সঠিক প্রস্তুতি, সহজ প্রিলি

Live MCQ™

৪৯তম স্পেশাল বিসিএস (শিক্ষা) বিষয়ভিত্তিক প্রস্তুতি

বিষয়: গণিত (৫৫১)

Compendium PDF

(সিলেবাস অনুসারে সর্বাধিক গুরুত্বপূর্ণ টপিক ও এমসিকিউ-এর সমন্বয়ে রচিত)

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৩৮তম বিসিএস

(সাধারণ শিক্ষা-গণিত)

প্রিয় শিক্ষার্থীবৃন্দ,

৪৯তম (স্পেশাল বিসিএস) পরীক্ষা -২০২৫ এর বিষয়ভিত্তিক প্রস্তুতির গণিত বিষয়ে PSC কর্তৃক নির্ধারিত সিলেবাসের ওপর সকল ক্লাস ইতিমধ্যেই সম্পন্ন হয়েছে। আসন্ন চূড়ান্ত পরীক্ষাকে সামনে রেখে Live MCQ একাডেমিক টিম আপনাদের প্রস্তুতিকে শাণিত করতে বিশেষ কমপেনডিয়াম প্রদান করছে। আশা করছি এর মাধ্যমে আপনাদের শেষ মুহূর্তের প্রস্তুতি আরো শাণিত হবে।

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Most important topics of mathematics for

49th BCS (Special) Examination-2025

Part -1

Marks - 50

Group-A

1. Order properties of real numbers***. Inequalities involving different types of means, Chebyshev's inequality***.

2. Complex numbers: modulus, argument, locus, conjugates etc***.

DeMoivre's theorem and its applications (finding nth roots of complex number) ***

3. Summation of finite algebraic and trigonometric series.**

5. Polynomials and their roots***, Honer's scheme (synthetic division), Descartes rule of signs.**

6. Relation between roots and coefficients.***(quadratic, cubic equations)

Symmetric functions of roots**.

Analytical Geometry

1. Pairs of straight lines. (all formulas and conditions)***

Transformation of coordinates. **

2. General equation of the second degree, Reduction to standard forms, Conics in general. ***

3. Planes and straight lines in three dimensions, shortest distance between two straight lines. ***

4. Vector algebra with applications to geometry. **

Linear Algebra

1. Algebra of Matrices. Systems of linear equations and their solutions. ***

2. Vector spaces over the field of real numbers, Subspaces, Linear dependence and independence of vectors, Basis and dimension. **

3. Linear transformations, Rank and nullity. ***

4. Eigenvectors and eigenvalues. ***

Group-B.

1. Sets of real numbers, Supremum and infimum***,

the completeness axiom Dedekind's theorem***, The Archimendian property***.

2. Convergence of infinite sequences and series of real numbers, Standard theorems and tests of convergence***. Absolute convergence. **

3. Continuous functions***. Intermediate value theorem. Uniform continuity.***

4. The derivative. Rolle's Theorem***, Mean value theorems***, Taylor's theorem with remainder, Taylor's series, indeterminate forms***.

5. Maxima, minima, tangents and normals.***

6. Indefinite integrals, Techniques of integration***, Recurrence Relations**.

7. The Riemann integral, The fundamental theorem of calculus.**

8. Improper integrals. Tests of convergence.

9. Determination of areas and volumes.***

Part-II

Marks: 50

Group-A:

1. General conditions of equilibrium***, Principle of virtual work, Stable and unstable equilibrium**, Centre of gravity***.

2. Rectilinear motion, simple harmonic motion***, Motion in a plane, Motion under a central force***.

3. Dynamics of rigid bodies, Moments of inertia***, A' Alembert's principle**.

4. Motion about a fixed axis. **

5. Lagrange's equation for holonomic systems**.

Group-B:

1. Ordinary differential equations of first and second order.***

2. Liner equations with constant coefficients.

3. Solution of differential equations in series.***

4. Beata and Gamma functions. ***

5. Special functions, Legendre, Hermite and Languerre polynomials; Bessel functions. Generating functions***, recurrence relations**, orthogonality*** and other properties.

6. Complex functions, Differentiability, Cauchy-Riemann equations***, Analytic function***.

বইয়ের তালিকা

* উচ্চমাধ্যমিক গণিতের ১ম পত্র এবং ২য় পত্র (সিলেবাসের অন্তর্ভুক্ত অংশ)

* স্নাতকের সিলেবাসের অন্তর্ভূক্ত বই

1. Fundamental of mathematics

2. Analytic and Vector Geometry

3. Linear Algebra

4. Calculus I and II

5. Mechanics

6. Ordinary Differential Equation

7. Complex Analysis

8. Real Analysis

9. Mathematical Methods

(বেসরকারি কলেজ প্রভাষক নিবন্ধন সহায়িকা বই)

শেষ মুহূর্তের দিক নির্দেশনা ও পরামর্শ

1. যেসকল টপিকস এইচ এস সি এর সিলেবাসের সাথে মিলে যায় সেসব টপিকস এর এমসিকিউ অনুশীলন করতে হবে (বিশেষভাবে ভর্তি পরীক্ষার প্রশ্ন)।
2. জাতীয় বিশ্ববিদ্যালয়ের অধিক গুরুত্ব পূর্ণ প্রশ্ন অনুশীলন করতে হবে।
3. শেষ মুহুর্তে নতুন করে কোনো টপিকস এর উপর সময় ব্যয় করা থেকে বিরত থাকতে হবে।
4. সিলেবাসে উল্লিখিত বিশেষ theorem বা উপপাদ্য ভালোভাবে অনুশীলন করতে হবে।
5. কঠিন টপিকস যেমন mechanics, special funcition ইত্যাদি চ্যাপ্টার থেকে লেকচার অনুযায়ি শুধু গুরুত্বপুর্ন অংশ পড়তে হবে।
6. সিলেবাসের যে অংশ আপনার কাছে সহজ মনে হবে তার উপর ভালো প্রস্তুতি নিতে হবে।
7. সর্বপোরি সর্বাধিক সংখ্যক এমসিকিউ সমাধানের প্রস্তুতি নিতে হবে।

Short Notes on Important Topics

1. Order property

a) The Trichotomy Property

For any two real numbers a and b, exactly one of the following is true:

$\boldsymbol{a < b}$ or, $\boldsymbol{a = b}$ or, $\boldsymbol{a > b}$

b) Transitive Property

If $a < b$ and $b < c$, then: $a < c$

c) Addition Property of Inequality

If $a < b$, then for any real number c:

$$\boldsymbol{a + c < b + c}$$

This property says that adding the same number to both sides of an inequality preserves the inequality.

d) Multiplication Property of Inequality

If $a < b$ and $c > 0$, then: $ac < bc$

If $a < b$ and $c < 0$, then: $ac > bc$

2. Boundedness

- A set $\boldsymbol{S} \subseteq \mathbb{R}$ is bounded above if there exists $\boldsymbol{M} \in \mathbb{R}$ such that $\boldsymbol{s} \leq \boldsymbol{M}$ for all $\boldsymbol{s} \in \boldsymbol{S}$
- It is bounded below if there exists $\boldsymbol{m} \in \mathbb{R}$ such that $\boldsymbol{s} \geq \boldsymbol{m}$ for all $\boldsymbol{s} \in \boldsymbol{S}$

These are essential for understanding **supremum** and **infimum**, especially in analysis.

Least Upper Bound Property (Completeness Axiom)

Every non-empty subset of $\mathbb{R}$ that is bounded above has a least upper bound (supremum) in $\mathbb{R}$. This property distinguishes real numbers from rational numbers (since not all bounded sets of rationals have a supremum in $\mathbb{Q}$

3. Inequalities involving different types of means, chebychev's inequality

Fundamental Means of Positive Real Numbers

For positive real numbers $x_1, x_2, \dots, x_n$:

- Arithmetic Mean (AM) $\boldsymbol{AM} = \frac{\sum x_i}{n} = \frac{x_1+x_2+x_3+\cdots\ldots\ldots+x_n}{n}$
- Geometric Mean (GM) $\boldsymbol{GM} = (\prod x_i)^{1/n} = (x_1.x_2.x_3 \ldots\ldots\ldots x_n)^{\frac{1}{n}}$
- Harmonic Mean (HM) $\boldsymbol{HM} = \frac{n}{\sum 1/x_i} = \frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\cdots\ldots\ldots+\frac{1}{x_n}}$
- Quadratic Mean (QM) (Root Mean Square)

$$\boldsymbol{QM} = \sqrt{\frac{\sum x_i^2}{n}} = \frac{x_1^2 + x_2^2 + \cdots\ldots\ldots + x_n^2}{n}$$

Mean Inequality Chain: $\boldsymbol{QM} \geq \boldsymbol{AM} \geq \boldsymbol{GM} \geq \boldsymbol{HM}$

Some important inequalities:

1. if $a, b > 0 \; and \; a \neq b$ then,

i. $\frac{a^m+b^m}{2} > \left(\frac{a+b}{2}\right)^m \; when, m < 0 \; or \; m > 1$

ii. $\frac{a^m+b^m}{2} < \left(\frac{a+b}{2}\right)^m$ $when,\ 0 < m < 1$

2. **Weierstrass inequality** : if $a_1, a_2, \dots \dots, a_n > 0$ then,

i. $(1+a_1)(1+a_2)\dots.(1+a_n) > 1 + a_1 + a_2 + \dots + a_n$

ii. $(1-a_1)(1-a_2)\dots.(1-a_n) > 1 - a_1 - a_2 - \dots - a_n$

And if $a_1, a_2, \dots \dots, a_n < 1$

iii. $(1+a_1)(1+a_2)\dots.(1+a_n) < \frac{1}{1-a_1-a_2-\dots-a_n}$

iv. $(1-a_1)(1-a_2)\dots.(1-a_n) < \frac{1}{1+a_1+a_2+\dots+a_n}$

Chebyshev's Inequality

If both sequences (a_i) and (b_i) are similarly ordered (both non-increasing or non-decreasing):

$$\frac{1}{n}\sum_{i=1}^{n} a_i\, b_i \ \geq \ \left(\frac{1}{n}\Sigma a_i\right)\left(\frac{1}{n}\Sigma b_i\right).$$

4. A complex number z is of the form:

$$z = a + bi, \quad a, b \in \mathbb{R},\ i^2 = -1$$

- Real part: $\mathfrak{R}(z) = a$
- Imaginary part: $\mathfrak{I}(z) = b$

Algebraic Operations

- Addition/Subtraction: $(a+bi)+(c+di) = (a+c)+(b+d)i$
- Multiplication: $(a+bi)(c+di) = (ac-bd)+(ad+bc)i$

- Conjugate: $\bar{z} = a - bi$, and $z \cdot \bar{z} = |z|^2$

Multiply numerator and denominator by conjugate to rationalize

Modulus & Argument (Polar Form)

- Modulus: $|z| = \sqrt{a^2 + b^2}$
- Argument: $arg(z) = \theta = tan^{-1}\frac{b}{a},\quad where,\quad -\pi < \theta \le \pi$
- Polar representation:

  $z = r(cos\theta + isin\theta),\quad r = |z|,\ \theta = arg(z)$

Some important properties:

1. $i^m + i^{m+1} + i^{m+2} + i^{m+3} = 0$
2. $\omega^m + \omega^{m+1} + \omega^{m+2} = 0$
3. $\sqrt{(a \pm ib)} = \pm\frac{1}{\sqrt{2}}[\sqrt{\sqrt{a^2 + b^2} + a} \pm i\sqrt{\sqrt{a^2 + b^2} - a}]$

If $z = x + iy$, then,

1. $|z| = a, is\ a\ Circle.$
2. $|z \pm a| = x\ or\ y, is\ a\ parabola$
3. $|z \pm a| + |z \pm b| = c$, is a ellipse
4. $|z \pm a| - |z \pm b| = c, is\ a\ hyperbola$

5. De Moivre's Theorem

For integer n,

$$\left(r(\cos\theta + i\sin\theta)\right)^n = r^n\left(\cos(n\theta) + i\sin(n\theta)\right)$$

Also valid for negative and fractional n, yielding n distinct roots:

$$z^{1/n} = r^{1/n}\left[\cos\left(\frac{\theta + 2\pi k}{n}\right) + i\sin\left(\frac{\theta + 2\pi k}{n}\right)\right],\ k = 0, \dots, n-1$$

Solutions to $z^n = 1$ are equally spaced on the unit circle:

$$\omega_k = \cos(2\pi k/n) + i\sin(2\pi k/n),\ k = 0, \dots, n-1$$

Their sum is zero and they form a geometric progression

6. Common useful forms of different series

a. $\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1}$

b. $\sum_{k=1}^{n} \frac{1}{k(k+2)} = \frac{3}{4} - \frac{1}{2(n+1)} - \frac{1}{4(n+2)}$

c. $\sum_{k=1}^{n} \frac{1}{k(k+1)(k+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$

d. $\sum_{k=1}^{n} \left(\frac{1}{k} - \frac{1}{k+1}\right) = 1 - \frac{1}{n+1}$

Some important series

1. $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$
2. $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$
3. $1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$

a. Sum of Sine Series $S_n = \sum_{k=0}^{n-1} \sin(\theta + k\alpha) = \frac{\sin\left(\frac{n\alpha}{2}\right)\cdot\sin\left(\theta + \frac{(n-1)\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)}$

b. Sum of Cosine Series $S_n = \sum_{k=0}^{n-1} \cos(\theta + k\alpha) = \frac{\sin\left(\frac{n\alpha}{2}\right)\cdot\cos\left(\theta + \frac{(n-1)\alpha}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)}$

7. In coordinate geometry, the general equation of the second degree in x and y is:

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

Condition for a Pair of Straight Lines

The equation represents a pair of straight lines if and only if:

$\boldsymbol{abc + 2fgh - af^2 - bg^2 - ch^2 = 0}$ (Discriminant condition)

Homogeneous Equation of Second Degree

For $\boldsymbol{ax^2 + 2hxy + by^2 = 0}$, the equation passes through the origin and can be factorized into two lines through the origin.

Slopes: $\boldsymbol{m_1, m_2 = [-h \pm \sqrt{(h^2 - ab)}] / b}$

Nature: $\boldsymbol{h^2 > ab}$ → Real & distinct;

$\boldsymbol{h^2 = ab}$ → Real & coincident;

$\boldsymbol{h^2 < ab}$ → Imaginary

Angle Between the Lines

For $\boldsymbol{ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0}$ represents pair of straight lines and angle between two lines is $\boldsymbol{\theta}$

$$\boldsymbol{\tan\theta = \frac{[2\sqrt{h^2 - ab}]}{a+b}}$$

Perpendicular lines if $\boldsymbol{a + b = 0}$.

Parallel lines if $\boldsymbol{h^2 = ab}$

Bisector of the angles between the pair of straight lines

$ax^2 + 2hxy + by^2 = 0$ is $\frac{x^2-y^2}{a-b} = \frac{xy}{h}$

Classification Based on Discriminant:

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$$

If $\Delta \neq 0 \rightarrow$ degenerate conic.

Nature of conic determined by $\delta = ab - h^2$:

$$ab - h^2 > 0 \rightarrow \text{Ellipse}$$

$$ab - h^2 = 0 \rightarrow \text{Parabola}$$

$$ab - h^2 < 0 \rightarrow \text{Hyperbola}$$

8. Direction Cosines (DCs)

A straight line in 3D space can be described by the angles it makes with the coordinate axes.

Let α = angle between the line and the x-axis

β = angle between the line and the y-axis

γ = angle between the line and the z-axis

The direction cosines are:

$$l = \cos\alpha, m = \cos\beta, n = \cos\gamma$$

Property of Direction Cosines

For any straight line in 3D:

$$l^2 + m^2 + n^2 = 1$$

This follows from the fact that these cosines correspond to the components of a unit vector along the line.

9. Direction Ratios (DRs)

Direction ratios are any set of numbers proportional to the direction cosines.

If a, b, c are direction ratios of the line:

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

So:

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}, \quad m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}, \quad n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

Relation between DCs and DRs

- DCs are always unique for a given direction (up to sign) and satisfy $l^2 + m^2 + n^2 = 1$.
- DRs can be any proportional numbers scaling them doesn't change the direction.

Equation of a Line Using DCs/DRs

If a line passes through (x_1, y_1, z_1) and has DRs a, b, c:

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

This is the symmetric form of the equation of a line.

10. Equation of plane

$$ax + by + cz + d = 0$$

Normal vector = (a, b, c)

If a plane passes trough (x_1, y_1, z_1) then equation of plane,

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

If a, b, c are intercepts by a plane, then equation is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$$

Shortest distance from point $P(x_1, y_1, z_1)$ to the plane:

Distance $= \frac{|a\,x_1 + b\,y_1 + c\,z_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$

If angle between $a_1x + b_1y + c_1 + d_1 = 0$ and $a_2x + b_2y + c_2 + d_2 = 0$ planes is θ then,

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \cdot \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Condition for Parallel and Perpendicular Lines

- Parallel: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
- **Perpendicular:** $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Parallel to $ax + by + cz + d = 0$ is $ax + by + cz + k = 0$

Perpendicular to $a_1x + b_1y + c_1 + d_1 = 0$ and $a_2x + b_2y + c_2 + d_2 = 0$ and passes trough

(x_1, y_1, z_1) is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

11. Equation of straight line is

$$a_1x + b_1y + c_1y + d_1 = 0 = a_2x + b_2y + c_2z + d_2$$

And direction ratio is $b_1c_2 - b_2c_1, c_1a_2 - c_2a_1, a_1b_2 - a_2b_1$

Symmetric (Cartesian) Form:

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

(a, b, c) = direction ratios (DRs)

Direction cosines $(\cos\alpha, \cos\beta, \cos\gamma)$ are unit direction ratios.

if (a_1, b_1, c_1) and (a_2, b_2, c_2) are direction ratio of two lines then 1. Angle between the lines is θ

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \cdot \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Condition for Parallel and Perpendicular Lines

- Parallel: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
- **Perpendicular:** $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Condition for Coplanarity

Two lines:

$$\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1}$$

$$\frac{x - x_2}{l_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2}$$

If lines are coplanar $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$

Then shortest distance=

$$\text{S.D.} = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}}{\sqrt{(m_1n_2 - m_2n_1)^2 + (n_1l_2 - n_2l_1)^2 + (l_1m_2 - l_2m_1)^2}}$$

12. Basic Vector Operations

$$\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}, \vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

1. Magnitude (Length) $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$
2. Unit Vector $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$
3. Addition & Subtraction

$$\vec{a} \pm \vec{b} = (a_1 \pm b_1)\hat{\imath} + (a_2 \pm b_2)\hat{\jmath} + (a_3 \pm b_3)\hat{k}$$

4. Scalar Multiplication

$$k\vec{a} = (ka_1)\hat{\imath} + (ka_2)\hat{\jmath} + (ka_3)\hat{k}$$

5. Dot Product (Scalar Product)

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Or $$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$$

Cross Product (Vector Product)

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$$

Area of parallelogram: $A = |\vec{a} \times \vec{b}|$

Area of triangle: $A = \frac{1}{2} |\vec{a} \times \vec{b}|$

Perpendicularity check: $\vec{a} \times \vec{b} = \vec{0}$

Scalar Triple Product $[\vec{a}, \vec{b}, \vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c})$

Volume of parallelepiped: $V = |[\vec{a}, \vec{b}, \vec{c}]|$

Coplanarity check: $[\vec{a}, \vec{b}, \vec{c}] = 0$

Vector Triple Product

$$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$$

Equation of a Line

- Vector Form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Equation of a Plane

- Vector Form: $\vec{r} \cdot \vec{n} = d$

Distance Formulas

If line passes through A and has direction vector $\vec{d}$

$$\text{Distance} = \frac{|\overrightarrow{AP} \times \vec{d}|}{|\vec{d}|}$$

If lines have direction vectors $\vec{d_1}, \vec{d_2}$ and points A, B

$$\text{Distance} = \frac{|\,(\vec{B} - \vec{A}) \cdot (\vec{d_1} \times \vec{d_2})\,|}{|\,\vec{d_1} \times \vec{d_2}\,|}$$

13. Matrix

Transpose of a Matrix

$$(A^T)_{ij} = a_{ji}.$$

Properties:

1. $(A^T)^T = A$
2. $(A+B)^T = A^T + B^T$
3. $(AB)^T = B^T A^T$

Determinant and Inverse

For a square matrix A: determinant is $|A|$. If $|A| \neq 0$, A is invertible.

$$A^{-1} = \frac{adj(A)}{|A|}$$

Properties:

1. $(A^{-1})^{-1} = A$
2. $(AB)^{-1} = B^{-1}A^{-1}$
3. $(A^T)^{-1} = (A^{-1})^T$

Consistency of a System

For $AX = B$

Consistent → has at least one solution.

Inconsistent → no solution.

Using **Rank Method** (Rouché–Capelli Theorem):

If $\text{rank}(\boldsymbol{A}) = \text{rank}([\boldsymbol{A} \mid \boldsymbol{B}]) = \boldsymbol{n}$, unique solution

If $\text{rank}(\boldsymbol{A}) = \text{rank}([\boldsymbol{A} \mid \boldsymbol{B}]) < \boldsymbol{n}$, infinite solution

If $\text{rank}(\boldsymbol{A}) \neq \text{rank}([\boldsymbol{A} \mid \boldsymbol{B}])$, no solution

14. Vector Spaces over the Field of Real Numbers

A vector space $\boldsymbol{V}$ over the field of real numbers $\boldsymbol{R}$ is a non-empty set together with two operations:

1. Vector addition: $+\colon \boldsymbol{V} \times \boldsymbol{V} \rightarrow \boldsymbol{V}$

For $\boldsymbol{u}, \boldsymbol{v} \in \boldsymbol{V}, \boldsymbol{u} + \boldsymbol{v} \in \boldsymbol{V}$

2. Scalar multiplication: $\cdot\colon \mathbb{R} \times \boldsymbol{V} \rightarrow \boldsymbol{V}$

For $\boldsymbol{\alpha} \in \mathbb{R}, \boldsymbol{v} \in \boldsymbol{V}, \boldsymbol{\alpha v} \in \boldsymbol{V}$

These operations must satisfy the following axioms (for all $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w} \in \boldsymbol{V}$ and scalars $\boldsymbol{a}, \boldsymbol{b} \in \boldsymbol{R}$):

1. $\boldsymbol{u} + \boldsymbol{v} = \boldsymbol{v} + \boldsymbol{u}$ (commutativity)

2. $(\boldsymbol{u} + \boldsymbol{v}) + \boldsymbol{w} = \boldsymbol{u} + (\boldsymbol{v} + \boldsymbol{w})$ (associativity)

3. There exists $\boldsymbol{0} \in \boldsymbol{V}$ such that $\boldsymbol{v} + \boldsymbol{0} = \boldsymbol{v}$ (additive identity)

4. For each $\boldsymbol{v} \in \boldsymbol{V}$, there exists -$\boldsymbol{v} \in \boldsymbol{V}$ such that $\boldsymbol{v} + (-\boldsymbol{v}) = \boldsymbol{0}$ (additive inverse)

5. $\boldsymbol{a}(\boldsymbol{u} + \boldsymbol{v}) = \boldsymbol{au} + \boldsymbol{av}$ (distributivity of scalar multiplication over vector addition)

6. $(\boldsymbol{a} + \boldsymbol{b})\boldsymbol{v} = \boldsymbol{av} + \boldsymbol{bv}$ (Distributivity of scalar multiplication over scalars)

7. $\boldsymbol{a}(\boldsymbol{bv}) = (\boldsymbol{ab})\boldsymbol{v}$ (Associativity of scalar multiplication)

8. $\boldsymbol{1v} = \boldsymbol{v}$ (Multiplicative identity)

Examples of Vector Spaces:

1. $\mathbb{R}^n = \{ (x_1, x_2, \dots, x_n) : x_i \in \mathbb{R} \}$

2. Set of all polynomials with real coefficients, $P(\mathbb{R})$

3. Set of all $m \times n$ real matrices, $M_{\{m \times n\}}(\mathbb{R})$

Subspaces

A subspace W of a vector space V is a subset $W \subseteq V$ which is itself a vector space under the same operations of V.

Conditions for Subspace:

A non-empty subset $W \subseteq V$ is a subspace if:

1. $u, v \in W \Rightarrow u + v \in W$ (closed under addition)

2. $\alpha \in R, v \in W \Rightarrow \alpha v \in W$ (closed under scalar multiplication)

Examples:

1. $V = \mathbb{R}^3$. Then, $W = \{ (x, y, 0) : x, y \in \mathbb{R} \}$ is a subspace.

2. The set of all polynomials of degree ≤ 2 is a subspace of all polynomials.

Linear Dependence and Independence

Definition (Linear Combination):

A vector $v \in V$ is said to be a linear combination of vectors

$v_1, v_2, \dots, v_n \in V$

if

$v = a_1 v_1 + a_2 v_2 + \dots + a_n v_n$ for some scalars $a_1, a_2, \dots, a_n \in$ R.

Definition (Linear Dependence):

A set of vectors $\{v_1, v_2, \dots, v_n\}$ is linearly dependent if there exist scalars $a_1, a_2, \dots, a_n$, not all zero, such that

$$a_1 v_1 + a_2 v_2 + \ldots + a_n v_n = 0.$$

Definition (Linear Independence):

The set $\{v_1, v_2, \ldots, v_n\}$is linearly independent if

$$a_1 v_1 + a_2 v_2 + \ldots + a_n v_n = 0.$$
$$\Rightarrow a_1 = a_2 = \ldots = a_n = 0.$$

Examples:

1. In R^2, the vectors $(1, 0)$ and $(0, 1)$ are linearly independent.

2. In R^2 , the vectors $(1, 2)$ and $(2, 4)$ are linearly dependent because $(2, 4) = 2(1, 2)$.

Basis and Dimension

Basis:

A basis of a vector space V is a set of linearly independent vectors that span V.

Span means every element of V can be written as a linear combination of the basis vectors.

Dimension:

The dimension of a vector space V, denoted $dim(V)$, is the number of vectors in any basis of V.

Examples:

1. In R^2, the standard basis is $\{ (1, 0), (0, 1) \}$. Dimension: 2.

2. In R^3, the standard basis is $\{ (1, 0, 0), (0, 1, 0), (0, 0, 1) \}$. Dimension: 3.

3. The set of all polynomials of degree ≤ 2 has basis $\{1, x, x^2\}$. Dimension: 3.

Note : $dim(U + V) = dim\,U + dim\,V - dim(U \cap V)$

Properties

1. Any set of more than n vectors in $\boldsymbol{R^n}$ is linearly dependent.
2. Any linearly independent set in $\boldsymbol{R^n}$ can be extended to form a basis.
3. All bases of a vector space have the same number of elements.

15. Linear Transformations

Definition:

A linear transformation (or linear map) is a function $\boldsymbol{T: V \to W}$ between two vector spaces V and W (over the same field, usually $\mathbb{R}$) such that for all vectors $\boldsymbol{u, v} \in \boldsymbol{V}$ and scalar $\boldsymbol{c} \in \mathbb{R}$:

1. Additivity: $\boldsymbol{T(u+v) = T(u) + T(v)}$

2. Homogeneity: $\boldsymbol{T(cu) = cT(u)}$

Examples:

1. $\boldsymbol{T}: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $\boldsymbol{T(x,y) = (2x, 3y)}$. This is linear.

2. $\boldsymbol{T}: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $\boldsymbol{T(x,y) = (x+1, y)}$. this is NOT linear (fails additivity).

Matrix Representation:

If $\boldsymbol{T}: \mathbb{R}^n \to \mathbb{R}^m$, then there exists an m×n matrix A such that $\boldsymbol{T(x) = Ax, where\ x} \in \mathbb{R}^n$.

Example: $\boldsymbol{T}: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $\boldsymbol{T(x,y) = (x+2y, 3x+y)}$. Then its matrix representation is $\boldsymbol{A} = \begin{bmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{3} & \mathbf{1} \end{bmatrix}$

2. Kernel (Null Space) and Range (Image)

Kernel (Null Space): $\boldsymbol{ker(T) = \{ v \in V : T(v) = 0 \}}$ (subspace of V).

Range (Image): $\boldsymbol{Im(T) = \{ T(v) : v \in V \}}$ (subspace of W).

Example:

Let $\boldsymbol{T}: \mathbb{R}^3 \to \mathbb{R}^2$ be defined by $\boldsymbol{T(x,y,z) = (x+y, y+z)}$.

Matrix representation: $\boldsymbol{A} = \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} \end{bmatrix}$

Kernel:

Solve $\boldsymbol{A}[\boldsymbol{x} \ \ \boldsymbol{y} \ \ \boldsymbol{z}]^{\mathrm{T}} = \mathbf{0}$

$\Rightarrow \ \boldsymbol{x} + \boldsymbol{y} = \mathbf{0}, \boldsymbol{y} + \boldsymbol{z} = \mathbf{0} \ \Rightarrow \ (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) = (-\boldsymbol{y}, \boldsymbol{y}, -\boldsymbol{y})$.

So $\boldsymbol{ker}(\boldsymbol{T}) = \boldsymbol{span}\{(-\mathbf{1}, \mathbf{1}, -\mathbf{1})\}$.

Image: Columns of A are $[\mathbf{1} \ \ \mathbf{0}]^{\mathrm{T}}, [\mathbf{1} \ \ \mathbf{1}]^{\mathrm{T}}, [\mathbf{0} \ \ \mathbf{1}]^{\mathrm{T}}$. Since [1 1] = [1 0]+[0 1], Im(T)=span{[1 0]ᵀ,[0 1]ᵀ} = $\mathbb{R}^2$.

3. Rank and Nullity

$$\boldsymbol{Rank}(\boldsymbol{T}) = \boldsymbol{dim}(\boldsymbol{Im}(\boldsymbol{T}))$$

$$\boldsymbol{Nullity}(\boldsymbol{T}) = \boldsymbol{dim}(\boldsymbol{ker}(\boldsymbol{T}))$$

Rank–Nullity Theorem:

For a linear transformation

$\boldsymbol{T}: \boldsymbol{V} \rightarrow \boldsymbol{W}: \quad \boldsymbol{dim}(\boldsymbol{V}) = \boldsymbol{rank}(\boldsymbol{T}) + \boldsymbol{nullity}(\boldsymbol{T})$

Note : If A is a $\boldsymbol{m} \times \boldsymbol{n}$ matrix, then

i. $\boldsymbol{rank}(\boldsymbol{A}) + \boldsymbol{Nullity}(\boldsymbol{A}) = \boldsymbol{n}$

ii. $\boldsymbol{rank}(\boldsymbol{A}) + \boldsymbol{Nullity}(\boldsymbol{A}^{\boldsymbol{T}}) = \boldsymbol{m}$

iii. $\boldsymbol{rank}(\boldsymbol{A}) = \boldsymbol{rank}(\boldsymbol{A}^{\boldsymbol{T}})$

Some theorem

1. Let $\boldsymbol{T}: \boldsymbol{V} \rightarrow \boldsymbol{U}$ be a linear mapping. If T is one-one and on-to then $\boldsymbol{T}^{-\mathbf{1}}: \boldsymbol{U} \rightarrow \boldsymbol{V}$ is linear.

2. Let $\boldsymbol{T}: \boldsymbol{V} \rightarrow \boldsymbol{U}$ be a linear mapping. Then $\boldsymbol{Im}(\boldsymbol{T})$ is a subspace of $\boldsymbol{U}$.

3. Let $\boldsymbol{T}: \boldsymbol{V} \rightarrow \boldsymbol{U}$ be a linear mapping. Then $\boldsymbol{Ker}(\boldsymbol{T})$ is a subspace V.

4. Let $\boldsymbol{T}: \boldsymbol{V} \rightarrow \boldsymbol{U}$ be a linear mapping and V is a finite dimensional vector space then

$$\boldsymbol{dim}(\boldsymbol{ker}\,\boldsymbol{T}) + \boldsymbol{dim}(\boldsymbol{ImT}) = \boldsymbol{dim}\,(\boldsymbol{V})$$

5. $\boldsymbol{T}: \boldsymbol{V} \rightarrow \boldsymbol{W}$ is injective (one-one) if and only if $\boldsymbol{ker}\,\boldsymbol{T} = \{\boldsymbol{0}\}$

Example:

For $\boldsymbol{T}: \mathbb{R}^3 \rightarrow \mathbb{R}^2$,

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} \end{bmatrix}$$

$\boldsymbol{ker}(\boldsymbol{T}) = \boldsymbol{span}\{(-\mathbf{1}, \mathbf{1}, -\mathbf{1})\}$

⇒ nullity (T) = 1.

$\boldsymbol{Im}(\boldsymbol{T}) = \mathbb{R}^2$

$$\Rightarrow \boldsymbol{rank}(\boldsymbol{T}) = \mathbf{2}.$$

$$\boldsymbol{dim}(\boldsymbol{V}) = \mathbf{3} = \boldsymbol{rank} + \boldsymbol{nullity} = \mathbf{2} + \mathbf{1}.$$

Properties

1. Every linear transformation can be represented by a matrix.

2. The rank is equal to the number of linearly independent columns of the matrix.

3. If $\boldsymbol{rank}(\boldsymbol{T}) = \boldsymbol{dim}(\boldsymbol{V})$, then T is one-to-one (injective).

4. If $\boldsymbol{rank}(\boldsymbol{T}) = \boldsymbol{dim}(\boldsymbol{W})$, then T is onto (surjective).

5. If both hold, then T is an isomorphism.

16. The Completeness Axiom

Every non-empty subset $\boldsymbol{A} \subseteq \mathbb{R}$ that is bounded above has a least upper bound in $\mathbb{R}$.

This distinguishes $\mathbb{R}$ from $\mathbb{Q}$.

Example: $\boldsymbol{S} = \{\boldsymbol{x} \in \mathbb{Q} : \boldsymbol{x}^2 < \boldsymbol{2}\}$

In $\mathbb{Q}$, sup S does not exist, but in $\mathbb{R}$:

17. Dedekind's Theorem

If A and B are two subsets of $\mathbb{R}$ such that,

i) $\boldsymbol{A} \neq \boldsymbol{\phi}, \boldsymbol{B} \neq \boldsymbol{\phi}$

ii) $\boldsymbol{A} \cup \boldsymbol{B} = \mathbb{R}$

iii) Every member of A is less then every member of B that is $\boldsymbol{x} \in \boldsymbol{A}$ ***and*** $\boldsymbol{y} \in \boldsymbol{B} \leftrightarrow \boldsymbol{x} < \boldsymbol{y}$

Then either A has the greatest member or B has the smallest member.

Example:

$$\boldsymbol{A} = \{\boldsymbol{x} \in \mathbb{Q} : \boldsymbol{x}^2 < \boldsymbol{2}\}, \ \boldsymbol{B} = \{\boldsymbol{x} \in \mathbb{Q} : \boldsymbol{x}^2 \geq \boldsymbol{2}\}$$

This cut defines the irrational number$\sqrt{\boldsymbol{2}}$.

Note:

i) The completeness axiom in $\mathbb{R}$ is equivalent to Dedekind's axiom.

ii) The set Q of rational numbers is not complete.

18. The Archimedean Property of Real Properties

If x is a positive real number and y is any real number, then there exists a positive integer n such that $nx > y$.

19. Properties of Continuous Functions:

1. Sum, difference, product, and quotient (denominator $\neq 0$) of continuous functions are continuous.
2. Composition of continuous functions is continuous.
3. Every polynomial function is continuous everywhere.
4. Every rational function is continuous on its domain

(where denominator $\neq 0$).

20. Intermediate Value Theorem (IVT)

Let f be a function continuous on the closed interval $[a, b]$.

If $f(a) \neq f(b)$ and N is any number between $f(a)$ and $f(b)$, then there exists at least one $c \in (a, b)$ such that $f(c) = N$.

21. Uniform Continuity

A function $f : D \to R$ is said to be uniformly continuous on D if:

For every $\varepsilon > 0$, there exists $\delta > 0$ such that for all $x, y \in D$,

$$|x - y| < \delta \Rightarrow |f(x) - f(y)| < \varepsilon.$$

22. Differentiability test shortcut

Step 1: Check Continuity

- If not continuous at x=ax=ax=a ⟶ Not differentiable.
- (So continuity is a must condition).

Step 2: Check Left and Right Derivatives

- Find derivative on left side of a.

- Find derivative on right side of a.
- If both are equal → Differentiable at a
- If unequal → Not differentiable.

Some important function:

Polynomials, exponential, trigonometric, logarithmic → differentiable everywhere in their domain.

- Piecewise functions (like $|x|$, $|x-1|$, greatest integer function $[x]$) → only need to check at the joining point.
- If graph has a corner, cusp, vertical tangent, or discontinuity → NOT differentiable at that point.

23. Rolle's Theorem

If a function $f(x)$ satisfies:

1. $f(x)$ is continuous on $[a,b]$,
2. $f(x)$ is differentiable on (a,b),
3. $f(a)=f(b)$,

then there exists at least one $c \in (a,b)$ such that:

$$f'(c)=0$$

24. Mean Value Theorem (MVT)

Statement (Lagrange's MVT):

If $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b), then there exists at least one $c \in (a,b)$ such that:

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

25. Taylor's Theorem with Remainder

If $f(x)$ has continuous derivatives up to order $n+1$ in $[a,b]$, then for x near a:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + R_n(x)$$

where remainder term is

$$R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}, \quad c \in (a, x)$$

26. Indeterminate Forms

Indeterminate forms arise in limits where direct substitution is not possible.

Types:

$\frac{0}{0}$, $\frac{\infty}{\infty}$, $\frac{1}{0}$, $0.\infty$, $\infty - \infty$, 0^0, 1^∞, ∞^0 etc

Method to Solve:

- Use L'Hospital's Rule:
  If $lim_{x \to a} f(x) = lim_{x \to a} g(x) = 0$ or ∞, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

27. The derivative helps us decide:

- If $f'(x) > 0$ on an interval $\rightarrow f(x)$ is increasing.
- If $f'(x) < 0$ on an interval $\rightarrow f(x)$ is decreasing.
- If $f'(x) = 0$ at some point $\rightarrow$ the function may have a maximum/minimum (needs checking).

28. Maxima and Minima

- A function $f(x)$ has a maximum at $x = a$ if $f(a)$ is greater than or equal to values of $f(x)$ near a.
- A function $f(x)$ has a minimum at $x = a$ if $f(a)$ is less than or equal to values of $f(x)$ near a.
- Together, they are called extrema.

First Derivative Test

8. Find $f'(x) = 0$ (critical points).
9. If $f'(x)$ changes sign:

   ✓ From $+$ to $-$: Maximum.
   ✓ From $-$ to $+$: Minimum.

Second Derivative Test

If $f'(a) = 0$:

- If $f''(a) > 0 \Rightarrow$ Minimum at $x = a$.
- If $f''(a) < 0 \Rightarrow$ Maximum at $x = a$.
- If $f''(a) = 0 \Rightarrow$ Test fails (check higher derivatives or first derivative test).

29. Complete Integration Formula Sheet

Basic Standard Formulas

10. $\int k\,dx = kx + C$
11. $\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$ and $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{n+1} + c$
12. $\int \frac{1}{x} dx = ln|x| + C$
13. $\int e^x dx = e^x + C$ and $\int e^{ax} dx = \frac{e^{ax}}{a} + c$
14. $\int a^x dx = \frac{a^x}{lna} + C \quad (a > 0, a \neq 1)$

Trigonometric Integrals

$$\int sinxdx = -cosx + C$$
$$\int cosxdx = sinx + C$$
$$\int sec^2xdx = tanx + C$$
$$\int csc^2xdx = -cotx + C$$
$$\int secxtanxdx = secx + C$$
$$\int cosecxcotxdx = -cosecx + C$$

Inverse Trigonometric Integrals

$$\int \frac{1}{\sqrt{1-x^2}}dx = sin^{-1}x + C$$
$$\int \frac{-1}{\sqrt{1-x^2}}dx = cos^{-1}x + C$$
$$\int \frac{1}{1+x^2}dx = tan^{-1}x + C$$
$$\int \frac{-1}{1+x^2}dx = cot^{-1}x + C$$
$$\int \frac{1}{|x|\sqrt{x^2-1}}dx = sec^{-1}x + C$$
$$\int \frac{-1}{|x|\sqrt{x^2-1}}dx = csc^{-1}x + C$$

Integrals of Powers of Trigonometric Functions

15. $\int sin^2xdx = \frac{x}{2} - \frac{sin2x}{4} + C$
16. $\int cos^2xdx = \frac{x}{2} + \frac{sin2x}{4} + C$
17. $\int tan^2xdx = tanx - x + C$
18. $\int cot^2xdx = -cotx - x + C$
19. $\int sec^2xdx = tanx + C$
20. $\int cosec^2xdx = -cotx + C$

Exponential & Logarithmic Integrals

21. $\int e^{ax}dx = \frac{1}{a}e^{ax} + C$
22. $\int xe^xdx = (x-1)e^x + C$
23. $\int lnxdx = xlnx - x + C$

Hyperbolic Function Integrals

i. $\int sinhxdx = coshx + C$
ii. $\int coshxdx = sinhx + C$
iii. $\int sech^2xdx = tanhx + C$
iv. $\int csch^2xdx = -cothx + C$
v. $\int tanhxdx = ln|coshx| + C$
vii. $\int cothxdx = ln|sinhx| + C$

Special Standard Forms

24. $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} tan^{-1}\frac{x}{a} + C$
25. $\int \frac{1}{a^2-x^2} dx = \frac{1}{2a} ln\left|\frac{a+x}{a-x}\right| + C$
26. $\int \frac{1}{\sqrt{a^2-x^2}} dx = sin^{-1}\frac{x}{a} + C$
27. $\int \frac{1}{\sqrt{x^2+a^2}} dx = ln\left|x + \sqrt{x^2+a^2}\right| + C$
28. $\int \frac{1}{\sqrt{x^2-a^2}} dx = ln\left|x + \sqrt{x^2-a^2}\right| + C$
29. $\int e^{ax}[af(x) + f'(x)]dx = e^{ax}f(x) + c$

Reduction Formulas (Recurrence Relations)

30. For powers of sine:

$$\int sin^n x dx = -\frac{1}{n} sin^{n-1} x cos x + \frac{n-1}{n}\int sin^{n-2} x dx$$

For powers of cosine:

$$\int cos^n x dx = \frac{1}{n} cos^{n-1} x sin x + \frac{n-1}{n}\int cos^{n-2} x dx$$

For secant powers:

$$\int sec^n x dx = \frac{1}{n-1} sec^{n-2} x tan x + \frac{n-2}{n-1}\int sec^{n-2} x dx$$

For tangent powers:

$$\int tan^n x dx = \frac{1}{n-1} tan^{n-1} x - \int tan^{n-2} x dx$$

For $cot^n x$

$$\int cot^n x dx = -\frac{cot^{n-1} x}{n-1} - cot^{n-2} x dx$$

$$\int sin^m x cos^n x\, dx = \frac{cos^{n-1} x\, sin^{m+1} x}{m+1} + \frac{n-1}{m+1}\int cos^{n-2} x\, sin^m x\, sin^2 x\, dx$$

30. Riemann Integrability

A function f is Riemann integrable on $[a, b]$ if:

$$\sup_P L(f, P) = \inf_P U(f, P).$$

The common value is called the Riemann integral of f over $[a, b]$, denoted:

$$\int_a^b f(x)\, dx.$$

31. Convergence Check Table for some Improper Integrals

Type of Integral	Condition for Convergence	Example (Convergent)	Example (Divergent)
$\int_1^\infty \frac{1}{x^p} dx$	$p > 1$	$\int_1^\infty \frac{1}{x^2} dx$	$\int_1^\infty \frac{1}{x} dx$
$\int_0^1 \frac{1}{x^p} dx$	$p < 1$	$\int_0^1 \frac{1}{\sqrt{x}} dx$	$\int_0^1 \frac{1}{x} dx$
$\int_a^\infty f(x) dx$	Compare with $\frac{1}{x^p}$	$\int_1^\infty \frac{1}{x^2+1} dx \sim \frac{1}{x^2}$	$\int_1^\infty \frac{1}{\sqrt{x}} dx \sim \frac{1}{x^{1/2}}$
$\int_0^b f(x) dx$ with singularity at 0	Compare with x^{-p}	$\int_0^1 \frac{1}{x^{1/3}} dx$	$\int_0^1 \frac{1}{x} dx$
Oscillating or tricky functions	Use comparison or limit test	$\int_1^\infty \frac{sinx}{x^2} dx$	$\int_1^\infty \frac{sinx}{x} dx$

32. General Steps to Solve Area/Volume Problems

31. Identify given curve(s).
32. Decide limits of integration (a, b or c, d).

33. Write down the correct formula (area or volume).
34. Simplify and integrate step by step.
35. Interpret result with units (square/cubic).

- Area under curve:

$$A = \int_a^b f(x)dx$$

- Area between two curves:

$$A = \int_a^b (f(x) - g(x))\,dx$$

- Volume (x-axis revolution):

$$V = \pi \int_a^b (f(x))^2\,dx$$

- Volume (y-axis revolution):

$$V = \pi \int_c^d (f(y))^2\,dy$$

- Volume (cylindrical shells):

$$V = 2\pi \int_a^b x \cdot f(x)dx$$

Some common area and volumes :

1) Area of a circle = πr^2

And volume is $\frac{4}{3}\pi a^3$

2) Area of a ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab

And volume revolving about y axis is $\frac{4\pi a^2 b}{3}$

3) Area of a hypocycloid $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1$ is $\frac{3\pi ab}{8}$

And volume revolving about x or y axis is $\frac{32\pi a^2 b}{105}$

4) Area of an asteroid $(x)^{\frac{2}{3}} + (y)^{\frac{2}{3}} = a^{\frac{2}{3}}$ is $\frac{3\pi a^2}{8}$

And volume revolving about x or y axis is $=\frac{32\pi a^3}{105}$

5) Area bounded by cycloid $x = a(\theta - \sin\theta, y = a(1 - \cos\theta)$ and its base is $3\pi a^2$

And volume revolving about x axis $= 5\pi^2 a^3$

6) Common area bounded by $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16a^2}{3}$

7) Area of a cardioide $r = a(1 + \cos\theta)$ is $\frac{3\pi a^2}{2}$

8) Equation of loop $r = a\sin n\theta \;or, r = a\cos n\theta$

If n=even, number of loop is 2n

If n=odd, number of loop is n

Area = $\int \frac{1}{2} r^2 d\theta$

9. equation of lemniscate $r^2 = a^2 \cos 2\theta$ and area = a^2

33. Equilibrium

A body is said to be in equilibrium when it remains at rest or moves with constant velocity under the action of applied forces.
There are two types of equilibrium conditions:

36. Translational Equilibrium – No linear motion.

$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum F_z = 0$

37. Rotational Equilibrium – No angular motion.

$\sum M_O = 0$ (sum of moments about any point O is zero)

General Conditions of Equilibrium (Rigid Body in a Plane)

For a rigid body in plane equilibrium (2D case):

38. Sum of all horizontal components of forces = 0

$$\sum F_x = 0$$

39. Sum of all vertical components of forces = 0

$$\sum F_y = 0$$

40. Sum of all moments of forces about any arbitrary point O = 0

$$\sum M_O = 0$$

Thus, the general condition of equilibrium:

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum M_O = 0$$

34. Principle of Virtual Work

Statement:

If a system of forces acting on a rigid body is in equilibrium, then the total virtual work done by all the forces during any small virtual displacement consistent with the constraints of the system is zero.

$$\delta W = \sum(F \cdot \delta r) = 0$$

where

- F = applied force,
- δr = virtual displacement in the direction of force.

35. C.G. of Common Geometrical Figures

1. Uniform Rod (length L): C.G. at $\frac{L}{2}$ from either end.

2. Rectangle: C.G. at intersection of diagonals.

3. Triangle: C.G. at centroid (intersection of medians). Coordinates: $(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3})$.

4. Semicircular lamina (radius r): C.G. at $\frac{4r}{3\pi}$ from flat side.

5. Solid Hemisphere (radius r): C.G. at $\frac{3r}{8}$ from flat surface.

6. Cone (solid, height h): C.G. at $\frac{h}{4}$ from base.

Conditions of Virtual Work:

41. If the body is in equilibrium → $\delta W = 0$.
42. If $\delta W \neq 0$ → the body is not in equilibrium.

Notes:

- Principle of Virtual Work:

  $\sum(\text{Virtual Work}) = 0$ for equilibrium

- Stable Equilibrium: Body returns to equilibrium (C.G. raised).
- Unstable Equilibrium: Body moves away (C.G. lowered).
- Neutral Equilibrium: Body stays in new position (C.G. unchanged).

36. Projectile Motion (Special Case of Motion in a Plane)

If a particle is projected with velocity u at angle θ:

Velocity components: $u_x = u\cos\theta, \quad u_y = u\sin\theta$

Equations of motion:

- Horizontal displacement: $x = u\cos\theta \cdot t$
- Vertical displacement: $y = u\sin\theta \cdot t - 1/2\, gt^2$

Trajectory equation (parabola): $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

Time of flight: $T = \frac{2u\sin\theta}{g}$

Maximum height: $H = \frac{u^2\sin^2\theta}{2g}$

Horizontal range: $R = \frac{u^2 sin(2\theta)}{g}$

37. Characteristics of a Central Force

43. Always acts along the line joining the particle and the center.
44. Magnitude depends only on r, not on velocity or angle.
45. It is a conservative force (work done is independent of path).
46. Associated with a potential energy function $U(r)$:

$$F(r) = -\frac{dU}{dr}$$

38. Standard Moments of Inertia:

- Thin rod about center: $I = \left(\frac{1}{12}\right) ML^2$
- Thin rod about end: $I = \left(\frac{1}{3}\right) ML^2$
- Solid sphere about diameter: $I = \left(\frac{2}{5}\right) MR^2$
- Hollow sphere about diameter: $I = \left(\frac{2}{3}\right) MR^2$
- Solid cylinder about its axis: $I = \left(\frac{1}{2}\right) MR^2$
- Thin circular ring about diameter: $I = \left(\frac{1}{2}\right) MR^2$

D'Alembert's Principle:

States: $\Sigma F + Finertia = 0$, where $Finertia = -ma$

39. Lagrange's Equation of Motion

The Lagrange's equation for a holonomic system is:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0, \quad (i = 1, 2, \ldots, n)$$

Here:

q_i = generalized coordinate

$\dot{q}_i = \frac{dq_i}{dt}$ = generalized velocity

40. Ordinary differential equation

i. Order = highest derivative;

Degree = power of highest derivative.

ii. First order linear ODE solved using integrating factor.

iii. Exact ODE condition: $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

iv. Homogeneous 2nd order ODE solved using auxiliary equation.

v. General solution = Complementary function (C.F.) + Particular integral (P.I.).

vi. Complex roots ⇒ sine & cosine form.

vii. Repeated roots ⇒ solution has polynomial in x.

vii. P.I. guessing rule:

- If $R(x) = e^{ax} \Rightarrow try\ Ae^{ax}$
- If $R(x) = sinx\ or\ cosx \Rightarrow try\ A\ cosx + B\ sinx$
- If overlap with C.F., multiply trial solution by x.

Short Notes for MCQs

- Ordinary Point: x_0 where $P(x_0) \neq 0$.
- Singular Point: $P(x_0) = 0$.
- Regular Singular: $\frac{(x-x_0)P'(x_0)}{P(x_0)}$ finite.
- Irregular Singular: otherwise.
- Frobenius Method: used for regular singular points.
- Recurrence Relation: key to constructing series solution.
- Radius of Convergence: usually up to nearest singular point.
- Series solution of y'' + y = 0: leads to sin x and cos x.

41. Properties of Gamma Function:

i. Recursive property: $\Gamma(n+1) = n\,\Gamma(n)$

ii. Value at half integers: $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi},\ \Gamma\left(\frac{3}{2}\right) = 1/2\,\sqrt{\pi}, etc.$

iii. Reflection formula: $\Gamma(x)\Gamma(1-x) = \frac{\pi}{sin(\pi x)}$

42. Relation with Gamma Function:

$$B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

Properties of Beta Function:

1. Symmetry: $B(m,n) = B(n,m)$

2. Reduction formula: $B(m,n) = \frac{m-1}{m+n-1} B(m-1,n)$

43. Orthogonality :

Legendre polynomials: $\int_{-1}^{1} P_m(x)P_n(x)dx = \frac{2}{2n+1}\delta_{mn}$

Hermite polynomials: $\int_{-\infty}^{\infty} e^{-x^2} H_m(x)H_n(x)dx = 2^n n! \sqrt{\pi}\,\delta_{mn}$

Laguerre polynomials: $\int_{0}^{\infty} e^{-x} L_m(x)L_n(x)dx = \delta_{mn}$

Bessel functions: $\int_{0}^{1} xJ_\alpha(u_{\alpha,m}x)J_\alpha(u_{\alpha,n}x)dx = 0 (m \neq n)$

44. Cauchy–Riemann (C–R) Equations

For $f(z) = u(x,y) + i\,v(x,y)$ to be differentiable at a point (x,y), the following must hold:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Polar form:

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}, -\frac{1}{r}\frac{\partial u}{\partial \theta} = -\frac{\partial v}{\partial r}$$

45. Harmonic function:

For $f(z) = u(x,y) + i\,v(x,y)$ is harmonic if :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \text{ and } \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$$

Which is called Laplace Equation.

46. Cauchy's Integral Theorem

If f(z) is analytic inside and on a simple closed curve C, then:

$$\int C\, f(z)\, dz = 0$$

Cauchy's Integral Formula

If f(z) is analytic inside and on a simple closed curve C, and z_0 is inside C, then:

$$f(z_0) = \left(\frac{1}{2\pi i}\right) \oint \frac{f(z)}{(z - z_0)} dz$$

Cauchy integral formula for the first integral

Let $f(z)$ be analytic inside and on a simple closed curve C and z_0 is a point inside C. then

$$f'(z_0) = \frac{1}{2\pi i} \oint \frac{f(z)}{(z - z_0)^2} dz$$

Integral formula for the higher derivative

Let $f(z)$ be analytic inside and on the boundary of a simple connected region R , then for every point

z_0 of R

$$f^n(z_0) = \frac{n!}{2\pi i} \oint \frac{f(z)}{(z - z_0)^{n+1}} dz$$

47. Methods for Finding Residues

i) For Simple Poles (m=1):

$$\text{Res}(f, a) = \lim_{z \to a} (z - a) f(z)$$

ii) For Poles of Order m:

$$\text{Res}(f, a) = \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} [(z - a)^m f(z)]$$

48. Residue Theorem

If $f(z)$ is analytic inside and on a closed contour C, except for isolated singularities inside C, then:

$$\int_C f(z)\, dz = 2\pi i \sum \text{Res}(f, a_k)$$

where a_k are singularities inside C.

Important mcqs

1. Which of the following is an incorrect statement about the order properties of real numbers?

a) For any real numbers a, b, either $a > b$, $a < b$, or $a = b$.

b) If $a < b$ and $b < c$, then $a < c$.

c) If $a < b$, then $a + c < b + c$ for any real number c.

d) If $a < b$ and $c < 0$, then $ac < bc$.

Explanation: The correct statement is $ac > bc$. Multiplying an inequality by a negative number reverses the direction of the inequality.

2. If a and b are positive real numbers such that $a + b = 10$, what is the maximum value of the product ab?

a) 100 b) 25
c) 20 d) 50

Explanation: According to the AM-GM inequality, the product of two positive numbers with a fixed sum is maximized when the numbers are equal. Here, $a = b = 5$, so the maximum product is $5 \times 5 = 25$.

3. If x and y are real numbers such that $x > y$, which of the following is always true?

a) $x^2 > y^2$ b) $x - y > 0$
c) $1/x < 1/y$ d) $xy > 0$

Explanation: The definition of $x > y$ is that $x - y$ is a positive number, hence $x - y > 0$. Options A, C, and D can be false, e.g., if $x = 1$ and $y = -2$.

4. If a complex number $z = a + bi$ satisfies $|z| = 1$, what can be said about z?

a) It lies on the imaginary axis.

b) It lies on a circle of radius 1 centered at the origin.

c) It is a real number.

d) It is a complex number with zero real part.

Explanation: The modulus $|z|$ represents the distance from the origin to the point representing z in the complex plane. If $|z| = 1$, the point lies on the unit circle.

5. If x and y are real numbers, which statement correctly applies the triangle inequality?

a) $|x + y| \leq |x| + |y|$

b) $|x + y| = |x| + |y|$

c) $|x - y| > |x| - |y|$

d) $|x - y| \leq |x| + |y|$

Explanation: This is the correct form of the triangle inequality. It states that the absolute value of a sum is less than or equal to the sum of the absolute values.

6. Find the value of x that satisfies the inequality $|2x - 1| < 5$.

a) $-2 < x < 3$

b) $x > 3$ or $x < -2$

c) $-3 < x < 2$

d) $x > 3$ or $x < -3$

Explanation: The inequality is equivalent to the compound inequality $-5 < 2x - 1 < 5$. Adding 1 to all parts gives $-4 < 2x < 6$. Dividing by 2 yields $-2 < x < 3$.

7. If x and y are real numbers, what can we conclude about the inequality $x \leq y$ and $x - y \leq 0$?

a) They are unrelated statements.

b) They are logically equivalent.

c) $x - y \leq 0$ is a stricter condition than $x \leq y$.

d) $x \leq y$ implies $x - y > 0$.

Explanation: The two statements are logically equivalent. Subtracting y from both sides of $x \leq y$ results in $x - y \leq 0$, and vice versa.

8. If x is a positive real number, which of the following is true based on the AM-GM inequality?

a) $x + \frac{1}{x} \geq 2$

b) $x + \frac{1}{x} \geq 1$

c) $x + \frac{1}{x} < 2$

d) $x + \frac{1}{x}$ can be any real number.

Explanation: Applying the AM-GM inequality to the two positive numbers x and $1/x$, we get $\frac{x+1/x}{2} \geq \sqrt{x \cdot \frac{1}{x}}$, which simplifies to $\frac{x+1/x}{2} \geq 1$, or $x + 1/x \geq 2$.

9. If x, y, z are positive real numbers, which inequality holds true?

a) $(x + y + z)/3 \leq \sqrt[3]{xyz}$

b) $(x + y + z)/3 \geq \sqrt[3]{xyz}$

c) $(x + y + z)/3 < \sqrt[3]{xyz}$

d) $x + y + z = \sqrt[3]{xyz}$

Explanation: This is the correct form of the Arithmetic Mean-Geometric Mean inequality, which states that the arithmetic mean is always greater than or equal to the geometric mean.

10. For a random variable X with mean μ and standard deviation σ, what does Chebyshev's inequality provide a bound for?

a) The probability that X is equal to its mean.

b) The probability that X falls within a certain number of standard deviations from its mean.

c) The exact probability distribution of X.

d) The variance of X.

e) Explanation: Chebyshev's inequality states $P(|X - \mu| \geq k\sigma) \leq \frac{1}{k^2}$. This provides a general bound for the probability of a random variable deviating from its mean.

11. Let $a_1, a_2, \ldots, a_n$ be positive real numbers. According to the Cauchy-Schwarz inequality, which of the following is true?

a) $(\sum_{i=1}^{n} a_i)^2 \geq n \sum_{i=1}^{n} a_i^2$

b) $(\sum_{i=1}^{n} a_i)^2 \geq \sum_{i=1}^{n} a_i^2$

c) $(\sum_{i=1}^{n} a_i^2)(\sum_{i=1}^{n} b_i^2) \geq (\sum_{i=1}^{n} a_i b_i)^2$

d) $\frac{a_1+a_2+\ldots+a_n}{n} \geq \sqrt[n]{a_1 a_2 \ldots a_n}$

Explanation: The Cauchy-Schwarz inequality for two sequences of real numbers a_i and b_i is $(\sum a_i^2)(\sum b_i^2) \geq (\sum a_i b_i)^2$.

12. Using Chebyshev's inequality, for a random variable X with mean 10 and standard deviation 2, what is a lower bound for $P(|X - 10| < 4)$?

a) 0.75

b) 0.25

c) 0.5

d) 1

Explanation: We have $|X - 10| < 4$, which means $|X - \mu| < 2\sigma$, so $k = 2$. The inequality gives $P(|X - 10| \geq 4) \leq \frac{1}{2^2} = 0.25$. Therefore, the probability of being within the range is $P(|X - 10| < 4) = 1 - P(|X - 10| \geq 4) \geq 1 - 0.25 = 0.75$.

13. If $x_1, x_2, \ldots, x_n$ are positive numbers and $y_1, y_2, \ldots, y_n$ are positive numbers, which of the following is an application of Chebyshev's inequality for sums?

a) If both sequences are monotonic in the same direction, then $\frac{1}{n}\sum x_i y_i \geq \left(\frac{1}{n}\sum x_i\right)\left(\frac{1}{n}\sum y_i\right)$.

b) If both sequences are monotonic in opposite directions, then $\frac{1}{n}\sum x_i y_i \geq \left(\frac{1}{n}\sum x_i\right)\left(\frac{1}{n}\sum y_i\right)$.

c) $(\sum x_i^2)(\sum y_i^2) \geq (\sum x_i y_i)^2$

d) $\frac{\sum x_i}{n} \geq \sqrt[n]{\prod x_i}$

Explanation: This is the correct statement of the sum form of Chebyshev's inequality for sequences with the same monotonicity.

14. Which of the following is a direct consequence of the Cauchy-Schwarz inequality?

a) Finding the roots of a polynomial.

b) Proving that the arithmetic mean is greater than or equal to the geometric mean.

c) Establishing a lower bound for the sum of products of two sequences of real numbers.

d) Solving systems of linear equations.

Explanation: The Cauchy-Schwarz inequality provides an upper bound for the square of the sum of products of two sequences. This allows us to establish a lower bound for certain expressions.

15. Which of the following complex numbers is equal to $(\sqrt{3}+i)^6$?

a) $64i$

b) -64

c) 64

d) $-64i$

Explanation: First, convert to polar form: $z = 2(\cos(\pi/6) + i\sin(\pi/6))$. By DeMoivre's theorem, $z^6 = 2^6(\cos(6\pi/6) + i\sin(6\pi/6)) = 64(\cos(\pi) + i\sin(\pi)) = 64(-1+0i) = -64$.

16. What is the modulus of the complex number $z = \frac{1}{2} - i\frac{\sqrt{3}}{2}$?

a) 1

b) 2

c) $\sqrt{3}$

d) $\frac{1}{2}$

Explanation: The modulus is $|z| = \sqrt{(\text{Re}(z))^2 + (\text{Im}(z))^2} = \sqrt{(1/2)^2 + (-\sqrt{3}/2)^2} = \sqrt{1/4 + 3/4} = \sqrt{1} = 1$.

17. If $z = r(\cos\theta + i\sin\theta)$, what is the result of z^n using DeMoivre's theorem?

a) $nr(\cos(n\theta) + i\sin(n\theta))$

b) $r^n(\cos(n\theta) + i\sin(n\theta))$

c) $r^n(\cos\theta + i\sin\theta)$

d) $r(\cos(n\theta) + i\sin(n\theta))$

Explanation: DeMoivre's theorem states that for a complex number in polar form, the modulus is raised to the power, and the argument is multiplied by the power.

18. What is the argument of the complex number $z = -1 + i$?

a) $\pi/4$

b) $-\pi/4$

c) $3\pi/4$

d) $5\pi/4$

Explanation: The complex number $-1+i$ is in the second quadrant. The reference angle is $\arctan(|1/-1|) = \pi/4$. The argument in the second quadrant is $\pi - \pi/4 = 3\pi/4$.

19. Find the roots of the equation $z^3 = 8$ in the complex plane.

a) $z = 2, 2\omega, 2\omega^2$, where $\omega = e^{i2\pi/3}$ is a complex cube root of unity.

b) $z = 2$

c) $z = 8i, -8i, 8$

d) $z = 8, 8i, -8i$

Explanation: Using DeMoivre's theorem for roots, the roots are given by $z = \sqrt[3]{8}e^{i(2\pi k)/3}$ for $k = 0,1,2$. This gives $2e^{i0}, 2e^{i2\pi/3}, 2e^{i4\pi/3}$, which correspond to the given options.

20. Which of the following is a direct consequence of DeMoivre's theorem?

a) The power rule for derivatives.

b) Formulas for $\cos(n\theta)$ and $\sin(n\theta)$ in terms of $\cos\theta$ and $\sin\theta$.

c) The fundamental theorem of algebra.

d) The properties of limits.

Explanation: Expanding $(\cos\theta + i\sin\theta)^n$ using the binomial theorem and equating the real and imaginary parts to $\cos(n\theta) + i\sin(n\theta)$ gives the formulas for multiple angles.

21. The sum of the first n natural numbers is:

a) $\frac{n(n+1)}{2}$ b) $\frac{n(n-1)}{2}$

c) n^2 d) $2n+1$

Answer: a

Explanation: By formula, $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$.

22. The sum of the squares of the first n natural numbers is:

a) $\frac{n(n+1)}{2}$ b) $\frac{n(n+1)(2n+1)}{6}$

c) $\frac{n^2(n+1)^2}{4}$ d) n^3

Answer: b

Explanation: Formula: $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.

23. If roots of $x^2 + px + q = 0$ are equal, then discriminant condition is:

a) $p^2 - 4q > 0$ b) $p^2 - 4q = 0$

c) $p^2 - 4q < 0$ d) None

Answer: b

Explanation: Equal roots occur when discriminant = 0.

24. The sum $\sum_{r=1}^{n} \cos r\theta$ equals:

a) $\frac{\sin\left(\frac{n\theta}{2}\right)\cos\left(\frac{(n+1)\theta}{2}\right)}{\sin(\theta/2)}$

b) $\frac{\sin\left(\frac{(n+1)\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)}{\sin(\theta/2)}$

c) $\frac{\cos\left(\frac{(n+1)\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)}{\cos(\theta/2)}$

d) None

Answer: a

Explanation: Derived from geometric progression of complex numbers: $\sum e^{ir\theta}$.

25. The sum of the first n odd natural numbers is:

a) n^2 b) $2n$

c) $n(n+1)$ d) n^3

Answer: a

Explanation: $1 + 3 + 5 + \ldots + (2n-1) = n^2$.

26. The sum $\sum_{r=1}^{n} \sin r\theta$ equals:

a) $\frac{\sin\left(\frac{n\theta}{2}\right)\sin\left(\frac{(n+1)\theta}{2}\right)}{\sin(\theta/2)}$ b) $\frac{\sin\left(\frac{n\theta}{2}\right)\cos\left(\frac{(n+1)\theta}{2}\right)}{\sin(\theta/2)}$

c) $\frac{\sin\left(\frac{(n+1)\theta}{2}\right)\sin\left(\frac{n\theta}{2}\right)}{\cos(\theta/2)}$ d) None

Answer: b

Explanation: Obtained using imaginary part of geometric series.

27. The polynomial $x^3 - 6x^2 + 11x - 6$ has roots:

a) 1, 2, 3 b) 2, 3, 6

c) 1, 3, 6 d) 2, 4, 6

Answer: a

Explanation: Factorization: $x^3 - 6x^2 + 11x - 6 = (x-1)(x-2)(x-3)$.

28. Descartes' rule of signs: the polynomial $x^4 - 3x^3 + 2x - 1$ has maximum possible positive roots:

a) 4 b) 3

c) 2 d) 1

Answer: d

Explanation: Signs: +, −, +, − → 3 variations. Possible positive roots = 3 or 1.

29. In synthetic division, dividing $x^3 + 2x^2 - 5x + 6$ by $x - 2$ leaves remainder:

a) 0 b) 2

c) 6 d) 10

Answer: c

Explanation: Using Horner's scheme, substitute root = 2 → $2^3 + 2(2^2) - 5(2) + 6 = 8 + 8 - 10 + 6 = 12$. Correction: remainder = 12.

30. If a polynomial has degree 5, then maximum number of real roots is:

a) 3 b) 4

c) 5 d) Infinite

Answer: c

Explanation: A polynomial of degree n has at most n real roots.

31. The polynomial $x^3 - x^2 - x + 1$ factors as:

a) $(x-1)(x^2+1)$

b) $(x-1)^2(x+1)$

c) $(x+1)(x^2-1)$

d) Irreducible

Answer: b

Explanation: Factorization gives $(x-1)^2(x+1)$.

32. If roots of $ax^2 + bx + c = 0$ are α, β, then $\alpha + \beta =$:

a) $\frac{b}{a}$ b) $-\frac{b}{a}$

c) $\frac{c}{a}$ d) $-\frac{c}{a}$

Answer: b

Explanation: By formula: $\alpha + \beta = -\frac{b}{a}$.

33. For $x^2 - 5x + 6 = 0$, product of roots is:

a) 6 b) -6

c) 5 d) -5

Answer: a

Explanation: By relation, product = $\frac{c}{a} = 6$.

34. If roots of $x^2 + px + q = 0$ are reciprocal, then:

a) $p = 0$

b) $q = 1$

c) $p^2 = 4q$

d) None

Answer: b

Explanation: Product of roots = q. Reciprocal roots ⇒ product = 1. So $q = 1$.

35. If roots of $x^2 + 2x + 3 = 0$ are α, β, then $\alpha^2 + \beta^2 =$:

a) 2 b) 4

c) -2 d) 10

Answer: c

Explanation: $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = (-2)^2 - 2(3) = 4 - 6 = -2$.

36. For cubic equation $x^3 + px^2 + qx + r = 0$, sum of products of roots two at a time =

a) $-\boldsymbol{p}$ b) $\boldsymbol{q}$

c) $-\boldsymbol{q}$ d) $\boldsymbol{r}$

Answer: b

Explanation: $\boldsymbol{\alpha\beta+\beta\gamma+\gamma\alpha=q}$.

37. Roots of $\boldsymbol{x^3-3x^2+3x-1}$ are:

a) 1, 1, 1 b) 0, 1, 2

c) -1, -1, -1 d) 1, 2, 3

Answer: a

Explanation: Factorization: $\boldsymbol{(x-1)^3}$.

38. If roots are $\boldsymbol{\alpha,\beta}$, then $\boldsymbol{\alpha^3+\beta^3}$ =:

a) $\boldsymbol{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}$

b) $\boldsymbol{\alpha\beta(\alpha+\beta)}$

c) $\boldsymbol{(\alpha+\beta)(\alpha\beta)}$

d) None

Answer: a

Explanation: Direct expansion identity.

39. For quadratic $\boldsymbol{x^2-7x+10=0}$, roots are:

a) 2, 5 b) -2, -5

c) 1, 10 d) 7, 10

Answer: a

Explanation: Factorization: $\boldsymbol{(x-2)(x-5)=0}$.

40. If polynomial has roots $\boldsymbol{\alpha,\beta,\gamma}$, then symmetric sum $\boldsymbol{\alpha^2\beta+\beta^2\gamma+\gamma^2\alpha}$ is expressed as:

a) in terms of $\boldsymbol{p,q,r}$ of cubic

b) $\boldsymbol{-pr-q^2}$

c) complicated but reducible

d) None

Answer: b

Explanation: Using relations, reduces to $\boldsymbol{-pr-q^2}$.

41. The equation $ax^2+2hxy+by^2=0$ represents:

(A) A pair of straight lines

(B) A parabola

(C) A circle

(D) An ellipse

Answer: (A)

Explanation: Homogeneous quadratic equation in two variables represents a pair of straight lines through the origin.

42. For the equation $x^2-5xy+6y^2=0$, the lines are:

(A) Parallel (B) Coincident

(C) Real and distinct (D) Imaginary

Answer: (C)

Explanation: Discriminant = $(h^2-ab)=\left(\frac{-5}{2}\right)^2-(1)(6)=25/4-6=1/4>0$. Hence real and distinct.

43. Angle between the lines $x^2-5xy+6y^2=0$ is:

(A) 90° (B) 45°

(C) 30° (D) 60°

Answer: (D)

Explanation: Angle θ satisfies $\tan\theta=\frac{2\sqrt{h^2-ab}}{a+b}$. Substitution gives $\tan\theta=\frac{1}{\sqrt{3}}\Longrightarrow\theta=30^\circ$. Angle between lines = $180^\circ-$

$30^\circ = 150^\circ$, acute angle = 30°. (Options may vary; usually 30° is taken.)

44. If the origin is shifted to (h, k), then $x = X + h$, $y = Y + k$. This is an example of:
(A) Rotation of axes
(B) Translation of axes
(C) Reflection
(D) Inversion
Answer: (B)
Explanation: Shifting origin is translation of coordinates.

45. The equation of pair of straight lines making angle 45° between them and bisectors coinciding with coordinate axes is:
(A) $x^2 - y^2 = 0$
(B) $x^2 + y^2 = 0$
(C) $x^2 - 2xy + y^2 = 0$
(D) $xy = 0$
Answer: (A)
Explanation: Lines are $y = x$ and $y = -x$.

46. The general equation of the second degree is:
(A) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
(B) $ax^2 + by^2 = 0$
(C) $x^2 + y^2 = r^2$
(D) $x^2 - y^2 = 0$
Answer: (A)
Explanation: General conic equation.

47. Condition for general second degree equation to represent a circle is:
(A) $a = b, h = 0$
(B) $a = b, h \neq 0$
(C) $a \neq b, h = 0$
(D) None
Answer: (A)
Explanation: Circle: $x^2 + y^2 + 2gx + 2fy + c = 0$. So coefficients of x^2, y^2 equal and no xy-term.

48. The eccentricity of a parabola is:
(A) 0 (B) 1
(C) Less than 1 (D) Greater than 1
Answer: (B)
Explanation: Parabola has eccentricity $e = 1$.

49. The standard form of ellipse equation is:
(A) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$
(B) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
(C) $y^2 = 4ax$
(D) $x^2 + y^2 = r^2$
Answer: (A)
Explanation: Standard ellipse form.

50. If the eccentricity $e > 1$, the conic is:
(A) Ellipse (B) Circle

(C) Parabola (D) Hyperbola

Answer: (D)

Explanation: Hyperbola has eccentricity greater than 1.

51. Equation of a plane is of the form:

(A) $ax + by + cz + d = 0$

(B) $x^2 + y^2 + z^2 = r^2$

(C) $ax^2 + by^2 = 0$

(D) None

Answer: (A)

Explanation: General equation of a plane.

52. A line parallel to vector $\vec{d} = (a, b, c)$ and passing through (x_1, y_1, z_1) is:

(A) $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$

(B) $ax + by + cz + d = 0$

(C) $x^2 + y^2 + z^2 = r^2$

(D) None

Answer: (A)

Explanation: Symmetric form of line.

53. Two skew lines are:

(A) Non-intersecting and non-parallel

(B) Parallel

(C) Intersecting

(D) Coincident

Answer: (A)

Explanation: Skew lines exist in 3D, not parallel, not intersecting.

54. Shortest distance between two skew lines can be found using:

(A) Dot product (B) Cross product

(C) Triple scalar product (D) None

Answer: (C)

Explanation: Shortest distance formula uses scalar triple product.

55. The distance of point (x_1, y_1, z_1) from plane $ax + by + cz + d = 0$ is:

(A) $\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$

(B) $\sqrt{(x_1^2 + y_1^2 + z_1^2)}$

(C) $\frac{x_1+y_1+z_1}{a+b+c}$

(D) None

Answer: (A)

Explanation: Standard distance formula.

56. If $\vec{a} \cdot \vec{b} = 0$, then vectors are:

(A) Parallel (B) Perpendicular

(C) Equal (D) Collinear

Answer: (B)

Explanation: Dot product zero ⇒ perpendicular vectors.

57. If $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|$, then angle between vectors is:

(A) 0° (B) 45°

(C) 60° (D) 90°

Answer: (D)

Explanation: Cross product magnitude is maximum when vectors are perpendicular.

58. The area of parallelogram formed by vectors $\vec{a}, \vec{b}$ is:

(A) $|\vec{a} \cdot \vec{b}|$

(B) $|\vec{a} \times \vec{b}|$

(C) $\frac{1}{2}|\vec{a} \times \vec{b}|$

(D) None

Answer: (B)

Explanation: Magnitude of cross product gives area of parallelogram.

59. The scalar triple product $\vec{a} \cdot (\vec{b} \times \vec{c})$ gives:

(A) Area of parallelogram

(B) Volume of parallelepiped

(C) Length of vector

(D) None

Answer: (B)

Explanation: Scalar triple product gives volume of parallelepiped.

60. If $\vec{a}, \vec{b}, \vec{c}$ are coplanar, then:

(A) $\vec{a} \cdot (\vec{b} \times \vec{c}) = 0$

(B) $\vec{a} + \vec{b} + \vec{c} = 0$

(C) $\vec{a} \times \vec{b} = \vec{c}$

(D) None

Answer: (A)

Explanation: Coplanarity condition is scalar triple product = 0.

61. If $\boldsymbol{A}$ and $\boldsymbol{B}$ are $\mathbf{2} \times \mathbf{2}$ matrices with $\boldsymbol{AB} = \boldsymbol{BA}$, then which of the following is always true?

a) $\boldsymbol{A} + \boldsymbol{B}$ is invertible

b) $\boldsymbol{AB}$ is diagonalizable

c) $\boldsymbol{AB} = \boldsymbol{BA}$ implies they can be simultaneously triangularized

d) Both are idempotent

Answer: c)

Explanation: Commuting matrices over ℂ can be simultaneously triangularized. Options (a), (b), and (d) are not guaranteed.

62. If $\boldsymbol{det}(\boldsymbol{A}) = \mathbf{5}$, then $\boldsymbol{det}(\mathrm{adj}(\boldsymbol{A}))$ for a $\mathbf{3} \times \mathbf{3}$ matrix $\boldsymbol{A}$ is:

a) $\mathbf{25}$ b) $\mathbf{125}$

c) $\mathbf{5}$ d) $\mathbf{0}$

Answer: a)

Explanation: For an $\boldsymbol{n} \times \boldsymbol{n}$ matrix, $\boldsymbol{det}(\mathrm{adj}(\boldsymbol{A})) = \boldsymbol{det}(\boldsymbol{A})^{\boldsymbol{n}-\mathbf{1}}$. Here $\boldsymbol{n} = \mathbf{3}$, so $\boldsymbol{det}(\mathrm{adj}(\boldsymbol{A})) = \mathbf{5}^{\mathbf{2}} = \mathbf{25}$.

63. The system of equations

$$\boldsymbol{x} + \boldsymbol{y} + \boldsymbol{z} = \mathbf{3}, \quad \mathbf{2}\boldsymbol{x} + \mathbf{2}\boldsymbol{y} + \mathbf{2}\boldsymbol{z} = \mathbf{6}, \quad \boldsymbol{x} - \boldsymbol{y} + \boldsymbol{z} = \mathbf{1}$$

has:

a) Unique solution b) Infinite solutions

c) No solution d) Exactly two solutions

Answer: b)

Explanation: The second equation is a multiple of the first. Rank of augmented = rank of coefficient < 3 ⇒ infinite solutions.

64. If $\boldsymbol{A}$ is a $\mathbf{2} \times \mathbf{2}$ real matrix with $\mathrm{tr}(\boldsymbol{A}) = \mathbf{0}$ and $\boldsymbol{det}(\boldsymbol{A}) = \mathbf{1}$, then:

a) Eigenvalues are real and equal

b) Eigenvalues are purely imaginary conjugates

c) Eigenvalues are real and distinct

d) Matrix is nilpotent

Answer: b)

Explanation: Characteristic polynomial: $\lambda^2 + 1 = 0$. Roots: $\pm i$.

65. The system $AX = 0$ has a non-trivial solution iff:

a) A is invertible

b) $det(A) \neq 0$

c) $det(A) = 0$

d) All diagonal entries are 0

Answer: c)

Explanation: A homogeneous system has a non-trivial solution if and only if the determinant is zero.

66. Dimension of the vector space of all 3×3 symmetric matrices over $\mathbb{R}$:

a) 6 b) 9

c) 3 d) 5

Answer: a)

Explanation: For $n \times n$ symmetric matrices, dimension = $\frac{n(n+1)}{2}$. Here: $\frac{3(4)}{2} = 6$.

67. The set $\{(1,0,0),(0,1,0),(1,1,0)\}$ in $\mathbb{R}^3$ is:

a) Linearly independent

b) Linearly dependent

c) Basis of $\mathbb{R}^3$

d) Orthogonal set

Answer: b)

Explanation: Since $(1,1,0) = (1,0,0) + (0,1,0)$, the set is dependent.

68. The dimension of the solution space of $x + y + z = 0$ in $\mathbb{R}^3$ is:

a) 1 b) 2

c) 3 d) 0

Answer: b)

Explanation: One linear condition reduces dimension by 1. So dimension = $3 - 1 = 2$.

69. If $\{u, v, w\}$ is linearly independent, then $\{u + v, v + w, w + u\}$ is:

a) Always dependent

b) Always independent

c) Independent only if $u + v + w \neq 0$

d) None

Answer: a)

Explanation: $(u + v) + (v + w) + (w + u) = 2(u + v + w)$. Non-trivial relation exists ⇒ dependent.

70. The maximum number of linearly independent vectors in $\mathbb{R}^n$ is:

a) n^2 b) $2n$

c) n d) Infinite

Answer: c)

Explanation: By definition, dimension of $\mathbb{R}^n$ = n. So max independent set size = n.

71. If $T: \mathbb{R}^3 \to \mathbb{R}^3$ has rank 2, then nullity is:

a) 0 b) 1

c) 2 d) 3

Answer: b)

Explanation: Rank–nullity theorem: $dimV = \text{rank}(T) + \text{nullity}(T)$. So $3 = 2 + 1$.

72. If $\boldsymbol{T(x,y) = (x+2y, 3x+6y)}$, then rank of $\boldsymbol{T}$ is:

a) 1 b) 2

c) 0 d) Infinite

Answer: a)

Explanation: Image vectors are multiples of $\boldsymbol{(1,3)}$. So rank = 1.

73. Linear transformation $\boldsymbol{T}: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $\boldsymbol{T(x,y) = (y,x)}$ has matrix:

a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Answer: a)

Explanation: $\boldsymbol{T(1,0) = (0,1)}$, $\boldsymbol{T(0,1) = (1,0)}$. Matrix = exchange matrix.

74. If rank of matrix $\boldsymbol{A}$ is 3, order of $\boldsymbol{A}$ is $\boldsymbol{3 \times 5}$, then nullity is:

a) 0 b) 2

c) 3 d) 5

Answer: b)

Explanation: Nullity = number of columns – rank = $\boldsymbol{5 - 3 = 2}$.

75. The transformation $\boldsymbol{T}: \mathbb{R}^2 \to \mathbb{R}^2$ given by $\boldsymbol{T(x,y) = (2x+3y, 4x+6y)}$ is:

a) One-to-one but not onto

b) Onto but not one-to-one

c) Both one-to-one and onto

d) Neither one-to-one nor onto

Answer: d)

Explanation: Image vectors are multiples of $\boldsymbol{(2,4)}$. Rank=1<2 ⇒ not onto, nullity=1 ⇒ not injective.

76. Eigenvalues of matrix $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$:

a) 2,3 b) 5,0

c) 1,2 d) None

Answer: a)

Explanation: Already diagonal matrix, eigenvalues = diagonal entries.

77. For matrix $\boldsymbol{A} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, eigenvalues are:

a) ± 1 b) $\pm i$

c) 0,1 d) 2,–2

Answer: b)

Explanation: Characteristic polynomial: $\boldsymbol{\lambda^2 + 1 = 0}$. Roots = $\pm i$.

78. If $\boldsymbol{A}$ is $\boldsymbol{n \times n}$ with all eigenvalues = 0, then:

a) $\boldsymbol{A}$ is invertible

b) $\boldsymbol{A}$ is diagonalizable

c) $\boldsymbol{A}$ is singular

d) None

Answer: c)

Explanation: Determinant = product of eigenvalues = 0 ⇒ singular.

79. Eigenvalues of triangular matrices are:

a) Always zero

b) Always 1

c) Equal to diagonal entries

d) Equal to sum of rows

Answer: c)

Explanation: For triangular matrices, eigenvalues are diagonal elements.

80. If eigenvalues of $\boldsymbol{A}$ are 2 and 3, then eigenvalues of $\boldsymbol{A^2}$ are:

a) 4,9 b) 5,6

c) 2,3 d) None

Answer: a)

Explanation: If λ is eigenvalue of A, then λ^2 is eigenvalue of A^2.

81. The set $A = \{\frac{1}{n} : n \in \mathbb{N}\}$ has

A) Supremum 1, Infimum 0

B) Supremum 0, Infimum 1

C) Supremum 1, Infimum 1

D) Supremum does not exist

Explanation: $\frac{1}{n}$ tends to 0 but never reaches 0. Supremum = 1 (largest element), Infimum = 0 (greatest lower bound).

82. If $B = \{x \in \mathbb{R} : x^2 < 2\}$, then $sup(B)$ is:

A) $\sqrt{2}$ B) 2

C) -$\sqrt{2}$ D) Does not exist

Explanation: The largest boundary point satisfying $x^2 < 2$ is $\sqrt{2}$.

83. Dedekind's theorem asserts:

A) Every bounded sequence converges

B) Every cut in $\mathbb{Q}$ corresponds to a real number

C) Every Cauchy sequence converges

D) Every bounded set has maximum

Explanation: Dedekind completeness says: every Dedekind cut defines a real number.

84. Which property shows that real numbers are unbounded above?

A) Completeness axiom

B) Archimedean property

C) Supremum principle

D) Bolzano–Weierstrass theorem

Explanation: Archimedean property: for every $x \in \mathbb{R}$, there exists $n \in \mathbb{N}$ with $n > x$.

85. The infimum of $\{n - \sqrt{2} : n \in \mathbb{N}\}$ is:

A) -$\sqrt{2}$

B) 0

C) 1

D) Does not exist

Explanation: The smallest distance occurs when $n = 2$, so min value is $2 - \sqrt{2} > 0$. Hence infimum = 0.

86. Which of the following sets has no maximum?

A) $[0, 1)$ B) $[0, 1]$

C) $(2, 3]$ D) $\{1\}$

Explanation: $[0, 1)$ has supremum 1 but no element equal to 1.

87. Supremum of $\{(-1)^n \left(1 - \frac{1}{n}\right)\}$ is:

A) 1 B) -1

C) 0 D) Does not exist

Explanation: Even terms approach 1, odd terms approach -1. Supremum = 1.

88. Which is true for real numbers?

A) Every bounded above set has a maximum

B) Every bounded above set has a least upper bound

C) Every bounded set is finite

D) Every unbounded set diverges

Explanation: Completeness axiom guarantees existence of supremum, not necessarily maximum.

89. The Archimedean property fails in:

A) $\mathbb{R}$ B) $\mathbb{Q}$

C) $\mathbb{N}$ D) Non-standard real field

Explanation: In hyperreal numbers,

infinitesimals violate Archimedean property. So ans is D

90. The set $\{1/n : n \in \mathbb{N}\}$ has:

A) Minimum but no maximum

B) Maximum but no minimum

C) Both min and max

D) Neither min nor max

Explanation: Largest = 1, but smallest value never reached (inf = 0, no min).

Ans is B

91. The sequence $a_n = \frac{n}{n+1}$ converges to:

A) 0 B) 1

C) ∞ D) -1

Explanation: Divide numerator/denominator by n: $a_n = \frac{1}{1+1/n} \to 1$.

92. The series $\sum \frac{1}{n^2}$ is:

A) Divergent

B) Convergent

C) Conditionally convergent

D) Oscillatory

Explanation: p-series with $p = 2 > 1$ converges.

93. The alternating harmonic series $\sum (-1)^{n+1} \frac{1}{n}$ is:

A) Absolutely convergent

B) Conditionally convergent

C) Divergent

D) Uniformly convergent

Explanation: Harmonic diverges, but alternating converges (Leibniz test). Not absolutely convergent.

94. Ratio test applied to $\sum \frac{n!}{n^n}$:

A) Diverges B) Converges

C) Oscillates D) Test fails

Explanation: Ratio $\to e^{-1} < 1$, so converges.

95. If $\sum a_n$ converges absolutely, then:

A) $\sum a_n$ diverges B) $\sum a_n$ converges

C) Test fails D) Series oscillates

Explanation: Absolute convergence ⇒ convergence.

96. A continuous function on a closed interval is:

A) Always bounded B) Always unbounded

C) Always constant D) Always periodic

Explanation: Weierstrass theorem: continuous functions on closed bounded sets are bounded.

97. Intermediate Value Theorem applies to:

A) Continuous functions on open intervals

B) Continuous functions on closed intervals

C) Differentiable functions only

D) Strictly increasing functions

Explanation: IVT requires continuity on $[a, b]$.

98. Uniform continuity implies:

A) Differentiability B) Continuity

C) Monotonicity D) Boundedness

Explanation: Uniform continuity ⇒ continuity, but not vice versa.

99. Example of continuous but not uniformly continuous function on $(0, 1)$:

A) $f(x) = \sqrt{x}$ B) $f(x) = \frac{1}{x}$

C) $f(x) = x^2$ D) $f(x) = sinx$

Explanation: $1/x$ is continuous but unbounded near 0.

100. If $f: [0,1] \to \mathbb{R}$ is continuous, then it is:

A) Differentiable

B) Uniformly continuous

C) Strictly increasing

D) Polynomial

Explanation: Heine–Cantor theorem.

101. Rolle's theorem requires:

A) Differentiability on $[a,b]$

B) Continuity on $[a,b]$ and differentiability on (a,b)

C) Function positive on $[a,b]$

D) Function bounded

Explanation: Plus $f(a) = f(b)$. Ans is B

102. If $f(x) = x^3$, Rolle's theorem on $[-1,1]$ gives:

A) $f'(c) = 0$ at $c = 0$

B) No solution

C) At $c = \pm 1$

D) Diverges

Explanation: $f(-1) = -1, f(1) = 1$ not equal $\Rightarrow$ Rolle's theorem not applicable. Correction: no solution.

103. Mean Value Theorem ensures:

A) $f'(c) = \frac{f(b)-f(a)}{b-a}$

B) $f'(c) = 0$

C) $f(a) = f(b)$

D) Function constant

Explanation: Lagrange's MVT. Ans is A

104. Taylor's expansion of e^x at 0 is:

A) $1 + x + \frac{x^2}{2!} + \cdots$ B) $sinx$ series

C) Divergent D) Undefined

Explanation: Standard Maclaurin series. Ans is A

105. Using L'Hôpital's rule, $lim_{x\to 0} \frac{sinx}{x} =$:

A) 0 B) 1

C) ∞ D) Does not exist

Explanation: Differentiate top/bottom: $cos0/1 = 1$.

106. The remainder in Taylor's theorem is given by:

A) Lagrange form B) Newton's form

C) Divergence form D) Euler's form

Explanation: $R_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-a)^{n+1}$. Ans A

107. A critical point occurs where:

A) $f'(x) = 0$ or undefined B) $f''(x) = 0$

C) $f(x) = 0$ D) $f'(x) = 1$

Explanation: Stationary or non-differentiable points. Ans A

108. Second derivative test: If $f'(c) = 0, f''(c) > 0$, then c is:

A) Local max B) Local min

C) Inflection D) Saddle point

Explanation: Positive second derivative $\Rightarrow$ concave up $\Rightarrow$ minimum. Ans B

109. Equation of tangent to $y = x^2$ at $(1,1)$:

A) $y = 2x - 1$ B) $y = x^2$

C) $y = x + 1$ D) $y = 3x - 2$

Explanation: Slope = $2x$=2, tangent: $y - 1 = 2(x-1)$.

110. Normal to $y = x^2$ at $(1, 1)$:

A) $y = -\frac{1}{2}x + \frac{3}{2}$ B) $y = 2x - 1$

C) $y = x + 1$ D) $y = 3x - 2$

Explanation: Normal slope = negative reciprocal = $-1/2$. Equation: $y - 1 = -\frac{1}{2}(x - 1)$.

111. $\int e^{2x} cos(3x)\, dx$ =?

A) $\frac{e^{2x}}{13}(2cos(3x) + 3sin(3x)) + C$

B) $\frac{e^{2x}}{13}(2cos(3x) - 3sin(3x)) + C$

C) $\frac{e^{2x}}{5}(2cos(3x) + 3sin(3x)) + C$

D) None

Answer: A

Explanation: Apply integration by parts or use the standard formula $\int e^{ax} cos(bx)dx = \frac{e^{ax}(acosbx+bsinbx)}{a^2+b^2}$.

112. Recurrence relation: $I_n = \int (lnx)^n dx$. Then

A) $I_n = x(lnx)^n - nI_{n-1}$

B) $I_n = x(lnx)^n + nI_{n-1}$

C) $I_n = \frac{(lnx)^{n+1}}{n+1}$

D) None

Answer: A

Explanation: Integration by parts with $u = (lnx)^n$, $dv = dx$ gives the recurrence.

113. $\int \frac{x^3}{(1+x^2)^2} dx$ equals

A) $\frac{1}{2}ln(1 + x^2) - \frac{x^2}{2(1+x^2)} + C$

B) $\frac{1}{2}ln(1 + x^2) + \frac{x^2}{2(1+x^2)} + C$

C) $ln(1 + x^2) - \frac{x^2}{1+x^2} + C$

D) None

Answer: A

Explanation: Split numerator: $x^3 = x(x^2 + 1 - 1)$. Simplify and integrate.

114. $\int tan^n x dx$ recurrence relation:

A) $I_n = \frac{1}{n-1}tan^{n-1}x - I_{n-2}$

B) $I_n = \frac{1}{n-1}tan^{n-1}x + I_{n-2}$

C) $I_n = tan^{n-2}x - I_{n-1}$

D) None

Answer: A

Explanation: Standard reduction formula using $tan^n x = tan^{n-2}x(sec^2 x - 1)$.

115. $\int \frac{dx}{x^2+a^2}$ equals

A) $tan^{-1}\frac{x}{a} + C$ B) $\frac{1}{a}tan^{-1}\frac{x}{a} + C$

C) $ln(x^2 + a^2) + C$ D) None

Answer: B

Explanation: Standard form: $\int \frac{dx}{x^2+a^2} = \frac{1}{a}arctan\left(\frac{x}{a}\right) + C$.

116. If f is continuous on $[a, b]$, then $\int_a^b f'(x)dx$ =?

A) $f(b) - f(a)$ B) $f(b) + f(a)$

C) 0 D) Undefined

Answer: A

Explanation: By Fundamental Theorem of Calculus (FTC).

117. If $F(x) = \int_0^x sin(t^2)dt$, then $F'(x)$ =?

A) $cos(x^2)$ B) $sin(x^2)$

C) $2xsin(x^2)$ D) None

Answer: B

Explanation: FTC: derivative of integral is the integrand evaluated at upper limit.

118. Riemann integrable functions must be

A) Continuous everywhere

B) Bounded with finite set of discontinuities

C) Differentiable

D) Monotone only

Answer: B

Explanation: Bounded + discontinuities of measure zero ensures Riemann integrability.

119. For $f(x) = x^2$ on [0,1], the Riemann sum with n subintervals, right endpoints is

A) $\frac{n(2n+1)(n+1)}{6n^3}$ B) $\frac{(n+1)(2n+1)}{6n^2}$

C) $\frac{n^2}{3}$ D) None

Answer: B

Explanation: Compute sum $\frac{1}{n}\sum_{k=1}^{n}\left(\frac{k}{n}\right)^2$.

120. $\frac{d}{dx}\int_1^{x^2} \ln t\, dt =?$

A) $\ln(x^2)$ B) $2x\ln(x^2)$

C) $2x\ln(x)$ D) None

Answer: B

Explanation: Chain rule + FTC: derivative = $f(x^2) \cdot 2x$.

121. $\int_1^{\infty} \frac{1}{x^p} dx$ converges iff

A) $p < 1$ B) $p > 1$

C) $p = 1$ D) Always diverges

Answer: B

Explanation: p-test for improper integrals.

122. $\int_0^1 \frac{1}{\sqrt{x}} dx$ equals

A) Divergent B) 1

C) 2 D) 3

Answer: C

Explanation: Evaluate: $\int_0^1 x^{-1/2}\, dx = \left[2\sqrt{x}\right]_0^1 = 2$.

123. $\int_0^{\infty} e^{-x^2}\, dx$ converges to

A) $\sqrt{\pi}$

B) $\frac{\sqrt{\pi}}{2}$

C) 1

D) Diverges

Answer: B

Explanation: Known Gaussian integral: $\int_{-\infty}^{\infty} e^{-x^2}\, dx = \sqrt{\pi}$. Half is $\frac{\sqrt{\pi}}{2}$.

124.
$\int_1^{\infty} \frac{\sin x}{x} dx$ is

A) Divergent

B) Conditionally convergent

C) Absolutely convergent

D) None

Answer: B

Explanation: Dirichlet's test → conditionally convergent, not absolutely.

125. $\int_0^1 \ln x dx$ is

A) Divergent B) -1

C) 0 D) Undefined

Answer: B

Explanation: Integration by parts or expansion gives $[x\ln x - x]_0^1 = -1$.

126. $\int_0^\infty \frac{dx}{1+x^2}$ equals

A) π

B) $\pi/2$

C) 1

D) Divergent

Answer: B

Explanation: Standard arctan form:

$arctanx|_0^\infty = \pi/2$.

127. The integral test is applied to

A) Continuous, positive, decreasing functions

B) Any bounded function

C) Oscillatory functions

D) None

Answer: A

Explanation: Integral test requires positive decreasing continuous function.

128. $\int_0^1 \frac{dx}{x^p}$ converges iff

A) $p < 1$

B) $p > 1$

C) $p = 1$

D) Diverges for all p

Answer: A

Explanation: Near 0, convergence if exponent < 1.

129. $\int_0^\infty \frac{sin^2 x}{x^2} dx$ equals

A) $\pi/2$

B) $\pi/4$

C) 1

D) Divergent

Answer: B

Explanation: Known result using Fourier analysis or Parseval.

130. $\int_0^\infty \frac{e^{-ax}-e^{-bx}}{x} dx$ =? where $a, b > 0$.

A) $ln\frac{b}{a}$

B) $\frac{1}{a} - \frac{1}{b}$

C) Diverges

D) None

Answer: A

Explanation: Standard Frullani's integral formula.

131. Area under $y = x^2$ from 0 to 1:

A) 1/3

B) 1/2

C) 2/3

D) 1

Answer: A

Explanation: $\int_0^1 x^2\, dx = 1/3$.

132. Area bounded by $y = sinx$, x-axis between 0 and π:

A) 2

B) 1

C) π

D) 0

Answer: A

Explanation: $\int_0^\pi sin\, x dx = 2$.

133. Volume of solid obtained by rotating $y = \sqrt{x}$, $0 \le x \le 1$, about x-axis:

A) $\pi/2$

B) $\pi/3$

C) $\pi/5$

D) None

Answer: A

Explanation: $V = \pi \int_0^1 (\sqrt{x})^2\, dx = \pi \int_0^1 x\, dx = \pi/2$.

134. Area enclosed by $y = x^2$ and $y = \sqrt{x}$:

A) 1/6

B) 1/3

C) 1/2

D) None

Answer: B

Explanation: Solve intersection $x^2 = \sqrt{x} \rightarrow x = 0, 1$. Area = $\int_0^1 (\sqrt{x} - x^2)\, dx = 1/3$.

135. Volume of sphere radius r:

A) $\frac{4}{3}\pi r^3$

B) $2\pi r^3$

C) πr^2

D) None

Answer: A

Explanation: Standard result by disk method.

136. Surface area of sphere radius r:

A) $2\pi r^2$ B) $4\pi r^2$

C) πr^3 D) None

Answer: B

137. Area under one arch of $y = cosx$, 0 to π:

A) 0 B) 2

C) π D) 1

Answer: 0

Explanation: $\int_0^{\pi} cos\,xdx = sinx|_0^{\pi} = 0$.

138. Centroid of semicircle of radius r above x-axis is at height

A) $\frac{4r}{3\pi}$ B) $\frac{2r}{3}$

C) $\frac{r}{2}$ D) None

Answer: A

Explanation: Known centroid formula for semicircle.

139. Volume of solid obtained by rotating region under $y = x$, 0≤x≤1, about x-axis:

A) $\pi/2$ B) $\pi/3$

C) $\pi/4$ D) None

Answer: B

Explanation: $V = \pi \int_0^1 x^2\,dx = \pi/3$.

140. If curve $y = f(x)$ encloses area A between a and b, then rotated about x-axis volume is

A) $\pi \int_a^b f(x)dx$

B) $2\pi \int_a^b f(x)dx$

C) $\pi \int_a^b (f(x))^2\,dx$

D) None

Answer: C

Explanation: Disk method formula.

141. For a system of coplanar concurrent forces to be in equilibrium, which condition must hold?

A) ΣFx = 0 and ΣFy = 0 B) ΣFx = 0 only

C) ΣFy = 0 only D) ΣM = 0

Answer: A

Explanation: For concurrent forces in a plane, only two independent equations are required: the algebraic sum of horizontal and vertical components must vanish.

142. For a rigid body under coplanar forces to be in equilibrium:

A) ΣFx = 0, ΣFy = 0, ΣM = 0

B) ΣFx = 0, ΣFy = 0

C) ΣM = 0 only

D) None

Answer: A

Explanation: All three equilibrium conditions are necessary in the plane: no resultant force and no resultant moment.

143. Which of the following cannot be a condition of equilibrium?

A) ΣFx = 0 B) ΣFy = 0

C) ΣFz = 0 D) ΣE = 0

Answer: D

Explanation: Equilibrium requires net force and moment balance, not energy balance.

144. If a ladder rests against a smooth wall and rough ground, equilibrium requires:

A) Vertical reaction at wall

B) Friction at wall

C) Friction at ground

D) None

Answer: C

Explanation: Wall is smooth → no friction at wall. Ground provides both vertical and horizontal support via normal + friction.

145. A beam is in equilibrium under three forces. Then:

A) They are parallel

B) They are concurrent or parallel

C) They must form a triangle

D) They must all be horizontal

Answer: B

Explanation: Three coplanar forces in equilibrium must be concurrent or parallel (Lami's theorem).

146. Principle of virtual work states:

A) Net work = 0

B) Net virtual work of forces for permissible displacement = 0

C) Real displacement work = 0

D) Energy = 0

Answer: B

Explanation: For equilibrium, total virtual work done by forces in any virtual displacement is zero.

147. Stable equilibrium means:

A) Small displacement increases potential energy

B) Small displacement decreases potential energy

C) Potential energy unchanged

D) Kinetic energy increases

Answer: A

Explanation: Stable equilibrium corresponds to minimum potential energy.

148. A particle is in unstable equilibrium if:

A) Potential energy is maximum

B) Potential energy is minimum

C) Kinetic energy is zero

D) Net force is zero

Answer: A

Explanation: At unstable equilibrium, PE is maximum; small displacement reduces PE and drives the particle away.

149. Neutral equilibrium occurs when:

A) $dU/dx = 0$ and $d^2U/dx^2 > 0$

B) $dU/dx = 0$ and $d^2U/dx^2 < 0$

C) $dU/dx = 0$ and $d^2U/dx^2 = 0$

D) None

Answer: C

Explanation: Neutral equilibrium corresponds to constant potential energy.

150. The principle of virtual work can be applied to:

A) Only particles

B) Only rigid bodies

C) Both particles and rigid bodies

D) Fluids only

Answer: C

Explanation: It is a general equilibrium principle valid for any mechanical system.

151. The centroid of a uniform semicircular lamina lies at:

A) 2R/π from the center along axis

B) 4R/(3π) from base

C) R/2 from base

D) None

Answer: B

Explanation: For semicircle, centroid is at distance 4R/(3π) from the base along symmetry axis.

152. The center of gravity of a homogeneous solid hemisphere lies:

A) At 3R/8 from base along axis

B) At R/2 from base

C) At R/4 from base

D) At 2R/3 from base

Answer: A

Explanation: For a solid hemisphere, C.G. lies at 3R/8 above the flat base.

153. The center of gravity of a thin rod of length L lies:

A) At L/3 from one end

B) At L/2 from either end

C) At 2L/3 from one end

D) None

Answer: B

Explanation: Uniform rod → CG at its midpoint.

154. The centroid of a triangle lies at:

A) Midpoint of base

B) Point of concurrency of medians

C) Orthocenter

D) Incenter

Answer: B

Explanation: Centroid is intersection of medians, divides them 2:1.

155. The center of gravity of a thin circular arc subtending angle 2θ at center lies at:

A) Rsinθ/θ　　B) Rcosθ/θ

C) Rsinθ/(2θ)　　D) None

Answer: A

Explanation: Distance from center = (Rsinθ)/θ.

156. A particle in SHM has displacement x = A cos(ωt). Its velocity is:

A) -Aωsin(ωt)　　B) Aωcos(ωt)

C) -Acos(ωt)　　D) $A\omega^2\cos(\omega t)$

Answer: A

Explanation: Differentiate displacement: v = dx/dt = -Aωsin(ωt).

157. Time period of SHM depends on:

A) Mass and amplitude

B) Mass and force constant

C) Amplitude only

D) None

Answer: B

Explanation: $T = 2\pi\sqrt{\frac{m}{k}}$. Amplitude has no effect.

158. For SHM, acceleration is proportional to:

A) Displacement

B) Velocity

C) Square of displacement

D) Displacement with negative sign

Answer: D

Explanation: $a = -\omega^2 x$.

159. If amplitude is doubled, maximum kinetic energy becomes:

A) Same B) Twice

C) Four times D) Half

Answer: C

Explanation: Max KE = ½kA² ∝ A² → doubling A makes KE 4 times.

160. Frequency of a simple pendulum depends on:

A) Mass of bob

B) Length of string

C) Amplitude

D) Both A and B

Answer: B

Explanation: $T = 2\pi\sqrt{\frac{L}{g}}$; independent of mass and small amplitude.

161. A projectile at 45° has maximum range R. If angle = 30°, range is:

A) R/2 B) $\sqrt{3}R/2$

C) 3R/4 D) None

Answer: C

Explanation: Range $R = (u^2 sin2\theta)/g. Ratio: sin60/sin90 = \sqrt{3}/2 \approx 0.866 = 3/4.$

162. For uniform circular motion, acceleration is:

A) Zero

B) Along tangent

C) Towards center

D) Away from center

Answer: C

Explanation: Centripetal acceleration is directed to center: $a = v^2/r$.

163. In two-dimensional motion, independence of motion principle states:

A) x and y motions are coupled

B) x and y motions independent

C) y depends on x

D) None

Answer: B

Explanation: Horizontal and vertical components are independent.

164. The velocity vector of projectile at top is:

A) Vertical B) Horizontal

C) Zero D) Oblique

Answer: B

Explanation: At highest point, vertical velocity = 0, horizontal remains.

165. Maximum height H and range R of projectile are related by:

A) $H = R/4\, tan\theta$ B) $H = R/2\, tan\theta$

C) $H = Rtan\theta/4$ D) $H = R/8\, tan\theta$

Answer: C

Explanation: $H = u^2 sin^2\theta/2g, R = u^2 sin2\theta/g \rightarrow relation\ H = (Rtan\theta)/4.$

166. Motion under central force implies:

A) Angular momentum conserved

B) Linear momentum conserved

C) Energy not conserved

D) Force always tangential

Answer: A

Explanation: Central force is always radial → torque = 0 → angular momentum conserved.

167. A planet moves faster when:

A) Near aphelion

B) Near perihelion

C) Everywhere same

D) At half orbit

Answer: B

Explanation: By Kepler's 2nd law, planet sweeps equal areas in equal times; speed max at perihelion.

168. Path of particle under inverse square central force is:

A) Circle/ellipse/parabola/hyperbola

B) Always circle

C) Always parabola

D) Straight line

Answer: A

Explanation: General conic sections depending on energy.

169. Effective potential for central force includes:

A) Only potential energy

B) Potential + centrifugal term

C) Only kinetic energy

D) None

Answer: B

Explanation: $\boldsymbol{Veff(r) = U(r) + \frac{L^2}{2mr^2}}$.

170. In central force motion, areal velocity is:

A) Constant B) Zero

C) Increasing D) Decreasing

Answer: A

Explanation: Conservation of angular momentum → constant areal velocity.

171. The moment of inertia of a uniform solid sphere of mass $\boldsymbol{M}$ and radius $\boldsymbol{R}$ about a diameter is:

A) $\frac{2}{5}\boldsymbol{MR^2}$ B) $\frac{3}{5}\boldsymbol{MR^2}$

C) $\frac{1}{2}\boldsymbol{MR^2}$ D) $\frac{5}{7}\boldsymbol{MR^2}$

Answer: A

Explanation: For a solid sphere, $\boldsymbol{I = \frac{2}{5}MR^2}$. Derived by integration in spherical coordinates.

172. Parallel axis theorem states:

A) $\boldsymbol{I = I_{CM} + Md^2}$ B) $\boldsymbol{I = I_{CM} - Md^2}$

C) $\boldsymbol{I = I_{CM}/Md^2}$ D) None

Answer: A

Explanation: The theorem shifts the axis parallel to CM by distance $\boldsymbol{d}$.

173. A disc rolls without slipping on a horizontal surface. Its kinetic energy is:

A) Purely translational

B) Purely rotational

C) Translational + Rotational

D) Zero

Answer: C

Explanation: $\boldsymbol{K.E. = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2}$, with $\boldsymbol{\omega = v/R}$.

174. D'Alembert's principle converts a dynamic problem into:

A) A static problem with inertia force

B) A kinematic problem

C) A gravitational problem

D) None

Answer: A

Explanation: Introduces inertial force $-ma$, reducing dynamics to equilibrium.

175. A uniform rod of length L rotates about one end perpendicular to its length. Its moment of inertia is:

A) $\frac{1}{12}ML^2$ B) $\frac{1}{3}ML^2$

C) $\frac{1}{2}ML^2$ D) ML^2

Answer: B

Explanation: From integration: $I = \frac{1}{3}ML^2$.

176. A flywheel of moment of inertia I rotates with angular velocity ω. Its kinetic energy is:

A) $\frac{1}{2}I\omega$ B) $\frac{1}{2}I\omega^2$

C) $I\omega^2$ D) $I\omega$

Answer: B

Explanation: Rotational KE = $\frac{1}{2}I\omega^2$.

177. Angular acceleration α is related to torque τ by:

A) $\tau = I\alpha$ B) $\tau = m\alpha$

C) $\tau = \alpha/I$ D) None

Answer: A

Explanation: Newton's second law for rotation.

178. A wheel rotating at 30 rad/s is brought to rest in 10 s by uniform angular deceleration. α =?

A) 3 rad/s^2 B) -3 rad/s^2

C) 30 rad/s^2 D) -30 rad/s^2

Answer: B

Explanation: $\alpha = \Delta\omega/\Delta t = (0-30)/10 = -3$.

179. Work done by a torque over angle θ:

A) $W = \tau\theta$ B) $W = I\theta$

C) $W = \frac{1}{2}\tau\theta$ D) $W = \omega\tau$

Answer: A

Explanation: Work = torque × angular displacement.

180. A body rotating about fixed axis has angular velocity ω proportional to time. Then angular acceleration is:

A) Constant B) Zero

C) Increasing linearly D) Decreasing

Answer: A

Explanation: If $\omega = kt$, then $\alpha = d\omega/dt = k$, constant.

181. Lagrangian is defined as:

A) $L = T + V$ B) $L = T - V$

C) $L = V - T$ D) None

Answer: B

Explanation: L = kinetic energy – potential energy.

182. For a particle in conservative field, Euler–Lagrange equation gives:

A) Newton's second law

B) Newton's third law

C) Newton's first law

D) None

Answer: A

Explanation: Reduces to $F = ma$.

183. A pendulum of length l, angle θ. The generalized coordinate is:

A) Cartesian (x, y) B) Polar r

C) Angle θ D) Momentum

Answer: C

Explanation: Only one independent coordinate describes system.

184. Lagrange's equation for holonomic system:

A) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$ B) $\frac{\partial L}{\partial t} = 0$

C) $dL/dt = 0$ D) None

Answer: A

Explanation: Fundamental form of Lagrangian mechanics.

185. Advantage of Lagrangian formulation is:

A) No need to consider constraint forces

B) More complex than Newtonian

C) Limited to 1-D motion

D) None

Answer: A

Explanation: Generalized coordinates automatically incorporate holonomic constraints.

186. Solution of $\frac{dy}{dx} = ky$:

A) $y = kx$ B) $y = e^{kx}$

C) $y = Ce^{kx}$ D) $y = Cx^k$

Answer: C

Explanation: First-order separable equation.

187. The order of ODE: $\frac{d^3y}{dx^3} + y = 0$ is:

A) 1 B) 2

C) 3 D) 0

Answer: C

Explanation: Highest derivative = 3.

188. The ODE $\frac{dy}{dx} + P(x)y = Q(x)$ is solved by:

A) Separation of variables

B) Integrating factor

C) Series solution

D) Laplace transform

Answer: B

Explanation: Standard linear first-order form.

189. The differential equation of SHM:

A) $\ddot{x} + \omega^2 x = 0$ B) $\dot{x} + \omega x = 0$

C) $\ddot{x} - \omega^2 x = 0$ D) None

Answer: A

Explanation: Governs oscillatory motion.

190. Solution of $\ddot{x} + 9x = 0$:

A) $x = Ae^{3t} + Be^{-3t}$

B) $x = Acos3t + Bsin3t$

C) $x = At + B$

D) None

Answer: B

Explanation: Roots imaginary ⇒ harmonic solution.

191. The auxiliary equation of $y'' + y = 0$ is:

A) $m^2 + 1 = 0$ B) $m^2 - 1 = 0$

C) $m + 1 = 0$ D) None

Answer: A

Explanation: Substitution $y = e^{mx}$.

192. General solution of $y'' - 4y = 0$:

A) $Ae^{2x} + Be^{-2x}$ B) $Ae^{4x} + Be^{-4x}$

C) $Acos2x + Bsin2x$ D) None

Answer: A

Explanation: Auxiliary eqn: $m^2 - 4 = 0 \Rightarrow m = \pm 2$.

193. For repeated roots in auxiliary equation, solution contains:

A) $Ae^{mx} + Bxe^{mx}$ B) $Ae^{mx} + Be^{-mx}$

C) $Acos(mx)$ D) None

Answer: A

Explanation: Multiplicity ⇒ polynomial factor.

194. The particular integral of $(D^2+1)y = \sin x$ is:

A) $\frac{1}{2}x\cos x$ B) $\frac{1}{2}x\sin x$

C) $-\frac{1}{2}x\cos x$ D) None

Answer: C

Explanation: Resonance case handled by multiplying trial solution by x.

195. If auxiliary eqn roots are complex conjugates $a \pm ib$, solution is:

A) $e^{ax}(A\cos bx + B\sin bx)$

B) $Ae^{ax} + Be^{-ax}$

C) $Ae^{bx} + Be^{-bx}$

D) None

Answer: A

Explanation: Euler's formula.

196. Power series solution is valid near:

A) A regular point

B) A singular point

C) Only infinity

D) None

Answer: A

Explanation: Taylor expansion works near ordinary (regular) points.

197. The method of Frobenius is used for:

A) Regular points

B) Regular singular points

C) Irregular singular points

D) None

Answer: B

Explanation: Introduces solution of form $\sum a_n x^{n+r}$.

198. The indicial equation arises in:

A) Series expansion at singular point

B) Auxiliary equation

C) Separation of variables

D) None

Answer: A

Explanation: From lowest power in Frobenius substitution.

199. The Bessel equation is solved by:

A) Fourier series

B) Frobenius method

C) Integrating factor

D) None

Answer: B

Explanation: Bessel equation has a regular singularity at $x = 0$.

200. Legendre's differential equation appears in:

A) Heat conduction

B) Laplace's equation in spherical coordinates

C) Elasticity

D) None

Answer: B

Explanation: Separation of variables in spherical coordinates yields Legendre's polynomials.

201. The relation between Beta and Gamma functions is:

A) $B(m,n) = \frac{\Gamma(m)+\Gamma(n)}{\Gamma(m+n)}$

B) $B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

C) $B(m,n) = \frac{\Gamma(m+n)}{\Gamma(m)\Gamma(n)}$

D) $B(m,n) = \Gamma(mn)$

Answer: B

Explanation: By definition, $B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}dx = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

202. The value of $\Gamma(1/2)$ is:

A) 1 B) $\sqrt{\pi}$

C) π D) 2

Answer: B

Explanation: From Gaussian integral, $\Gamma(1/2) = \int_0^\infty e^{-x} x^{-1/2} dx = \sqrt{\pi}$.

203. If $\Gamma(n+1) = n!$, then $\Gamma(6) =$?

A) 5! B) 6!

C) 120 D) Both A and C

Answer: D

Explanation: $\Gamma(n+1) = n!$. For $n = 5$, $\Gamma(6) = 5! = 120$.

204. Evaluate $B(2,3)$.

A) $\frac{1}{6}$ B) $\frac{1}{12}$

C) $\frac{1}{3}$ D) $\frac{1}{2}$

Answer: A

Explanation: $B(2,3) = \frac{\Gamma(2)\Gamma(3)}{\Gamma(5)} = \frac{1\cdot 2}{24} = \frac{1}{12}$. Oops correction: $\Gamma(2) = 1, \Gamma(3) = 2, \Gamma(5) = 24$. So result = 2/24 = 1/12. Correct answer = B.

205. Which identity is correct?

A) $\Gamma(z+1) = z\Gamma(z)$

B) $\Gamma(z+1) = \Gamma(z)/z$

C) $\Gamma(z+1) = z^2\Gamma(z)$

D) None

Answer: A

Explanation: This is the functional relation of Gamma function.

206. The duplication formula is:

A) $\Gamma(z)\Gamma(z+1/2) = 2^{1-2z}\sqrt{\pi}\Gamma(2z)$

B) $\Gamma(2z) = 2^{2z}\Gamma(z)\Gamma(z+1/2)/\sqrt{\pi}$

C) Both A and B

D) None

Answer: C

Explanation: Both forms are equivalent rearrangements.

207. The integral $\int_0^\infty e^{-x^2} dx$ evaluates to:

A) 1

B) $\sqrt{\pi}/2$

C) π

D) ∞

Answer: B

Explanation: From Gaussian integral: $\int_0^\infty e^{-x^2} dx = \sqrt{\pi}/2$.

208. The value of $\Gamma(3/2)$ is:

A) $\frac{\sqrt{\pi}}{2}$ B) $\frac{3\sqrt{\pi}}{2}$

C) $\frac{\pi}{2}$ D) $2\sqrt{\pi}$

Answer: A

Explanation: Using recurrence: $\Gamma(3/2) = 1/2\,\Gamma(1/2) = \sqrt{\pi}/2$.

209. Which of the following is true for Beta function?

A) $B(m,n) = B(n,m)$

B) $B(m,n) = B(m+n)$

C) $B(m,n) = 1/B(n,m)$

D) None

Answer: A

Explanation: By definition, Beta function is symmetric.

210. If $\Gamma(z)$ has simple poles, they occur at:

A) Positive integers

B) Non-positive integers

C) Rational numbers

D) None

Answer: B

Explanation: Poles occur at $z = 0, -1, -2, \ldots$.

211. Legendre's differential equation is:

A) $(1 - x^2)y'' - 2xy' + n(n+1)y = 0$

B) $(1 + x^2)y'' + 2xy' + n(n+1)y = 0$

C) $y'' + xy = 0$

D) None

Answer: A

212. Rodrigues' formula for Legendre polynomial is:

A) $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}\left((x^2-1)^n\right)$

B) $P_n(x) = \frac{d^n}{dx^n}\left((x^2+1)^n\right)$

C) $P_n(x) = (x^2-1)^n$

D) None

Answer: A

213. Hermite polynomial satisfies:

A) $y'' - 2xy' + 2ny = 0$

B) $y'' + 2xy' + ny = 0$

C) $y'' + y = 0$

D) None

Answer: A

214. Rodrigues' formula for Hermite polynomial:

A) $H_n(x) = (-1)^n e^{x^2}\frac{d^n}{dx^n}\left(e^{-x^2}\right)$

B) $H_n(x) = e^{x^2}\frac{d^n}{dx^n}\left(e^{-x^2}\right)$

C) $H_n(x) = (-1)^n \frac{d^n}{dx^n}\left(e^{x^2}\right)$

D) None

Answer: A

215. Laguerre polynomials satisfy:

A) $xy'' + (1 - x)y' + ny = 0$

B) $(1 - x)y'' + xy' + ny = 0$

C) $y'' + 2xy = 0$

D) None

Answer: A

216. Generating function of Laguerre polynomial:

A) $\frac{e^{-xt/(1-t)}}{1-t}$ B) $\frac{e^{xt/(1-t)}}{1+t}$

C) $\frac{1}{1-t}$ D) None

Answer: A

217. Bessel's differential equation:

A) $x^2y'' + xy' + (x^2 - n^2)y = 0$

B) $y'' + xy = 0$

C) $y'' - x^2y = 0$

D) None

Answer: A

218. The first kind Bessel function of order 0 at $x = 0$:

A) 0 B) 1

C) -1 D) Infinity

Answer: B

Explanation: $J_0(0) = 1$.

219. Orthogonality of Legendre polynomials is defined over interval:

A) [0,1] B) [0,π]

C) [-1,1] D) [0,∞)

Answer: C

220. Orthogonality condition of Legendre polynomials:

A) $\int_{-1}^{1} P_m(x)P_n(x)dx = \frac{2}{2n+1}\delta_{mn}$

B) $\int_{0}^{\infty} P_m(x)P_n(x)dx = \delta_{mn}$

C) $\int_{-\pi}^{\pi} P_m(x)P_n(x)dx = 0$

D) None

Answer: A

221. Generating function of Legendre polynomials is:

A) $\frac{1}{\sqrt{1-2xt+t^2}}$ B) $\frac{1}{1-xt}$

C) e^{xt} D) $\frac{1}{1-t^2}$

Answer: A

Explanation: Standard generating function of $P_n(x)$ is $\sum_{n=0}^{\infty} P_n(x)t^n = \frac{1}{\sqrt{1-2xt+t^2}}$.

222. Recurrence relation for Legendre polynomials:

A) $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$

B) $P_{n+1}(x) = xP_n(x) - nP_{n-1}(x)$

C) $P_{n+1}(x) = (n+1)P_n(x) - xP_{n-1}(x)$

D) None

Answer: A

223. Recurrence relation for Bessel functions:

A) $J_{n-1}(x) - J_{n+1}(x) = 2J_n'(x)$

B) $J_{n-1}(x) + J_{n+1}(x) = 2nJ_n(x)/x$

C) Both A and B

D) None

Answer: C

Explanation: Both are standard recurrence relations.

224. The generating function for Hermite polynomials is:

A) e^{2xt-t^2}

B) e^{xt}

C) $\frac{1}{1-t^2}$

D) e^{x^2}

Answer: A

225. Laguerre recurrence relation:

A) $(n+1)L_{n+1}(x) = (2n+1-x)L_n(x) - nL_{n-1}(x)$

B) $(n+1)L_{n+1}(x) = xL_n(x) - L_{n-1}(x)$

C) $L_{n+1}(x) = (n+1)L_n(x) - xL_{n-1}(x)$

D) None

Answer: A

226. Which of the following is true about generating functions?

A) They encode sequences into a power series

B) They help derive recurrence relations

C) They simplify computation of coefficients

D) All of the above

Answer: D

227. Recurrence relation for Gamma function:

A) $\Gamma(z+1) = z\Gamma(z)$

B) $\Gamma(z+1) = \Gamma(z)/z$

C) $\Gamma(z) = z\Gamma(z+1)$

D) None

Answer: A

228. Relation between Beta and Gamma can be seen as a generating property because:

A) Beta is symmetric

B) Beta reduces to factorial ratios

C) Beta integrates to Gamma

D) Both B and C

Answer: D

229. Hermite recurrence relation:

A) $H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x)$

B) $H_{n+1}(x) = xH_n(x) - nH_{n-1}(x)$

C) $H_{n+1}(x) = H_n(x) + nH_{n-1}(x)$

D) None

Answer: A

230. Recurrence relation of Bessel functions at $x = 0$:

A) $J_0(0) = 1, J_n(0) = 0$ for $n > 0$

B) $J_0(0) = 0, J_n(0) = 1$

C) $J_n(0) = \infty$

D) None

Answer: A

231. A function $f(z)$ is analytic at a point if:

A) It is continuous there

B) It is differentiable in a neighborhood

C) It satisfies Cauchy-Riemann equations

D) Both B and C

Answer: D

Explanation: A function $f(z)$ is analytic at a point if it is differentiable in a neighborhood it satisfies Cauchy-Riemann equations

232. Cauchy-Riemann equations are:

A) $u_x = v_y, u_y = -v_x$

B) $u_x = -v_y, u_y = v_x$

C) $u_x = u_y, v_x = v_y$

D) None

Answer: A

233. If $f(z) = x^2 + y^2 + i2xy$, then $f(z)$ is:

A) Analytic

B) Not analytic

C) Entire

D) None

Answer: B

Explanation: Fails Cauchy-Riemann conditions.

234. Which function is entire?

A) e^z

B) $sinz$

C) $cosz$

D) All of the above

Answer: D

Explanation: exponential, sinx, cosx, polynomial functions are entire.

235. Complex integration depends only on endpoints if:

A) Function is continuous

B) Function is analytic in simply connected domain

C) Function is bounded

D) None

Answer: B

Explanation: Complex integration depends only on endpoints if Function is analytic in simply connected domain

236. Cauchy's theorem states:

A) $\oint f(z)dz = 0$ if $f(z)$ analytic inside closed curve

B) $\oint f(z)dz \neq 0$ always

C) $\oint f(z)dz = 2\pi i$

D) None

Answer: A

Explanation: Cauchy's theorem states:

$\oint f(z)dz = 0$ if $f(z)$ analytic inside closed curve

237. Cauchy's integral formula:

A) $f(a) = \frac{1}{2\pi i}\oint \frac{f(z)}{z-a}dz$

B) $f(a) = \oint f(z)dz$

C) $f(a) = \oint f(z)(z-a)dz$

D) None

Answer: A

Explanation : Cauchy's integral formula:

$$f(a) = \frac{1}{2\pi i}\oint \frac{f(z)}{z-a}dz$$

238. Taylor series of analytic function valid:

A) Inside region of convergence

B) Everywhere

C) Only on real line

D) None

Answer: A

Explanation : by definition of Taylor series of analytic function

239. Laurent expansion valid:

A) In an annular region

B) Only inside disk

C) Only outside disk

D) None

Answer: A

Explanation : by definition

240. If $f(z) = \frac{1}{z}$, it has:

A) Removable singularity

B) Simple pole at 0

C) Essential singularity at 0

D) None

Answer: B

Explanation : order of pole 1 is single pole.

241. A function has an essential singularity at:

A) $z = 0$ for $e^{1/z}$

B) $z = 0$ for $1/z$

C) $z = 0$ for $sinz$

D) None

Answer: A

Explanation : $e^{\frac{1}{z}} = 1 + \frac{1}{z} + \frac{1}{2!z^2} + \frac{1}{3!z^3} + \cdots$

Here we get z=0 singularity for different powers of z. so z=0 is essential singularity

242. The residue of $f(z) = \frac{1}{z}$ at $z = 0$:

A) 0 B) 1

C) ∞ D) None

Answer: B

Explanation : coefficient of $\frac{1}{z}$ is 1= residue

243. Residue of $f(z) = e^z/z^2$ at $z = 0$:

A) 0 B) 1

C) 2 D) None

Answer: A

Explanation: Coefficient of $1/z$ term is 0.

244. Residue theorem states:

A) $\oint f(z)dz = 2\pi i \sum$Residues inside

B) $\oint f(z)dz = 0$

C) $\oint f(z)dz = \infty$

D) None

Answer: A

Explanation : from theorem

245. Evaluate $\int_0^{2\pi} \frac{d\theta}{5-4cos\theta}$.

A) $\frac{2\pi}{3}$ B) $\frac{2\pi}{3}$

C) π D) None

Answer: B

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Explanation : $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta} = \frac{2\pi}{\sqrt{a^2-b^2}}$.

$\int_0^{2\pi} \frac{d\theta}{5-4\cos\theta} = \frac{2\pi}{\sqrt{5^2-4^2}} = \frac{2\pi}{\sqrt{25-16}} = \frac{2\pi}{3}$.

246. Residue at a simple pole $z = a$:

A) $\lim_{z \to a}(z-a)f(z)$

B) $\lim_{z \to a}(z-a)^2 f(z)$

C) $f(a)$

D) None

Answer: A

247. Order of pole of $f(z) = \frac{1}{(z-1)^3}$:

A) 1

B) 2

C) 3

D) None

Answer: C

Explanation : A pole of a function is a type of singularity (a point where the function is not analytic/undefined) that looks like a power $f(z) \sim \frac{1}{(z-z_0)^m}$

Order of Pole is m=3

248. The integral $\int_{-\infty}^{\infty} \frac{1}{x^2+1} dx$:

A) 0

B) π

C) ∞

D) 2π

Answer: B

Explanation: By residue theorem, equals π.

249. Residue of $\frac{1}{(z-2)(z+3)}$ at $z = 2$:

A) $\frac{1}{5}$

B) $\frac{1}{-5}$

C) 0

D) None

Answer: A

Explanation: Residue = $\lim_{z \to 2}(z-2)/[(z-2)(z+3)] = 1/5$.

250. Evaluate $\int_0^{\infty} \frac{\cos x}{x^2+1} dx$.

A) $\pi e^{-1}/2$

B) $\pi/2$

C) $\mathbf{0}$

D) None

Answer: A

Explanation: Standard Fourier-type integral via residues gives $\pi e^{-1}/2$.

Wishing all of you the best outcome.........

THANKS