply one million, two hundred thousand, one hundred seventy, by five hundred twenty thousand, two hundred five, and then divide the product by one billion, your result will be closest to:
   a. 0.6 b. 6 c. 600 d. 6,000 e. 6,000,000

18. Which of the following is the best approximation of the length of one side of a square with an area of 12 square inches?
   a. 3.1 inches b. 3.2 inches c. 3.3 inches d. 3.5 inches e. 3.6 inches

19. Which of the following numbers is closest to the square root of $\frac{1}{2}$?
   a. 0.25 b. 0.50 c. 0.60 d. 0.70 e. 0.80

GROUP 6: SERIES PROBLEMS

20. Which of the following is the next number in the series: 3, 6, 4, 9, 5, 12, 6, __?
   a. 7 b. 9 c. 12 d. 15 e. 24

21. What is the next number in the series: 1, 1, 2, 4, 5, 25, __?
   a. 8 b. 12 c. 15 d. 24 e. 26

GROUP 7: TRICKY QUESTIONS

22. A decrease of 1 from which one of the following numbers will result the greatest decrease in the product of 11 x 12 x 13 x 14 x 15?
   a. 11 b. 12 c. 13 d. 14 e. 15

23. The product of 8,754,896 and 48,933 equals =
   a. 428,403,325,965 b. 428,403,325,966 c. 428,403,325,967
   d. 428,403,325,968 e. 428,403,325,969

24. For all numbers x and y, x # y = xy + x. What is the value of 5 # 4?
   a. 9 b. 24 c. 25 d. 36 e. 41

25. If @ x denotes the greatest integer that is less than x, then @ (– 0.1) + @ (0.1) =

   a. –2 b. –1 c. 0 d. 1 e. 2

PART IV: TAKE-HOME ASSIGNMENT

1. If x is an odd number, what is the sum of next two odd numbers greater than 3x+1?
   a. 3x + 3   b. 6x+8   c. 6x+6   d. 6x+5   e. 6x+4

2. The sum of four consecutive odd integers must be:
   a. even, but not necessarily divisible by 4
   b. divisible by 4, but not necessarily by 8
   c. divisible by 8, but not necessarily by 16
   d. divisible by 16
   e. None of the above

3. If x and y are integers, for which of the following ordered pairs (x,y), is 2x + y an odd number?
   a. 0, 2   b. 1, 2   c. 2, 1   d. 2, 4   e. 3, 0

4. The sum of five odd numbers is always:
   a. even   b. divisible by three   c. divisible by five
   d. a prime number   e. None of these

5. If a is 3 less then b/2, which of the following gives b in terms of a?
   a. $\frac{a+3}{2}$ b. 2a + 3   c. 2(a – 3)   d. 2(3 – a)   e. 2(a + 3)

6. If a number is divisible by 51, then it is also divisible by:
   a. 23   b. 11   c. 7   d. 5   e. 3

7. When 16 and 9 are divided by n, the remainder is 2. What is the value of n?
   a. 3   b. 4   c. 5   d. 6   e. 7

8. Which of the following numbers is divisible by 24?
   a. 76,300   b. 78,132   c. 80,424   d. 81,234   e. 83,636

9. All numbers divisible by both 4 and 15 are also divisible by which of the following?
   a. 6   b. 8   c. 18   d. 24   e. 45

10. How many prime numbers are there between 56 and 100?
    a. 8   b. 9   c. 10   d. 11   e. None of these

11. If p is a prime number greater than 3, which of the following is NOT a factor of 6p?
    a. p^2 b. 6p   c. 3p   d. 2p   e. 3

12. If p and q are positive integers each greater than 1, and 17 (p+1) = 29 (q+1), what is the least possible value of p+q?
    a. 36   b. 42   c. 44   d. 46   e. None of these

13. What is the average of the following numbers: 91.4, 91.5, 91.6, 91.7, and 92.3?
    a. 91.6   b. 91.7   c. 91.8   d. 92.0   e. 92.1

14. In a class with six boys and four girls, the students all took the same test. The boys' scores were 74, 82, 48, 84, 88, and 95, while the girls' scores were 80, 82, 86, and 86. Which of the following statements is true?
    a. The boys' average was 0.1 higher than the class average.
    b. The girls' average was 0.1 lower than the boys' average.
    c. The class average was 2.0 higher than the boys' average.
    d. The boys' average was 1.0 higher than the class average.
    e. The girls' average was 1.0 lower than the boys' average.

15. If a, b, and c are all divisible by 8, then their average must be:
    a. divisible by 8   b. divisible by 4   c. divisible by 2
    d. an integer   e. None of these

16. $\frac{1}{3}$ of a number is 3 more than $\frac{1}{4}$ of the number. What is the average of 0 and this number?
a. 18 b. 24 c. 30 d. 36 e. 48

17. Which of the following is closest to the square root of $\frac{3}{5}$?
a. $\frac{1}{2}$ b. $\frac{2}{3}$ c. $\frac{3}{4}$ d. $\frac{6}{5}$ e. 1

18. Which of the following is the best approximation of the length of one side of a square auditorium with an area of 0.0121 square km?
a.0.90 km b. 0.61 km c. 0.11 km d. 0.006 km e. None

19. Which of the following is the best approximation of the product (2.005) × (10.0025) × (0.0205)?
a. 0.02 b. 0.40 c. 40 d. 0.025 e. None

20. After applying square root operation, which of the following has the closest value comparing to itself?
a. 0.3 b. 0.5 c. 0.7 d. 0.8 e. 0.9

21. What is the next number in the following series: 1, 4, 2, 8, 4, 16, 8, __?
a. 8 b. 20 c. 24 d. 32 e. None

22. What is the next term in the series: 9, 8, 6, 3, __?
a. 0 b. −2 c. 1 d. −3 e. −1

23. What is the next term in the series: 1, 1, 2, 3, 5, 8, 13, __?
a. 17 b. 21 c. 13 d. 9 e. 24

24. Which of the following is the largest?
a. $(2 + 2 + 2)^2$ b. $(2 \times 2 \times 2)^2$ c. $2+2^2+2^4$ d. $\{(2+2)^2\}^2$ e. 4^3

25. If A + B = 12, and B + C = 16, what is the value of A + C?
a. –4 b. –28 c. 4 d. 28 e. Cannot be determined

26. If # x = x^2 – x for all whole numbers, then # (# 3) =?
a. 27 b. 30 c. 58 d. 72 e. 121

27. For all real numbers except 0, x # y # z = $\frac{x+y}{z}$. What is the value of 9 # 3 # 2?
a. 1 b. 3 c. 6 d. 9 e. 12

28. For any value of x and y, x ❖ y = 2x + y. If 2 ❖ a = a ❖ 3, then what is the value of a?
a. 0 b. –1 c. 1 d. 1.5 e. 4

29. How many digits at most you may get in the number which is obtained by multiplying two 3-digit numbers?

a. 4 b. 5 c. 6 d. 7 e. 9

PART V: REVIEW LESSON FOR NEXT LECTURE

Rounding off:

Rounding off a number to a decimal place means finding the multiple of representative of that decimal place which is closest to the original number. Thus, rounding off a number to the nearest hundredth means finding the multiple of 100 which is closest to the original number. Rounding off to the nearest tenth means finding the multiple of 1/10 which is closest to the original number. After a number has been rounded off to a particular decimal place, all the digits to the right of that particular decimal place will be zero.

In any number, the non-zero digits from the most left is expressed as 1st significant figure or 1st place of accuracy and similarly, 2nd digit from the left is called 2nd significant figure or 2nd place of accuracy.
For example: Round off 3.210 to the nearest tenth.

Answer: 3.2

Round off 5320 to 1st significant figure.
Answer: 5000

Factorization:

To solve algebraic equations, factorization is necessary. Factorization is breaking down an expression into two or more expressions, the product of which is the original expression. For example, 6 can be factored into 2 and 3 because $2\times3 = 6$. Then, if $x^2 + dx + e$ is factorable, it will be factored into two expressions in the form $(x + d)$ and $(x + e)$. If the expression $(x + d)$ is multiplied by the expression $(x + e)$, their product is $x^2 + (d + e)x + de$. For example, $(x + 3)(x + 2)$ equals $x^2 + 5x + 6$. To factor an expression such as $x^2 + 6x + 8$, find a, d, and e such that $d + e = 6$ and $de = 8$. Of the various factors of 8, we find that $d = 4$ and $e = 2$. Thus, $x^2 + 6x + 8$ can be factorized into the expressions $(x +4)$ and $(x +2)$. Below are some factorized expressions:

$x^2 + 2x + 1 = (x + 1)(x + 1)$
$x^2 + 4x + 4 = x^2 +2x + 2x + 4 = (x + 2)(x + 2)$
$x^2 - 4x + 3 = x^2 - 3x - x + 3 = (x - 3)(x - 1)$
$x^2 + 10x + 16 = x^2 + 8x + 2x + 16 = (x + 8)(x + 2)$
$x^2 - 5x + 6 = x^2 - 3x - 2x + 6 = (x - 2)(x - 3)$

Inequalities:

These problems deal with numbers that are less than, greater than, or equal to other numbers. The following rules apply to all inequalities:

< means less than, thus $3 < 4$
> means greater than, thus $5 > 2$
≤ means less than or equal to, thus $3 \le 4$ and $3 \le 3$
≥ means greater than or equal to, thus $5 \ge 2$ and $2 \ge 2$

If equal quantities are added to both sides of an inequality, the direction of the inequality does not change.
If $x < y$, then $x + z < y + z$ and $x - z < y - z$.
If $x > y$, then $x + z > y + z$ and $x - z > y - z$.

Subtracting parts of an inequality from an equation /a number reverses the order of the inequality.
If $x < y$, then $z - x > z - y$.
If $x > y$, then $z - x < z - y$.

Multiplying or dividing an inequality by a number greater than zero does not change the order of the inequality.
If $x > y$, and $a > 0$, then $xa > ya$ and $x/a > y/a$.
If $x < y$, and $a > 0$, then $xa < ya$ and $x/a < y/a$.

Multiplying or dividing an inequality by a number less than zero changes the order of the inequality.
If $x > y$, and $a < 0$, then $xa < ya$ and $x/a < y/a$.
If $x < y$, and $a < 0$, then $xa > ya$ and $x/a > y/a$.
If $-3 < 2$ is multiplied through by -2 it becomes $6 > -4$ and the order of the inequality is reversed.

The product of two numbers with same signs is positive.
If $x > 0$ and $y > 0$, then $xy > 0$.
If $x < 0$ and $y < 0$, then $xy > 0$.

The product of two numbers with different signs is negative.
If $x < 0$ and $y > 0$, then $xy < 0$.

Linear inequalities in one unknown: In these problems, a first power variable is given in an inequality and this variable must be solved in terms of the inequality. Examples of linear inequalities in one unknown are: $2x + 7 > 4 + x$, $8y - 3 < 2y$, etc.

STEP 1: By ordinary algebraic addition and subtraction (as if it were an equation), get all of the constant terms on one side of the inequality and all of the variable terms on the other side. In the inequality $2x + 4 < 8x + 10$, subtract 4 and 8x from both sides and get $-6x < 12$.

STEP 2: Divide both sides by the coefficient of the variable. **[Important:** If the coefficient of the variable is negative, you must reverse the inequality sign. For example, in $-6x < 12$, dividing by -6 gives $x > 2$ (The inequality is reversed).**]** In case of $3x < 12$, dividing by 3 gives $x < 4$.

- Solve for y in the inequality $4y + 7 \geq 9 - 2y$.
  Solution: Subtracting $-2y$ and 7 from both sides gives $6y \geq 2$. Dividing both sides by 6 gives $y \geq 1/3$.
- Solve for a in the inequality $10 - 2a < 0$.
  Solution: Subtracting 10 from both sides gives $-2a < -10$. Dividing both sides by -2 gives $a > \frac{-10}{-2}$, or $a > 5$. **[Note:** The inequality sign has been reversed because of the division by a negative number.**]**

Exponents:

An exponent is an easy way to express repeated multiplication. For example, $5x5x5x5 = 5^4$. The 4 is the exponent. In the expression $7^3 = 7 \times 7 \times 7$, 3 is the exponent. 7^3 means 7 is multiplied by itself three times. If the exponent is 0, the expression always has a value of 1. Thus, $6^0 = 15^0 = 1$, etc. If the exponent is 1, the value of the expression is the number base. Thus, $4^1 = 4$ and $9^1 = 9$.

In the problem $5^3 \times 5^4$, we can simplify by counting the factors of 5. Thus $5^3 \times 5^4 = 5^{3+4} = 5^7$. When we multiply and the base number is the same, we keep the base number and add the exponents. For example, $7^4 \times 7^8 = 7^{12}$. In short, $a^m \times a^n = a^{m+n}$

For division, we keep the same base number and subtract exponents. Thus, $8^8 \div 8^2 = 8^{8-2} = 8^6$. In short, $a^m \div a^n = a^{m-n}$

A negative exponent indicates the reciprocal of the expression with a positive exponent, thus, $3^{-2} = 1/3^2$. In short, $a^{-m} = 1/a^m$.

Roots:

The square root of a number is a positive number which when multiplied by itself gives the original number. For example, $\sqrt{16} = 4$, since $4 \times 4 = 16$

To simplify a square root, we factorize the number.

$\sqrt{32} = \sqrt{(16 \times 2)} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$

$\sqrt{72} = \sqrt{(36 \times 2)} = \sqrt{36} \times \sqrt{2} = 6\sqrt{2}$

We can add expressions with the square roots only if the numbers inside the square root sign are the same. For example,

$\sqrt{7} + 2\sqrt{7} = 5\sqrt{7}$

$\sqrt{18} + \sqrt{2} = \sqrt{(9 \times 2)} + \sqrt{2} = \sqrt{9} \times \sqrt{2} + \sqrt{2} = 4\sqrt{2}$.

L.C.M. and H.C.F.

You have to note that L.C.M. is the abbreviation of lowest common multiple and H.C.F is the abbreviation of highest common factor.

(i) The product of two natural numbers is equal to the product of their H.C.F and their L.C.M.
1st number □ 2nd number = L.C.M □ H.C.F

(ii) H.C.F of fractions = $\dfrac{H.C.F.\text{ of Numerators}}{L.C.M.\text{ of Denominators}}$

(iii) L.C.M of fractions = $\dfrac{L.C.M.\text{ of Numerators}}{H.C.F.\text{ of Denominators}}$

Face Value of Currencies:

Value = Rate x Number of items

Example: Amitabh has $3.00 in nickels and dimes in his pocket. If he has twice as many nickels as he has dimes, how many coins does he have altogether?

Here, Rate □ Number of coins = Value

	cents/coin	Coins	Cents
Nickels	5	2c	10c
Dimes	10	c	10c

Now we recall the additional bit of information that the total value of the nickels and dimes is $3.00, or 300 cents. Thus, $5 \times 2c + 10c = 300$; $20c = 300$; so, $c = 15$, the number of dimes. Rafi has twice as many nickels, so, $2c = 30$.

The total number of coins is $c + 2c = 3c = 45$.

Remember:

1 dollar = 100 cents

1 nickel = 5 cents

1 dime = 10 cents

1 quarter = 25 cents

Half dollar = 50 cents