

IBA

Name :

Batch:

MATH LECTURE - 04

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PART I: CLASS PRACTICE

GROUP 1: RATIO & PROPORTION

- On a blueprint of a park, 1 foot represents 1 mile. If an error of $\frac{1}{4}$ inch is made in reading the blueprint, what will be the corresponding error on the actual park? [1 mile = 5280 feet]
a. 110 feet b. 220 feet c. 330 feet d. 440 feet e. None of these
- On a blueprint, $\frac{1}{4}$ inch represents 1 foot. If a window is supposed to be 60 inches wide, how wide would its representation be on the blueprint?
a. $1\frac{1}{6}$ inches b. $1\frac{1}{4}$ inches c. $1\frac{1}{3}$ inches d. 15 inches e. $18\frac{2}{3}$ inches
- A jar contains black and white marbles. If there are 20 marbles in the jar, all of the following could be the ratio of black to white marbles EXCEPT:
a. 9:1 b. 7:3 c. 1:1 d. 1:4 e. 1:10
- If $\frac{1}{3}$ of the girls at a school equals $\frac{1}{5}$ of the total number of students, then what is the ratio of girls to boys at that school?
a. 5:3 b. 3:2 c. 2:5 d. 1:3 e. 1:5

GROUP 2: UNITARY METHOD

- If 8 men can cut down 28 trees in one day, how many trees can 20 men cut down in one day?
a. 28 trees b. 100 trees c. 160 trees d. 70 trees e. 80 trees
- 2 men or 3 women can survive with some food for 10 days. 4 men and 3 women together can survive with the same food for how many days?
a. $\frac{8}{3}$ b. $\frac{3}{10}$ c. 3 d. $\frac{10}{3}$ e. $\frac{11}{2}$
- 7 workers can dig a canal in 14 weeks by working 8 hours daily. In how many weeks, 14 workers can do the same work by working 7 hours daily?
a. 12 b. 9 c. 8 d. 7 e. None of these
- A group of people can live on X liter of water for D days in a trip. Then Q people can live on Y liter of water for how many days?
a. $\frac{DPY}{XQ}$ b. $\frac{DPX}{YQ}$ c. $\frac{DXY}{PQ}$ d. $\frac{DPQ}{XY}$ e. None of these

GROUP 3: MIXTURE

- Coffee A normally costs 75 paise per pound. It is mixed with Coffee B, which normally costs 80 paise per pound, to form a mixture, which costs 78 paise per pound. If there are 10 pounds of the mix, how many pounds of Coffee A were used in the mix?
a. 3 b. 4 c. 4.5 d. 5 e. 6
- How many pounds of chocolate worth \$1.20 per pound must be mixed with 10 pounds of chocolate worth 90 cents per pound to produce a mixture worth \$1.00 per pound?
a. 3 b. 5 c. 10 d. 15 e. 20
- A 30% solution of Grape Juice is mixed with 10 grams of water to form a 20% solution. How many grams of the original solution did we start with?
a. 10 b. 15 c. 20 d. 25 e. 30

12. How many liters of water should be added to a 30 liter mixture of milk and water containing milk and water in the ratio of 7:3 so that the resultant mixture has 40% water in it?
 a. 7 b. 10 c. 5 d. 12 e. None of these
13. A chemistry experiment calls for a 30% sulfuric acid solution. The lab supply room has only 50% and 20% sulfuric acid solutions on hand which will be used to prepare the desired solution. How much 50% sulfuric acid solution should be mixed in the mix to obtain 12 liters of a 30% solution?
 a. 3 liters b. 4 liters c. 6 liters d. 8 liters e. None

GROUP 4: SIMPLE & COMPOUND INTEREST

14. How much simple interest will \$2000 earn in 18 months at an annual rate of 6%?
 a. Tk. 90 b. 120 c. 140 d. 160 e. 180
15. A moneylender charged Tk. 25 as simple interest on a loan of Tk. 150 for $\frac{1}{6}$ years. What was the rate of interest per annum?
 a. 125 b. 50 c. 75 d. 25 e. 100
16. In how many years taka 1800 will become taka 2250 if the simple interest rate is 5% p.a.?
 a. 3 b. 4 c. 5 d. 6 e. None
17. Nuha puts Tk. 100 in the bank for two years at 5% interest compounded annually. At the end of the two years, what will be her balance?
 a. Tk. 100.00 b. Tk. 105.00 c. Tk. 105.25 d. Tk. 110.00 e. Tk. 110.25
18. A sum of money was put into bank. After 1 year the money grew to \$330. If the rate of interest was 10% compounded annually, what was the initial amount put into the bank?
 a. \$250 b. \$280 c. \$300 d. \$310 e. None of these

GROUP 5: AVERAGE / MEAN, MEDIAN

19. In order to graduate, Abrar needs an average of 65 percent for his five major subjects. His first four grades were 55, 60, 65, and 65. What grade does he need in the fifth subject in order to graduate?
 a. 65 b. 70 c. 75 d. 80 e. 85
20. The average of 7 numbers is 30. If the average of the first three numbers is 25 and that of the last three is 35, the fourth number is:
 a. 25 b. 34 c. 36 d. 39 e. None of these
21. The average age of a committee of 8 members is 40 years. A member aged 55 years retired and another member aged 39 years took his place. The average age of the present committee is:
 a. 29 years b. 38 years c. 21 years d. 25 years e. None of these
22. The average age of 5 students from 26th batch is 8 years. If the age of Neelim is included, it is increased by 7 years. Find the age of Neelim.
 a. 40 years b. 45 years c. 50 years d. 55 years e. None of these
23. The average height of a group of 25 students goes up by 2 cm when a new student of height 165 cm replaces an old student from the group. The height of the boy who went out of the group was:
 a. 115 cm b. 112 cm c. 114 cm d. 116 cm e. None of these
24. The average age of a husband and wife, who were married 7 years ago, was 25 years. At present, the average age of the family including the husband, the wife and a child who was born during the interval is 22 years. How old is the child now?
 a. 2 years b. 3 years c. 4 years d. 6 years e. None of these

| Test Score | Number of students |
|------------|--------------------|
| 90 | 2 |
| 85 | 1 |
| 80 | 1 |
| 60 | 3 |

25. The test scores of 7 students are shown above. Let 'M' and 'm' be the median and mean scores respectively. What is the value of M – m?
 a. 3 b. 4 c. 5 d. 6 e. 7

PART II: TAKE HOME ASSIGNMENT

1. Which of the following expresses the ratio of 3 inches to 2 yards?
a. 3:2 b. 1:9 c. 1:4 d. 1:8 e. 1:24
2. If coconuts are twice as expensive as bananas and bananas are one-third as expensive as grapefruits, what is the ratio of the price of a coconut to that of a grapefruit?
a. 2:3 b. 3:2 c. 6:1 d. 1:6 e. None of these
4. If the ratio of Tarique's allowance to Neelim's allowance is 3:2, and the ratio of Tarique's allowance to Zafar's allowance is 3:4, what is the ratio of Neelim's allowance to Zafar's allowance?
a. 8:9 b. 2:5 c. 1:2 d. 3:4 e. 9:8
5. On a map, 1 inch represents 1000 miles. If the area of a country is actually 16 million square miles, what is the area of the country's representation on the map?
a. 4 sq. inches b. 16 sq. inches c. 4,000 sq. inches
d. 16,000 sq. inches e. 4×10^6 sq. Inches
6. If 10 men can survive for 24 days on 15 cans of rations, how many cans will be needed for 8 men to survive for 36 days?
a. 15 cans b. 16 cans c. 17 cans d. 18 cans e. 19 cans
7. If four men need tk. 24.00 worth of food for a three-day camping trip, how much will two men need for a two-week trip?
a. tk. 12.00 b. tk. 24.00 c. tk. 28.00 d. tk. 42 e. tk. 56
8. If a worker can pack $\frac{1}{6}$ of a carton of canned food in 15 minutes and there are 40 workers in a factory, how many cartons should be packed in the factory in $1\frac{1}{2}$ hours?
a. 16 b. 40 c. 45 d. 90 e. None of these
9. A bag of chicken feed will feed 18 chickens for 54 days. For how many days will it feed 12 chickens?
a. 36 b. 37 c. 53 d. 72 e. 81
10. Ten pints of 15% salt solution is mixed with 15 pints of 10% salt solution. What is the concentration of the resulting solution?
a. 10% b. 12% c. 12.5% d. 13% e. 15%
11. A grocer mixes peanuts that cost \$2.50 per pound and walnuts that cost \$3.90 per pound to make 100 pounds of a mixture that costs \$3.20 per pound. How many pounds of \$2.50 per pound nut is put into the mixture?
a. 30 b. 40 c. 45 d. 50 e. 60
12. How many cubic centimeters of water must be added to 100 cc of 80% solution of Boric Acid to reduce it to a 50% solution?
a. 40 b. 45 c. 50 d. 55 e. 60
13. A man adds two quarts of pure alcohol to a 30% solution of alcohol in water. If the new concentration is 40%, how many quarts of the original solution were there?
a. 12 b. 15 c. 18 d. 20 e. 24
14. How many ounces of water must be added to 48 ounces of alcohol to make a solution that is 25% alcohol?
a. 140 b. 144 c. 148 d. 152 e. 156
15. A sum of money at compound interest amounts to thrice itself in 3 years. In how many years will it be 9 times itself?
a. 18 b. 12 c. 9 d. 6 e. 10

16. Tk. 75 is charged as a simple interest on a loan which is taken for 3 years at an interest rate of 5% per annum. What was the amount of loan in taka?
 a. 225 b. 350 c. 475 d. 500 e. 550
17. In how many years taka 1500 will become taka 1860 if the simple interest rate is 6% p.a.?
 a. 3 b. 4 c. 5 d. 6 e. None
18. Safa who is weak in maths, puts Tk. 1000 in the bank for two years at 10% interest compounded annually. At the end of the two years, what will be her balance?
 a. Tk. 210 b. Tk. 1100 c. Tk. 1200 d. Tk. 1210 e. Tk. None
19. A sum of \$5000 was taken as a loan. After 1 year the money grew to \$5500 by compounding annually. What was the rate of interest per annum?
 a. 5% b. 8% c. 10% d. 12% e. None of these
20. The average allowance per head of the entire staff of an office including the officers and the clerks is Tk. 60. The average allowance per head of the officers is Tk. 400 and that of the clerks is Tk. 56. If there are twelve officers, find the number of clerks in the office.
 a. 1020 b. 1376 c. 1074 d. 1078 e. None of these
21. The average expenditure of a man for the first five months is Tk. 120 and for the next seven months, it is Tk. 130. Find his monthly average income if he saves Tk. 290 during the year.
 a. Tk. 140 b. Tk. 150 c. Tk. 350 d. Tk. 450 e. None of these
22. The average of the runs made by 10 players in a cricket team of 11 players is 43. If the eleventh player's runs are also considered, the average of 11 players' run decreases by 1. How many runs did the eleventh player score?
 a. 22 b. 32 c. 44 d. 48 e. None of these
23. The average temperature of a town in the first four days of a month was 58° . The average for the second, third, fourth and fifth days was 66° . If the temperatures on the first and the fifth days were in the ratio 7:11, find the temperature on these days.
 a. $56^\circ, 88^\circ$ b. $60^\circ, 70^\circ$ c. $80^\circ, 65^\circ$ d. $35^\circ, 55^\circ$ e. None of these
24. Samiha buys 16 cookies, Zafar buys 12 cookies, and Nihat buys 'x' cookies. The average number of cookies the three of them bought is between 19 and 23, inclusive. What is the smallest number of cookies Nihat could have bought?
 a. 35 b. 31 c. 30 d. 29 e. 28
25. If the average of 5 consecutive even integers is n, what is the median of these 5 integers?
 a. 0 b. 2 c. n d. $n - 2$ e. $n - 4$

PART III: REVIEW LESSON FOR THE NEXT LECTURE

Speed, Distance and Time

Distance = Speed \times Time

Example: Peter can walk a mile in 10 minutes. He can travel a mile on his bicycle in 2 minutes. How far away is his uncle's house if Peter can walk there and return on his bicycle back in 1 hour exactly?

Solution: To solve a rate problem such as the one above, follow these steps:

Step 1: Determine the names of the quantities that represent input, output, and rate in the problem you are doing. In the example, Peter's input is time and his output is distance. His rate will be distance per unit of time, which is commonly called speed.

Step 2: Write down the fundamental relationship in terms of the quantities mentioned, making each the heading of the column. In the example, set up the table like this:

$$\text{Speed} \times \text{Time} = \text{Distance}$$

Step 3: Directly below the name of each quantity, write the unit of measurement in terms of the answer you want. Your choice of unit should be the most convenient one, but remember, once you have chosen a unit, you must convert all quantities to that unit.

We must select a unit of time. Since a minute was the unit used in this problem, it is the most logical choice. Similarly, we will choose a mile for our unit of distance. Speed (which is the ratio of distance to time) will therefore be expressed in miles per minute, usually abbreviated as mile/min. Thus, our chart now looks like this:

| Speed × Time = Distance | | |
|-------------------------|---------|-------|
| mile/min | Minutes | Miles |

Step 4: The problem will mention various situations in which some quantity of input is used to get a certain quantity of output. Represent each of these situations on a different line of the table, leaving blanks for unknown quantities.

In the sample problem, four situations are mentioned: Peter can walk a mile in 10 minutes; he can cover a mile on his bicycle in 2 minutes; he walks to his uncle's house; and he returns on his bicycle home. In the diagram, with the appropriate boxes filled, the problem will look like this:

| Speed × Time = Distance | | | |
|-------------------------|----------|---------|-------|
| | mile/min | Minutes | Miles |
| 1. Walking | | 10 | 1 |
| 2. Bicycling | | 2 | 1 |
| 3. Walking | | | |
| 4. Bicycling | | | |

Step 5: From the chart and from the relationship at the top of the chart, quantities for filling some of the empty spaces may become obvious. Fill in these values directly.

In the example, on the first line of the chart, we see that the walking speed times 10 equals 1. Thus, the walking speed is 0.1 mi./min. ($\text{mi./min} = 1 \text{ mi./}10 \text{ min.} = 0.1$). Similarly, on the second one, we see that the bicycle speed (0.05) shown on line 2. Adding this information to our table, we get:

| Speed × Time = Distance | | | |
|-------------------------|----------|---------|-------|
| | mile/min | Minutes | Miles |
| 1. Walking | 0.1 | 10 | 1 |
| 2. Bicycling | 0.5 | 2 | 1 |
| 3. Walking | 0.1 | | |
| 4. Bicycling | 0.5 | | |

Step 6: Next, fill in the blanks with algebraic expressions to represent the quantities indicated, being careful to take advantage of simple relationships stated in the problem or appearing in the chart. Continuing the example, we represent the time spent traveling shown on line 3 by x . According to the fundamental relationship, the distance traveled on this trip must be $0.1x$. Similarly, if y represents the time shown on line 4, the distance traveled is $0.5y$. Thus our chart now looks like this:

| Speed × Time = Distance | | | |
|-------------------------|----------|---------|--------|
| | mile/min | Minutes | Miles |
| 1. Walking | 0.1 | 10 | 1 |
| 2. Bicycling | 0.5 | 2 | 1 |
| 3. Walking | 0.1 | x | $0.1x$ |
| 4. Bicycling | 0.5 | y | $0.5y$ |

Step 7: Now, from the statement of the problem, you should be able to set up enough equations to solve for all the unknowns. In the example, there are two facts, which we have not used yet. First, since Peter is going to his uncle's house and back, it is assumed that the distances covered on the two trips are equal. Thus we get the equation: $0.1x = 0.5y$. We are told that the total time to and from his uncle's house is one hour. Since we are using minutes as our unit of time, we convert the one hour to 60 minutes. Thus we get the equation: $x + y = 60$. Solving these two equations ($0.1x = 0.5y$ and $x + y = 60$) algebraically, we find that $x = 50$ and $y = 10$.

Step 8: Now that you have all the information necessary, you can calculate the answer required. In the sample problem, we are required to determine the distance to the uncle's house which is $0.1x$ or $0.5y$. Using $x = 50$ or $y = 10$ gives us the distance as 5 miles.

Example: In a sports car race, David gives Kenny a head start of 10 miles. David's car goes 80 miles per hour and Kenny's car goes 60 miles per hour. How long should it take David to catch up to Kenny if they both leave their starting marks at the same time?

Solution:

Speed \times Time = Distance

| | m/hr. | Hours | Miles |
|-------|-------|-------|-------|
| Kenny | 60 | X | 60x |
| David | 80 | X | 80x |

From the statement of the problem, we know that David gave Kenny a 10 miles head start. In other words, David's distance is 10 more miles than Kenny's distance. This can be stated algebraically as $60x + 10 = 80x$ that is, Kenny's distance + 10 miles = David's distance. Solving for x gives us $x = \frac{1}{2}$

Rowing Boat, Current & Speed

Another common math word problem is "boat-in-the-river", where speed of the boat and current can be found. These are actually just a variation of the dreaded uniform motion word problems. With the boat in the river problems, we assume that the boat has a uniform speed in still water and that the speed of the water (or the speed of the current in the river) is constant. You will be presented with a math word problem in which you have to solve the speed of the boat (in still water, implied), or the speed of the river current, or the time spent going upstream or the time going downstream or the distance travelled or some combination of these variables.

Downstream means with the direction of the river current and Upstream means against the direction of the river current.

With these problems, following variables may be used:

B = speed of the boat in still water

C = the speed of the current

T_d = time spent going downstream

T_u = time spent going upstream

D_d = distance gone downstream

D_u = distance gone upstream

As with other uniform motion problems, an important algebraic equation to remember is:

Distance = Speed \times Time

As in downstream, we are going with the direction of current, so it will work to increase our speed. Hence, we modify this slightly when we are going downstream to get the actual speed.

$$D_d = (B + C) \times T_d$$

So Downstream speed = $B + C$

That is, the speed (or rate) of the boat going downstream is the speed of the boat in still water plus the speed of the current.

But in downstream, we are going against the direction of current, so it will work to decrease our speed.

So we have to change our distance equation to reflect this by subtracting the speed of the current from the speed of the boat to get our speed.

$$D_u = (B - C) \times T_u$$

So Upstream speed = $B - C$

Train & Speed

This is just another version of speed, distance, time word problems and the same formula (**Distance = Speed \times Time**) will be used. Some important points to be kept in mind while solving this particular type of problems. The principle about objects moving in opposite directions toward each other is- the relative speed which comes into count is the sum of the speeds of that two objects. Hence, we add two speeds when they move into opposite direction.

Again, when two objects move in the same direction, the relative speed is the difference between the speeds of those objects. Thus we subtract the slower object's speed from the faster object's speed. Solving this type of problems, we always use relative speed into the formula, distance = speed \times time, when both the objects are moving.

Nothing complicated about the distance or length! Irrespective to direction, when two objects have significant lengths, you can always add the lengths of two objects to find out the distance/length that is to be put in the formula; e.g. two trains crossing each other or a train crossing a bridge or a train crossing a platform etc.

If one object doesn't have any significant length, you don't need to add anything with the length of the train. Here, distance in the formula will only be the length of the train. Such cases may be- a train crossing a certain point or a train crossing a man etc.

Name.....

Review Test on lecture 3
10 marks, 10 minutes

Batch.....

- Shyam gave half of his money to Fahim, $\frac{2}{5}$ to Ali and $\frac{3}{10}$ to Nadif and had only \$20 left. How much money did he have in the beginning?
a. \$4000 b. \$400 c. \$40 d. €400 e. None of these
- What is 0.05%, expressed as a fraction?
a. $\frac{2}{5}$ b. $\frac{1}{20}$ c. $\frac{4}{25}$ d. $\frac{1}{200}$ e. $\frac{1}{2000}$
- A certain company increased its price by 30%. But later, it was forced to cut back its price by 20%. What was the net change in price?
a. A net decrease in price of more than 10% b. A net decrease in price of 10% or less
c. No net change in price d. A net increase in price of 10% or less
e. A net increase in price of more than 10%
- Mahin bought a whip listed at Tk. 400. He was given 15% and 10% discounts respectively. How much did he pay for the whip?
a. 306 b. 350 c. 360 d. 316 e. None of these
- Mr. Sayem, who owns $\frac{2}{3}$ of a restaurant, sells half of his share for \$ 33,333 to Rashik. How much is the value of the entire restaurant?
a. \$33,333 b. \$50,000 c. \$66,666 d. \$75,000 e. \$99,999
- A dress shop marked down all merchandise as follows:

| Group | Regular price | Sale price |
|-------|---------------|------------|
| A | \$60 | \$50 |
| B | \$65 | \$55 |
| C | \$70 | \$60 |
| D | \$75 | \$65 |
| E | \$80 | \$70 |

Which group of the merchandise was offered at the greatest rate of discount from its original price?
a. A b. B c. C d. D e. E
- A merchant marks a certain lamp up 50% above cost. Then he gives a customer a 20% discount. If the final selling price of the lamp was \$110.4, what was the approximate cost price?
a. \$78 b. \$86.20 c. \$92 d. \$92.50 e. \$99
- What is 'a' percent of 'b' divided by 'b' percent of 'a'?
a. $\frac{a}{b}$ b. $\frac{b}{a}$ c. $a \times b$ d. a e. 1
- Of the 120 people in a room, $\frac{3}{5}$ are women. If $\frac{2}{3}$ of the people are married, what is the maximum number of women in the room who could be unmarried?
a. 80 b. 72 c. 48 d. 40 e. 32
- After applying successive discounts of 10% and 5% on an article, it was sold at Tk. 513. Find the marked price of the article.
a. 590 b. 600 c. 603.5 d. None of these e. Cannot be determined

Answer Sheet

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-
-
-
-
-
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-
-

SCORE.....

REMARKS