Chapter Eight

Activity page no. 575

Trigonometry

Exercise-8.3

After completing the chapter, the students will be able to—

1. find the ratios of negative angles (– θ).
2. find the trigonometric ratios of different angles.
3. find the trigonometric ratios of the angle (n. $\frac{\pi}{2} \pm \theta$) for integer n(n≤4).
4. solve simple trigonometric equations.

Exercise Questions

■ 15 Exercise Questions

Exercise Questions and Solutions

Practice the Solutions of this part properly. It will help you to solve the Creative Questions easily.

1. If sinA = $\frac{1}{\sqrt{2}}$ then find the value of sin2A.

ⓐ $\frac{1}{\sqrt{2}}$ ⓑ $\frac{1}{2}$ ⓒ 1 ⓓ $\sqrt{2}$ **ⓒ**

Explanation: sinA = $\frac{1}{\sqrt{2}}$

sin A = sin 45° ∴ A = 45°

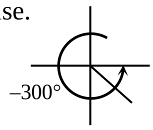
Now, sin2A = sin (2 × 45°) = sin 90° = 1

2. In which quadrant the angle 300° lie?

ⓐ First ⓑ Second

ⓒ Third ⓓ Fourth **ⓐ**

Explanation: The direction of negative angle is clockwise and the direction of positive angle is anti-clockwise.



3. If sinθ + cosθ = 1 then find the value of θ.

i. 0° ii. 30° iii. 90°

Which is correct of the following**?**

ⓐ i and ii ⓑ i and iii

ⓒ ii and iii ⓓ i,ii and iii **ⓑ**

Explanation:

i. The equation is satisfied by 0° ∴ θ = 0°

ii. " " does not " " 30°.

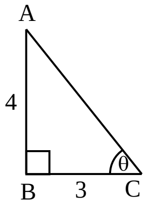
iii. " " is " " 90° ∴ θ = 90°

4. From the figure-

i. tanθ = $\frac{4}{3}$

ii. sinθ = $\frac{5}{3}$

iii. $\cos^2\theta = \frac{9}{25}$



Which is correct of the following?

ⓐ i and ii ⓑ i and iii

ⓒ ii and iii ⓓ i,ii and iii **ⓑ**

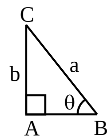
Explanation: Here, opposite side = 4 unit, adjacent side = 3 unit and hypotenuse = $\sqrt{4^2 + 5^2}$ = 5 unit

i. correct, because tanθ = $\frac{\text{opposite side}}{\text{adjacent side}} = \frac{4}{3}$

ii. not correct, because sinθ = $\frac{\text{opposite side}}{\text{hypotenuse}} = \frac{4}{5}$

iii. correct, because $\cos^2\theta = \frac{(\text{adjacent side})^2}{(\text{hypotenuse})^2} = \frac{9}{25}$

Answer to the questions 5 and 6 on the basis of the following figure:



5. sinB + cosC = solve?

ⓐ $\frac{2b}{a}$ ⓑ $\frac{2a}{b}$

ⓒ $\frac{a^2 + b^2}{ab}$ ⓓ $\frac{ab}{a^2 + b^2}$ **ⓐ**

Explanation: sinB + cosC = $\frac{AC}{BC} + \frac{AC}{BC}$

$= \frac{b}{a} + \frac{b}{a} = \frac{2b}{a}$

6. Which is the value of tanB?

ⓐ $\frac{a}{a^2 - b^2}$ ⓑ $\frac{b}{a^2 - b^2}$

ⓒ $\frac{a}{\sqrt{a^2 - b^2}}$ ⓓ $\frac{b}{\sqrt{a^2 - b^2}}$ **ⓓ**

Explanation: tanB = $\frac{AC}{AB} = \frac{b}{\sqrt{a^2 - b^2}}$

[Since, AB = $\sqrt{BC^2 - AC^2} = \sqrt{a^2 + b^2}$]

[**N.B.:-** The equation given in the text book is not correct**]**

7. Find the value of:

(i) sin 7π

Solution: $\sin 7\pi = \sin\left(14.\frac{\pi}{2}+0\right)$. Here, n = 14 is an even number. So, sin will remain unchanged and the sign of sin will be negative, since the angle lies in the third quadrant.

$\therefore \sin\left(14.\frac{\pi}{2}+0\right) = -\sin 0° = 0$

∴ The required value = 0

(ii) $\cos\frac{11\pi}{2}$

Solution: $\cos 11.\frac{\pi}{2} = \cos\left(11.\frac{\pi}{2}+0\right)$

Here, n = 11 is an odd number. So, cos will be changed into sin, and the sign of cos will be positive, since the angle lies in the forth quadrant.

$\therefore \cos\frac{11\pi}{2} = \cos\left(11.\frac{\pi}{2}+0\right) = \sin 0 = 0$

∴ The required value = 0

(iii) cot11π

Solution: $\cot 11\pi = \cot\left(22.\frac{\pi}{2}+0\right)$

Here, n = 22 is an even number. So, cot will remain unchanged and sign of cot will be positive, since the angle lies in the third quadrant. cot11π

$= \cot\left(22.\frac{\pi}{2}+0\right) = \cot 0 =$ Undefined.

(iv) $\tan\left(-\frac{23\pi}{6}\right)$

Solution: $-\tan\frac{23\pi}{6}$ [Since, tan (– θ) = – tanθ]

$= -\tan\left(4\pi - \frac{\pi}{6}\right)$

$= -\tan\left(8\times\frac{\pi}{2} - \frac{\pi}{6}\right)$

Here, n = 8 is an even number, so tan will remain unchanged and the sign of tan will be negative, since the angle lies in the fourth quadrant.

$= \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

∴ The required value = $\frac{1}{\sqrt{3}}$

(v) $\operatorname{cosec}\frac{19\pi}{3}$

Solution: $\operatorname{cosec}\frac{19\pi}{3} = \operatorname{cosec}\left(6\pi+\frac{\pi}{3}\right)$

$= \operatorname{cosec}\left(12\times\frac{\pi}{2}+\frac{\pi}{3}\right)$

Here, n = 12, even number, so cosec will remain unchanged and the sign of cosec will be positive, since the angle lies in the fourth quadrant.

$\therefore \operatorname{cosec}\frac{19\pi}{3} = \operatorname{cosec}\left(12\times\frac{\pi}{2}+\frac{\pi}{3}\right) = \operatorname{cosec}\frac{\pi}{3}$

$= \frac{2}{\sqrt{3}}\left[\because \operatorname{cosec}\frac{\pi}{3} = \frac{2}{\sqrt{3}}\right]$

Ans: $\frac{2}{\sqrt{3}}$

(vi) $\sec\left(-\frac{25\pi}{2}\right)$

Solution: $\sec\left(\frac{25\pi}{2}\right) = \sec\left(\frac{25\pi}{2}\right)$ [∵ sec (– θ) = sec θ)

$= \sec\left(12\pi+\frac{\pi}{2}\right)$

$= \sec\left(24.\frac{\pi}{2}+\frac{\pi}{2}\right)$

Here, n = 24 is an even number and the angle lies in the fourth quadrant.

$\therefore \sec\left(-\frac{25\pi}{2}\right) = \sec\frac{\pi}{2} =$ Undefined.

(vii) $\sin\frac{31\pi}{6}$

Solution: $\sin\frac{31\pi}{6} = \sin\left(5\pi+\frac{\pi}{6}\right)$

$= \sin\left(10.\frac{\pi}{2}+\frac{\pi}{6}\right)$

Here n = 10 is an even number. So, sin will remain unchanged and the sign of sin will be negative, since the angle lies in the third quadrant.

$= -\sin\frac{\pi}{6}$

$\therefore \sin\left(10.\frac{\pi}{2}+\frac{\pi}{6}\right) = -\frac{1}{2}$

∴ The required value = $-\frac{1}{2}$

(viii) $\cos\left(-\frac{25\pi}{6}\right)$

Solution: $\cos\frac{25\pi}{6}$ [∴ cos (– θ) = cosθ]

$= \cos\left(4\pi+\frac{\pi}{6}\right)$

$= \cos\left(8.\frac{\pi}{2}+\frac{\pi}{6}\right)$

Here, n = 8 is an even number. So, cos will remain unchanged and the sign of cos will be positive, since the angle lies in the first quadrant.

$= \cos\frac{\pi}{6}$

$\therefore \cos\left(8.\frac{\pi}{2}+\frac{\pi}{6}\right)$

$= \frac{\sqrt{3}}{2}$

∴ The required value = $\frac{\sqrt{3}}{2}$

8. Prove that,

(i) $\cos\frac{17\pi}{10} + \cos\frac{13\pi}{10} + \cos\frac{9\pi}{10} + \cos\frac{\pi}{10} = 0$

Solution: L.S. $= \cos\frac{17\pi}{10} + \cos\frac{13\pi}{10} + \cos\frac{9\pi}{10} + \cos\frac{\pi}{10}$

$= \cos\left(2\pi - \frac{3\pi}{10}\right) + \cos\left(\pi + \frac{3\pi}{10}\right) + \cos\left(\pi - \frac{\pi}{10}\right) + \cos\frac{\pi}{10}$

$= \cos\frac{3\pi}{10} - \cos\frac{3\pi}{10} - \cos\frac{\pi}{10} + \cos\frac{\pi}{10}$

$= 0$

= R.S

$\therefore \cos\frac{17\pi}{10} + \cos\frac{13\pi}{10} + \cos\frac{9\pi}{10} + \cos\frac{\pi}{10} = 0$ **(Proved)**

(ii) $\tan\frac{\pi}{12}\tan\frac{5\pi}{12}\tan\frac{7\pi}{12}\tan\frac{11\pi}{12} = 1$

Solution: L.S.

$= \tan\frac{\pi}{12}\tan\frac{5\pi}{12}\tan\frac{7\pi}{12}\tan\frac{11\pi}{12}$

$= \tan 15° \tan 75° \tan 105° \tan 165°$

$= \tan 15° \tan(90° - 15°) \tan(90° + 15°) \tan(180° - 15°)$

$= \tan 15° \cot 15° (-\cot 15°)(-\tan 15°)$

$= \tan^2 15° \cot^2 15° = \tan^2 15° \times \frac{1}{\tan^2 15°} = 1$ R.S

$\therefore \tan\frac{\pi}{12}\tan\frac{5\pi}{12}\tan\frac{7\pi}{12}\tan\frac{11\pi}{12} = 1$ **(Proved)**

(iii) $\sin^2\frac{\pi}{7} + \sin^2\frac{5\pi}{14} + \sin^2\frac{8\pi}{7} + \sin^2\frac{9\pi}{14} = 2$

Solution: L.S. $= \sin^2\frac{\pi}{7} + \sin^2\frac{5\pi}{14} + \sin^2\frac{8\pi}{7} + \sin^2\frac{9\pi}{14}$

$= \sin^2\frac{\pi}{7} + \left\{\sin\left(\frac{\pi}{2} - \frac{\pi}{7}\right)\right\}^2 + \left\{\sin\left(\pi + \frac{\pi}{7}\right)\right\}^2 + \left\{\sin\left(\frac{\pi}{2} + \frac{\pi}{7}\right)\right\}^2$

$= \sin^2\frac{\pi}{7} + \left(\cos\frac{\pi}{7}\right)^2 + \left(-\sin\frac{\pi}{7}\right)^2 + \left(\cos\frac{\pi}{7}\right)^2$

$= \sin^2\frac{\pi}{7} + \cos^2\frac{\pi}{7} + \sin^2\frac{\pi}{7} + \cos^2\frac{\pi}{7}$

$= 2\left(\sin^2\frac{\pi}{7} + \cos^2\frac{\pi}{7}\right) = 2 =$ R.S

$\therefore \sin^2\frac{\pi}{7} + \sin^2\frac{5\pi}{14} + \sin^2\frac{8\pi}{7} + \sin^2\frac{9\pi}{14} = 2$ **(Proved)**

(iv) $\sin\frac{7\pi}{3}\cos\frac{13\pi}{6} - \cos\frac{5\pi}{3}\sin\frac{11\pi}{6} = 1$

Solution:

L.S. $= \sin\frac{7\pi}{3}\cos\frac{13\pi}{6} - \cos\frac{5\pi}{3}\sin\frac{11\pi}{6}$

$= \sin\left(2\pi + \frac{\pi}{3}\right)\cos\left(2\pi + \frac{\pi}{6}\right) - \cos\left(2\pi - \frac{\pi}{3}\right)\sin\left(2\pi - \frac{\pi}{6}\right)$

$= \sin\frac{\pi}{3}\cos\frac{\pi}{6} - \cos\frac{\pi}{3}.\left(-\sin\frac{\pi}{6}\right)$

$= \sin\frac{\pi}{3}\cos\frac{\pi}{6} + \cos\frac{\pi}{3}\sin\frac{\pi}{6}$

$= \frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2} + \frac{1}{2}.\frac{1}{2}$

$= \frac{3}{4} + \frac{1}{4} = \frac{3+1}{4} = \frac{4}{4} = 1 =$ R.S

$\therefore \sin\frac{7\pi}{3}\cos\frac{13\pi}{6} - \cos\frac{5\pi}{3}\sin\frac{11\pi}{6} = 1$ **(Proved)**

(v) $\sin\frac{13\pi}{3}\cos\frac{13\pi}{6} - \sin\frac{11\pi}{6}\cos\left(-\frac{5\pi}{3}\right) = 1$

Solution: L.S. $= \sin\frac{13\pi}{3}\cos\frac{13\pi}{6} - \sin\frac{11\pi}{6}\cos\left(-\frac{5\pi}{3}\right)$

$= \sin\left(4\pi + \frac{\pi}{3}\right)\cos\left(2\pi + \frac{\pi}{6}\right) - \sin\left(2\pi - \frac{\pi}{6}\right)\cos\left(2\pi - \frac{\pi}{3}\right)$

[$\because \cos(-\theta) = \cos\theta$]

$= \sin\frac{\pi}{3}\cos\frac{\pi}{6} - \left(-\sin\frac{\pi}{6}\right).\cos\frac{\pi}{3}$

$= \sin\frac{\pi}{3}\cos\frac{\pi}{6} + \sin\frac{\pi}{6}\cos\frac{\pi}{3}$

$= \frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2} + \frac{1}{2}.\frac{1}{2}$

$= \frac{3}{4} + \frac{1}{4} = \frac{3+1}{4} = \frac{4}{4} = 1 =$ R.S

$\therefore \sin\frac{13\pi}{3}\cos\frac{13\pi}{6} - \sin\frac{11\pi}{6}\cos\left(-\frac{5\pi}{3}\right) = 1$ **(Proved)**

(vi) If $\tan\theta = \frac{3}{4}$ and $\sin\theta$ is negative then prove that, $\frac{\sin\theta + \cos\theta}{\sec\theta + \tan\theta} = \frac{14}{5}$

Solution: Given,

$\tan\theta = \frac{3}{4}$ and $\sin\theta$ is negative

or, $\frac{\sin\theta}{\cos\theta} = \frac{3}{4}$

or, $3\cos\theta = 4\sin\theta$

or, $9\cos^2\theta = 16\sin^2\theta$ [squaring both sides]

or, $9(1 - \sin^2\theta) = 16\sin^2\theta$

or, $9 - 9\sin^2\theta - 16\sin^2\theta = 0$

or, $-25\sin^2\theta = -9$

or, $\sin^2\theta = \frac{9}{25}$

or, $\sin\theta = \pm\frac{3}{5}$

$\therefore \sin\theta = -\frac{3}{5}$ [$\because \sin\theta$ is negative]

Again, $\tan\theta = \frac{3}{4}$

or, $\frac{\sin\theta}{\cos\theta} = \frac{3}{4}$

or, $3\cos\theta = 4\sin\theta$

$\therefore \cos\theta = \frac{4}{3} \times \left(-\frac{3}{5}\right) = -\frac{4}{5}$

and $\sec\theta = \frac{1}{\cos\theta} = \frac{1}{-\frac{4}{5}} = -\frac{5}{4}$

$\therefore$ L.S. $= \frac{\sin\theta + \cos\theta}{\sec\theta + \tan\theta}$

$= \frac{-\frac{3}{5} - \frac{4}{5}}{-\frac{5}{4} + \frac{3}{4}} = \frac{\frac{-3-4}{5}}{\frac{-5+3}{4}} = \frac{-\frac{7}{5}}{-\frac{2}{4}}$

$= \frac{-7}{5} \times \frac{4}{-2} = \frac{14}{5} =$ R.S

$\therefore \frac{\sin\theta + \cos\theta}{\sec\theta + \tan\theta} = \frac{14}{5}$ **(Proved)**

9. Find the value of:

(i) $\cos\frac{9\pi}{4}+\cos\frac{5\pi}{4}+\cos\frac{31\pi}{36}-\sin\frac{5\pi}{36}$

Solution: $\cos\frac{9\pi}{4}+\cos\frac{5\pi}{4}+\cos\frac{31\pi}{36}-\sin\frac{5\pi}{36}$

$=\cos\left(2\pi+\frac{\pi}{4}\right)+\cos\left(\pi+\frac{\pi}{4}\right)+\sin\left(\pi-\frac{5\pi}{36}\right)-\sin\frac{5\pi}{36}$

$=\cos\frac{\pi}{4}-\cos\frac{\pi}{4}+\sin\frac{5\pi}{36}-\sin\frac{5\pi}{36}$

$=0$

$\therefore$ The required value = 0

(ii) $\cot\frac{\pi}{20}\cot\frac{3\pi}{20}\cot\frac{5\pi}{20}\cot\frac{7\pi}{20}\cot\frac{9\pi}{20}$

Solution: $\cot\frac{\pi}{20}\cot\frac{3\pi}{20}\cot\frac{5\pi}{20}\cot\frac{7\pi}{20}\cot\frac{9\pi}{20}$

$=\cot\frac{\pi}{20}\cot\left(\frac{\pi}{2}-\frac{7\pi}{20}\right)\cot\frac{\pi}{4}\cot\frac{7\pi}{20}\cot\left(\frac{\pi}{2}-\frac{\pi}{20}\right)$

$=\cot\frac{\pi}{20}\tan\frac{7\pi}{20}\,1.\cot\frac{7\pi}{20}\tan\frac{\pi}{20}$

$=\cot\frac{\pi}{20}\cdot\frac{1}{\cot\frac{7\pi}{20}}\cdot\cot\frac{7\pi}{20}\frac{1}{\cot\frac{\pi}{20}}=1$

$\therefore$ The required value = 1

(iii) $\sin^2\frac{\pi}{4}+\sin^2\frac{3\pi}{4}+\sin^2\frac{5\pi}{4}+\sin^2\frac{7\pi}{4}$

Solution:

$\sin^2\frac{\pi}{4}+\sin^2\frac{3\pi}{4}+\sin^2\frac{5\pi}{4}+\sin^2\frac{7\pi}{4}$

$=\sin^2\frac{\pi}{4}+\sin^2\frac{3\pi}{4}+\sin^2\left(\frac{\pi}{2}+\frac{3\pi}{4}\right)+\sin^2\left(\frac{3\pi}{2}+\frac{\pi}{4}\right)$

$=\sin^2\frac{\pi}{4}+\sin^2\frac{3\pi}{4}+\cos^2\frac{3\pi}{4}+\cos^2\frac{\pi}{4}$

$=\left(\sin^2\frac{\pi}{4}+\cos^2\frac{\pi}{4}\right)+\left(\sin^2\frac{3\pi}{4}+\cos^2\frac{3\pi}{4}\right)$

$=1+1=2$

$\therefore$ The required value = 2

(iv) $\cos^2\frac{\pi}{8}+\cos^2\frac{3\pi}{8}+\cos^2\frac{5\pi}{8}+\cos^2\frac{7\pi}{8}$

Solution:

$\cos^2\frac{\pi}{8}+\cos^2\frac{3\pi}{8}+\cos^2\frac{5\pi}{8}+\cos^2\frac{7\pi}{8}$

$=\cos^2\frac{\pi}{8}+\cos^2\frac{3\pi}{8}+\cos^2\left(\frac{\pi}{2}+\frac{\pi}{8}\right)+\cos^2\left(\frac{\pi}{2}+\frac{3\pi}{8}\right)$

$=\cos^2\frac{\pi}{8}+\cos^2\frac{3\pi}{8}+\sin^2\frac{\pi}{8}+\sin^2\frac{3\pi}{8}$

$=\left(\cos^2\frac{\pi}{8}+\sin^2\frac{\pi}{8}\right)+\left(\cos^2\frac{3\pi}{8}+\sin^2\frac{3\pi}{8}\right)$

$=1+1$

$=2$

$\therefore$ The required value = 2

(v) $\sin^2\frac{17\pi}{18}+\sin^2\frac{5\pi}{8}+\cos^2\frac{37\pi}{18}+\cos^2\frac{5\pi}{8}$

Solution:

$\sin^2\frac{17\pi}{18}+\sin^2\frac{5\pi}{8}+\cos^2\frac{37\pi}{18}+\cos^2\frac{5\pi}{8}$

$=\left(\sin\frac{17\pi}{18}\right)^2+\left(\sin\frac{5\pi}{8}\right)^2+\left(\cos\frac{37\pi}{18}\right)^2+\left(\cos\frac{5\pi}{8}\right)^2$

$=\sin\left(\frac{17\pi}{18}\right)^2+\cos\left(\frac{37\pi}{18}\right)^2+\sin^2\frac{5\pi}{8}+\cos^2\frac{5\pi}{8}$

$=\left\{\sin\left(\pi-\frac{\pi}{18}\right)\right\}^2+\left\{\cos\left(2\pi+\frac{\pi}{18}\right)\right\}^2+\sin^2\frac{5\pi}{8}+\cos^2\frac{5\pi}{8}$

$=\sin^2\frac{\pi}{18}+\cos^2\frac{\pi}{18}+\sin^2\frac{5\pi}{8}+\cos^2\frac{5\pi}{8}$

$=\left(\sin^2\frac{\pi}{18}+\cos^2\frac{\pi}{18}\right)+\left(\sin^2\frac{5\pi}{18}+\cos^2\frac{5\pi}{8}\right)$

$=1+1$ [$\because \sin^2\theta+\cos^2\theta=1$]

$=2$

$\therefore$ The required value = 2

[**N.B.**:- The equation given in the text book is not correct]

10. If $\theta=\frac{\pi}{3}$ then justify the following identities:

(i) $\sin 2\theta = 2\sin\theta\cos\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

Solution: Given, $\theta=\frac{\pi}{3}$

Now, $\sin 2\theta = \sin 2.\frac{\pi}{3}$ [$\because \theta=\frac{\pi}{3}$]

$=\sin\frac{2\pi}{3}$

$=\sin\left(\pi-\frac{\pi}{3}\right)=\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$

And $2\sin\theta\cos\theta = 2\sin\frac{\pi}{3}\cos\frac{\pi}{3}$ [$\because \theta=\frac{\pi}{3}$]

$=2.\frac{\sqrt{3}}{2}.\frac{1}{2}=\frac{\sqrt{3}}{2}$

Again, $\frac{2\tan\theta}{1+\tan^2\theta}=\frac{2\tan\frac{\pi}{3}}{1+\tan^2\frac{\pi}{3}}$ [$\because \theta=\frac{\pi}{3}$]

$=\frac{2.\sqrt{3}}{1+(\sqrt{3})^2}$

$=\frac{2\sqrt{3}}{1+3}=\frac{2\sqrt{3}}{4}=\frac{\sqrt{3}}{2}$

$\therefore \sin 2\theta = 2\sin\theta\cos\theta=\frac{2\tan\theta}{1+\tan^2\theta}$ **(Proved)**

(ii) $\sin 3\theta = 3\cos\theta - 4\sin^3\theta$

Solution: Given, $\theta=\frac{\pi}{3}$

L.S. $=\sin 3\theta$

$=\sin\left(3.\frac{\pi}{3}\right)$ [$\because \theta=\frac{\pi}{3}$]

$=\sin\pi$

$=\sin\left(2.\frac{\pi}{2}+0\right)$

$=\sin 0$

$=0$

R.S $=3\sin\theta-4\sin^3\theta$

$=3\sin\frac{\pi}{3}-4\sin^3\frac{\pi}{3}$ [$\because \theta=\frac{\pi}{3}$]

$= 3.\frac{\sqrt{3}}{2} - 4.\left(\frac{\sqrt{3}}{2}\right)^3$

$= \frac{3\sqrt{3}}{2} - 4.\ \frac{3\sqrt{3}}{8}$

$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2}$

$= 0$

$\therefore$ $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ **(Proved)**

(iii) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.

Solution: Given, $\theta = \frac{\pi}{3}$

L.S. $= \cos 3\theta$

$= \cos 3.\frac{\pi}{3}$ $[\because \theta = \frac{\pi}{3}]$

$= \cos\pi$

$= \cos\left(2.\frac{\pi}{2} + 0\right)$

$= -\cos 0° = -1$

R.S $= 4\cos^3\theta - 3\cos\theta$

$= 4\cos^3\frac{\pi}{3} - 3\cos\frac{\pi}{3}$ $[\because \theta = \frac{\pi}{3}]$

$= 4.\left(\frac{1}{2}\right)^3 - 3.\frac{1}{2}$

$= 4.\frac{1}{8} - \frac{3}{2} = \frac{1}{2} - \frac{3}{2}$

$= \frac{1-3}{2} = \frac{-2}{2} = -1$

$\therefore$ $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. **(Proved)**

(iv) $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

Solution: Given, $\theta = \frac{\pi}{3}$

L.S. $= \tan 2\theta$

$= \tan 2.\frac{\pi}{3}$ $[\because \theta = \frac{\pi}{3}]$

$= \tan\frac{2\pi}{3}$

$= \tan\left(\pi - \frac{\pi}{3}\right)$

$= -\tan\frac{\pi}{3}$

$= -\sqrt{3}$

R.S $= \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2\tan\frac{\pi}{3}}{1-\tan^2\frac{\pi}{3}}$ $[\because \theta = \frac{\pi}{3}]$

$= \frac{2.\sqrt{3}}{1-(\sqrt{3})^2}$

$= \frac{2\sqrt{3}}{1-3} = \frac{2\sqrt{3}}{-2} = -\sqrt{3}$

$\therefore$ $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$ **(Proved)**

11. Find the value of α (alpha) satisfying the given conditions:

(i) $\cot\alpha = -\sqrt{3}$; $\frac{3\pi}{2} < \alpha < 2\pi$

Solution: In tohe fourth quadrant,

$\cot\alpha = -\sqrt{3}$

or, $\cot\alpha = -\cot\frac{\pi}{6}$

$= \cot\left(2\pi - \frac{\pi}{6}\right)$

$= \cot\left(\frac{12\pi - \pi}{6}\right) = \cot\frac{11\pi}{6}$

$\therefore \alpha = \frac{11\pi}{6}$ is acceptable, since $\frac{3\pi}{2} < \alpha < 2\pi$

$\therefore$ The required value, $\alpha = \frac{11\pi}{6}$

(ii) $\cos\alpha = -\frac{1}{2}$; $\frac{\pi}{2} < \alpha < \frac{3\pi}{2}$

Solution: In the second quadrant, $\cos\alpha = -\frac{1}{2}$

or, $\cos\alpha = \cos\left(\pi - \frac{\pi}{3}\right)$

or, $\alpha = \frac{3\pi - \pi}{3}$

$\therefore$ $\alpha = \frac{2\pi}{3}$

which satisfies the condition $\frac{\pi}{2} < \alpha < \frac{3\pi}{2}$

Again, in the third quadrant, $\cos\alpha = -\frac{1}{2}$

or, $\cos\alpha = \cos\left(\pi + \frac{\pi}{3}\right)$

or, $\alpha = \frac{3\pi + \pi}{3}$

$\therefore$ $\alpha = \frac{4\pi}{3}$

which satisfies the condition $\frac{\pi}{2} < \alpha < \frac{3\pi}{2}$

$\therefore$ The required value, $\alpha = \frac{2\pi}{3}$ and $\frac{4\pi}{3}$

(iii) $\sin\alpha = -\frac{\sqrt{3}}{2}$; $\frac{\pi}{2} < \alpha < \frac{3\pi}{2}$

Solution: $\sin\alpha = -\frac{\sqrt{3}}{2}$

or, $\sin\alpha = -\sin\frac{\pi}{3}$

or, $\sin\alpha = \sin\left(\pi + \frac{\pi}{3}\right)$ [$\because$ sin is negative in third quadrant]

or, $\alpha = \pi + \frac{\pi}{3}$

$\therefore$ $\alpha = \frac{4\pi}{3}$, which satisfies the condition $\frac{\pi}{2} < \alpha < \frac{3\pi}{2}$

$\therefore$ The required value $= \frac{4\pi}{3}$

(iv) $\cot\alpha = -1; \pi < \alpha < 2\pi$

Solution: $\cot\alpha = -1$

or, $\cot\alpha = -\cot\frac{\pi}{4}$

or, $\cot\alpha = \cot\left(2\pi - \frac{\pi}{4}\right)$; [$\because$ cot is negative in fourth quadrant]

or, $\alpha = 2\pi - \frac{\pi}{4}$

$\therefore \alpha = \frac{7\pi}{4}$, which satisfies the condition $\pi < \alpha < 2\pi$

$\therefore$ The required value $= \frac{7\pi}{4}$

12. Solve: $\left(\text{when } 0 < \theta < \frac{\pi}{2}\right)$

(i) $2\cos^2\theta = 1 + 2\sin^2\theta$

Solution: Given that,

$2\cos^2\theta = 1 + 2\sin^2\theta$

or, $2\cos^2\theta - 2\sin^2\theta = 1$

or, $2(1-\sin^2\theta) - 2\sin^2\theta = 1$ [$\because \cos^2\theta = 1 - \sin^2\theta$]

or, $2 - 2\sin^2\theta - 2\sin^2\theta = 1$

or, $2 - 4\sin^2\theta = 1$

or, $-4\sin^2\theta = -1$

or, $\sin^2\theta = \frac{1}{4}$

or, $\sin\theta = \pm\frac{1}{2}$

Since $0 < \theta < \frac{\pi}{2}$, so $\sin\theta = -\frac{1}{2}$ is not acceptable.

$\therefore \sin\theta = \frac{1}{2}$

or, $\sin\theta = \sin\frac{\pi}{6}$ [$\because \sin\frac{\pi}{6} = \frac{1}{2}$]

$\therefore \theta = \frac{\pi}{6}$

$\therefore$ The required solution, $\theta = \frac{\pi}{6}$

(ii) $2\sin^2\theta - 3\cos\theta = 0$

Solution: Given that,

$2\sin^2\theta - 3\cos\theta = 0$

or, $2(1-\cos^2\theta) - 3\cos\theta = 0$

or, $2 - 2\cos^2\theta - 3\cos\theta = 0$

or, $-(2\cos^2\theta + 3\cos\theta - 2) = 0$

or, $2\cos^2\theta + 3\cos\theta - 2 = 0$

or, $2\cos^2\theta + 4\cos\theta - \cos\theta - 2 = 0$

or, $2\cos\theta(\cos\theta + 2) - 1(\cos\theta + 2) = 0$

or, $(2\cos\theta - 1)(\cos\theta + 2) = 0$

Here, $\cos\theta + 2 \neq 0$ because, if $\cos\theta + 2 = 0$ then $\cos\theta = -2$, which is not acceptable, because the value of $\cos\theta$ can never be greater than 1 and less than -1.

$\therefore 2\cos\theta - 1 = 0$ when $0° < \theta < \frac{\pi}{2}$

or, $2\cos\theta = 1$

or, $\cos\theta = \frac{1}{2}$

or, $\cos\theta = \cos\frac{\pi}{3}$

$\therefore \theta = \frac{\pi}{3}$

$\therefore$ The required solution, $\theta = \frac{\pi}{3}$

(iii) $6\sin^2\theta - 11\sin\theta + 4 = 0$

Solution: Given,

$6\sin^2\theta - 11\sin\theta + 4 = 0$

or, $6\sin^2\theta - 8\sin\theta - 3\sin\theta + 4 = 0$

or, $2\sin\theta(3\sin\theta - 4) - 1(3\sin\theta - 4) = 0$

or, $(2\sin\theta - 1)(3\sin\theta - 4) = 0$

Here, $3\sin\theta - 4 \neq 0$ because, if $3\sin\theta - 4 = 0$ then $\sin\theta = \frac{4}{3}$, which is not acceptable. Because the value of $\sin\theta$ can never be greater than 1 and less than -1.

$\therefore 2\sin\theta - 1 = 0$

or, $\sin\theta = \frac{1}{2}$

or, $\sin\theta = \sin\frac{\pi}{6}$

$\therefore \theta = \frac{\pi}{6}$

$\therefore$ The required solution, $\theta = \frac{\pi}{6}$

(iv) $\tan\theta + \cot\theta = \frac{4}{\sqrt{3}}$

Solution: Given,

$\tan\theta + \cot\theta = \frac{4}{\sqrt{3}}$

or, $\tan\theta + \frac{1}{\tan\theta} = \frac{4}{\sqrt{3}}$

or, $\frac{\tan^2\theta + 1}{\tan\theta} = \frac{4}{\sqrt{3}}$

or, $\tan^2\theta + 1 = \frac{4\tan\theta}{\sqrt{3}}$

or, $\sqrt{3}\tan^2\theta + \sqrt{3} = 4\tan\theta$

or, $\sqrt{3}\tan^2\theta - 4\tan\theta + \sqrt{3} = 0$

or, $\sqrt{3}\tan^2\theta - 3\tan\theta - \tan\theta + \sqrt{3} = 0$

or, $\sqrt{3}\tan\theta(\tan\theta - \sqrt{3}) - 1(\tan\theta - \sqrt{3}) = 0$

or, $(\tan\theta - \sqrt{3})(\sqrt{3}\tan\theta - 1) = 0$

Either, $\tan\theta - \sqrt{3} = 0$ or, $\sqrt{3}\tan\theta - 1 = 0$

or, $\tan\theta = \sqrt{3}$ or, $\tan\theta = \frac{1}{\sqrt{3}}$

or, $\tan\theta = \tan\frac{\pi}{3}$ or, $\tan\theta = \tan\frac{\pi}{6}$

$\therefore \theta = \frac{\pi}{3}$ [$\because \tan\frac{\pi}{3} = \sqrt{3}$]

$\therefore \theta = \frac{\pi}{6}$ [$\because \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$]

which satisfies the condition, $0 < \theta < \frac{\pi}{2}$

$\therefore$ The required solution, $\theta = \frac{\pi}{6}, \frac{\pi}{3}$

(v) $2\sin^2\theta + 3\cos\theta = 3$

Solution: Given,

$2\sin^2\theta + 3\cos\theta = 3$

or, $2(1-\cos^2\theta) + 3\cos\theta - 3 = 0$

or, $2 - 2\cos^2\theta + 3\cos\theta - 3 = 0$

or, $-2\cos^2\theta + 3\cos\theta - 1 = 0$

or, $-(2\cos^2\theta - 3\cos\theta + 1) = 0$

or, $2\cos^2\theta - 3\cos\theta + 1 = 0$

or, $2\cos^2\theta - 2\cos\theta - \cos\theta + 1 = 0$

or, $2\cos\theta(\cos\theta - 1) - 1(\cos\theta - 1) = 0$

or, $(2\cos\theta - 1)(\cos\theta - 1) = 0$

$\therefore$ Either, $2\cos\theta - 1 = 0$

or, $\cos\theta = \frac{1}{2}$

or, $\cos\theta = \cos\frac{\pi}{3}$

$\therefore \theta = \frac{\pi}{3}$

or, $\cos\theta - 1 = 0$

or, $\cos\theta = 1$

or, $\cos\theta = \cos 0$

$\therefore \theta = 0$

But $0 < \theta < \frac{\pi}{2}$

$\therefore \theta \neq 0°$

$\therefore$ The required solution, $\theta = \frac{\pi}{3}$

13. Solve: (when $0 < \theta < 2\pi$)

(i) $2\sin^2\theta + 3\cos\theta = 0$

Solution: Given,

$2\sin^2\theta + 3\cos\theta = 0$

or, $2(1-\cos^2\theta) + 3\cos\theta = 0$

or, $2 - 2\cos^2\theta + 3\cos\theta = 0$

or, $2\cos^2\theta - 3\cos\theta - 2 = 0$ [Multiplying both sides by (-1)]

or, $2\cos^2\theta - 4\cos\theta + \cos\theta - 2 = 0$

or, $2\cos\theta(\cos\theta - 2) + 1(\cos\theta - 2) = 0$

or, $(2\cos\theta + 1)(\cos\theta - 2) = 0$

But, $\cos\theta - 2 \neq 0$ because, if $\cos\theta - 2 = 0$ then $\cos\theta = 2$, which is impossible.

So, $2\cos\theta + 1 = 0$

or, $\cos\theta = -\frac{1}{2}$

or, $\cos\theta = -\frac{1}{2} = -\cos\frac{\pi}{3}$

or, $\cos\theta = \cos(\pi - \frac{\pi}{3}), \cos(\pi + \frac{\pi}{3})$ [according to the condition $0 < \theta < 2\pi$]

or, $\cos\theta = \cos\frac{2\pi}{3}, \cos\frac{4\pi}{3}$

$\therefore \theta = \frac{2\pi}{3}, \frac{4\pi}{3}$, which satisfies the condition $0 < \theta < 2\pi$

$\therefore$ The required solution: $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

(ii) $4(\cos^2\theta + \sin\theta) = 5$

Solution: Given,

$4(\cos^2\theta + \sin\theta) = 5$

or, $4(1 - \sin^2\theta + \sin\theta) = 5$

or, $4 - 4\sin^2\theta + 4\sin\theta = 5$

or, $4\sin^2\theta - 4\sin\theta + 1 = 0$ [multiplying both sides by (–1)]

or, $(2\sin\theta - 1)^2 = 0$

or, $2\sin\theta - 1 = 0$ [taking square root]

or, $\sin\theta = \frac{1}{2}$

or, $\sin\theta = \sin\frac{\pi}{6}, \sin(\pi - \frac{\pi}{6})$ [according to the condition]

or, $\sin\theta = \sin\frac{\pi}{6}, \sin\frac{5\pi}{6}$

$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}$, which satisfies the condition $0 < \theta < 2\pi$

$\therefore$ The required solution: $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

(iii) $\cot^2\theta + \operatorname{cosec}^2\theta = 3$.

Solution: Given,

$\cot^2\theta + \operatorname{cosec}^2\theta = 3$

or, $\cot^2\theta + 1 + \cot^2\theta = 3$

or, $2\cot^2\theta = 2$

or, $\cot^2\theta = 1$

or, $\cot\theta = \pm 1$

By taking $\cot\theta = 1$ we get,

$\cot\theta = \cot\frac{\pi}{4}, \cot(\pi + \frac{\pi}{4})$ [according to the condition]

or, $\cot\theta = \cot\frac{\pi}{4}, \cot\frac{5\pi}{4}$

$\therefore \theta = \frac{\pi}{4}, \frac{5\pi}{4}$

Again taking, $\cot\theta = -1$ we get,

$\cot\theta = -\cot\frac{\pi}{4}$

or, $\cot\theta = \cot(\pi - \frac{\pi}{4}), \cot(2\pi - \frac{\pi}{4})$ [according to the condition]

or, $\cot\theta = \cot\frac{3\pi}{4}, \cot\frac{7\pi}{4}$

$\therefore \theta = \frac{3\pi}{4}, \frac{7\pi}{4}$, which satisfies the condition $0 < \theta < 2\pi$

$\therefore$ The required solution: $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

(iv) $\tan^2\theta + \cot^2\theta = 2$

Solution: Given,

$\tan^2\theta + \cot^2\theta = 2$

or, $\tan^2\theta + \frac{1}{\tan^2\theta} = 2$

or, $\tan^4\theta + 1 = 2\tan^2\theta$ [multiplying both sides by $\tan^2\theta$]

or, $\tan^4\theta - 2\tan^2\theta + 1 = 0$

or, $(\tan^2\theta - 1)^2 = 0$

or, $\tan^2\theta - 1 = 0$

or, $\tan^2\theta = 1$

or, $\tan\theta = \pm 1$

Now, taking $\tan\theta = 1$ we get,

$\tan\theta = \tan\frac{\pi}{4}, \tan(\pi + \frac{\pi}{4})$ [according to the condition]

or, $\tan\theta = \tan\frac{\pi}{4}, \tan\frac{5\pi}{4}$

$\therefore \theta = \frac{\pi}{4}, \frac{5\pi}{4}$

Again by taking tanθ = –1 we get,

$\tan\theta = -\tan\frac{\pi}{4}$

or, $\tan\theta = \tan(\pi - \frac{\pi}{4}),\ \tan(2\pi - \frac{\pi}{4})$ [According to the condition]

or, $\tan\theta = \tan\frac{3\pi}{4}, \tan\frac{7\pi}{4}$

$\therefore\ \theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

$\therefore$ The required solution : $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

(v) $\sec^2\theta + \tan^2\theta = \frac{5}{3}$

Solution: Given,

$(\sec^2\theta + \tan^2\theta) = \frac{5}{3}$

or, $3(1 + \tan^2\theta + \tan^2\theta) = 5$

or, $3 + 6\tan^2\theta - 5 = 0$

or, $6\tan^2\theta = 2$

or, $\tan^2\theta = \frac{1}{3}$

$\therefore\ \tan\theta = \pm\frac{1}{\sqrt{3}}$

Now, by taking $\tan\theta = \frac{1}{\sqrt{3}}$ we get,

$\tan\theta = \tan\frac{\pi}{6}, \tan(\pi + \frac{\pi}{6})$ [According to the condition]

or, $\tan\theta = \tan\frac{\pi}{6},\ \tan\frac{7\pi}{6}$

$\therefore\ \theta = \frac{\pi}{6}, \frac{7\pi}{6}$

Again, by taking $\tan\theta = -\frac{1}{\sqrt{3}}$ we get,

or, $\tan\theta = -\tan\frac{\pi}{6}$

or, $\tan\theta = \tan(\pi - \frac{\pi}{6}),\ \tan(2\pi - \frac{\pi}{6})$ [According to the condition]

or, $\tan\theta = \tan\frac{5\pi}{6},\ \tan\frac{11\pi}{6}$

$\therefore\ \theta = \frac{5\pi}{6}, \frac{11\pi}{6}$

$\therefore$ The required solution: $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

(vi) $5\operatorname{cosec}^2\theta - 7\cot\theta\operatorname{cosec}\theta - 2 = 0$

Solution: Given,

$5\operatorname{cosec}^2\theta - 7\cot\theta\operatorname{cosec}\theta - 2 = 0$

or, $\frac{5}{\sin^2\theta} - \frac{7\cos\theta}{\sin^2\theta} - 2 = 0$

or, $5 - 7\cos\theta - 2\sin^2\theta = 0$

or, $5 - 7\cos\theta - 2(1 - \cos^2\theta) = 0$

or, $5 - 7\cos\theta - 2 + 2\cos^2\theta = 0$

or, $2\cos^2\theta - 7\cos\theta + 3 = 0$

or, $2\cos^2\theta - 6\cos\theta - \cos\theta + 3 = 0$

or, $2\cos\theta(\cos\theta - 3) - 1(\cos\theta - 3) = 0$

or, $(2\cos\theta - 1)(\cos\theta - 3) = 0$

Either, $2\cos\theta - 1 = 0$ or, $\cos\theta - 3 = 0$

or, $\cos\theta = \frac{1}{2}$ $\qquad\therefore\ \cos\theta = 3$

$\therefore\ \cos\theta = \frac{1}{2}$ or 3

But, the value of cosθ can never be greater than 1.

$\therefore\ \cos\theta = \frac{1}{2}$

$\cos\theta = \cos\frac{\pi}{3}, \cos(2\pi - \frac{\pi}{3})$ [according to the condiion]

$\therefore\ \theta = \frac{\pi}{3}, \frac{5\pi}{3}$, which lie in the given limit $0 < \theta < 2\pi$

$\therefore$ The required solution : $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

(vii) $2\sin x\cos x = \sin x\ (0 \le x \le 2\pi)$.

Solution: $2\sin x\cos x = \sin x$

or, $(2\sin x\cos x)^2 = (\sin x)^2$ [squaring]

or, $4\sin^2x\cos^2x = \sin^2x$

or, $4\sin^2x(1 - \sin^2x) = \sin^2x$

or, $4\sin^2x - 4\sin^4x - \sin^2x = 0$

or, $-4\sin^4x + 3\sin^2x = 0$

or, $-\sin^2x(4\sin^2x - 3) = 0$

or, $\sin^2x(4\sin^2x - 3) = 0$

Either, $\sin^2x = 0$

or, $4\sin^2x - 3 = 0$.

or, $\sin x = 0$

or, $4\sin^2x = 3$

or, $\sin x = \sin 0°, \sin(\pi - 0), \sin(2\pi - 0)$

or, $\sin^2x = \frac{3}{4}$

or, $\sin x = \pm\sqrt{\frac{3}{4}}$

or, $\sin x = \pm\frac{\sqrt{3}}{2}$

$\therefore\ x = 0,\ \pi, 2\pi$

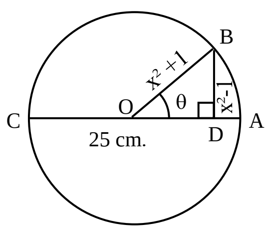
or, $\sin x = \sin\frac{\pi}{3}, \sin\left(\pi - \frac{\pi}{3}\right), \sin\left(2\pi - \frac{\pi}{3}\right)$

$\therefore\ \sin x = \sin\frac{\pi}{3}, \sin\frac{2\pi}{3}, \sin\frac{5\pi}{3}$

$\therefore\ x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}$; which lie in the limit $0 \le x \le 2\pi$

$\therefore$ The required solution: $\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{5\pi}{3}$

Question ►14



a. In figure ABC is a circular wheel and length of arc AB is 25 cm. then find the value of θ.
b. What is the speed of the wheel if it revolve five times in a second?
c. In the figure, if ∠BOD = θ then prove that, tanθ + secθ = x using the value of sinθ.

Solution to the question no. 14

a Let, length of the arc AB of the wheel, S = 25 cm.
From the figure we get, radius, r = 25 cm.
Now we know, S = rθ

or, $\theta = \frac{s}{r}$

or, $\theta = \frac{25}{25}$ radian

= 1 radiun

$= \frac{180°}{\pi}$

= 57.30°

∴ The required value of θ is 57.30°

Ans: 57.30°

b In 1 revolution the wheel covers the distance of 2πr
= 2 × 3.1416 × 25 cm.
= 157.08 cm.
= 1.5708 m. (approx.)
= 1.57 m. (approx.)
∴ In 1 revolution, the wheel covers the distance = 1.57 m. (approx)

1 hour = 60 minutes = 60 × 60 seconds
= 3600 seconds
The wheel ABC takes 1 second for 5 revolutions.
∴ The wheel revolve in 1 hour = (3600 × 5) times
= 18000 times.
∴ The wheel covers the distance in 1 hour
= 18000 × 1.57 m.
= 28260 m.
= 28.26 km. (approx.)
The speed of the wheel is 28.26 km/hour (approx.)
Ans. 28.26 km. (approx.)

c From the figure we get, $\sin\theta = \frac{BD}{BO}$

or, $\sin\theta = \frac{x^2 - 1}{x^2 + 1}$

$\therefore \cos\theta = \sqrt{1 - \sin^2\theta}$

$= \sqrt{1 - \left(\frac{x^2 - 1}{x^2 + 1}\right)^2}$

$= \sqrt{1 - \frac{(x^2 - 1)^2}{(x^2 + 1)^2}}$

$= \sqrt{\frac{(x^2 + 1)^2 - (x^2 - 1)^2}{(x^2 + 1)^2}}$

$= \sqrt{\frac{4x^2}{(x^2 + 1)^2}} = \frac{2x}{x^2 + 1}$

Now, $\tan\theta + \sec\theta = \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}$

$= \frac{\frac{x^2 - 1}{x^2 + 1}}{\frac{2x}{x^2 + 1}} + \frac{1}{\frac{2x}{x^2 + 1}}$

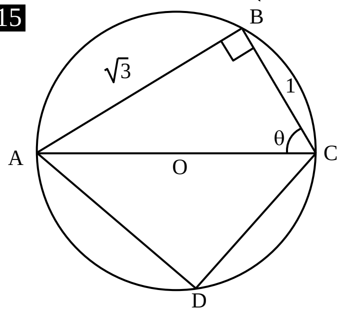
$= \left(\frac{x^2 - 1}{x^2 + 1} \times \frac{x^2 + 1}{2x}\right) + \left(1 \times \frac{x^2 + 1}{2x}\right)$

$= \frac{x^2 - 1}{2x} + \frac{x^2 + 1}{2x}$

$= \frac{x^2 - 1 + x^2 + 1}{2x} = \frac{2x^2}{2x} = x$

∴ tanθ + sec θ = x **(Proved)**

Question ▶15



a. In the figure, O is the centre of the circle, then find the circular value of ∠B and the value of AC.
b. Prove that, tanA + tanB + tanC + tanD = 0
c. sec θ + cos θ = P, then find the value of P and solve the equation.

Solution to the question no. 15

a In the figure, ∠B = 90°

We know, $1° = \frac{\pi}{180}$ radian

$\therefore 90° = \left(\frac{\pi}{180}.90\right)$,,

$= \frac{\pi}{2}$,,

∴The circular value of ∠B is $\frac{\pi}{2}$ radian **(Ans.)**

Again, in ΔABC, ∠B = 90°
∴ By applying the theorem of Pythagoras we get,
$AC^2 = AB^2 + BC^2$
or, $AC^2 = (\sqrt{3})^2 + 1^2$
or, $AC^2 = 3 + 1$
or, $AC^2 = 4$
∴ AC = 2 units **(Ans.)**

b The quadrilateral ABCD is inscribed in the circle ABCD with centre O.
∴ ∠A + ∠C = 180°
and ∠B + ∠D = 180°
Now, L.S. = tanA + tanB + tanC + tanD
= tanA + tan(180° – D) + tan (180° – A) + tanD
= tanA + tan(2 × 90° – D) + tan (2 × 90° – A) + tanD
= tanA – tanD – tanA + tanD [Since, tan is negative in the second quadrant]
= 0 = R.S
∴ tanA + tanB + tanC + tanD = 0 **(Proved)**

c Given, sec θ + cos θ = P ... ... (i)

Here, $\sec\theta = \frac{AC}{BC} = \frac{2}{1} = 2$ [∵ AC = 2 and BC = 1]

Again, $\cos\theta = \frac{BC}{AC} = \frac{1}{2}$

Putting the value of secθ and cosθ in equation (i) we get,

$2 + \frac{1}{2} = P$

or, $\frac{4+1}{2} = P$ or, $P = \frac{5}{2}$

$\therefore$ The required value of P is $\frac{5}{2}$

Now, from (i) we get,

$\sec\theta + \cos\theta = \frac{5}{2}$

or, $\frac{1}{\cos\theta} + \cos\theta = \frac{5}{2}$

or, $\frac{1 + \cos^2\theta}{\cos\theta} = \frac{5}{2}$

or, $2\cos^2\theta + 2 = 5\cos\theta$

or, $2\cos^2\theta - 5\cos\theta + 2 = 0$

or, $2\cos^2\theta - 4\cos\theta - \cos\theta + 2 = 0$

or, $2\cos\theta(\cos\theta - 2) - 1(\cos\theta - 2) = 0$

or, $(2\cos\theta - 1)(\cos\theta - 2) = 0$

Either, $2\cos\theta - 1 = 0$ or, $\cos\theta - 2 = 0$

or, $2\cos\theta = 1$ $\therefore \cos\theta = 2$

or, $\cos\theta = \frac{1}{2}$ But, $\cos\theta \neq 2$

Because, the value of $\cos\theta$ can never be greater than 1.

or, $\cos\theta = \cos\frac{\pi}{3}$

$\therefore \theta = \frac{\pi}{3}$

$\therefore$ The required solution: $\theta = \frac{\pi}{3}$

Creative Multiple Choice Questions

117 Multiple Choice Questions ■ 86 simple multiple questions ■ 14 Multiple Completion ■ 17 Situation Set ■ 21 Board questions ■ 28 Cadet College questions

Board Exam MCQs with Answers

Board Exam questions are very important for the exam preparation. So practice these questions again and again properly.

1. In which quadrant – 240° angle is located? [Dhaka Board-'15]

ⓐ 1st ⓑ 2nd
ⓒ 3rd ⓓ 4th **ⓑ**

2. What is the value of sin 120°? [Dhaka Board-'15]

ⓐ $\frac{\sqrt{3}}{2}$ ⓑ $\frac{1}{2}$
ⓒ $\frac{1}{\sqrt{2}}$ ⓓ $-\frac{1}{2}$ **ⓐ**

3. tan(–1140°) =? [Rajshahi Board-'15]

ⓐ $-\sqrt{3}$ ⓑ $-\frac{1}{\sqrt{3}}$
ⓒ $\frac{1}{\sqrt{3}}$ ⓓ $\sqrt{3}$ **ⓐ**

4. If $\sin\theta + \cos\theta = \sqrt{2}$ then θ =? [Rajshahi Board-'15]

ⓐ 30° ⓑ 45°
ⓒ 60° ⓓ 90° **ⓑ**

5. $12\sin^2\theta - 14\sin\theta + 4 = 0$; then θ = ?
$\left[0 < \theta < \frac{\pi}{2}\right]$ [Dinajpur Board-'15]

ⓐ 0° ⓑ 30°
ⓒ 45° ⓓ 60° **ⓑ**

6. If $\sin\theta = \frac{b}{a}$ (a > b > 0) then— [Comilla Board-'15]

i. $\tan\theta = \frac{b}{\sqrt{a^2 - b^2}}$

ii. $\cot\theta = \frac{\sqrt{a^2 - b^2}}{b}$

iii. $\sec\theta = \frac{\sqrt{a^2 - b^2}}{a}$

Which of the following is true?

ⓐ i & ii ⓑ i & iii
ⓒ ii & iii ⓓ i, ii & iii **ⓐ**

7. In which quadrant lies the angle – 230°? [Comilla Board-'15]

ⓐ 1st ⓑ 2nd
ⓒ 3rd ⓓ 4th **ⓑ**

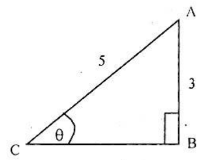
8. What is the value of cos (–330°)? [Comilla Board-'15]

ⓐ $\frac{-\sqrt{3}}{2}$ ⓑ $\frac{-1}{2}$
ⓒ $\frac{1}{2}$ ⓓ $\frac{\sqrt{3}}{2}$ **ⓓ**

9. Which one is true? [Chittagong Board-'15]

ⓐ $\sin\left(-\frac{\pi}{6}\right) = \sin\frac{\pi}{6}$ ⓑ $\tan\left(-\frac{\pi}{6}\right) = \tan\frac{\pi}{6}$
ⓒ $\cos\left(-\frac{\pi}{6}\right) = \cos\frac{\pi}{6}$ ⓓ $\mathrm{cosec}\left(-\frac{\pi}{6}\right) = \mathrm{cosec}\frac{\pi}{6}$ **ⓓ**

In view of the given figure answer the questions No 10 and 11:



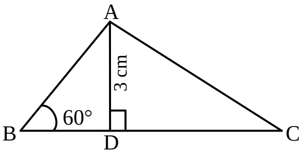
10. sinA + cosC = what? [Chittagong Board-'15]

ⓐ $\frac{3}{4}$ ⓑ $\frac{4}{5}$
ⓒ $\frac{5}{4}$ ⓓ $\frac{8}{5}$ **ⓓ**

11. Which one is the value of cotθ? [Chittagong Board-'15]

ⓐ $\frac{4}{3}$ ⓑ $\frac{5}{4}$
ⓒ $\frac{3}{4}$ ⓓ $\frac{3}{5}$ **ⓐ**

Answer question 12 and 13:



ΔABC of $\angle$A= 90°

12. BD =? [Sylhet Board-'15]

ⓐ $\frac{1}{\sqrt{3}}$ ⓑ $\sqrt{3}$
ⓒ $2\sqrt{3}$ ⓓ $3\sqrt{3}$ **ⓑ**

13. AC = ? [Sylhet Board-'15]

ⓐ $\frac{3}{2}$ cm ⓑ $2\sqrt{3}$ cm
ⓒ $3\sqrt{2}$ mc ⓓ 6 cm **ⓓ**

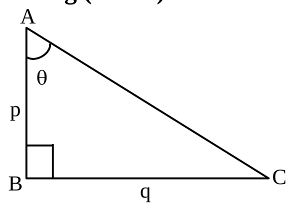
14. If sinθ + cosθ = 1 then θ =? [Sylhet Board-'15]

i. 0° ii. 30°
iii. 90°

Which of the following is true?

ⓐ i & ii ⓑ i & iii
ⓒ ii & iii ⓓ i, ii & iii **ⓑ**

Answer the following (15–16)



15. i. $\tan\theta = \frac{p}{q}$ [Sylhet Board-'15]

ii. $\cos\theta = \frac{p}{\sqrt{p^2 + q^2}}$ iii. $\sin\theta = \frac{q}{\sqrt{p^2 + q^2}}$

Which of the following is true?

ⓐ i ⓑ i & iii
ⓒ ii & iii ⓓ i, ii & iii **ⓒ**

16. The angle 430° is situated at— [Sylhet Board-'15]

ⓐ 1st quadrant ⓑ 2nd
ⓒ 3rd ⓓ 4th **ⓐ**

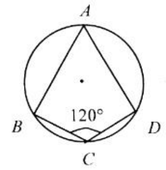
17. If sinθ = Error!, 0 < θ < Error!, then what is the value of θ? [Jessore Board-'15]

ⓐ $\frac{5\pi}{3}$ ⓑ $\frac{4\pi}{3}$
ⓒ $\frac{2\pi}{3}$ ⓓ $\frac{\pi}{3}$ **ⓑ**

18. The angle-665° lies on which quadrant? [Jessore Board-'15]

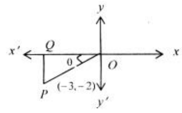
ⓐ First ⓑ Second
ⓒ Third ⓓ Fourth **ⓐ**

19.



In the figure, sinA = what? [Barisal Board-'15]

ⓐ 0 ⓑ **Error!**
ⓒ $\frac{1}{2}$ ⓓ **Error!** **Error!**



Answer to the questions No. 20 & 21 from the above figure:

20. In ΔPOQ, tanθ = what? [Barisal Board-'15]

ⓐ $-\frac{3}{2}$ ⓑ $-\frac{2}{3}$
ⓒ $\sqrt{3}$ ⓓ $\frac{2}{3}$ **ⓓ**

21. In ΔPOQ, cotθ + cosec²θ = what? [Barisal Board-'15]

ⓐ $-\frac{19}{4}$ ⓑ $-\frac{7}{4}$
ⓒ $\frac{7}{4}$ ⓓ $\frac{19}{4}$ **ⓓ**

Cadet Colleges MCQs with Answers

Cadet Colleges questions are also important for your excellent preparation. They will help you to give a clear idea about the question as well as chapterwise exclusive questions and answers. So, practice them with proper attention.

22. In which quadrant the angle 300° lie? *[Mirzapur Cadet-15]*

ⓐ First ⓑ Second
ⓒ Third ⓓ Fourth **ⓓ**

23. Which value of $\cos\left(-\frac{25\pi}{6}\right)$? *[Mymensingh Girls' Cadet-15]*

ⓐ $-\frac{1}{2}$ ⓑ $\frac{3}{2}$ ⓒ $\frac{1}{2}$ ⓓ $\frac{\sqrt{3}}{2}$ **ⓓ**

24. In which quadrant the angle $(550+\theta)^0$ lie? *[Rajshahi Cadet-15]*

ⓐ First ⓑ Second ⓒ Third ⓓ Fourth **ⓓ**

25. If $\theta = \frac{5\pi}{3}$ then the value of $(\cos^2\theta - 2)$ is — *[Rajshahi Cadet-15]*

ⓐ $\frac{1}{3}$ ⓑ $\frac{4}{7}$
ⓒ $\frac{7}{4}$ ⓓ $-\frac{7}{4}$ **ⓓ**

26. What is the value of $\cot\left(\frac{7\pi}{4}\right)$? *[Feni Girls' Cadet-15]*

ⓐ –1 ⓑ $-\frac{1}{\sqrt{3}}$
ⓒ $\frac{1}{\sqrt{3}}$ ⓓ 1 **ⓐ**

27. $\sin^2\phi - \cos^2\phi = \cos\phi$ then value of ϕ = ? *[Feni Girls' Cadet-15]*

ⓐ $\frac{\pi}{3}$ ⓑ $-\frac{\pi}{2}$ ⓒ $\frac{2\pi}{5}$ ⓓ $\frac{2\pi}{3}$ **ⓐ**

28. The value of $\tan\left(\frac{11\pi}{6}\right)$ is – *[Faujdarhat Cadet-15]*

ⓐ $-\frac{1}{\sqrt{3}}$ ⓑ $\frac{1}{\sqrt{3}}$
ⓒ $\sqrt{3}$ ⓓ $-\sqrt{3}$ **ⓐ**

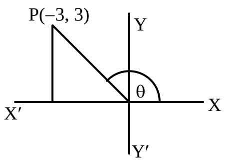
29. What is the value of cos $\left(\frac{-25\pi}{6}\right)$? *[Jhenidah Cadet-15]*

ⓐ $\frac{2}{\sqrt{3}}$ ⓑ $\frac{1}{2}$
ⓒ $\frac{\sqrt{3}}{2}$ ⓓ $\frac{1}{\sqrt{2}}$ **ⓒ**

30. $\sin^2(-\theta) + \cos^2(-\theta)$ = ? *[Jhenidah Cadet-15]*

ⓐ 0 ⓑ 1
ⓒ $-\theta$ ⓓ θ **ⓑ**

31. What is the value of θ for the following — *[Jhenidah Cadet-15]*



ⓐ $\frac{\pi}{4}$ ⓑ $\frac{5\pi}{4}$
ⓒ $\frac{3\pi}{4}$ ⓓ $\frac{2\pi}{4}$ **ⓒ**

32. What is the value of sin(–θ)? *[Barisal Cadet-15]*

ⓐ sinθ ⓑ –sinθ
ⓒ cosθ ⓓ –cosθ **ⓑ**

33. What is the value of sin$\left(\frac{11\pi}{2} + \theta\right)$? *[Barisal Cadet-15]*

ⓐ sinθ ⓑ –sinθ
ⓒ cosθ ⓓ –cosθ **ⓓ**

34. Which one is the sum of exterior angle of an equilateral triangle? *[Barisal Cadet-15]*

ⓐ 180° ⓑ 120°
ⓒ 270° ⓓ 360° **ⓓ**

35. The value of cos $\left(2\pi + \frac{\pi}{6}\right)$ is — *[Mirzapur Cadet-14]*

ⓐ $-\frac{\sqrt{3}}{2}$ ⓑ $-\frac{1}{\sqrt{3}}$
ⓒ $\frac{\sqrt{3}}{2}$ ⓓ $\frac{1}{\sqrt{2}}$ **ⓒ**

36. $\sin^2\varnothing - \cos^2\varnothing = \cos\varnothing$ then value of $\varnothing$ = ? *[Mymensingh Girls' Cadet-14]*

ⓐ $\frac{\pi}{3}$ ⓑ $-\frac{\pi}{2}$
ⓒ $\frac{2\pi}{5}$ ⓓ $\frac{2\pi}{3}$ **ⓐ**

37. Value of cos (990°) — *[Pabna Cadet-14]*

ⓐ 0 ⓑ 1
ⓒ –1 ⓓ $\frac{1}{\sqrt{2}}$ **ⓐ**

38. What is the value of x in sin2x + cos2x = 1? *[Joypurhat Girls' Cadet-14]*

ⓐ 0° ⓑ 30°
ⓒ 45° ⓓ 90° **ⓐ**

39. If sin3x = 1 and cos3x = 0, then the value of x is — *[Joypurhat Girls' Cadet-14]*

ⓐ 0° ⓑ 30°
ⓒ 45° ⓓ 90° **ⓑ**

40. What is the value of sin1500°? *[Rangpur Cadet-14]*

ⓐ 0 ⓑ $\frac{1}{2}$
ⓒ 1 ⓓ $\frac{\sqrt{3}}{2}$ **ⓓ**

41. If tanθ = $\frac{3}{4}$ and $180^0 < \theta < 270^0$, what is the value of cosθ? *[Rangpur Cadet-14]*

ⓐ $-\frac{4}{5}$ ⓑ $-\frac{3}{5}$
ⓒ $-\frac{3}{4}$ ⓓ $\pm\frac{4}{5}$ **ⓐ**

42. If sinθ + cosθ = 1, possible values of 'θ'. *[Feni Girls' Cadet-14]*

ⓐ 0° & 45° ⓑ 0° & 90°
ⓒ 0° & 120° ⓓ 30° & 45° **ⓑ**

43. If cosθ = $\frac{1}{\sqrt{2}}$, possible values of 'θ'. *[Feni Girls' Cadet-14]*

ⓐ 45° & 300° ⓑ 45° & 315°
ⓒ 45° & 120° ⓓ 30° & 45° **ⓑ**

44. In which quadrant the angle 280° lie? *[Faujdarhat Cadet-14]*

ⓐ 1st ⓑ 2nd
ⓒ 3rd ⓓ 4th **ⓓ**

45. If sinθ = $\frac{1}{\sqrt{2}}$, possible values of '0' — *[Sylhet Cadet-14]*

ⓐ 45°, 300° ⓑ 45°, 135°
ⓒ 45°, 120° ⓓ 30°, 45° **ⓑ**

46. If tanθ = $-\sqrt{3}$, then the value of θ is — *[Jhenidah Cadet-14]*

ⓐ $\frac{2\pi}{3}$ ⓑ $\frac{5\pi}{3}$
ⓒ Both a & b ⓓ None of the above **ⓓ**

47. If θ is any angle, then which of the following is correct? *[Jhenidah Cadet-14]*

ⓐ $-\infty \le \tan\theta \le \infty$ ⓑ $-\infty \le \cot\theta \le \infty$
ⓒ $-1 \le \sin\theta \le 1$ ⓓ All of the above **ⓒ**

48. Solution of the equation 2 sinx cosx = sin x is — *[Barisal Cadet-14]*

ⓐ $\frac{5\pi}{40}$ ⓑ $\frac{5\pi}{4}$
ⓒ $\frac{5\pi}{3}$ ⓓ $\frac{5\pi}{11}$ **ⓒ**

49. i. sin(4π + A) = sinA
ii. tan(3π + A) = tanA
iii. cos(2π + A) = cosA

Which one is correct? *[Rajshahi Cadet-15]*

ⓐ i and ii ⓑ ii and iii
ⓒ i and iii ⓓ i, ii and iii **ⓓ**

Topicwise MCQs with Answers

Pay your earnest attention to the topic-related information for making your concept clear

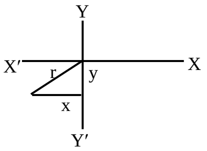
★★8.12 Trigonometrical Ratios of (– θ) | Text page-160

TOP TIPS

- Trigonometric ratios of the angle **(–θ):**

sin(–θ) = – sinθ	cosec(–θ) = – cosecθ
cos(–θ) = cosθ	sec(–θ) = secθ
tan(–θ) = – tanθ	cot(–θ) = – cotθ

50.



In the figure, what is the value of sec (– θ) ?(easy)

ⓐ $\frac{r}{x}$ ⓑ $-\frac{r}{x}$ ⓒ $\frac{x}{r}$ ⓓ $-\frac{x}{r}$ **ⓐ**

51. If cos (– θ) = $\frac{\sqrt{3}}{2}$, what is the value of θ in degree? (medium)

ⓐ – 30 ⓑ 0 ⓒ 30 ⓓ 60 **ⓒ**

52. If sin (– θ) = $\frac{1}{2}$, what is the value of θ in radian? (medium)

ⓐ $-\frac{\pi}{6}$ ⓑ $-\frac{\pi}{3}$ ⓒ $\frac{\pi}{6}$ ⓓ $\frac{\pi}{3}$ **ⓐ**

53. What is the value of tan $\left(-\frac{\pi}{6}\right)$? (easy)

ⓐ $\frac{1}{\sqrt{3}}$ ⓑ $\frac{1}{3}$ ⓒ $\frac{1}{\sqrt{2}}$ ⓓ $-\frac{1}{\sqrt{3}}$ **ⓓ**

54. What is the value of cot $\left(-\frac{\pi}{3}\right)$? (easy)

ⓐ $\frac{1}{\sqrt{3}}$ ⓑ $\frac{1}{\sqrt{2}}$ ⓒ $-\frac{1}{\sqrt{2}}$ ⓓ $-\frac{1}{\sqrt{3}}$ **ⓓ**

55. If tan(– θ) = – tanθ —

i. tan (– 60°) = – $\sqrt{3}$ ii. $\tan^2$ (– 60°) = 3

iii. $\sec^2$ (– 60°) = 4

Which of the following is correct? (medium)

ⓐ i and ii ⓑ i and iii

ⓒ ii and iii ⓓ i, ii and iii **ⓓ**

On the basis of following information, answer the questions (7-9):

cosec (– θ) = $\frac{2}{\sqrt{3}}$.

56. What is the value of θ in degree? (medium)

ⓐ – 60 ⓑ 0 ⓒ 45 ⓓ 60 **ⓐ**

57. What is the value of sinθ ? (easy)

ⓐ $-\frac{\sqrt{3}}{2}$ ⓑ $-\frac{2}{\sqrt{3}}$ ⓒ $\frac{\sqrt{3}}{2}$ ⓓ $\frac{2}{\sqrt{3}}$ **ⓐ**

58. cosec^2 (– θ) + $\sin^2\theta$ = what? (medium)

ⓐ $-\frac{25}{12}$ ⓑ $-\frac{12}{25}$

ⓒ $\frac{12}{25}$ ⓓ $\frac{25}{12}$ **ⓓ**

★★★8.13 Trigonometric Ratios of $\left(\frac{\pi}{2} - \theta\right)$ and $\left(\frac{\pi}{2} + \theta\right)$ | Text page-161, 163

TOP TIPS

- Trigonometric ratios of $\left(\frac{\pi}{2} - \theta\right)$ and $\left(\frac{\pi}{2} + \theta\right)$

sin(90° – θ) = cosθ	cosec(90° – θ) = secθ
cos(90°– θ) = sinθ	sec(90° – θ) = cosecθ
tan(90°– θ) = cotθ	cot(90° – θ) = tanθ

sin(90° + θ) = cosθ	cosec(90° + θ) = secθ
cos(90° + θ) = – sinθ	sec(90° + θ) = – cosecθ
tan(90° + θ) = – cotθ	cot(90° + θ) = – tanθ

59. What is the value of sin Error!? (easy)

ⓐ $\frac{2}{\sqrt{3}}$ ⓑ $\frac{\sqrt{3}}{2}$

ⓒ $\frac{3}{4}$ ⓓ $\frac{9}{4}$ **ⓑ**

60. sin $\left(\frac{\pi}{2} - \frac{\pi}{3}\right)$ = what? (medium)

ⓐ – sin $\frac{\pi}{3}$ ⓑ sin $\frac{\pi}{3}$

ⓒ cos $\frac{\pi}{3}$ ⓓ – cos $\frac{\pi}{3}$ **ⓒ**

61. What is the value of sec$\left(\frac{\pi}{2} - \frac{\pi}{4}\right)$? (easy)

ⓐ – $\sqrt{2}$ ⓑ $-\frac{1}{\sqrt{2}}$ ⓒ $\frac{1}{\sqrt{2}}$ ⓓ $\sqrt{2}$ **ⓓ**

62. Which one of the following is equal to sec$\left(\frac{\pi}{4}\right)$? (medium)

ⓐ tan$\frac{\pi}{4}$ ⓑ cosec$\left(\frac{\pi}{4}\right)$

ⓒ cot$\frac{\pi}{4}$ ⓓ cos$\left(\frac{\pi}{4}\right)$ **ⓑ**

63. If tan $\left(\frac{\pi}{2} + \theta\right)$ = $\sqrt{3}$, what is the value of cotθ ? (medium)

ⓐ – $\sqrt{3}$ ⓑ 0 ⓒ $\sqrt{3}$ ⓓ 1 **ⓐ**

64. If sin $\left(\frac{\pi}{2} + \frac{\pi}{4}\right)$ = $\frac{1}{\sqrt{2}}$, what is the value of cos $\left(-\frac{\pi}{4}\right)$? (medium)

ⓐ $\frac{1}{\sqrt{2}}$ ⓑ $\frac{\sqrt{3}}{2}$ ⓒ $\frac{2}{\sqrt{3}}$ ⓓ $\sqrt{2}$ **ⓐ**

65. What is the value of cosec$\left(\frac{\pi}{2} + \frac{\pi}{4}\right)$? (medium)

ⓐ 2 ⓑ $\frac{1}{\sqrt{2}}$ ⓒ $\sqrt{2}$ ⓓ $\sqrt{3}$ **ⓒ**

66. What is the value of tan $\left(\frac{\pi}{2} - \theta\right)$? (easy)

ⓐ tan θ ⓑ cotθ

ⓒ – tanθ ⓓ – cotθ **ⓑ**

67. What is the value of cos $\left(\frac{\pi}{2}-\frac{\pi}{6}\right)$? (easy)

ⓐ $\frac{1}{\sqrt{3}}$ ⓑ $\frac{1}{2}$ ⓒ $-\frac{1}{\sqrt{3}}$ ⓓ $-\frac{2}{\sqrt{3}}$ **ⓑ**

68. sec $\frac{3\pi}{4}$ is equal to –

i. $\sec\left(\frac{\pi}{2}+\frac{\pi}{4}\right)$. ii. $-\text{cosec}\,\frac{\pi}{4}$.

iii. $-\sqrt{2}$.

Which of the following is correct? (medium)

ⓐ i and ii ⓑ i and iii

ⓒ ii and iii ⓓ i, ii and iii **ⓓ**

On the basis of following information answer the questions (69-70):

$A = \tan\left(\frac{\pi}{2}+x\right) = -\frac{1}{\sqrt{3}}$

69. Which one of the following is equal to A ?(easy)

ⓐ – cotx ⓑ – tanx

ⓒ cotx ⓓ tanx **ⓐ**

70. What is the value of x in degree?(medium)

ⓐ – 60 ⓑ – 30

ⓒ 30 ⓓ 60 **ⓓ**

★★★ 8.14 Trigonometrical Ratios of (π + θ) and (π –θ) | Text page -164, 165 **TOP TIPS**

- Trigonometrical ratios of (180° – θ), (0° < θ < 90°)

sin(180° – θ) = sinθ	cosec(180° – θ) = cosecθ
cos(180° – θ) = – cosθ	sec(180° – θ) = – secθ
tan(180° – θ) = – tanθ	cot(180° – θ) = – cotθ

- Trigonometrical ratios of (180° + θ) , (0° < θ < 90°)

sin(180° + θ) = – sinθ	cosec(180° + θ) = –cosecθ
cos(180° + θ) = – cosθ	sec(180° + θ) = – secθ
tan(180° + θ) = tanθ	cot(180° + θ) = cotθ

71. What is the value of sin $\left(\frac{4\pi}{3}\right)$? (medium)

ⓐ $\frac{-\sqrt{3}}{2}$ ⓑ $\frac{\sqrt{3}}{2}$ ⓒ $\frac{1}{2}$ ⓓ 1 **ⓐ**

72. If $\tan(\pi+x)=\frac{1}{\sqrt{3}}$, what is the value of x in radian? (medium)

ⓐ $\frac{\pi}{6}$ ⓑ $\frac{\pi}{4}$ ⓒ $\frac{\pi}{3}$ ⓓ $\frac{\pi}{2}$ **ⓐ**

73. tan (π – 30°) = what? (easy)

ⓐ – tan 30° ⓑ tan 30°

ⓒ cot 30° ⓓ tan 60° **ⓐ**

74. If tanθ = $\sqrt{3}$,

i. $\tan(\pi+\theta)=\sqrt{3}$

ii. $\tan(\pi-\theta)=-\sqrt{3}$

iii. $\theta=\frac{\pi}{6}$

Which of the following is correct? (medium)

ⓐ i and ii ⓑ i and iii

ⓒ ii and iii ⓓ i, ii and iii **ⓐ**

Explanation: iii. is not right. Because,

$\tan\theta=\sqrt{3}=\tan\frac{\pi}{3}\ \therefore\ \theta=\frac{\pi}{3}$

75. tan $\frac{5\pi}{6}$ is equal to —

i. $\tan\left(\pi+\frac{\pi}{6}\right)$ ii. $-\tan\frac{\pi}{6}$ iii. $-\frac{1}{\sqrt{3}}$

Which of the following is correct? (medium)

ⓐ i and ii

ⓑ i and iii

ⓒ ii and iii

ⓓ i, ii and iii **ⓒ**

On the basis of following information, answer the questions (27-28):

$A = \cos(\pi+x) = -\frac{\sqrt{3}}{2}$.

76. Which one of the following is equal to A?(easy)

ⓐ – cosx ⓑ cosx

ⓒ sinx ⓓ secx **ⓐ**

77. x = what? (medium)

ⓐ $\frac{\pi}{2}$ radian ⓑ $\frac{\pi}{3}$ radian

ⓒ $\frac{\pi}{4}$ radian ⓓ $\frac{\pi}{6}$ radian **ⓓ**

★★ 8.15 Trigonometrical ratios of $\left(\frac{3\pi}{2}-\theta\right)$ and $\left(\frac{3\pi}{2}+\theta\right)$ | Text page-166, 167 **TOP TIPS**

- Trigonometrical ratios of (270° – θ), (0° < θ < 90°)

sin(270° – θ) = –cosθ	cosec(270° – θ) = –secθ
cos(270° – θ) = –sinθ	sec(270° – θ) = –cosecθ
tan(270° – θ) = cotθ	cot(270° – θ) = tanθ

- Trigonometrical ratios of (270° + θ), (0° < θ < 90°)

sin(270° + θ) = –cosθ	cosec(270° + θ) = –secθ
cos(270° + θ) = sinθ	sec(270° + θ) = cosecθ
tan(270° + θ) = –cotθ	cot(270° + θ) = –tanθ

78. If cotθ = $\frac{1}{\sqrt{3}}$, what is the value of tan $\left(\frac{3\pi}{2}-\theta\right)$? (medium)

ⓐ $-\frac{1}{\sqrt{3}}$ ⓑ $-\sqrt{3}$ ⓒ $\frac{1}{\sqrt{3}}$ ⓓ $\sqrt{3}$ **ⓒ**

79. If cosθ = $\frac{\sqrt{3}}{2}$, what is the value of sin $\left(\frac{3\pi}{2}+\theta\right)$? (medium)

ⓐ $-\frac{\sqrt{3}}{2}$ ⓑ $\frac{\sqrt{3}}{2}$ ⓒ $\frac{2}{\sqrt{3}}$ ⓓ $\frac{1}{\sqrt{3}}$ **ⓐ**

80. If sin $\left(\frac{3\pi}{2}-\theta\right)=-\frac{1}{\sqrt{2}}$

i. $\sin\left(\frac{3\pi}{2}-\theta\right)=-\cos\theta$.

ii. $\cos\theta=\frac{1}{\sqrt{2}}$. iii. $\theta=\frac{\pi}{4}$.

Which of the following is correct? (medium)

ⓐ i and ii ⓑ i and iii

ⓒ ii and iii ⓓ i, ii and iii **ⓓ**

★★★8.16 Trigonometrical Ratios of $(2\pi - \theta)$ and $(2\pi + \theta)$ | Text page-166, 167

TOP TIPS

- Trigonometrical ratios of $(2\pi - \theta)$ and $(2\pi + \theta)$
- Trigonometrical ratios of $(360° - \theta)$, $(0° < \theta < 90°)$

$\sin(360° - \theta) = -\sin\theta$	$\text{cosec}(360° - \theta) = -\text{cosec}\theta$
$\cos(360° - \theta) = \cos\theta$	$\sec(360° - \theta) = \sec\theta$
$\tan(360° - \theta) = -\tan\theta$	$\cot(360° - \theta) = -\cot\theta$

- Trigonometrical ratios of $(360° + \theta)$, $(0° < \theta < 90°)$

$\sin(360° + \theta) = \sin\theta$	$\text{cosec}(360° + \theta) = \text{cosec}\theta$
$\cos(360° + \theta) = \cos\theta$	$\sec(360° + \theta) = \sec\theta$
$\tan(360° + \theta) = \tan\theta$	$\cot(360° + \theta) = \text{tot}\theta$

81. What is the value of $\sin\left(2\pi - \frac{\pi}{3}\right)$? (medium)

ⓐ $-\frac{\sqrt{3}}{2}$ ⓑ $\frac{\sqrt{3}}{2}$ ⓒ 1 ⓓ $\sqrt{3}$ **ⓐ**

82. What is the value of $\sin\left(2\pi + \frac{\pi}{6}\right)$? (easy)

ⓐ $-\frac{1}{2}$ ⓑ $-\frac{1}{\sqrt{2}}$ ⓒ $\frac{1}{2}$ ⓓ $\frac{1}{\sqrt{2}}$ **ⓒ**

83. What is the value of $\cos\left(2\pi + \frac{\pi}{6}\right)$?(easy)

ⓐ $-\frac{\sqrt{3}}{2}$ ⓑ $-\frac{1}{\sqrt{3}}$ ⓒ $\frac{\sqrt{3}}{2}$ ⓓ $\frac{1}{\sqrt{2}}$ **ⓒ**

84. What is the value of $\tan\left(360° + \frac{\pi}{4}\right)$?(easy)

ⓐ -1 ⓑ $-\frac{1}{\sqrt{3}}$ ⓒ $\frac{1}{\sqrt{3}}$ ⓓ 1 **ⓓ**

85. What is the value of $\cot\left(2\pi - \frac{\pi}{6}\right)$? (easy)

ⓐ $\sqrt{3}$ ⓑ $\frac{1}{\sqrt{3}}$ ⓒ $-\frac{1}{\sqrt{3}}$ ⓓ $-\sqrt{3}$ **ⓓ**

86. What is the value of $\sec\left(2\pi - \frac{\pi}{4}\right)$? (easy)

ⓐ $-\sqrt{2}$ ⓑ $-\frac{2}{\sqrt{3}}$ ⓒ $\frac{2}{\sqrt{3}}$ ⓓ $\sqrt{2}$ **ⓓ**

87. What is the value of $\text{cosec}\left(2\pi + \frac{\pi}{4}\right)$? (easy)

ⓐ $-\frac{2}{\sqrt{3}}$ ⓑ $-\sqrt{2}$ ⓒ $\sqrt{2}$ ⓓ $\frac{2}{\sqrt{3}}$ **ⓒ**

88. If $\sec(2\pi - \theta) = \sqrt{2}$ –

i. $\sec(2\pi - \theta) = -\sec\theta$ ii. $\theta = \frac{\pi}{4}$

iii. $\sec\theta = \text{cosec}\theta$

Which of the following is correct? (easy)

ⓐ i and ii ⓑ i and iii
ⓒ ii and iii ⓓ i, ii and iii **ⓒ**

★★★ 8.17 Trigonometrical Ratios of any angle | Text page-168

TOP TIPS

- If n is any integer, the value of the angle $(n \times 90° \pm \theta)$ can be determined as follows:
- We are to divide the given angle into two parts whose one part is n multiple of 90^0 or $\frac{\pi}{2}$ i.e.$\left(n \times 90° \text{ ev, } n \times \frac{\pi}{2}\right)$ and the other part is an acute angle.
- If n is even, the ratio remain the same that is sine remain sine, cosine remain cosine etc.
- If n is odd, the ratio will be changed that is, sin, cos, tan, cot, sec, cosec will be changed into cos, sin, cot, tan, cosec and sec.
- After knowing the position in the quadrant of $(n \times 90° \pm \theta)$, we have to put the sign of ratio given in that quadrant in the step of ratio.

89. $\sin\left(\frac{9\pi}{2} + \theta\right)$ = what? (medium)

ⓐ $\sin\theta$ ⓑ $\cos\theta$ ⓒ $-\sin\theta$ ⓓ $-\cos\theta$ **ⓑ**

90. Which one of the following is the value of $\sec\left(-\frac{25\pi}{3}\right)$? (medium)

ⓐ -2 ⓑ $-\frac{1}{2}$ ⓒ $\frac{1}{2}$ ⓓ 2 **ⓓ**

91. What is the value of $\sin(19\pi + \theta)$? (medium)

ⓐ $\sin\theta$ ⓑ $\cos\theta$
ⓒ $-\sin\theta$ ⓓ $-\cos\theta$ **ⓒ**

92. What is the value of $\cot\left(\frac{21\pi}{2} - \theta\right)$? (medium)

ⓐ $\tan\theta$ ⓑ $\cot\theta$ ⓒ $-\tan\theta$ ⓓ $-\cot\theta$ **ⓐ**

93. $\tan\left(17\pi - \frac{\pi}{4}\right)$ = what? (medium)

ⓐ 1 ⓑ -1 ⓒ $\sqrt{3}$ ⓓ $-\sqrt{3}$ **ⓑ**

94. What is the value of $\sin\left(\frac{17\pi}{2} + \frac{\pi}{4}\right)$? (medium)

ⓐ $-\frac{1}{\sqrt{2}}$ ⓑ $-\sqrt{2}$ ⓒ $\frac{1}{\sqrt{2}}$ ⓓ $\sqrt{2}$ **ⓒ**

95. What is the value of $\text{cosec}\left(\frac{15\pi}{6}\right)$? (medium)

ⓐ -2 ⓑ $-\frac{\sqrt{3}}{2}$ ⓒ 1 ⓓ 2 **ⓒ**

96. If $\theta = \frac{7\pi}{3}$, what is the value of $\sec^2\theta - 1$? (hard)

ⓐ -3 ⓑ $-\sqrt{3}$ ⓒ $\sqrt{3}$ ⓓ 3 **ⓓ**

97. $\cos\frac{\pi}{15} + \cos\frac{16\pi}{15}$ = what? (hard)

ⓐ -1 ⓑ $-\frac{1}{\sqrt{2}}$ ⓒ 0 ⓓ 1 **ⓒ**

98. What is the value of $\cos^2\frac{\pi}{15} + \cos^2\frac{13\pi}{30}$? (hard)

ⓐ -2 ⓑ -1 ⓒ 0 ⓓ 1 **ⓓ**

99. If $\theta = \frac{5\pi}{3}$, what is the value of $\cos^2\theta - 2$? (hard)

ⓐ $\frac{4}{7}$ ⓑ $\frac{7}{4}$ ⓒ $-\frac{7}{4}$ ⓓ $-\frac{4}{7}$ **ⓒ**

100. If $\theta = \frac{3\pi}{2}$, what is the value of $1 + \sin^2\theta$? (hard)

ⓐ -1 ⓑ 0 ⓒ $\sqrt{2}$ ⓓ 2 **ⓓ**

101. In case of $\tan\left(\frac{n\pi}{2} + \theta\right)$ –

i. If n = 1, $-\cot\theta$ is obtained.

ii. If n = 9, $\tan\left(\frac{n\pi}{2} + \theta\right) = -\cot\theta$

iii. If n = 12, $\tan\left(\frac{n\pi}{2} + \theta\right) = \cot\theta$

Which of the following is correct? (easy)

ⓐ i and ii ⓑ i and iii ⓒ ii and iii ⓓ i, ii and iii **ⓐ**

102. If $\theta = \frac{3\pi}{2}$ —

i. $\tan(\theta + 60°) = -\frac{1}{\sqrt{3}}$

ii. $\sec(\theta - 45°) = -\sqrt{2}$

iii. $\text{cosec}\left(\theta - \frac{\pi}{6}\right) = -2$

Which of the following is correct? (medium)

ⓐ i and ii ⓑ i and iii ⓒ ii and iii ⓓ i, ii and iii — ⓐ

103. If θ = 360° —

i. $\cos\left(\theta - \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ ii. $\cot\left(\theta + \frac{\pi}{6}\right) = \sqrt{3}$

iii. $\tan\left(\theta - \frac{\pi}{4}\right) = 1$

Which of the following is correct? (medium)

ⓐ i and ii ⓑ i and iii

ⓒ ii and iii ⓓ i, ii and iii — ⓐ

104. If θ = π —

i. $\tan^2\left(\theta - \frac{\pi}{3}\right) = 3$ ii. $\sec^2\left(\theta + \frac{\pi}{4}\right) = 2$

iii. $\cos\left(\theta + \frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

Which of the following is correct? (medium)

ⓐ i and ii ⓑ i and iii

ⓒ ii and iii ⓓ i, ii and iii — ⓓ

105. If $\theta = \frac{14\pi}{2}$ —

i. $\text{cosec}\left(\theta - \frac{\pi}{3}\right) = \frac{2}{\sqrt{3}}$ ii. $\sin\left(\theta - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

iii. $\tan\left(\theta + \frac{\pi}{4}\right) = 1$

Which of the following is correct? (hard)

ⓐ i and ii ⓑ i and iii

ⓒ ii and iii ⓓ i, ii and iii — ⓓ

On the basis of following information, answer the questions (106-108):

$\tan\left(\frac{n\pi}{2} + \theta\right) = \sqrt{3}$, where n is odd.

106. For n = 3, which of the following is correct? (easy)

ⓐ $-\cot\theta$ ⓑ $\cot\theta$

ⓒ $\tan\theta$ ⓓ $-\tan\theta$ — ⓐ

107. What is the value of θ in radian? (medium)

ⓐ $\frac{\pi}{2}$ ⓑ $\frac{\pi}{3}$ ⓒ $\frac{\pi}{6}$ ⓓ $-\frac{\pi}{6}$ — ⓓ

108. If $\theta = -\frac{\pi}{6}$, what is the value of n? (hard)

ⓐ 0 ⓑ 1 ⓒ 2 ⓓ 6 — ⓑ

On the basis of following information, answer the questions (109-111):

If $\theta = \frac{\pi}{2}$ and $\psi = \pi$

109. What is the value of $\sin\left(\theta + \frac{\pi}{4}\right) + \cos\left(\psi - \frac{\pi}{4}\right)$? (hard)

ⓐ 0 ⓑ $\frac{1}{\sqrt{2}}$ ⓒ $\sqrt{2}$ ⓓ $-\sqrt{2}$ — ⓐ

110. Which one of the following is the value of $\text{cosec}\left(\theta + \frac{\pi}{6}\right) - \tan\left(\psi + \frac{\pi}{4}\right)$? (hard)

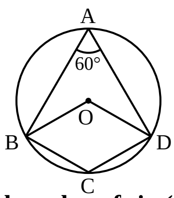
ⓐ 0 ⓑ $\frac{2-\sqrt{3}}{\sqrt{3}}$ ⓒ $\frac{1}{\sqrt{3}}$ ⓓ $\sqrt{3}$ — ⓑ

111. What is the value of $\sec\left(\theta - \frac{\pi}{3}\right) + \cot\left(\psi + \frac{\pi}{4}\right)$? (hard)

ⓐ 0 ⓑ 1 ⓒ $\frac{2}{\sqrt{3}}$ ⓓ $\frac{2}{\sqrt{3}} + 1$ — ⓓ

On the basis of following information, answer the questions (112-114):

The quadrilateral ABCD is inscribed in a circle centered at O.



112. What is the value of sin (A + C)? (medium)

ⓐ –1 ⓑ 0 ⓒ $\frac{1}{2}$ ⓓ 1 — ⓑ

113. What is the value of sin ∠ BOD? (medium)

ⓐ $-\frac{\sqrt{3}}{2}$ ⓑ 0 ⓒ $\frac{\sqrt{3}}{2}$ ⓓ 1 — ⓒ

114. What is the value of cos (A + B + C + D) ? (medium)

ⓐ –1 ⓑ 0 ⓒ $\frac{1}{2}$ ⓓ 1 — ⓓ

On the basis of following information, answer the questions (115-117):

If $A = \frac{13\pi}{2}$ and $B = \frac{19\pi}{2}$

115. $\text{cosec}\left(A + \frac{\pi}{3}\right)$ = what? (medium)

ⓐ 2 ⓑ 1 ⓒ –1 ⓓ –2 — ⓐ

116. $\cot\left(B + \frac{\pi}{6}\right)$ = what? (medium)

ⓐ $-\frac{1}{\sqrt{3}}$ ⓑ $-\frac{1}{\sqrt{2}}$ ⓒ $\frac{1}{\sqrt{2}}$ ⓓ $\frac{1}{\sqrt{3}}$ — ⓐ

117. What is the value of $\sin\left(A + \frac{\pi}{4}\right) + \cos\left(B + \frac{\pi}{4}\right)$? (medium)

ⓐ 0 ⓑ $\frac{2}{2}$ ⓒ $\sqrt{2}$ ⓓ 1 — ⓒ

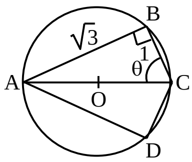
Essay-Type Practice Part

23 Creative Questions ■ 05 Board questions ■ 06 Cadet College questions ■ 02 Activity ■ 05 Additional questions ■ 05 Questions with hints

Board Exam Creative Questions with Answers

Board Exam questions are very important for the exam preparation. So practice these questions again and again properly.

Ques ▶ 1



[Dhaka Board-'15]

a. If O is the centre of the circle in the fig, then find AC.
b. Prove that, tanA + tanB + tanC + tanD = 0
c. If secθ + cosθ = x, Find the value of x and solve the equation.

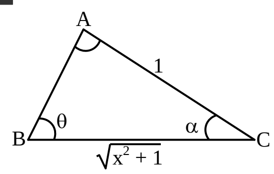
Ans to the Ques. No-1

a In the figure $\angle B$ is semi-circle angle.
We know, semi-circle angle measures 1 right angle.
$\therefore \angle B = 90°$
Again in ΔABC, $\angle B = 90°$
$\therefore$ ΔABC is a right angled triangle.
According to Pythagorus theorem,
$AC^2 = AB^2 + BC^2$
Or, $AC^2 = (\sqrt{3})^2 + 1^2$
Or, $AC^2 = 3 + 1$
Or, $AC^2 = 4$
$\therefore$ AC = 2 units **(Ans.)**

b ABCD is a quadrilateral inscribed in circle ABCD with centre O.
$\therefore \angle A + \angle C = 180°$ [Sum of opposite angles of a cyclic quadrilateral is equal to 180°]
And $\angle B + \angle D = 180°$
Now, L.H.S = tanA + tanB + tanC + tanD
= tanA + tan(180° – D) + tan (180° – A) + tanD
= tanA + tan(2 × 90° – D) + tan (2 × 90° – A) + tanD
= tanA – tanD – tanA + tanD
= 0
= R.H.S
$\therefore$ L.H.S = R.H.S. **(Proved)**

c Given, sec θ + cos θ = x ... ... (i)
Here, $\sec\theta = \frac{AC}{BC} = \frac{2}{1} = 2$ [$\because$ AC = 2; BC = 1]
And, $\cos\theta = \frac{BC}{AC} = \frac{1}{2}$
Putting values in (i)
$2 + \frac{1}{2} = x$
Or, $\frac{4+1}{2} = x$ Or, $x = \frac{5}{2}$
Now from (i) → $\sec\theta + \cos\theta = \frac{5}{2}$
Or, $\frac{1}{\cos\theta} + \cos\theta = \frac{5}{2}$
Or, $\frac{1 + \cos^2\theta}{\cos\theta} = \frac{5}{2}$
Or, $2\cos^2\theta + 2 = 5\cos\theta$
Or, $2\cos^2\theta - 5\cos\theta + 2 = 0$
Or, $2\cos^2\theta - 4\cos\theta - \cos\theta + 2 = 0$
Or, $2\cos\theta(\cos\theta - 2) - 1(\cos\theta - 2) = 0$
Or, $(2\cos\theta - 1)(\cos\theta - 2) = 0$
Either $\cos\theta - 2 = 0$ or, $2\cos\theta - 1 = 0$
or, $\cos\theta = 2$ or, $\cos\theta = \frac{1}{2}$
But $\cos\theta \neq 2$ or, $\cos\theta = \cos\frac{\pi}{3}$
Since it can't be more than 2 $\therefore \theta = \frac{\pi}{3}$

Ques ▶ 2

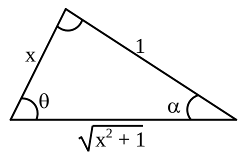


[Rajshahi Board-'15]

a. Find $\sin(\theta + \alpha)$. 2
b. From the stem, show that $(\sin\alpha + \cos\alpha)^2 = 1 + 2\sin\alpha.\cos\alpha$. 4
c. If $x + \sqrt{x^2+1} = \sqrt{3}$ find θ. 4

Ans to the Ques. No-2

a



$\therefore \sin(\theta + \alpha) = \sin\theta\cos\alpha + \cos\theta\sin\alpha$
$= \frac{1}{\sqrt{1+x^2}} \times \frac{1}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1+x^2}} \times \frac{x}{\sqrt{1+x^2}}$
$= \frac{1}{1+x^2} + \frac{x^2}{1+x^2}$
$= \frac{1+x^2}{1+x^2}$
= 1 **(Ans.)**

b L.H.S = $(\sin\alpha + \cos\alpha)^2$
$= \left(\frac{x}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1+x^2}}\right)^2$
$= \left(\frac{1+x}{\sqrt{1+x^2}}\right)^2$
$= \frac{(1+x)^2}{1+x^2}$
R.H.S = $1 + 2\sin\alpha\cos\alpha$
$= 1 + 2.\frac{x}{\sqrt{1+x^2}} \cdot \frac{1}{\sqrt{1+x^2}}$
$= 1 + \frac{2x}{1+x^2}$
$= \frac{1 + x^2 + 2x}{1+x^2} = \frac{(1+x)^2}{1+x^2}$
$\therefore$ L.H.S. = R.H.S. **(Proved)**

c $x + \sqrt{x^2+1} = \sqrt{3}$

$\Rightarrow (\sqrt{x^2+1})^2 = (\sqrt{3} - x)^2$

$\Rightarrow x^2 + 1 = 3 + x^2 - 2\sqrt{3}x$

$\Rightarrow 1 = 3 - 2\sqrt{3}x$

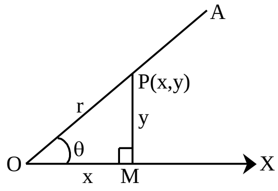
$\Rightarrow 2\sqrt{3}x = 2$

$\Rightarrow$ x = **Error!**

$\therefore \sin\theta = \frac{1}{\sqrt{1+x^2}} = \frac{1}{\sqrt{1+\frac{1}{3}}} = \frac{1}{\sqrt{\frac{4}{3}}}$ = **Error!**

$\theta = 60°$ **(Ans.)**

Ques ▶ 3



[Comilla Board-'15]

a. If x = y then prove that, $r = \sqrt{2}x$.

b. Prove with respect to stem, $sce^2\theta - \tan^2\theta = 1$.

c. If $\frac{2y^2}{x^2+y^2} - \frac{3x}{\sqrt{x^2+y^2}} = 0$ find the value of θ. (where $0° < \theta < \frac{\pi}{2}$).

Ans to the Ques. No-3

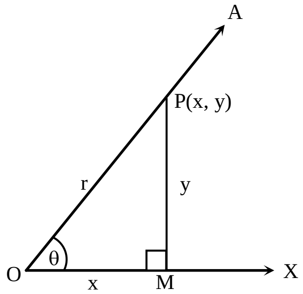
a In right angled ΔPOM, $OP^2 = OM^2 + PM^2$

Or, $r^2 = x^2 + y^2$

Or, $r^2 = x^2 + x^2$ [∵ x = y]

Or, $r^2 = 2x^2$ ∴ $r = \sqrt{2}\,x$ **(Proved)**

b



ΔPOM is a triangle.

whose, base, OM = x; Perpendicular PM = y & hypotenuse OP = r.

It is required to prove that, $\sec^2\theta - \tan^2\theta = 1$

From the figure,

$\sec\theta = \frac{\text{hypotenuse}}{\text{base}} = \frac{r}{x}$

$\tan\theta = \frac{\text{perpendicular}}{\text{base}} = \frac{y}{x}$

$\therefore \sec^2\theta - \tan^2\theta = \left(\frac{r}{x}\right)^2 - \left(\frac{y}{x}\right)^2$

$= \frac{r^2}{x^2} - \frac{y^2}{x^2} = \frac{r^2 - y^2}{x^2}$

$= \frac{x^2 + y^2 - y^2}{x^2}$ from $r^2 = x^2 + y^2$]

$= \frac{x^2}{x^2} = 1$

$\therefore \sec^2\theta - \tan^2\theta = 1$ **(Proved)**

c From the figure,

$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{y}{r} = \frac{y}{\sqrt{x^2+y^2}}$ [From @]

$\cos\theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{x}{r} = \frac{x}{\sqrt{x^2+y^2}}$ [From @]

Given, $\frac{2y^2}{x^2+y^2} - \frac{3x}{\sqrt{x^2+y^2}} = 0$

Or, $2\left(\frac{y}{\sqrt{x^2+y^2}}\right)^2 - 3\frac{x}{\sqrt{x^2+y^2}} = 0$

Or, $2\sin^2\theta - 3\cos\theta = 0$

Or, $2(1 - \cos^2\theta) - 3\cos\theta = 0$

Or, $2 - 2\cos^2\theta - 3\cos\theta = 0$

Or, $2\cos^2\theta + 3\cos\theta - 2 = 0$

Or, $2\cos^2\theta + 4\cos\theta - \cos\theta - 2 = 0$

Or, $2\cos\theta(\cos\theta + 2) - 1(\cos\theta + 2) = 0$

Or, $(\cos\theta + 2)(2\cos\theta - 1) = 0$

$\therefore \cos\theta + 2 = 0$

Or, $\cos\theta = -2$ [not accepted]

$[-1 \le \cos\theta \le 1]$

$\therefore 2\cos\theta - 1 = 0$

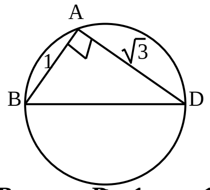
Or, $2\cos\theta = 1$

Or, $\cos\theta = \frac{1}{2}$

Or, $\cos\theta = \cos 60°$

$\therefore \theta = 60°$ **(Ans.)**

Ques ▶ 4



and $P = \frac{\cot B + \text{cosec} B - 1}{\cot B - \text{cosec} B + 1}$; $Q = \frac{1 + \sin D}{\cos D}$ [Jessore Board-'15]

a. Find the radius of cirde ABCD.

b. Prove that, cos (B – D) = cosB cosD + sinB sinD.

c. Show that, P = Q

Ans to the Ques. No-4

a In circle ABCD, ∠BAD is a right angle,

∴ ∠BAD is semi-circle angle.

So, BD is its diameter,

In, ΔABD,

$BD^2 = AB^2 + AD^2$

$= 1^2 + (\sqrt{3}^2)$

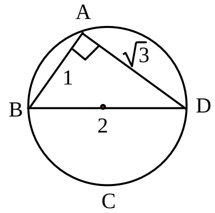
$= 1 + 3$

Or, $BD = \sqrt{4}$

$= 2$

∴ radius, $r = \frac{BD}{2} = 1$ **(Ans.)**

b In ΔABD,



$\cos B = \frac{AB}{BD} = \frac{1}{2}$

$\cos D = \frac{AD}{BD} = \frac{\sqrt{3}}{2}$

$\sin B = \frac{AD}{BD} = \frac{\sqrt{3}}{2}$

$\sin D = \frac{AB}{BD} = \frac{1}{2}$

Again, $\cos B = \frac{1}{2} = \cos\frac{\pi}{3}$

$\therefore B = \frac{\pi}{3}$

$\cos D = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$

$\therefore D = \frac{\pi}{6}$

L.H.S = cos(B – D)

$= \cos\left(\frac{\pi}{3} - \frac{\pi}{6}\right)$

$= \cos\frac{\pi}{6}$

$= \frac{\sqrt{3}}{2}$

R.H.S = cosB.cosD + sinB.sinD

$= \frac{1}{2}.\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}.\frac{1}{2}$ [Putting values]

$= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$

= **Error!**

$= \frac{\sqrt{3}}{2}$

∴ cos(B – D) = cosB.cosD + sinB sinD **(Proved)**

c From 'b'.

$B = \frac{\pi}{3}$

$D = \frac{\pi}{6}$

L.H.S = P

$= \frac{\cot B + \operatorname{cosec} B - 1}{\cot B - \operatorname{cosec} B + 1}$

= **Error!**
= **Error!**
= **Error!**
= **Error!**
= **Error!**

$= \sqrt{3}$

R.H.S. = Q

$= \frac{1 + \sin D}{\cos D}$

= **Error!**
= **Error!**

$= \frac{\frac{3}{2}}{\frac{\sqrt{3}}{2}}$

$= \sqrt{3}$

∴ P = Q **(Showed)**

Ques ▶5 A = 1 – sinθ, B = secθ – tanθ & C = 1 + sinθ. [Barisal Board-'15]

a. Show that, B = A secθ.

b. $B = (\sqrt{3})^{-1}$ Determine the value of θ, where θ is an acute angle.

c. Prove that, $AC^{-1} = B^2$.

Ans to the Ques. No-5

a Given, A = 1 – sinθ

B = secθ – tanθ

$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$

$= \frac{1 - \sin\theta}{\cos\theta}$

$= (1 - \sin\theta).\frac{1}{\cos\theta}$

= A.secθ [putting values]

∴ B = A.secθ **(Showed)**

b Given,

A = 1 – sinθ

$B = (\sqrt{3})^{-1}$

Or, $\sec\theta - \tan\theta = \frac{1}{\sqrt{3}}$ [∵ B = secθ – tanθ]

Or, $\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$

Or, $\frac{1 - \sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$

Or, $\sqrt{3}(1 - \sin\theta) = \cos\theta$

Or, $3(1 - 2\sin\theta + \sin^2\theta) = \cos^2\theta$ [squaring both sides]

Or, $3 - 6\sin\theta + 3\sin^2\theta = 1 - \sin^2\theta$

Or, $4\sin^2\theta - 6\sin\theta + 2 = 0$

Or, $2\sin^2\theta - 3\sin\theta + 1 = 0$

Or, $2\sin^2\theta - 2\sin\theta - \sin\theta + 1 = 0$

Or, $2\sin\theta(\sin\theta - 1) - 1(\sin\theta - 1) = 0$

Or, $(\sin\theta - 1)(2\sin\theta - 1) = 0$

Either, $\sin\theta = 1 = \sin\frac{\pi}{2}$

∴ $\theta = \frac{\pi}{2}$ [not acceptale since θ is acute]

Or, $2\sin\theta - 1 = 0$

Or, $\sin\theta = \frac{1}{2} = \sin\frac{\pi}{3}$

∴ $\theta = \frac{\pi}{3}$

∴ required solution, $\theta = \frac{\pi}{3}$ **(Ans.)**

c L.H.S = AC^{-1}

$= \frac{A}{C}$

$= \frac{1 - \sin\theta}{1 + \sin\theta}$ [∵ A = 1 – sinθ & c =1 + sinθ]

$= \frac{(1 - \sin\theta)(1 - \sin\theta)}{(1 + \sin\theta)(1 - \sin\theta)}$

$= \frac{(1 - \sin\theta)^2}{1 - \sin^2\theta}$

$= \frac{(1 - \sin\theta)^2}{\cos^2\theta}$

$= \left(\frac{1 - \sin\theta}{\cos\theta}\right)^2$

$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$

$= (\sec\theta - \tan\theta)^2$

$= B^2$ [∴ B = secθ – tanθ]

= R.H.S.

∴ $AC^{-1} = B^2$ **(Proved)**

Cadet Colleges Creative Questions with Answers

Cadet Colleges questions are also important for your excellent preparation. They will help you to give a clear idea about the question as well as chapterwise exclusive questions and answers. So, practice them with proper attention.

Ques ▶6 Given $\operatorname{cosec}A - \cot A = \frac{1}{x}$, where A is an acute angle. *[Rajshahi Cadet-14]*

a. Find the value of (cosecA + cotA) 2

b. Show that, $\sec A = \frac{x^2 + 1}{x^2 - 1}$ 4

c. $\frac{2x}{1 + x^2} + \frac{x^2 - 1}{x^2 + 1} = \sqrt{2}$, find the value of A. 4

Solution to the question no. 6

a We have,

$\operatorname{cosec} A - \cot A = \frac{1}{x}$

$\therefore$ (cosecA – cotA)cosecA + cotA) = $\frac{1}{2}$(cosecA + cotA)

or, $cosec^2 - cot^2A = \frac{1}{x}$(cosecA + cotA)

or, $1 = \frac{1}{x}$(cosecA + cotA), since $1 + cot^2A = cosec^2A$

or, cosecA + cotA = x, multiplying both sides by x

$\therefore$ cosecA + cotA = x

b From (a) we have,

x = cosecA + cotA

or, $x = \frac{1}{sinA} + \frac{cosA}{sinA}$

or, $x = \frac{1 + cosA}{sinA}$

or, $x^2 = \frac{(1 + cosA)^2}{sin^2A}$

or, $x^2 = \frac{(1cosA)^2}{1 - cos^2A}$

or, $x^2 = \frac{(1 + cosA)(1 + cosA)}{(1 + cosA)(1 - cosA)}$

or, $x^2 = \frac{1 + cosA}{1 - cosA}$

$\therefore \frac{x^2 + 1}{x^2 - 1} = \frac{1 + cosA + 1 - cosA}{1 + cosA - 1 + cosA}$

or, $\frac{x^2 + 1}{x^2 - 1} = \frac{2}{2cosA}$

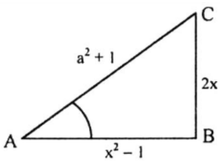
or, $\frac{x^2 + 1}{x^2 - 1} = \frac{1}{cosA}$

or, $\frac{x^2 + 1}{x^2 - 1} = secA$

$\therefore secA = \frac{x^2 + 1}{x^2 - 1}$ **(Showed)**

c From (b) we have,

$secA = \frac{x^2 + 1}{x^2 - 1} \Rightarrow cosA = \frac{x^2 - 1}{x^2 + 1}$



$\therefore sinA = \frac{2x}{1 + x^2}$

$\therefore \frac{2x}{x^2 + 1} + \frac{x^2 - 1}{x^2 + 1} = \sqrt{2}$

or, $(sinA + cosA)^2 = (\sqrt{2})^2$

or, $sinA + cosA = \sqrt{2}$

or, $(sinA + cosA)^2 + 2sinA. cosA = 2$

or, 1 + 2sinA . cosA = 2

or, 2sinA . cosA = 1

or, $4sin^2A . cos^2A = 1$, squaring both sides

or, $4(1 - cos^2A)cos^2A = 1$

or, $4cos^2A - 4cos^2A = 1$

or, $4cos^2A - 4cos^2A + 1 = 0$

or, $(2cos^2A)^2 - 2 . 2cos^2A . 1 + 1^2 = 0$

or, $(2 cos^2A - 1)^2 = 0$

or, $2cos^2A = 1$

or, $cos^2A = \frac{1}{2}$

or, $cosA = \frac{1}{\sqrt{2}} = cos45°$

$\therefore$ A = 45°

So, the value of A = 45°

Ques ▶ 7 We have $sin^2\theta + cos^2\theta = 1$, then–

[Joypurhat Cadet-14]

a. Find sinθ in terms of tanθ 2

b. Prove above trigonometric formula. 4

c. Solve : sinθ + cosθ = 1, where 0° ≤ θ ≤ 90° 4

Solution to the question no. 7

a $sin\theta = sin\theta . \frac{cos\theta}{cos\theta}$

$= \frac{sin\theta}{cos\theta} . cos\theta$

$= tan\theta . \sqrt{cos^2\theta}$

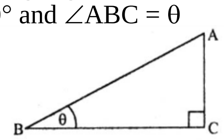
$= tan\theta . \sqrt{\frac{1}{sec^2\theta}}$

$= tan\theta . \sqrt{\frac{1}{1 + tan^2\theta}}$

$= \frac{tan\theta}{\sqrt{1 + tan^2\theta}}$

$\therefore sin\theta = \frac{tan\theta}{\sqrt{1 + tan^2\theta}}$

b In the adjoining figure, ΔABC is a right triangle with ∠C = 90° and ∠ABC = θ



$\therefore$ BC is the base and AC is the perpendicular of the triangle.

$\therefore sin\theta = \frac{AC}{AB} \Rightarrow sin^2\theta = \frac{AC^2}{AB^2}$ .... (i)

And $cos\theta = \frac{BC}{AB} \Rightarrow cos^2\theta = \frac{BC^2}{AB^2}$ .... (ii)

Now from (i) and (ii) we have,

$sin^2\theta + cos^2\theta = \frac{AC^2}{AB^2} + \frac{BC^2}{AB^2}$

$= \frac{AC^2 + BC^2}{AB^2}$

$= \frac{AB^2}{AB^2}$ according the theorem of Pythagoras

= 1.

$\therefore sin^2\theta + cos^2\theta = 1$. **(Proved)**

c sinθ + cosθ = 1

or, sinθ = 1 – cosθ

or, $sin^2\theta = 1 - 2cos\theta + cos^2\theta$, squaring both sides

or, $1 - cos^2\theta = 1 - 2cos\theta + cos^2\theta$

or, $2cos^2\theta - 2cos\theta = 0$

or, 2cosθ(cosθ – 1) = 0

or, cosθ = 0 = cos90° ⇒ θ = 90°

or, cosθ = 1 = cos90° ⇒ θ = 0°

since 0 ≤ θ ≤ 90°.

$\therefore$ The desired solution : θ = 0° or, 90°

Ques ▶ 8 If an angle by θ° & 0^c in sexagesimal & circular systems. *[Sylhet Cadet-14]*

a. If θ = 75° 30′, express as circular system. 2

b. Evaluate, $sin^2\frac{\pi}{7} + sin^2\frac{5\pi}{14} + sin^2\frac{8\pi}{7} + sin^2\frac{9\pi}{14}$. 4

c. Solve, $(cot^2\theta + cosec^2\theta) = 3$; where 0° ≤ θ ≤ 360°. 4

Solution to the question no. 8

a Here, θ = 75° 30′

$= 75.5' = 75.5 \times \frac{\pi}{180}$

$= \frac{151\pi}{2 \times 180} = \frac{151\pi}{360}$

$\therefore \theta = \frac{151\pi}{360}$ radian

b We have,

$\sin^2\frac{\pi}{7} + \sin^2\frac{5\pi}{14} + \sin^2\frac{8\pi}{7} + \sin^2\frac{9\pi}{14}$

$= \sin^2\frac{\pi}{7} + \sin^2\left(\frac{\pi}{2} - \frac{\pi}{7}\right) + \sin^2\left(\pi + \frac{\pi}{7}\right) + \sin^2\left(\frac{\pi}{2} + \frac{\pi}{7}\right)$

$=\sin^2\frac{\pi}{7} + \cos^2\frac{\pi}{7} + \sin^2\frac{\pi}{7} + \cos^2\frac{\pi}{7}$

$= 1 + 1 = 2$

$\therefore \sin^2\frac{\pi}{7} + \sin\frac{5\pi}{7} + \sin^2\frac{8\pi}{7} + \sin^2\frac{9\pi}{7} = 2.$

c Here we have,

$\cot^2 + \text{cosec}^2\theta = 3$

or, $\cot^2\theta + 1 + \cot^2\theta = 3$

or, $2\cot^2\theta = 3 - 1$

or, $2\cot^2\theta = 2$

or, $\cot^2\theta = 1$, dividing both sides by 2

or, $\cot\theta = 1 = \cot 45°$ or, $\cot 225°$

$\therefore$ θ = 45° or, 225° which is in 0 < θ < 360

So, the desired solution is : x = 45° or, 225°

Ques▶9 Use 3.1416 as the approximate value or π in solving problems. *[Barisal Cadet-14]*

a. The angles of a triangle are in arithmetical progress and largest angle is twice the smallest angle. What is the radian measurement of the angles? 2

b. Solve : (when $0 < \theta < 2\pi$); $3(\sec^2\theta + \tan^2\theta) = 5$. 4

c. Prove that : $\sin^2\frac{\pi}{7} + \sin^2\frac{5\pi}{14} + \sin^2\frac{8\pi}{7} + \sin^2\frac{9\pi}{14} = 2$ 4

<u>Solution to the question no. 9</u>

a Suppose, In a Triangle, the measurement of the angles are, 2θ, 3θ and 4θ.

According to the triangle,

2θ + 3θ + 4θ = 180°

or, 9θ = 180°

or, θ = 20°

∴ θ = 20°

Tirst angle in triangle $= 40° \times \frac{\pi}{180°} = \frac{2\pi}{9}$

Second angle in friangle $= 60° \times \frac{\pi}{180°} = \frac{\pi}{3}$

Third angle in triangle $= 80° \times \frac{\pi}{180°} = \frac{4\pi}{9}$

b $3(\text{Sec}^2\theta + \tan^2\theta) = 5$

or, $3\,\text{Sec}^2\theta + 3 + \text{an}^2\theta = 5$

or, $3(1 + \tan^2\theta) + 3\tan^2\theta = 5$ $[\text{Sec}^2\theta - \tan^2\theta = 1]$

or, $3 + 3\tan^2\theta + 3\tan^2\theta = 5$

or, $3 + 6\tan^2\theta = 5$

or, $6 + \text{an}^2\theta = 2$

or, $\tan^2\theta = \frac{2}{6}$

or, $\tan^2\theta = \frac{1}{3}$

or, $\tan\theta = \frac{1}{\sqrt{3}}$ [root in either side]

or, $\tan\theta = \tan\left(\frac{\pi}{6}\right)$

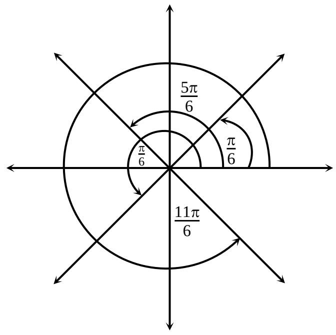
or, $\theta = \frac{\pi}{6}$

So, It is in the first quadrant. and range is 0 < θ < 2π.

$\tan\theta = \tan\left(r - \frac{r}{6}\right)$

or, $\tan\theta = \tan\frac{5\pi}{6}$

$\theta = \frac{5\pi}{6}$



or, $\tan\theta = \tan\left(\pi + \frac{\pi}{6}\right)$

or $\tan\theta = \tan\frac{7\pi}{6}$

Again, $\tan\theta = \tan\left(2\pi - \frac{\pi}{6}\right)$

$\therefore \theta = \frac{11\pi}{6}$

So, $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11r}{6}.$

c $\text{Sin}^2\frac{\pi}{7} + \sin^2\frac{5\pi}{14} + \text{Sin}^2\frac{87}{7} + \text{Sin}^2\frac{\pi}{14} = 2$

L.H.S $= \text{Sin}^2\frac{\pi}{7} + \text{Sin}^2\frac{5\pi}{14} + \text{Sin}^2\frac{8\pi}{7} + \text{Sin}^2\frac{9\pi}{14}$

$= \text{Sin}^2\frac{\pi}{7} + \left\{\text{Sin}\left(\frac{\pi}{2} - \frac{\pi}{7}\right)\right\}^2 + \left\{\text{Sin}\left(\frac{\pi}{2} + \frac{\pi}{7}\right)\right\}^2$

$= \left(\text{Sin}^2\frac{\pi}{7} + \text{Cos}^2\frac{\pi}{7}\right) + \left(\text{Sin}^2\frac{\pi}{7} + \text{Cos}^2\frac{\pi}{7}\right)$

$= 1 + 1$ $[\text{Cos}^2\theta + \text{Sin}^2\theta = 1] = 2$

= R.H.S

L.H.S = R.H.S.

Ques▶10 $6\sin^2\theta - 11\sin\theta + 4 = 0; 0 < \theta < \frac{\pi}{2}$. *[Rangpur Cadet-14]*

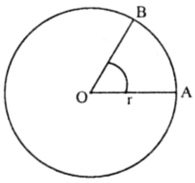
a. What is a radian angle? 2

b. Find the value of θ. 4

c. Find the values of $\sin\left(\frac{1023\pi}{2} + \theta\right)$ and $\tan\left(\frac{17\pi}{2} - \theta\right)$ 4

<u>Solution to the question no. 10</u>

a **Radian angle :** Radian angle is an angle which is subtended at the centre of a circle by its are of length equal its radius.



In the adjoining figure, there is a circle with centre O, radius r and are AB = r.
$\therefore \angle AOB$ is a radian angle in the figure.

b Here we have,
$6\sin^2\theta - 11\sin\theta + 4 = 0$
or, $6\sin^2\theta - 8\sin\theta - 3\sin\theta + 4 = 0$
or, $2\sin\theta\,(3\sin\theta - 4) - 1(3\sin\theta - 4) = 0$
or, $(2\sin\theta - 1)(3\sin\theta - 4) = 0$
$\therefore 2\sin\theta - 1 = 0 \Rightarrow \sin\theta = \frac{1}{2} = \sin\theta = \frac{1}{2} = \sin 30° \Rightarrow \theta = 30°$
or, $3\sin\theta - 4 = 0 \Rightarrow \sin\theta = \frac{4}{3}$ which is not acceptable.
$\therefore$ The value of $\theta = 30°$.

c Here given that,
$\sin\left(\frac{1023\pi}{2} + \theta\right)$
$= \sin\left(\frac{1023 + 3}{2}\pi + \theta\right)$
$= \sin\left\{1020\frac{\pi}{2} + \left(\frac{3}{2}\pi + \theta\right)\right\} = \sin\left(\frac{3\pi}{2} + \theta\right) = -\cos\theta.$
Again, $\tan\left(\frac{17\pi}{2} - \theta\right)$
$= \tan\left(\frac{16\pi + \pi}{2} - \theta\right)$
$= \tan\left\{\frac{16\pi}{2} + \left(\frac{\pi}{2} + \theta\right)\right\} = \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta.$

Ques ▶11 Trigonometry means the measurement of angles of a triangle. It is one of the vital branch of mathematics. There are a lot of applications of trigonometry in our daily life. *[Jhenidah Cadet-14]*

a. What is π? Find the angle subtended at the centre by the arc of length 11cm of a circle with radius 7 cm. 2
b. If $\tan\theta = \frac{5}{12}$ and $\cos\theta$ is the opposite sign of $\tan\theta$, then find the value of $\frac{\sin\theta + \cos(-\theta)}{\sec(-\theta) + \tan\theta}$ 4
c. Prove that $-1 \le \sin\theta \le 1$ and $-1 \le \cos\theta \le 1$, where θ is any angle. 4

Solution to the question no. 11

a π is the ratio of circumference to diameter of a circle.
Given,
r = 7 cm.
s = 11 cm.
We know,
$s = r\theta$
$\Rightarrow \theta = \frac{s}{r}$
$\Rightarrow \pi = \frac{11}{7}$
$\therefore \theta = 1.57$ rad (Ans)

b Given,
$\tan\theta = \frac{5}{12}$
$\Rightarrow \tan^2\theta = \left(\frac{5}{12}\right)^2$ by [Squaring Side]
$\Rightarrow \sec^2\theta - 1 = \frac{25}{144}$
$\Rightarrow \sec^2\theta = \frac{25}{144} + 1$
$\Rightarrow \sec^2\theta = \frac{25 + 144}{144}$
$\Rightarrow \sec^2\theta = \frac{169}{144}$
$\therefore \sec\theta \pm \frac{13}{12}$
$\Rightarrow \frac{1}{\sec\theta} \pm \frac{13}{12}$
According to condition, $\tan\theta$ is positive
So, $\cos\theta$ and $\sec\theta$ is negetive,

$\therefore \cos\theta = \frac{-12}{13}$ $\therefore \frac{\sin\theta + \cos(-\theta)}{\sec(-\theta) + \tan\theta}$ $\because \cos(-\theta) = \cos\theta$ $\sec(-\theta) = \sec\theta)$	Again $\sin\theta = \sqrt{1 - \cos^2\theta}$ $= \sqrt{1 - \left(\frac{-12}{13}\right)^3}$ $= \sqrt{\frac{169 - 144}{169}}$ $\therefore \sin\theta = \frac{5}{13}$

Putting the value
$= \frac{\frac{5}{13} - \frac{12}{13}}{-\frac{13}{12} + \frac{5}{12}}$
$= \frac{\frac{5 - 12}{13}}{\frac{-13 + 5}{12}}$
$= \frac{-7}{13} \times \frac{\overset{3}{\cancel{12}}}{\underset{2}{\cancel{-8}}}$
$= \frac{21}{26}$ **Ans.**

c $|\sin\theta| \le 1$, $|\cos\theta| \le 1$
Proof : $\sin^2\theta + \cos^2\theta = 1$
$\therefore \sin^2\theta \le 1$
$|\sin\theta| \le 1$, $|\cos\theta| \le 1$

Creative Essay type Questions with Answers Based on Activity

Activity promote higher thinking and to-the-point answering. Practise the questions attentively.

Tutor

Question ▶ 12 **sin $\left(\frac{11\pi}{2} \pm \theta\right)$, cos(11π ± θ), tan $\left(17\frac{\pi}{2} \pm \theta\right)$, cot (18π ± θ) are some ratios.** ◀*Activity; page-169*

a. Express sin $\left(\frac{11\pi}{2} \pm \theta\right)$ in terms of θ. 2

b. Show that, cos (11π ± θ) = sin $\left(\frac{11\pi}{2} \pm \theta\right)$ 4

c. Prove that, $\frac{\sin\left(\frac{11\pi}{2} \pm \theta\right)}{\cos(11\pi \pm \theta)} = \frac{\tan\left(\frac{17\pi}{2} + \theta\right)}{\cot(18\pi - \theta)}$ 4

Solution to the question no. 12

a $\sin\left(\frac{11\pi}{2} \pm \theta\right)$

Here, n = 11 is an odd number. So, sin will be changed into cos.

Again, $\left(11.\frac{\pi}{2} + \theta\right)$ lies in the 4th quadrant, so the sign of sin will be negative.

$\therefore \sin\left(11.\frac{\pi}{2} + \theta\right) = -\cos\theta.$

Again, $\left(11.\frac{\pi}{2} - \theta\right)$ lies in the 3rd quadrant, so the sign of sin will be negative.

$\therefore \sin\left(11.\frac{\pi}{2} - \theta\right) = -\cos\theta.$

$\therefore \sin\left(11\frac{\pi}{2} \pm \theta\right) = -\cos\theta$ **(Ans.)**

b In cos(11π ± θ) = cos $\left(22.\frac{\pi}{2} \pm \theta\right)$, n = 22 is a positive number. So, cos will remain unchanged.

Again, $\left(22.\frac{\pi}{2} + \theta\right)$ lies in the 3rd quadrant, so the sign of cos will be negative.

$\therefore \cos\left(22.\frac{\pi}{2} + \theta\right) = -\cos\theta.$

Again, $\left(22.\frac{\pi}{2} - \theta\right)$ lies in the 2nd quadrant, so the sign of cos will be negative.

$\therefore \cos\left(22.\frac{\pi}{2} - \theta\right) = -\cos\theta.$

From 'a' we get,

$\sin\left(\frac{11\pi}{2} \pm \theta\right) = -\cos\theta$

$\therefore \cos(11\pi \pm \theta) = \sin\left(\frac{11\pi}{2} \pm \theta\right)$ **(Shown)**

c Here, for $\tan(17.\frac{\pi}{2} + \theta)$, n = 17 is an odd number. So, tan will be changed into cot.

$(17.\frac{\pi}{2} + \theta)$ lies in the 2nd quadrant, so the sign of tan will be negative.

$\therefore \tan(17.\frac{\pi}{2} + \theta) = -\cot\theta$

Again, in cot(18π – θ), n = 18, which is an even number. So cot will remain unchanged.

$(36.\frac{\pi}{2} - \theta)$ lies in the 4th quadrant, so the sign of cot will be negative.

$\therefore \cot(18\pi - \theta) = -\cot\theta$

$$\text{L.S.} = \frac{\sin\left(\frac{11\pi}{2} \pm \theta\right)}{\cos(11\pi \pm \theta)} = \frac{-\cos\theta}{-\cos\theta} = 1$$

$$\text{R.S.} = \frac{\tan\left(\frac{17\pi}{2} + \theta\right)}{\cot(18\pi - \theta)} = \frac{-\cot\theta}{-\cot\theta} = 1$$

$$\therefore \frac{\sin\left(\frac{11\pi}{2} \pm \theta\right)}{\cos(11\pi \pm \theta)} = \frac{\tan\left(\frac{17\pi}{2} + \theta\right)}{\cot(18\pi - \theta)}$$ **(Proved)**

Question ▶ 13 **135°, 150°, 120° are three angles.** ◀ *Activity; page-164*

a. Express the angles in radians. 2

b. Find the value of secant, cosecant and cotangent with the help of the three angles. 4

c. What is the value of $\sec^2\left(\frac{3\pi}{4}\right) + \text{cosec}^2\left(\frac{5\pi}{6}\right) - \cot^2\left(\frac{2\pi}{3}\right)$? 4

Solution to the question no. 13

a $135° = \left(135 \times \frac{\pi}{180}\right)^c = \left(\frac{3\pi}{4}\right)^c$

$150° = \left(150 \times \frac{\pi}{180}\right)^c = \left(\frac{5\pi}{6}\right)^c$

$120° = \left(120 \times \frac{\pi}{180}\right)^c = \left(\frac{2\pi}{3}\right)^c$

b $\sec(135°) = \sec\left(\frac{3\pi}{4}\right) = \sec\left(\frac{\pi}{2} + \frac{\pi}{4}\right)$ [we get from 'a']

$= -\text{cosec}\frac{\pi}{4}$

$= -\sqrt{2}$

$\therefore$ The required value $= -\sqrt{2}$

$\text{cosec}(150°) = \text{cosec}\left(\frac{5\pi}{6}\right) = \text{cosec}\left(\frac{\pi}{2} + \frac{\pi}{3}\right)$ [we get from 'a']

$= \sec\frac{\pi}{3} = 2$

$\therefore$ The required value = 2

$\cot(120°) = \cot\left(\frac{2\pi}{3}\right)$

$= \cot\left(\frac{\pi}{2} + \frac{\pi}{6}\right)$ [we get from 'a']

$= -\tan\frac{\pi}{6} = -\frac{1}{\sqrt{3}}$

$\therefore$ The required value $= -\frac{1}{\sqrt{3}}$

c $\sec^2\left(\frac{3\pi}{4}\right) + \text{cosec}^2\left(\frac{5\pi}{6}\right) - \cot^2\left(\frac{2\pi}{3}\right)$

$= \left\{\sec\left(\frac{3\pi}{4}\right)\right\}^2 + \left\{\text{cosec}\left(\frac{5\pi}{6}\right)\right\}^2 - \left\{\cot\left(\frac{2\pi}{3}\right)\right\}^2$

$= (-\sqrt{2})^2 + (2)^2 - \left(-\frac{1}{\sqrt{3}}\right)^2$ [we get from 'b']

$= 2 + 4 - \frac{1}{3}$

$= \frac{6 + 12 - 1}{3} = \frac{17}{3} = 5\frac{2}{3}$ **(Ans.)**

Additional Creative Questions with Answers

Practice this part very well. Try to answer the questions all by yourself first. Read the answer and make sure your answer has been resembling with it.

Question ▶ 14 **If A = 60°, then**

a. Find the values of sin 50A, sin 2A. 2

b. Verify the following formulae.

$\sin 2A = 2\sin A\cos A = \frac{2\tan A}{1 + \tan^2 A}$

and prove that sin3A = 0 4

c. If $\cos\theta + \sin\theta = \frac{2\sqrt{2}}{\sqrt{3}}\sin 2A$, what is the value of θ when 0° < θ < 90°? 4

Solution to the question no. 14

a Given, A = 60°

$\therefore \sin 50A = \sin(50 \times 60)°$
$= \sin(3000)°$
$= \sin(33 \times 90° + 30°)$
$= \cos 30° = \frac{\sqrt{3}}{2}$

Again, $\sin 2A = \sin(2 \times 60)° = \sin(120)°$
$= \sin(90° + 30°)$
$= \cos 30° = \frac{\sqrt{3}}{2}$.

b From 'a' we get, L.S. = $\sin 2A = \frac{\sqrt{3}}{2}$

M.S.= 2 sinA cosA = 2 sin60° cos 60° [∵ A = 60°]

$= 2 \times \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{2}$

R.S.= $\frac{2\tan A}{1 + \tan^2 A} = \frac{2\tan 60°}{1 + \tan^2 60°}$ [∵ A = 60°]

$= \frac{2\sqrt{3}}{1 + (\sqrt{3})^2} = \frac{2\sqrt{3}}{1 + 3}$

$= \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

$\therefore \sin 2A = 2\sin A\cos A = \frac{2\tan A}{1 + \tan^2 A}$ **(Proved)**

and, sin3A= sin (3× 60°) [∴ A = 60°]
= sin 180°
= sin (2 × 90° + 0°)
= sin0° = 0

∴ sin3A = 0 **(Proved)**

c Given,

$\cos\theta + \sin\theta = \frac{2\sqrt{2}}{\sqrt{3}}\sin 2A$

or, $\cos\theta + \sin\theta = \frac{2\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = \sqrt{2}$ [putting the value of sin2A from 'b']

or, $\sin\theta = \sqrt{2} - \cos\theta$

or, $\sin^2\theta = 2 - 2\sqrt{2}\cos\theta + \cos^2\theta$

or, $1 - \cos^2\theta = 2 - 2\sqrt{2}\cos\theta + \cos^2\theta$

or, $2\cos^2\theta - 2\sqrt{2}\cos\theta + 1 = 0$

or, $(\sqrt{2}\cos\theta - 1)^2 = 0$

or, $\sqrt{2}\cos\theta - 1 = 0$

or, cosθ = **Error!**= cos 45°

∴ θ = 45°

∴ The required solution, θ = 45°

Question ▶ 15 **$\cot\theta = \frac{12}{5}$ and cosθ is negative,**

a. Find the values of cosθ and secθ. 2

b. If $\left\{\frac{\sin\theta + \cos(-\theta)}{\sec(-\theta) + \tan\theta}\right\}\frac{26}{51} = k$, then find the value of k. 4

c. If $\sec A + \cos A = \frac{5}{2}k$, then what is the value of A (where 0° < A < 90°)? 4

Solution to the question no. 15

a Given, $\cot\theta = \frac{12}{5}$ ∴ $\tan\theta = \frac{5}{12}$

We know, $\sec^2\theta = 1 + \tan^2\theta$

$= 1 + \left(\frac{5}{12}\right)^2 = 1 + \frac{25}{144} = \frac{169}{144}$

$\therefore \sec\theta = \pm\frac{13}{12}$

$\therefore \cos\theta = \pm\frac{12}{13}$

But, cosθ is negative, $\therefore \cos\theta = -\frac{12}{13}$

and, $\sec\theta = \frac{1}{\cos\theta} = -\frac{13}{12}$

b Given, $\tan\theta = \frac{5}{12}$

or, $\frac{\sin\theta}{\cos\theta} = \frac{5}{12}$

or, $\sin\theta = \frac{5}{12}\cos\theta = \frac{5}{12} \times \frac{-12}{13}$

or, $\sin\theta = -\frac{5}{13}$

Now, given expression $\left\{\frac{\sin\theta + \cos(-\theta)}{\sec(-\theta) + \tan\theta}\right\}\frac{26}{51}$

$= \left(\frac{\sin\theta + \cos\theta}{\sec\theta + \tan\theta}\right)\frac{26}{51}$

[∵ cos (–θ) = cosθ and sec (–θ) = secθ]

$$= \left(\frac{-\frac{5}{13} - \frac{12}{13}}{-\frac{13}{12} + \frac{5}{12}}\right) \frac{26}{51}$$

$$= \left(\frac{\frac{-5-12}{13}}{\frac{-13+5}{12}}\right) \frac{26}{51}$$

$$= \left(\frac{-17}{13} \times \frac{\cancel{12}^{3}}{-\cancel{8}_{2}}\right) \times \frac{26}{51}$$

$$= \frac{51}{26} \times \frac{26}{51}$$

$= 1.$

$\therefore$ The required value of k = 1

c Given, $\sec A + \cos A = \frac{5}{2} \times k$

or, $\frac{1}{\cos A} + \cos A = \frac{5}{2}$ [Putting the value of k obtained from 'b']

or, $\frac{1 + \cos^2 A}{\cos A} = \frac{5}{2}$

or, $1 + \cos^2 A = \frac{5}{2} \cos A$

or, $2 + 2\cos^2 A = 5\cos A$ [multiplying both sides by 2]

or, $2\cos^2 A - 5\cos A + 2 = 0$

or, $2\cos^2 A - 4\cos A - \cos A + 2 = 0$

or, $2\cos A(\cos A - 2) - 1(\cos A - 2) = 0.$

or, $(2\cos A - 1)(\cos A - 2) = 0$

Either, $2\cos A - 1 = 0$

or, $\cos A = \frac{1}{2}$

or, $\cos A = \cos 60°$

or, $A = 60°$

or, $\cos A - 2 = 0$

But, $\cos A = 2$ is not exceptable, because the value of cosA can never be greater than 1 and less then –1.

$\therefore$ The required solution, $A = 60°$.

Question ▶ 16 $\sin^2\frac{\pi}{7} + \sin^2\frac{5\pi}{14} + \sin^2\frac{8\pi}{7} + \sin^2\frac{9\pi}{14}$ **and** $\tan\frac{\pi}{4} + \cot\frac{\pi}{20}\cot\frac{3\pi}{20}\cot\frac{5\pi}{20}\cot\frac{7\pi}{20}\cot\frac{9\pi}{20}$ **are two trigonometric expressions whose values are p and q respectively.**

a. Find the values of $\sin\frac{5\pi}{6}$ and $\sin\frac{10\pi}{6}$. 2

b. Find the value of q. 4

c. Show that, the numerical value of p – q will be 0. 4

<u>Solution to the question no. 16</u>

a $\sin\frac{5\pi}{6} = \sin\left(\frac{\pi}{2} + \frac{\pi}{3}\right)$

$= \cos\frac{\pi}{3} = \frac{1}{2}$ **(Ans.)**

Again, $\sin\frac{10\pi}{6} = \sin\left(\frac{\pi}{2} \times 3 + \frac{\pi}{6}\right)$

$= -\cos\frac{\pi}{6} = -$ **Error! (Ans.)**

b According to the question,

$q = \tan\frac{\pi}{4} + \cot\frac{\pi}{20}\cot\frac{3\pi}{20}\cot\frac{5\pi}{20}\cot\frac{7\pi}{20}\cot\frac{9\pi}{20}$

$= 1 + \cot\frac{\pi}{20}\cot\left(\frac{\pi}{2} - \frac{7\pi}{20}\right)\cot\frac{5\pi}{20}\cot\frac{7\pi}{20}\cot\left(\frac{\pi}{2} - \frac{\pi}{20}\right)$

$= 1 + \cot\frac{\pi}{20}\tan\frac{7\pi}{20}.\cot\frac{\pi}{4}.\cot\frac{7\pi}{20}\tan\frac{\pi}{20}$

$= 1 + \cot\frac{\pi}{20}.\frac{1}{\cot\frac{7\pi}{20}}.1.\cot\frac{7\pi}{20}.\tan\frac{\pi}{20}$

$= 1 + \cot\frac{\pi}{20}.\frac{1}{\cot\frac{7\pi}{20}}.\cot\frac{7\pi}{20}.\tan\frac{\pi}{20}$

$= 1 + \cot\frac{\pi}{20}.\tan\frac{\pi}{20}$

$= 1 + \frac{1}{\tan\frac{\pi}{20}}.\tan\frac{\pi}{20} = 1 + 1 = 2$

$\therefore$ The value of q is 2 **(Ans.)**

c According to the question,

$p = \sin^2\frac{\pi}{7} + \sin^2\frac{5\pi}{14} + \sin^2\frac{8\pi}{7} + \sin^2\frac{9\pi}{14}$

$= \sin^2\frac{\pi}{7} + \sin^2\left(\frac{\pi}{2} - \frac{\pi}{7}\right) + \sin^2\left(\pi + \frac{\pi}{7}\right) + \sin^2\left(\frac{\pi}{2} + \frac{\pi}{7}\right)$

$= \sin^2\frac{\pi}{7} + \left\{\sin\left(\frac{\pi}{2} - \frac{\pi}{7}\right)\right\}^2 + \left\{\sin\left(\pi + \frac{\pi}{7}\right)\right\}^2 + \left\{\sin\left(\frac{\pi}{2} + \frac{\pi}{7}\right)\right\}^2$

$= \sin^2\frac{\pi}{7} + \cos^2\frac{\pi}{7} + \left(-\sin\frac{\pi}{7}\right)^2 + \cos^2\frac{\pi}{7}$

$= \sin^2\frac{\pi}{7} + \cos^2\frac{\pi}{7} + \sin^2\frac{\pi}{7} + \cos^2\frac{\pi}{7}$

$= 1 + 1$ [$\because \sin^2\theta + \cos^2\theta = 1$]

$= 2$

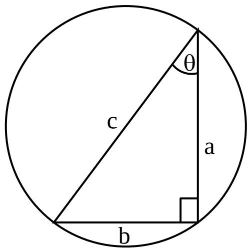
Now, p – q

$= 2 - 2$ [from 'b' we get q = 2]

$= 0$

$\therefore$ The numerical value of p – q will be 0. **(Shown)**

Question ▶ 17 A triangle is inscribed in a circle shown below-



a. Wright down the tangent and secant of the angle θ in terms of the ratio of sides. 2

b. If the sides of the triangle are related by the equation $b + c = a\sqrt{3}$, find the value of θ. 4

c. If the value of a is 1 m, what is the length of the arc cut by a? 4

<u>Solution to the question no. 17</u>

a We know, $\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{b}{a}$

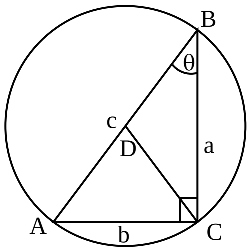
and, $\sec\theta = \frac{\text{hypotanuse}}{\text{adjacent side}} = \frac{c}{a}$

b From the figure,

$\frac{a}{c} = \cos\theta$

$\frac{b}{c} = \sin\theta$

Now, $b + c = a\sqrt{3}$



or, $\frac{b}{c} + 1 = \sqrt{3}\,\frac{a}{c}$ [dividing both sides by c]

or, $\sin\theta + 1 = \sqrt{3}\cos\theta$

or, $\sin^2\theta + 2\sin\theta + 1 = 3\cos^2\theta$

or, $\sin^2\theta + 2\sin\theta + 1 - 3\cos^2\theta = 0$

or, $\sin^2\theta + 2\sin\theta + 1 - 3(1 - \sin^2\theta) = 0$

or, $\sin^2\theta + 2\sin\theta + 1 - 3 + 3\sin^2\theta = 0$

or, $4\sin^2\theta + 2\sin\theta - 2 = 0$

or, $2\sin^2\theta + \sin\theta - 1 = 0$

or, $2\sin^2\theta + 2\sin\theta - \sin\theta - 1 = 0$

or, $2\sin\theta(\sin\theta + 1) - (\sin\theta + 1) = 0$

or, $(\sin\theta + 1)(2\sin\theta - 1) = 0$

Either, $\sin\theta + 1 = 0$

or, $\sin\theta = -1$ (which is not exceptable).

Or, $2\sin\theta - 1 = 0$

or, $2\sin\theta = 1$

or, $\sin\theta = \frac{1}{2}$

or, $\sin\theta = \sin 30°$

$\therefore \theta = 30°$ **(Ans.)**

c Now, from the above figure, $\cos\theta = \frac{a}{c}$

Here, $\theta = 30°$

or, $c = \frac{a}{\cos\theta}$

$a = 1$m.

$c = \frac{1}{\cos 30°}$

$c = \frac{1}{\frac{\sqrt{3}}{2}}$ $c = \frac{2}{\sqrt{3}}$ m.

Now from the figure above, in ΔABC, D is the mid-point of AB.

(D is the centre of the circle and AB is the diameter).

$AD = \frac{c}{2} = \frac{1}{\sqrt{3}} = CD = r$. (r is the radius of the circle)

So, $\angle DCB = \angle DBC = 30°$ [Opposite angles of two equal sides of a triangle are equal]

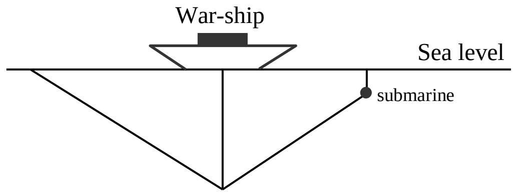
So, $\angle BDC = \theta = 120° = \frac{120 \times \pi}{180} = \frac{2\pi}{3}$

Now, we know, $s = r\theta$

or, $s = \frac{1}{\sqrt{3}} \times \frac{2\pi}{3}$

$\therefore s = 1.21$ m. **(Ans.)**

Question ▶ 18 **Suppose, you are the captain of a submarine. The submarine runs under 10 m. from the sea level. Suddenly you notice a war-ship 500m far from the submarine and then you dive the submarine deep into sea water.**



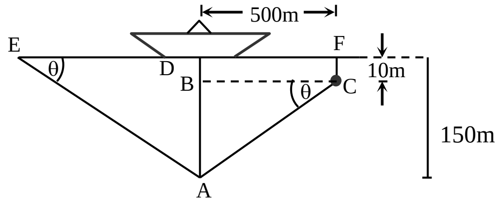
a. In what angle you have to dive the submarine to go 150m deep into sea water from sea level? 2

b. If the angles in sexagesional and circular system are respectively D and R, then find a relation between them. 4

c. After passing the war-ship, if the submarine dive in 1 radian angle with the water level to the upper direction, what will be the distance between the submarine and the war-ship after reaching the sea level? 4

Solution to the question no. 18

a



In the figure, the submarine is at the point C and the war-ship is at the point D.

Here, BC = 500m, AD = 150 m.

AB = AD – BD = AD – CF = 150 – 10 = 140m

Now, in the triangle ΔABC, $\tan\angle ACB = \frac{AB}{BC} = \frac{140}{500}$

or, $\tan\angle ACB = 0.28$

$\therefore$ $\angle ACB = 15.64°$ **(Ans.)**

b We know, 1 radian $= \frac{2}{\pi}$ right angle

or, 1 radian $= \frac{2}{\pi} \times 90°$

or, R radian $= \left(\frac{180}{\pi} \times R\right)^\circ$

Since the same angle in two systems are D° and R^C.

So, $D° = \left(\frac{180}{\pi} \times R\right)^\circ$

or, $D = \frac{180}{\pi} \times R$

$\therefore \frac{D}{180} = \frac{R}{\pi}$ **(Ans.)**

c From the figure, AD = 150 m

If the submarine dive in the angle ∠AED with the sea level and reach at the point E, then

$\angle AED = 1^c = \frac{2}{\pi}$ right angles = 57.3°

In the triangle ΔAED,

$\tan\theta = \frac{AD}{DE}$

or, $DE = \frac{AD}{\tan\theta}$

or, $DE = \frac{150}{\tan 57.3°}$

$\therefore$ DE = 96.31m

So, the distance between the submarine and the war-ship will be 96.31 m. **(Ans.)**

Creative Questions with hints

Answer these questions yourself. See the super tips which will help you to answer the questions easily.

Question ▶19 $7\sin^2\theta + 3\cos^2\theta = 4$ and θ is positive acute angle.

a. Find the value of $\cos^2\theta$ from the given equation. 2

b. Prove that, $\tan\theta = \pm\frac{1}{\sqrt{3}}$ 4

c. If $\tan\theta$ is positive, prove that, $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta} = \frac{1}{2}$ 4

Ans. a. $\cos^2\theta = \frac{3}{5}$;

Question ▶20 $\sin^2\frac{17\pi}{18} + \sin^2\frac{5\pi}{8} + \cos^2\frac{37\pi}{18} + \cos^2\frac{5\pi}{8}$ and $\sin^2\frac{\pi}{7} + \sin^2\frac{5\pi}{14} + \sin^2\frac{8\pi}{7} + \sin^2\frac{9\pi}{14}$ are two trigonometric equations.

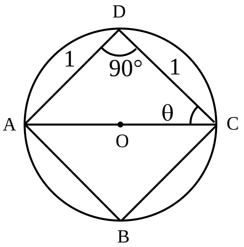
a. Show that, $\sin^2\frac{17\pi}{18} = \sin^2\frac{\pi}{18}$. 2

b. Find the value of first equation. 4

c. Show that, the values of two equations are equal. 4

Ans. b. 2

Question ▶21



In circle ABCD, O is the centre and AC is the diameter.

a. Find the length of AC. 2

b. Prove that, cosA + cosB + cosC + cosD = 0 4

c. If $\sec\theta + \cos\theta = p$, find p and solve the equation. 4

Ans. a. $\sqrt{2}$ unit; **c.** $p = \frac{3}{\sqrt{2}}$, $\theta = \frac{\pi}{4}$

Question ▶22 The angles of a triangle are in arithmetic progression and largest angle is double than the smallest angle.

a. If the smallest angle is A, find another two angles. 2

b. Determine the angles in radian and sexagesional system. 4

c. Show that, a solution of $2\sin^2\theta + 3\cos\theta = 3$ is equal to the another angle except the greatest and the smallest one, where $0° < \theta < 90°$. 4

Ans. a. $\frac{3A}{2}$, 2A; **b.** 40°, 60° and 80°.

Question ▶23 Any radius of a circle is 14 m. A radian angle $\frac{\pi}{6}$ is produce in the centre of tower with the height is equal with the circumference of the circle.

a. Express the angle at the centre subtended by the arc 28 m. in radian. 2

b. What is the distance from the centre of the circle to the tower? 4

c. A wheel with circumference equal to one-tenths of the diameter of the circle revolve 5 times I a second. How many times is required for it to pass 21 km. distance? 4

Ans. a**.** 2 radian; b**.** 50.79 m. (about); c**.** 25 minutes.

For More Creative Questions and Answers type the following address on the browser's address bar panjeree.com/e09/hmtq08.pdf

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In this part important information of the chpater, at which it is needed to cast a look before exam or you must remember, such subject matters have been mentioned here at a glance. So that you can keep the important information in mind easily; specially you can make you self-confident revising these in a quick view.

■ At a glance, the formulae which are used in this chapter:

- **The trigonometric ratios of the angle (– θ)**

sin (–θ) = – sin θ	cosec (–θ) = –cosec θ
cos (–θ) = cos θ	sec (–θ) = sec θ
tan (–θ) = – tan θ	cot (–θ) = –cot θ

- **The trigonometric ratios of the angle (90° – θ) or complementary angle (0° < θ < 90°)**

sin (90° – θ) = cos θ	cosec (90°– θ) = sec θ
cos (90° – θ) = sin θ	sec (90° – θ) = cosec θ
tan (90° – θ) = cot θ	cot (90° – θ) = tan θ

- **The trigonometric ratios of the angle (90° + θ) (0° < θ < 90°)**

sin (90° + θ) = cos θ	cosec (90°+θ) = sec θ
cos (90° + θ) = – sin θ	sec (90° + θ) = – cosec θ
tan (90° + θ) = – cot θ	cot (90° + θ) = – tan θ

- **The trigonometric ratios of the angle (180° – θ) (0° < θ < 90°)**

sin (180° – θ) = sin θ	cosec (180° –θ) = cosec θ

cos (180° – θ) = – cos θ	sec (180° – θ) = – sec θ
tan (180° – θ) = – tan θ	cot (180° – θ) = – cot θ

- **The trigonometric ratios of the angle (180° + θ) (0° < θ < 90°)**

sin (180° + θ) = – sin θ	cosec (180° + θ) = – cosec θ
cos (180° + θ) = – cos θ	sec (180° + θ) = – sec θ
tan (180° + θ) = tan θ	cot (180° + θ) = cot θ

- **The trigonometric ratios of the angle (270° - θ) (0° < θ < 90°)**

sin (270° – θ) = – cos θ	cosec (270° – θ) = – sec θ
cos (270° – θ) = –sin θ	sec (270° – θ) = –cosec θ
tan (270° – θ) = cot θ	cot (270° – θ) = tan θ

- **The trigonometric ratios of the angle (270° + θ) (0° < θ < 90°)**

sin (270° + θ) = – cos θ	cosec (270° + θ) = – sec θ
cos (270° + θ) = sin θ	sec (270° + θ) = cosec θ
tan (270° + θ) = – cot θ	cot (270° + θ) = – tan θ

- **The trigonometric ratios of the angle (360° - θ) (0° < θ < 90°)**

sin (360° – θ) = – sin θ	cosec (360° – θ) = –cosec θ
cos (360° – θ) = cos θ	sec (360° – θ) = sec θ
tan (360° – θ) = – tan θ	cot (360° – θ) = – cot θ

- **The trigonometric ratios of the angle (360° + θ) (0° < θ < 90°)**

sin (360° + θ) = sin θ	cosec (360° + θ) = cosec θ
cos (360° + θ) = cos θ	sec (360° + θ) = sec θ
tan (360° + θ) = tan θ	cot (360° + θ) = cot θ

- **The trigonometric ratios of the angle (n × 90° ±θ)**

If n is any integer, then the value of the angle (n × 90° ± θ) can be defined by the following way:

The given angle have to be divided into two parts such that one part of that will equal to an acute angle and the other part will equal to 90° or n multiple of $\frac{\pi}{2}$.

That is, the given angle = n × 90° ± θ

or, ,, ,, ,, $= n \times \frac{\pi}{2} \pm \theta$

2(a) If n is even number, the trigonometric ratio will remain unchanged.
That is, the ratio sin will remain sin.
and cos will remain cos, etc.
(b) If n is odd number, the trigonometric ratio will be changed,
The ratio sin will be changed into cos
,, ,, cos ,, ,, ,, ,, sin
,, ,, tan ,, ,, ,, ,, cot
,, ,, cot ,, ,, ,, ,, tan
,, ,, sec ,, ,, ,, ,, cosec
,, ,, cosec ,, ,, ,, ,, sec

Suggestion: Highway Ensuring a Brilliant Result

It is not that you will find all the questions common but the practice of these questions will guide you in solving different and difficult question patterns.

Suggestions | Creative Multiple Choice Questions

	Question No.
★★★	2, 3, 4, 11, 14, 20, 21, 41, 42, 48, 51, 54, 55
★★	5, 6, 17, 22, 23, 33, 45, 47, 49, 56, 57-60

Suggestions | Creative Broad Questions

	Question No.
★★★	4, 7, 8, 12, 14
★★	3, 11, 15, 18